Fourth SM family, breaking of mass democracy, and the CKM mixings

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We consider the violation of the democratic mass matrix in the framework of the four-family standard model. Predictions of fourth-family fermion masses as well as quark and lepton CKM mixings are presented. Production and decay modes of new fermions are discussed. [S0556-2821(96)00819-3]

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I. INTRODUCTION

It is known that the standard model (SM) describes the fundamental particles and their interactions well up to a few hundred GeV scale. On the other hand, there are a number of problems which have not been clarified by the SM. In particular, (i) the masses and mixings of fundamental fermions, (ii) the existence of fermion families and their number, (iii) the arbitrary assignment of left-handed particles to weak isospin doublets and right handed particles to the singlets, and (iv) the SM does not really unify the strong, weak, and electromagnetic interactions, because each of them is described by its own gauge groups. (The appearance of a photon as a mixture of gauge bosons corresponding to hypercharge and third component of weak isospin does not change the above statement.) To solve these problems, different approaches beyond the SM have been proposed: the extension of electroweak symmetry, grand-unified theories (GUT's), supersymmetry (SUSY), preonic models, etc. However, some of the above-mentioned problems might have an answer in the SM. For example, the democratic mass matrix (DMM) approach [1–5] has been developed to solve the first problem. In general, the authors tend to apply this approach to the first three-family fermion only. As a consequence, the extension of the SM becomes unavoidable and/or wrong results have been obtained: in [1–4] an $SU(2)_L \times SU(2)_R \times U(1)$ extension of the electroweak gauge group was considered, in [1,2] the mass of the top quark was predicted to be 11–14 GeV, etc. In our previous work [6], we have argued that the DMM approach leads to the idea of the existence of a fourth-fermion family in the SM.

In the present work, we consider violations of full democracy which give the possibility to obtain nonzero masses for the first three-family fermions and to estimate the Cabibbo-Kobayashi-Maskawa (CKM) mixings. In Sec. II, we give arguments in favor of the existence of the fourth SM family. Then, we present a parametrization of the mass matrix which allows one to give masses to the first three-family fermions. In Sec. III, we obtain CKM matrices for quark and leptonic sectors and compare the first one with experimental data. Decay modes of the fourth-family fermions are discussed in Sec. IV and their productions at hadronic machines in Sec. V. Finally, in Sec. VI, we make some concluding remarks.

II. NECESSITY OF THE FOURTH SM FAMILY

Before the symmetry breaking, fermions with the same quantum numbers (electric charge, weak isospin, etc.) are indistinguishable. Therefore, in fermion-Higgs interactions, the Lagrangian terms corresponding to fermions with the same quantum numbers should come with equal strength. Consequently, after the spontaneous symmetry breaking one deals with singular mass matrices in which all entries are equal to $a_F \eta$, where η is the vacuum expectation value of the Higgs field and a_F is the strength of fermion-Higgs in-

TABLE I. Masses of elementary fermions in units of GeV/c^2. The lepton entries are the observed pole masses. The light quark (u,d,s) masses are estimates of so-called ''current quark masses.'' Heavy quark (c,b) masses are the ''running masses'' [7]. For the t-quark mass we take the Collider Detector at Fermilab (CDF) result [8]. The D0 Collaboration gives higher value $m_t = 200 \pm 25$ GeV/c^2 [9].

Neutrinos	Charged leptons	Up quarks	Down quarks
ν_e: $<5.1\times10^{-9}$	e: 0.510 999 06(15)$\times10^{-3}$	u: $(2-8)\times10^{-3}$	d: $(5-15)\times10^{-3}$
ν_μ: $<0.27\times10^{-3}$	μ: 0.105 658 389(34)	c: 1.0–1.6	s: 0.1–0.3
ν_τ: <0.031	τ: 1.777 1(5)	t: 174 ± 23	b: 4.1–4.5
ν_4: >45	ℓ_4: >44.3	u_4: >85	d_4: >85

TABLE II. Parameters and corresponding mass values for quark sector at $a=g$.

