Goldstino couplings to matter

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(Received 5 April 1996)

A nonlinearly realized supersymmetric action describing the invariant couplings of the Goldstino to matter is constructed. Using the Akulov-Volkov Lagrangian, any operator can be made part of a supersymmetric-invariant action. Of particular interest are interaction terms which include the product of the Akulov-Volkov Lagrangian with the ordinary matter Lagrangian as well as the coupling of the product of the covariant derivative of the Goldstino field to the matter supersymmetry current. The latter is the lowest dimensional operator linear in the Goldstino field. A Goldstino Goldberger-Treiman relation is established and shown to be satisfied by the effective action. [S0556-2821(96)01321-5]

PACS number(s): 11.30.Na, 11.30.Pb, 14.80.Ly

I. INTRODUCTION

If supersymmetry (SUSY) is to be realized in nature, it must be as a broken symmetry. The breaking mechanism which maintains the preponderance of the dynamical constraints of the symmetry and hence is theoretically most attractive, is a spontaneous one. Indeed, many of the currently investigated attempts to construct realistic models of electroweak symmetry breaking incorporating SUSY use spontaneous supersymmetry breaking in one form or another. This includes both the so-called hidden sector [1] and visible sector classes of models [2].

A general consequence of the spontaneous breakdown of global supersymmetry is the appearance of a Nambu-Goldstone fermion, the Goldstino [3,4]. The leading term in the action describing its self-dynamics at energy scales below $4\pi/\kappa$, where $1/\kappa$ is the Goldstino decay constant, is uniquely fixed by the Akulov-Volkov effective Lagrangian [3] which takes the form

$$\mathcal{L}_{\mathrm{AV}} = -\frac{1}{2\kappa^2}\det A, \tag{1.1}$$

where

$$A_\mu^\nu \equiv \delta_\mu^\nu + i\kappa^2 \lambda \overleftrightarrow{\partial}_\mu \sigma^\nu \overline{\lambda}. \tag{1.2}$$

Here, $\lambda(\overline{\lambda})$ is the Goldstino Weyl spinor field. This effective Lagrangian provides a valid description of the Goldstino self-interactions independent of the particular (nonperturbative) mechanism by which the SUSY is dynamically broken [5]. Moreover, if the spontaneously broken supersymmetry is gauged, with the erstwhile Goldstino degrees of freedom absorbed to become the longitudinal (spin 1/2) modes of the gravitino via the super-Higgs mechanism, then the action formed from the Akulov-Volkov Lagrangian also describes the dynamics of those modes. This is completely analogous to using the gauged nonlinear sigma model to represent the dynamics of the longitudinal degrees of freedom of the $W_\pm$ and Z_0 vector bosons independent of the particular mechanism employed to break the electroweak symmetry.

Nonlinear realizations of symmetry transformations allow a model-independent analysis of the dynamical consequences of spontaneous symmetry-breaking using the Nambu-Goldstone degrees of freedom. This is true whether the part of the theory responsible for the symmetry breaking is strongly or weakly interacting. If weakly interacting, such as in the standard electroweak model with a light Higgs scalar, then the low energy physics can be directly calculated in perturbation theory. On the other hand, if the symmetry breaking sector of the model is strongly interacting, then explicit direct calculations of dynamical consequences can prove quite difficult. Since there are many such models of SUSY breaking presently being studied, it is worthwhile to determine, in a completely model-independent way, the various consequences of the supersymmetry breaking.

Using nonlinear realizations of supersymmetry for both the Goldstino and non-Goldstino degrees of freedom, Samuel and Wess [6] constructed supersymmetric-invariant couplings of the Goldstino to matter. Their construction entailed a somewhat elaborate procedure in which the Goldstino field and all matter fields are promoted to become superfields whose lowest components are the ordinary fields themselves and whose higher components involve the product of Goldstino fields and derivatives of the lowest components. For the special case of the Goldstino-promoted superfields, the θ or $\overline{\theta}$ components also contain the Goldstino decay constant as an additive component. Using these superfields, every ordinary operator can then be cast as a part of a manifestly supersymmetric action. While this procedure is elegant and complete, it does require the introduction of a considerable amount of additional (super)structure.

In this paper, we present an alternate construction of a nonlinearly realized supersymmetric-invariant action. Our procedure works directly with the ordinary (component) Goldstino and matter fields and does not require the introduction of the entire superfield structures. Thus, in a simple and straightforward manner, we can make any ordinary operator part of a manifestly supersymmetric action. After introducing the nonlinear SUSY transformations and covariant derivative, we construct the SUSY- (and internal symmetry-) invariant action terms using the special properties of the Akulov-Volkov Lagrangian. We focus on two particular interaction terms. One involves the coupling of the Akulov-Volkov Lagrangian to the ordinary matter action. Once the ordinary matter action is appropriately normalized, the coef-

ficient of this term is fixed solely by the Goldstino decay constant. Another action term, which is the lowest-dimensional operator linear in the Goldstino field, involves the coupling of its (SUSY-covariant) derivative to the ordinary matter supersymmetry current. This coupling is then used to show that a Goldberger-Treiman relation [7] associated with the spontaneous supersymmetry breaking is indeed satisfied.

II. NONLINEAR SUSY TRANSFORMATIONS

The self-dynamics of the Goldstino can be encapsulated in the Akulov-Volkov Lagrangian, Eq. (1.1). The supersymmetry transformations are nonlinearly realized on the Goldstino field by

$$\delta^Q(\xi,\bar{\xi})\lambda^\alpha=\frac{1}{\kappa}\xi^\alpha-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\lambda^\alpha,$$

$$\delta^Q(\xi,\bar{\xi})\bar{\lambda}_{\dot\alpha}=\frac{1}{\kappa}\bar{\xi}_{\dot\alpha}-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\bar{\lambda}_{\dot\alpha}, \tag{2.1}$$

where $\xi^\alpha,\bar{\xi}_{\dot\alpha}$ are Weyl spinor SUSY transformation parameters. The Akulov-Volkov Lagrangian then transforms as a total divergence

$$\delta^Q(\xi,\bar{\xi})\mathcal{L}_{\rm AV}=-i\kappa\partial_\rho[(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\mathcal{L}_{\rm AV}] \tag{2.2}$$

and hence, the associated action

$$I_{\rm AV}=\int d^4x\mathcal{L}_{\rm AV} \tag{2.3}$$

is SUSY invariant:

$$\delta^Q(\xi,\bar{\xi})I_{\rm AV}=0. \tag{2.4}$$

The supersymmetry algebra can also be nonlinearly realized on the matter (non-Goldstino) fields, generically denoted by ϕ_i, where i can represent any Lorentz or internal symmetry labels, as

$$\delta^Q(\xi,\bar{\xi})\phi_i=-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\phi_i. \tag{2.5}$$

This is referred to as the standard realization [6]. Forming the SUSY Ward identity functional differential operator

$$\delta^Q(\xi,\bar{\xi})=\int d^4x\left[\delta^Q(\xi,\bar{\xi})\lambda^\alpha\frac{\delta}{\delta\lambda^\alpha}+\delta^Q(\xi,\bar{\xi})\bar{\lambda}_{\dot\alpha}\frac{\delta}{\delta\bar{\lambda}_{\dot\alpha}}+\sum_i\delta^Q(\xi,\bar{\xi})\phi_i\frac{\delta}{\delta\phi_i}\right], \tag{2.6}$$

one readily establishes the SUSY algebra

$$[\delta^Q(\xi,\bar{\xi}),\delta^Q(\eta,\bar{\eta})]=-2i\delta^P(\xi\sigma\bar{\eta}-\eta\sigma\bar{\xi})$$

$$[\delta^Q(\xi,\bar{\xi}),\delta^P(a)]=0. \tag{2.7}$$

As usual, the space-time translations are given by

$$\delta^P(a)\lambda^\alpha=a^\mu\partial_\mu\lambda^\alpha,$$

$$\delta^P(a)\bar{\lambda}_\alpha=a^\mu\partial_\mu\bar{\lambda}_\alpha,$$

$$\delta^P(a)\phi_i=a^\mu\partial_\mu\phi_i, \tag{2.8}$$

with a^μ the global space-time translation parameter and

$$\delta^P(a)=\int d^4x\left[\delta^P(a)\lambda^\alpha\frac{\delta}{\delta\lambda^\alpha}+\delta^P(a)\bar{\lambda}_{\dot\alpha}\frac{\delta}{\delta\bar{\lambda}_{\dot\alpha}}+\sum_i\delta^P(a)\phi_i\frac{\delta}{\delta\phi_i}\right] \tag{2.9}$$

is the space-time translation Ward identity functional differential operator.