Up quarks	$\gamma=-0.000\,76$	$\beta=-0.007\,8$	$\alpha=0.55$	
	$m_u=6.6$ MeV	$m_c=1.13$ GeV	$m_t=176$ GeV	$m_{u_4}=638.6$ GeV
Down quarks	$\gamma=-0.000\,43$	$\beta=-0.001\,358$	$\alpha=0.012\,9$	
	$m_d=14.9$ MeV	$m_s=158$ MeV	$m_b=4.13$ GeV	$m_{d_4}=639.7$ GeV

teraction. Here, F denotes the type of fermions, namely neutrinos, charged leptons, up quarks, and down quarks. According to this approach, in the case of n SM families, $(n-1)$ families are massless and the nth-family fermions have masses $na_F\eta$. In principle, a_F might vary in type of fermions, but in the SM frame, it seems natural to assume that a_ν, a_l, a_u, and a_d have the values of the same order. Taking the real mass spectrum of the third-family fermions into account (see Table I), necessarily leads to the assumption that at least a fourth family should exist. Indeed, in the three-SM-family case we expect the third-family fermion masses to be equal to $3a_F\eta$. Since $\eta=$ 249 GeV, we obtain $a_\nu<4.15\times10^{-5}$, $a_l=2.38\times10^{-3}$, $a_u=0.233\pm0.031$, $a_d=(5.5\text{–}6.0)\times10^{-3}$. It is seen that there are great differences among the strengths of the fermion-Higgs interaction. For this reason, we proposed the existence of the fourth SM family in [6].

As a second assumption, we take the same value of a_F for all types of fermions. Furthermore, taking the common value of a_F to be equal to SU(2) gauge coupling constant g yields the fourth-family fermion masses $m_4=4g\eta=8m_W=640$ GeV, if g is replaced by the electromagnetic coupling constant e then $m_4=$ 320 GeV. At this stage the first three families remain massless. It is interesting that our prediction for fourth-family fermion masses are close to critical fermion mass values established by using partial wave unitarity at high energies in [10].

In terms of the mass matrix, the above arguments mean

$$M^0=a\eta\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix}\rightarrow M=4a\eta\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix} \tag{1}$$

for all types of fundamental fermions. In nature, at least charged leptons and quarks from the first three families have nonzero masses which means that Eq. (1) needs an appropriate modification. Assuming a modification which has a minimum effect on full democracy, we propose the following form of M^0:

$$M^0=a\eta\begin{pmatrix}1&1+\gamma&1+\beta&1\\1+\gamma&1+2\gamma&1+\beta&1\\1+\beta&1+\beta&1+\alpha&1-\alpha\\1&1&1-\alpha&1+\alpha\end{pmatrix}. \tag{2}$$

The γ parameter generates masses for the first-family fermions and regulates Cabibbo mixing, β gives masses for the second-family fermions and arranges b-c transition, α generates the third-family fermion masses. At the limit of $\gamma=\beta=0$, this matrix becomes the matrix given in [6]. Eigenvalues of matrix (2) give us masses of corresponding fermions which are used to fix the values of parameters α, β, and γ. In Tables II and III we present these values for the up- and down-quark sectors with predicted values of the fourth quark masses taking a_F equal to g and e, respectively. In the leptonic sector, we know masses of charged leptons precisely but for neutrino masses and mixings experiments give only upper limits. For these reasons, we have considered two different propositions for neutrino masses: (i) $m_{\nu_i}\sim m_{l_i}$ and (ii) $m_{\nu_i}\sim m_{l_i}^2$ $(i=1,2,3)$. Parameters and corresponding masses for the leptonic sector with predicted mass values of the fourth-family leptons are presented in Table IV, where m_{ν_1} is taken to be 1 eV.

III. CKM MATRICES

In order to clarify what follows let us introduce three different bases for fundamental fermion description: (1) SM basis, which is formed by fermion states in the SM multiplets; (2) mass basis, which is formed by mass eigenstates; (3) weak basis, which connects up and down fermion sectors.

Transformations from the SM basis to the mass basis [see Eq. (1)] are made by 4×4 unitary matrices which are different for different types of fermions. In general, each transfor-

TABLE III. Parameters and corresponding mass values for quark sector at $a=e$.

Up quarks	$\gamma=-0.001\,52$	$\beta=-0.015\,6$	$\alpha=1.1$	
	$m_u=6.6$ MeV	$m_c=1.13$ GeV	$m_t=176$ GeV	$m_{u_4}=318.6$ GeV
Down quarks	$\gamma=-0.000\,856$	$\beta=-0.002\,716$	$\alpha=0.025\,8$	
	$m_d=14.9$ MeV	$m_s=158$ MeV	$m_b=4.13$ GeV	$m_{d_4}=319.7$ GeV

TABLE IV. Parameters and corresponding mass values for lepton sector at $a=g$.