Under the nonlinear SUSY standard realization, the derivative of a matter field transforms as

$$\begin{aligned}\delta^Q(\xi,\bar{\xi})(\partial_\nu\phi_i)&=\partial_\nu[\delta^Q(\xi,\bar{\xi})\phi_i]\\&=-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho(\partial_\nu\phi_i)\\&\quad-i\kappa\partial_\nu(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\phi_i.\end{aligned} \tag{2.10}$$

In order to eliminate the second term on the right-hand side (RHS) and thus restore the SUSY-covariance, we introduce a SUSY covariant derivative which transforms analogously to ϕ_i. To achieve this, we note that

$$\begin{aligned}\delta^Q(\xi,\bar{\xi})A^\nu_\mu&=-i\kappa[(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho A^\nu_\mu\\&\quad+\partial_\mu(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})A^\nu_\rho],\end{aligned} \tag{2.11}$$

from which it follows that

$$\begin{aligned}\delta^Q(\xi,\bar{\xi})(A^{-1})^\nu_\mu&=-(A^{-1})^\rho_\mu[\delta^Q(\xi,\bar{\xi})A^\sigma_\rho](A^{-1})^\nu_\sigma\\&=-i\kappa[(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho(A^{-1})^\nu_\mu\\&\quad-\partial_\rho(\lambda\sigma^\nu\bar{\xi}-\xi\sigma^\nu\bar{\lambda})(A^{-1})^\rho_\mu],\end{aligned} \tag{2.12}$$

where

$$(A^{-1})^\nu_\mu A^\rho_\nu=\delta^\rho_\mu. \tag{2.13}$$

We are thus led to define the nonlinearly realized SUSY-covariant derivative as

$$\mathcal{D}_\mu\phi_i=(A^{-1})^\nu_\mu\partial_\nu\phi_i, \tag{2.14}$$

so that under the standard realization of SUSY:

$$\delta^Q(\xi,\bar{\xi})(\mathcal{D}_\mu\phi_i)=-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho(\mathcal{D}_\mu\phi_i). \tag{2.15}$$

In addition to the SUSY and space-time translations, we can also define R transformations under which the Goldstino field transforms as [8]

$$\delta^R(\omega)\lambda^\alpha=i\omega\lambda^\alpha,$$

$$\delta^R(\omega)\bar{\lambda}_{\dot\alpha}=-i\omega\bar{\lambda}_{\dot\alpha}. \tag{2.16}$$

Forming the R-transformation Ward identity functional differential operator

$$\delta^R(\omega)=\int d^4x\left[\delta^R(\omega)\lambda^\alpha\frac{\delta}{\delta\lambda^\alpha}+\delta^R(\omega)\bar{\lambda}_{\dot\alpha}\frac{\delta}{\delta\bar{\lambda}_{\dot\alpha}}+\sum_i\delta^R(\omega)\phi_i\frac{\delta}{\delta\phi_i}\right], \tag{2.17}$$

it is readily established that the algebra

$$[\delta^R(\omega),\delta^Q(\xi,\bar{\xi})]=\delta^Q(-i\omega\xi,i\omega\bar{\xi}) \tag{2.18}$$

holds independent of the form of $\delta^R(\omega)\phi_i$. The action formed from the Akulov-Volkov Lagrangian is invariant under R-symmetry, supersymmetry and space-time translations. Moreover, the improved currents associated with these symmetries have been shown [8] to form the components of a supercurrent [9]. Thus, all conservation laws and anomalies are derivable from the supercurrent conservation law and the generalized trace anomaly [10,11].

Since the Goldstino field transforms as a singlet under any internal symmetry transformation, $\delta^G(\Lambda)\lambda^\alpha=0=\delta^G(\Lambda)\bar{\lambda}_{\dot\alpha}$, the Akulov-Volkov action is also invariant under internal symmetry transformations:

$$\delta^G(\Lambda)I_{\rm AV}=0, \tag{2.19}$$

where Λ parametrizes the transformation. Denoting the internal symmetry matter field transformation as $\delta^G(\Lambda)\phi_i$, then the Ward identity functional differential operator characterizing the internal symmetry transformation is

$$\delta^G(\Lambda)=\int d^4x\sum_i\delta^G(\Lambda)\phi_i\frac{\delta}{\delta\phi_i}. \tag{2.20}$$

Note that if the internal symmetry is gauged, the nonlinearly realized SUSY, gauge-covariant derivative, Eq. (2.14), is replaced with

$$\mathcal{D}_\mu\phi_i=(A^{-1})_\mu{}^\nu D_\nu\phi_i, \tag{2.21}$$

where $D_\mu\phi_i$ is the ordinary gauge-covariant derivative.