$m_{\nu_i}\sim m_{l_i}$	$\gamma=-1.272\,3\times10^{-10}$	$\beta=-1.465\,5\times10^{-9}$	$\alpha=1.083\,9\times10^{-8}$	
	$m_{\nu_1}=1$ eV	$m_{\nu_2}=206.7$ eV	$m_{\nu_3}=3477$ eV	$m_{\nu_4}=640$ GeV
$m_{\nu_i}\sim m_{l_i}^2$	$\gamma=-1.827\times10^{-9}$	$\beta=-2.695\times10^{-7}$	$\alpha=3.778\times10^{-5}$	
	$m_{\nu_1}=1$ eV	$m_{\nu_2}=42.75$ keV	$m_{\nu_3}=12.09$ MeV	$m_{\nu_4}=640$ GeV
Charged leptons	$\gamma=-0.000\,065\,02$	$\beta=-0.000\,749\,1$	$\alpha=0.005\,54$	
	$m_e=0.511$ MeV	$m_\mu=105.66$ MeV	$m_\tau=1.7771$ GeV	$m_{l_4}=639.9$ GeV

mation matrix has six angles and ten phases. Some of these parameters are absorbed and the remaining exhibit themselves in quark and lepton CKM matrices. In the case of four SM families there are six angles and three phases in the quark sector. If neutrinos are Dirac particles, the leptonic CKM matrix contains the same number of observable angles and phases.

At this stage, we neglect the phase parameters. The CKM matrix without phases is given as $O_{\rm CKM}=O_uO_d^T$ where O_u and O_d are (real) rotations which diagonalize up- and down-quarks mass matrices, respectively. With the parameters given in Table II, one obtains

$$O_{\rm CKM}=\begin{pmatrix} 0.9755 & -0.2198 & 0.0021 & 0.0001\\ 0.2196 & 0.9749 & 0.0334 & 0.0001\\ -0.0094 & -0.0321 & 0.9994 & -0.0017\\ -0.0001 & -0.0001 & 0.0017 & 1.0000 \end{pmatrix}. \tag{3}$$

This matrix should be compared with the experimental one

$$\begin{pmatrix} 0.9728-0.9757 & 0.218-0.224 & 0.002-0.005 & .\\ 0.180-0.228 & 0.800-0.975 & 0.032-0.048 & .\\ 0-0.13 & 0-0.56 & 0-0.9995 & .\\ . & . & . & . \end{pmatrix} \tag{4}$$

taken from the Particle Data Group (PDG) [7]. As can be seen, our predictions are in good agreement with experimental data. One may interpret this result as an indication of smallness of phase parameters. When we have much more accurate experimental data, these phases might give an opportunity to tune theoretical predictions. With the parameters given in Table III, the CKM matrix of quarks takes the form

$$O_{\rm CKM}=\begin{pmatrix} 0.9755 & -0.2199 & 0.0021 & 0.0001\\ 0.2197 & 0.9749 & 0.0334 & 0.0002\\ -0.0094 & -0.0321 & 0.9994 & -0.0057\\ -0.0002 & -0.0003 & 0.0057 & 1.0000 \end{pmatrix}. \tag{5}$$

The leptonic CKM matrix is $O_{\rm CKM}=O_\nu O_l^T$ where O_ν and O_l are rotations which diagonalize neutrino and charged lepton mass matrices, respectively. As we mentioned in a previous section, for neutrino masses and mixings, experiments give only upper limits. Therefore, leptonic CKM predictions have only an illustrative meaning. It is natural that in the case of proposition $m_{\nu_i}\sim m_{\ell_i}$ ($i=1,2,3$), the leptonic CKM matrix has been obtained close to the unit matrix for both $a=g$ and $a=e$. For $m_{\nu_i}\sim m_{\ell_i}^2$ and $a=g$ (see Table IV) the leptonic CKM matrix becomes

$$O_{\rm CKM}^l=\begin{pmatrix} 0.9979 & -0.0645 & 0.0000 & 0.0000\\ 0.0645 & 0.9968 & 0.0482 & 0.0000\\ -0.0031 & -0.0481 & 0.9988 & -0.0001\\ -0.0000 & -0.0000 & 0.0001 & 1.0000 \end{pmatrix}. \tag{6}$$

As is seen, the value of ν_μ-ν_e mixing is the largest one. With $m_{\nu_1}=1$ eV and $m_{\nu_2}=42$ keV this value predicts ν_μ-ν_e oscillations well above the experimental data [7]. Alternatively, the difference between squared masses of corresponding neutrino states should be less than 0.09 eV^2. Actually, leptonic CKM mixings are not so sensitive to the absolute value of m_{ν_1}. For example, taking m_{ν_1} equal to 10^{-6} eV leads practically to the same CKM matrix. In this case one obtains $m_{\nu_2}=4.275\times10^{-2}$ eV and $m_{\nu_3}=12.09$ eV which do not violate astrophysical and/or cosmological bounds. If we let the neutrinos radiatively decay, the mass values given in Table IV would also be acceptable. But this scenario cannot be realized with the leptonic CKM matrix (6), since the fourth lepton family is almost decoupled from the first three. This issue will be considered elsewhere, taking into account possible Majorana mass terms of neutrinos.