III. INVARIANT ACTIONS

We now construct actions containing the Goldstino and matter fields which are invariant under both SUSY and internal symmetry transformations. The Akulov-Volkov action is one such term which contains the Goldstino kinetic term and self-couplings. Using the Akulov-Volkov Lagrangian, we can form SUSY-invariant actions out of any Lorentz and internal symmetry singlet operator $\mathcal{O}=\mathcal{O}(\phi,\mathcal{D}\phi)$. To achieve this, we note that under the nonlinear standard realization of SUSY given by Eqs. (2.1), (2.5), and (2.10), such an operator transforms as

$$\delta^Q(\xi,\bar{\xi})\mathcal{O}(\phi,\mathcal{D}\phi)=-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\mathcal{O}(\phi,\mathcal{D}\phi). \tag{3.1}$$

Consequently, the action

$$I_{\mathcal{O}}=-2\kappa^2C_{\mathcal{O}}\int d^4x\mathcal{L}_{\rm AV}\mathcal{O}=C_{\mathcal{O}}\int d^4x(\det A)\mathcal{O}, \tag{3.2}$$

with $C_{\mathcal{O}}$ a constant, is SUSY invariant:

$$\delta^Q(\xi,\bar{\xi})I_{\mathcal{O}}=0. \tag{3.3}$$

Since $\mathcal{L}_{\rm AV}$ is defined so as to contain the additive constant term $-1/2\kappa^2$ (or equivalently, the det$A$ starts with the identity), the action $I_{\mathcal{O}}$ includes the piece $C_{\mathcal{O}}\int d^4x\mathcal{O}(x)$ for any operator $\mathcal{O}$.

One special case is afforded by using the internal symmetry-invariant ordinary matter Lagrangian $\mathcal{L}_\phi(\phi,\mathcal{D}\phi)$ where all derivatives are replaced by SUSY-covariant derivatives. Under SUSY

$$\delta^Q(\xi,\bar{\xi})\mathcal{L}_\phi(\phi,\mathcal{D}\phi)=-i\kappa(\lambda\sigma^\rho\bar{\xi}-\xi\sigma^\rho\bar{\lambda})\partial_\rho\mathcal{L}_\phi(\phi,\mathcal{D}\phi), \tag{3.4}$$

while under the internal group transformation, the Lagrangian is invariant:

$$\delta^G(\Lambda)\mathcal{L}_\phi(\phi,\mathcal{D}\phi)=0. \tag{3.5}$$

It follows that the action

$$I_{LL}=-2\kappa^2\int d^4x\mathcal{L}_\phi(\phi,\mathcal{D}\phi)\mathcal{L}_{\rm AV}(\lambda,\bar{\lambda}) \tag{3.6}$$

is both SUSY and internal symmetry invariant

$$\delta^Q(\xi,\bar{\xi})I_{LL}=0, \tag{3.7}$$

$$\delta^G(\Lambda)I_{LL}=0. \tag{3.8}$$

Note that in the absence of Goldstino fields, this action reduces to the ordinary matter action $I_\phi=\int d^4x\mathcal{L}_\phi(\phi,\partial_\mu\phi)$ so I_{LL} contains the ordinary matter action as well as couplings of the Goldstino to matter. Thus, once the normalization of the ordinary matter action is fixed, so are its couplings to the Goldstino field. As such this term requires no additional independent coupling constant. Further note that using the nonlinear realization, various higher-dimensional operators, such as the electron anomalous magnetic moment operator, can also be made part of a SUSY-invariant action. On the other hand, such a term cannot be included in a SUSY-invariant action if the supersymmetry is linearly realized [12].

Both $I_{\rm AV}$ and I_{LL} depend on $\lambda,\bar{\lambda}$ only through $A^{\mu\nu}$ and thus only through the bilinear combination $\lambda\lambda$ (and derivatives). While, by using the Goldstino field, any Lorentz and internal symmetry singlet can be incorporated into a supersymmetric-invariant action, the most natural setting is to consider those pure matter actions which allow for linear realizations of the supersymmetry. In that case, using the associated internal symmetry singlet supersymmetry currents, $Q^\mu_{\phi\alpha}(\phi,\partial_\mu\phi)$ and $\bar{Q}^\mu_{\phi\dot\alpha}(\phi,\partial_\mu\phi)$, we can construct another invariant action whose Goldstino dependence is odd in $\lambda,\bar{\lambda}$ and, in fact, starts off as linear in $\partial_\mu\lambda$. Letting $Q^\mu_{\phi\alpha}=Q^\mu_{\phi\alpha}(\phi,\mathcal{D}\phi)$ and $\bar{Q}^\mu_{\phi\dot\alpha}=\bar{Q}^\mu_{\phi\dot\alpha}(\phi,\mathcal{D}\phi)$ be the matter supercurrents where all space-time derivatives are replaced by nonlinearly realized SUSY-covariant derivatives, it follows that under the standard realization of SUSY

$$\delta^Q(\xi,\overline{\xi})Q^\mu_{\phi\alpha}=-i\kappa(\lambda\sigma^\rho\overline{\xi}-\xi\sigma^\rho\overline{\lambda})\partial_\rho Q^\mu_{\phi\alpha},$$

$$\delta^Q(\xi,\overline{\xi})\overline{Q}^\mu_{\phi\dot\alpha}=-i\kappa(\lambda\sigma^\rho\overline{\xi}-\xi\sigma^\rho\overline{\lambda})\partial_\rho\overline{Q}^\mu_{\phi\dot\alpha}, \tag{3.9}$$

while

$$\delta^G(\Lambda)Q^\mu_{\phi\alpha}=0=\delta^G(\Lambda)\overline{Q}^\mu_{\phi\dot\alpha}. \tag{3.10}$$

When used in conjunction with the SUSY transformations:

$$\delta^Q(\xi,\overline{\xi})(\mathcal{D}_\mu\lambda^\alpha)=-i\kappa(\lambda\sigma^\rho\overline{\xi}-\xi\sigma^\rho\overline{\lambda})\partial_\rho(\mathcal{D}_\mu\lambda^\alpha),$$

$$\delta^Q(\xi,\overline{\xi})(\mathcal{D}_\mu\overline{\lambda}_{\dot\alpha})=-i\kappa(\lambda\sigma^\rho\overline{\xi}-\xi\sigma^\rho\overline{\lambda})\partial_\rho(\mathcal{D}_\mu\overline{\lambda}_{\dot\alpha}), \tag{3.11}$$

we construct the invariant action

$$I_{\lambda Q}=-2\kappa^3C_Q\int d^4x\mathcal{L}_{\rm AV}(\mathcal{D}_\mu\lambda^\alpha Q^\mu_{\phi\alpha}+\overline{Q}^\mu_{\phi\dot\alpha}\mathcal{D}_\mu\overline{\lambda}^{\dot\alpha}), \tag{3.12}$$

where C_Q is a constant. This action satisfies

$$\delta^Q(\xi,\overline{\xi})I_{\lambda Q}=0, \tag{3.13}$$

$$\delta^G(\Lambda)I_{\lambda Q}=0. \tag{3.14}$$

Using the form of the Akulov-Volkov Lagrangian and the SUSY-covariant derivative, we see that

$$I_{\lambda Q}=\kappa C_Q\int d^4x[\partial_\mu\lambda^\alpha Q^\mu_{\phi\alpha}(\phi,\partial_\nu\phi)+\overline{Q}^\mu_{\phi\dot\alpha}(\phi,\partial_\nu\phi)\partial_\mu\overline{\lambda}^{\dot\alpha}]+\cdots, \tag{3.15}$$

which is coupling linear in the Goldstino field. In fact, this mass dimension-six operator contains the smallest power of κ coefficient of the various couplings of the Goldstino to matter. The appearance of the coupling of the Goldstino field to the divergence of the matter supersymmetry current is certainly anticipated. In fact, it is reminiscent of the situation in spontaneously broken chiral symmetry where the Nambu-Goldstone pion couples derivatively to the spontaneously broken matter chiral symmetry current.