It is clear to see that the predicted quark and leptonic CKM matrices do not contradict precision low-energy measurements. For example, the contribution from fourth-family quarks to $b \to s\gamma$ is negligible compared to that from the t quark, due to the smallness of corresponding CKM matrix elements. According to Eq. (6) the fourth family does not affect $\mu \to e\gamma$ decay.

IV. DECAYS OF THE FOURTH-FAMILY FERMIONS

According to the DMM approach considered here, the fourth-family fermion masses are close to each other and equal to $4a\eta$ with great accuracy, where $e < a < g$. It is interesting that comparison [7] between the experimental value of $\rho = m_W^2/m_Z^2\cos^2\theta_W$ with theoretical prediction, taking into account the contribution of the fourth SM family, also implies a small (compared with m_4) difference between fourth-family fermion masses: for $m_H = 300$ GeV we obtain

$$(m_{u_4} - m_{d_4})^2 + \tfrac{1}{3}(m_{\nu_4} - m_{\ell_4})^2 < (111\ \text{GeV})^2. \tag{7}$$

With the predicted fourth-family fermion masses, given in Tables II–IV and quark and lepton CKM matrices, one sees that the dominant decay modes of these fermions are the following: $\nu_4 \to \tau^- + W^+$, $l_4 \to \nu_\tau + W^-$, $u_4 \to b + W^+$, $d_4 \to t + W^-$. The last decay will be followed by $t \to b + W^+$. Therefore pair production of u_4 quarks will appear in the detector as two high-energy b jets associated with a W^-W^+ pair. In pair production of d_4 there is an additional W^+W^- pair. Since the first- and second-family masses are small, slight deviations in the fourth-family fermion masses will also allow three-body decays as $\nu_4 \to \ell_4^- + e^+ + \nu_e$ and decays like $\nu_4 \to \ell_4^- + \pi^+$, etc. However, branching ratios of these decays will be negligible [11].

In general, if we do not restrict ourselves with the form of mass matrix given by Eq. (2), the experimental restriction of Eq. (7) allows different scenarios for decays of the fourth-family fermions. For example, if $m_{u_4} > m_{d_4} + m_W$, then the dominant decay mode of u_4 will be $u_4 \to d_4 + W^+$. For the time being, these scenarios are out of our interest.

V. PRODUCTION OF THE FOURTH-FAMILY FERMIONS

The fourth-family fermions will be pairly produced at future TeV energy colliders. Production at linear e^+e^- colliders and $\gamma\gamma$ colliders ([12], and references therein) based on them had been considered in a previous work [6]. In [13], pair production of d_4 and u_4 quarks at γp colliders ([14], and references therein) based on linac-ring-type ep colliders had been studied. For this reason, here we concentrate on production of the fourth-SM-family quarks at hadron colliders.

A dominant subprocess is gluon-gluon fusion and the corresponding cross section is well known (see Eq. (5.17) in [15]). Integration over the gluon distributions is necessary to obtain the cross section of the process $p + p(\bar{p}) \to q_4 + \bar{q}_4 + X$. For gluon distribution we use [16]

$$xG(x,Q^2) = 0.265x^{-1/2}(1+20x)(1-x)^{5.5}. \tag{8}$$

Numerical results are given in Figs. 1 and 2 where we plot the dependence of the cross section on the mass of the fourth-family fermions for $\sqrt{s} = 2$, 4, 8, and 14 TeV. The first value corresponds to present Fermilab $p\bar{p}$ collider, the second value corresponds to possible modification of Fermilab machine by replacing existing magnets with a peak magnetic field 4.4 T with 8.8 T magnets. The last value corresponds to design energy of the CERN Large Hadron Collider (LHC) proton beams, $\sqrt{s} = 8$ TeV corresponds to first stage of LHC if it will be constructed in two stages. As can be seen from Fig. 1, the present parameters of the Fermilab $p\bar{p}$ collider are not sufficent to observe the fourth-family fermions. When luminosity will be increased to 10^{33} cm^{-2} s^{-1}, as planned, discovery limits become 380 GeV, if the discovery limit is defined as 100 events per working year. The possible increase of the Fermilab center-of-mass energy up to 4 TeV