Combining the various terms, we secure the SUSY- and internal symmetry-invariant action

$$I=I_{\rm AV}+I_{LL}+I_{\lambda Q}=\int d^4x\mathcal{L}_{\rm AV}-2\kappa^2\int d^4x\mathcal{L}_{\rm AV}\mathcal{L}_\phi-2\kappa^3C_Q\int d^4x\mathcal{L}_{\rm AV}(\mathcal{D}_\mu\lambda^\alpha Q^\mu_{\phi\alpha}+\overline{Q}^\mu_{\phi\dot\alpha}\mathcal{D}_\mu\overline{\lambda}^{\dot\alpha}) \tag{3.16}$$

which satisfies

$$\delta^Q(\xi,\overline{\xi})I=0 \tag{3.17}$$

and

$$\delta^G(\Lambda)I=0. \tag{3.18}$$

The action starts out as

$$I=\int d^4x[\mathcal{L}_{\rm AV}+\mathcal{L}_\phi+\kappa C_Q(\partial_\mu\lambda^\alpha Q^\mu_{\phi\alpha}+\overline{Q}^\mu_{\phi\dot\alpha}\partial_\mu\overline{\lambda}^{\dot\alpha})+\cdots], \tag{3.19}$$

so that

$$\frac{\delta I}{\delta\lambda^\alpha}=\frac{\delta I_{AV}}{\delta\lambda^\alpha}-\kappa C_Q\partial_\mu Q^\mu_{\phi\alpha}+\cdots$$

$$=-\frac{i}{2}\det A(A^{-1})^{\nu\mu}(\sigma_\nu\partial_\mu\overline{\lambda})_\alpha-\kappa C_Q\partial_\mu Q^\mu_{\phi\alpha}+\cdots$$

$$=-i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\overline{\lambda^\alpha}-\kappa C_Q\partial_\mu Q^\mu_{\phi\alpha}+\cdots \tag{3.20}$$

and the Goldstino field equation takes the form

$$\sigma^\mu_{\alpha\dot\alpha}\frac{1}{i}\partial_\mu\overline{\lambda^\alpha}=\kappa C_Q\partial_\mu Q^\mu_{\phi\alpha}+\cdots. \tag{3.21}$$

IV. GOLDBERGER-TREIMAN RELATION

As a consequence of their Nambu-Goldstone nature, the couplings of the Goldstino to matter are constrained to satisfy certain general relationships. One such constraint is the analogue of the Goldberger-Treiman relationship [7] familiar from pion physics and spontaneously broken chiral symmetry. When applied to spontaneously broken supersymmetry, the analogous relation ties form factors of the supersymmetry and Goldstino currents at zero-momentum transfer to the Goldstino decay constant and mass differences between matter boson and fermion states. The Lorentz decomposition of the supersymmetry current Q^μ_α taken between arbitrary single particle (scalar) Bose and (spin 1/2) Fermi states, $|p_1;B\rangle$ and $|p_2;F\rangle$, of masses m_B and m_F and carrying four-momenta p_1^μ and p_2^μ, respectively, takes the form

$$\langle p_1;B|Q^\mu_\alpha(0)|p_2;F\rangle=[A_1(q^2)q^\mu+A_2(q^2)k^\mu+A_3(q^2)\sigma^\mu\overline{\sigma}\cdot q]^\beta_\alpha\chi_\beta(p_2)_F+[A_4(q^2)\sigma^\mu+A_5(q^2)q^\mu\sigma\cdot q+A_6(q^2)k^\mu\sigma\cdot q]_{\alpha\dot\alpha}\overline{\chi^{\dot\alpha}}(p_2)_F, \tag{4.1}$$

where $q^\mu=(p_1-p_2)^\mu$ and $k^\mu=(p_1+p_2)^\mu$, and the fermion spinors satisfy

$$\sigma\cdot p_2\overline{\chi}(p_2)_F=-m_F\chi(p_2)_F,$$

$$\overline{\sigma}\cdot p_2\chi(p_2)_F=-m_F\overline{\chi}(p_2)_F. \tag{4.2}$$

Conservation of the supersymmetry current, $\partial_\mu Q^\mu_\alpha=0$, then relates the various form factors as

$$q^2[A_1(q^2)-A_3(q^2)]=(m_B^2-m_F^2)A_2(q^2). \tag{4.3}$$

Since the massless Goldstino directly couples to the supersymmetry current, some of these form factors are singular in the $q^2\to0$ limit. Thus, before taking this limit, we need to include the effect of the massless Goldstino pole. This pole is reflected in the nonvanishing matrix element of the supersymmetry current between the vacuum and single Goldstino state, $|q;\lambda\rangle$, of four-momentum q^μ which is given by

$$\langle0|Q^\mu_\alpha(0)|q;\lambda\rangle=\frac{1}{i\kappa}\sigma^\mu_{\alpha\dot\alpha}\overline{\chi^{\dot\alpha}_\lambda}, \tag{4.4}$$

where κ^{-1} is the Goldstino decay constant.

It follows that the combination $Q^{\mu}_{\alpha}-(1/i\kappa)\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\lambda}^{\dot{\alpha}}$ has vanishing matrix element between the vacuum and single Goldstino state. The matrix element of this combination taken between the single Bose state and single Fermion state can then be Lorentz decomposed just as in Eq. (4.1) where now the various form factors are all nonsingular in the $q^2\to 0$ limit.

Finally, the Goldstino current j^{G}_{α} is defined through the Goldstino field equation

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\frac{1}{i}\partial_{\mu}\overline{\lambda}^{\dot{\alpha}}=j^{G}_{\alpha}. \tag{4.5}$$

Taking its matrix element between the Bose and Fermi states leads to the Lorentz decomposition

$$\langle p_1;B|j^{G}_{\alpha}(0)|p_2;F\rangle=B_1(q^2)\chi_{\alpha}(p_2)_F+B_2(q^2)\times(\sigma\cdot q)_{\alpha\dot{\alpha}}\overline{\chi}^{\dot{\alpha}}(p_2)_F \tag{4.6}$$

and, thus,

$$\langle p_1;B|\overline{\lambda}^{\dot{\alpha}}(0)|p_2;F\rangle=-\frac{B_1(q^2)}{q^2}(\overline{\sigma}\cdot q)^{\dot{\alpha}\alpha}\chi_{\alpha}(p_2)_F+B_2(q^2)\overline{\chi}^{\dot{\alpha}}(p_2)_F. \tag{4.7}$$

Since the form factors of the combination $Q^{\mu}_{\alpha}-(1/i\kappa)\sigma^{\mu}_{\alpha\dot{\alpha}}\overline{\lambda}^{\dot{\alpha}}$ are regular as $q^2\to 0$, we see on comparison of Eqs. (4.1) and (4.7) that the $A_1(q^2)$ form factor is regular while the $A_3(q^2)$ form factor is singular. The singular piece is given by

$$\lim_{q^2\to 0} q^2A_3(q^2)=\frac{i}{\kappa}B_1(0). \tag{4.8}$$

Substituting into Eq. (4.3) and taking the $q^2\to 0$ limit, we secure the Goldstino Goldberger-Treiman relation

$$-\frac{i}{\kappa}B_1(0)=(m_B^2-m_F^2)A_2(0). \tag{4.9}$$

To establish that the effective action [Eq. (3.16)] satisfies this Goldstino Goldberger-Treiman relation, we note that a Noether construction of the conserved supersymmetry current starts out as

$$Q^{\mu}_{\alpha}=C_QQ^{\mu}_{\phi\alpha}+\cdots, \tag{4.10}$$

while the Goldstino field equation (3.21) provides the identification of the Goldstino current as

$$j^{G}_{\alpha}=\kappa C_Q\partial_{\mu}Q^{\mu}_{\phi\alpha}+\cdots. \tag{4.11}$$

For the ''matter'' supersymmetry current we use [9,13]

$$Q^{\mu}_{\phi\alpha}=\partial^{\mu}\overline{A}\psi_{\alpha}+\cdots, \tag{4.12}$$

where $\overline{A}(\psi)$ are the Bose (Fermi) fields creating (destroying) the Bose (Fermi) states in the matrix elements of the Lorentz decomposition of the supersymmetry and Goldstino currents. The matrix elements are readily computed, yielding

$$A_2(0)=-\frac{i}{2}C_Q,$$

$$B_1(0)=\frac{\kappa}{2}(m_B^2-m_F^2)C_Q, \tag{4.13}$$

and the Goldberger-Treiman relation (4.9) is indeed satisfied.

ACKNOWLEDGMENT

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-91ER40681 (Task B).

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