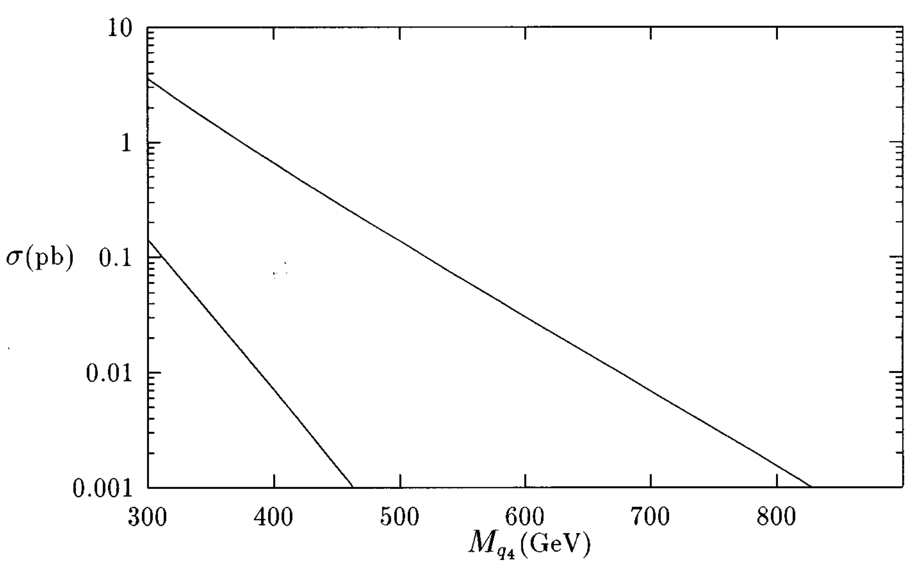

FIG. 1. Total cross section of fourth-family quark production at Fermilab. The lower line corresponds to $\sqrt{s} = 2$ TeV, the upper line to $\sqrt{s} = 4$ TeV.

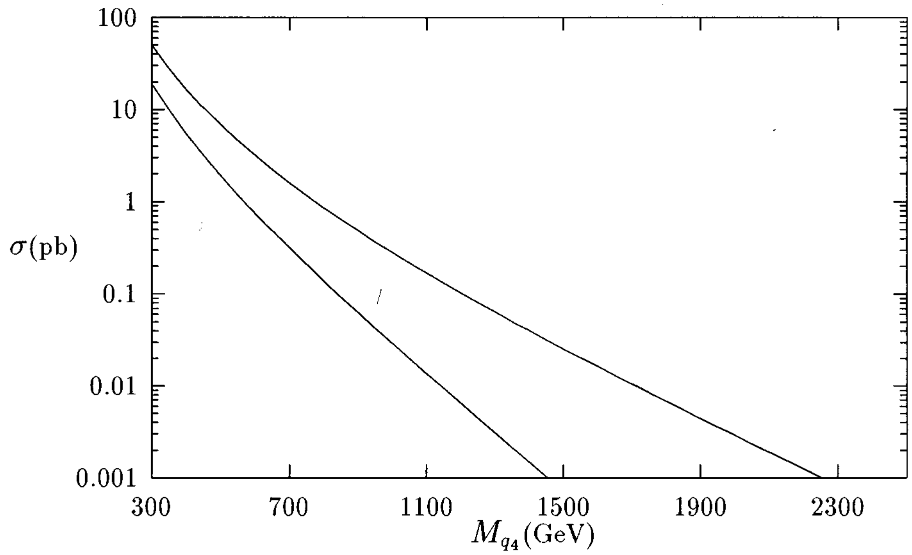

FIG. 2. Total cross section of fourth-family quark production at LHC. The lower line corresponds to $\sqrt{s}=8$ TeV, the upper line to $\sqrt{s}=14$ TeV.

will give an opportunity to observe the fourth-family quarks with masses up to 500 GeV for $\mathcal{L}=10^{32}$ cm^{-2} s^{-1} and 675 GeV for $\mathcal{L}=10^{33}$ cm^{-2} s^{-1}. Figure 2 shows that even the first stage of the LHC covers the interested region of the fourth-family masses.

VI. SUMMARY

We have shown that the existence of the fourth family is favorable in the framework of the SM. The masses of fourth-family fermions are close to each other within an accuracy of the order of a few GeV and lie between 300 and 700 GeV. Our predictions for the quark CKM matrix are in good agreement with experimental data. It seems that the experiments on $\nu_\mu \leftrightarrow \nu_e$ oscillations [17] have ruled out the scenario with $m_\nu \sim m_\ell$ and the choice $m_\nu \sim m_\ell^2$ is preferable. In our opinion, the search for the fourth family should be taken into account for the future colliders.

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