Radiative seesaw mechanism at the weak scale

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We investigate an alternative seesaw mechanism for neutrino mass generation. The neutrino mass is generated at the loop level but the basic concept of the usual seesaw mechanism is kept. One simple model is constructed to show how this mechanism is realized. The applications of this seesaw mechanism at weak scale to cosmology and neutrino physics are discussed. [S0556-2821(96)02521-0]

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The seesaw mechanism [1] is one of the best and simplest ways to understand why the neutrino, if massive, is much lighter than the corresponding charged lepton in the same generation. The central idea of the seesaw mechanism is to introduce a right-handed neutrino ν_R, which will couple to a lepton doublet through Yukawa coupling. The point is that in addition to the Yukawa interaction term there is another bare Majorana mass term M_R for ν_R. After gauge symmetry breaking the Yukawa term will result in a Dirac neutrino mass m_D. Therefore the neutrino mass matrix takes the form

$$\begin{pmatrix} 0 & m_D \\ m_D^+ & M_R \end{pmatrix}. \tag{1}$$

In the three-generation model m_D and M_R are three by three mass matrices. Diagonalizing the mass matrix one gets the neutrino mass eigenstates. If M_R is much bigger than m_D the mass of the light neutrinos, which are mostly left handed, is determined as $m_D^T M_R^{-1} m_D$. The heavy states, which are mostly right handed, have mass almost as M_R. Therefore one sees that even if the Dirac mass term is comparable to the charged lepton mass the light neutrino mass can be much smaller. The features one should notice in this mechanism are the following: M_R is a free scale usually taken from the weak scale to the grand unified theory (GUT) scale. And the heavy neutrinos are not stable, they decay through mixing to the light neutrinos. For large M_R the heavy neutrinos decay very fast, so they have no cosmological consequence. In this mechanism the lepton number symmetry is broken either explicitly or spontaneously. Although the smallness of the neutrino mass can be understood in this mechanism, the actual values of the neutrino mass and mixing are not predicted due to the unknown scale M_R and structure of m_D. As an indication, if one assumes that m_D is same as the charged lepton mass matrix and M_R is a unit matrix up to a scale, one gets the relations for the light neutrino masses $m_{\nu_i}=m_i^2/M_R$, where the index i denotes the ith generation. So it is the scale M_R that determines the order of the magnitude of the neutrino mass. If M_R is at the GUT scale, one obtains $m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau} \leq 10^{-3}$ eV. These tiny masses may only play a role for solar neutrino behavior. Another most interesting scale is the weak scale. There are a number of physical motivations to consider M_R at the weak scale. First of all for the weak scale M_R the new physics mechanism can be tested in future experiments, second it avoids introducing an intermediate scale between the weak and GUT scales. For M_R at the weak scale all three light neutrino masses are close to their upper bound, i.e., a few eV, 100 keV, and 10 MeV for electron, muon, and τ neutrinos. These neutrinos are strongly constrained from cosmological and astrophysical consideration depending on their decay modes [2]. Obviously they offer no solutions to the solar neutrino and atmospheric neutrino problems [3], but they may play a role in the dark matter issue by providing either a hot dark matter component in the mixed dark matter model [4] or a late decaying particle [5,6] in the cold dark matter model [7]. And no cold dark matter candidate is provided. Moreover there may be a problem for the seesaw model in consideration of the baryogenesis of the Universe. The problem is due to the $B-L$ (baryon number minus lepton number) symmetry violation. Once the $B-L$ violation process through the seesaw mechanism and the anomalous $B+L$ process induced by gauge interaction are in the thermal equilibrium at an early stage of the Universe [8], any primordial origin of baryon and lepton asymmetries generated earlier are washed out. It leads to very strong constrains on the neutrino mass [9]. An upper bound of a few eV for all three light neutrino masses are obtained in order to avoid this problem [10], which in turn implies the scale of M_R should be much larger than the weak scale. However, this problem can be evaded if the $B-L$ symmetry is spontaneously broken at the weak scale. Before the $B-L$ symmetry breaking only the $(B+L)$-violating process due to the gauge anomaly is active and after the $B-L$ symmetry breaking the anomalous $(B+L)$-violating process is already suppressed as the temperature of the Universe is low enough. These two processes will never be in thermal equilibrium through the evolution of the Universe. Hence the constrains on the strength of $B-L$ violation from the baryogenesis of the Universe is avoided [11].

In this work we consider a different scheme for the seesaw mechanism. The main consideration is to keep the basic concept of the seesaw mechanism, i.e., the light neutrino mass is suppressed by the large right-handed neutrino mass M_R, and require the neutrino mass only generated radiatively [12]. For this kind of scenario the neutrino mass is expressed as $m_\nu \sim (\lambda/16\pi^2) m_D^T M_R^{-1} m_D$. One sees that adding to the usual seesaw form is another suppression factor from the loop effect. λ is some combination of the coupling constants besides Yukawa coupling. This constant can be very small naturally if it is associated with the lepton number violation. Therefore the neutrino mass is at least 2 orders of magnitude smaller than that in the usual seesaw model for the same scale M_R. This scenario has some very interesting features. First of all ν_R can be stable by imposing some discrete symmetries, while still giving a light neutrino nonzero mass. In fact, in order to avoid a tree-level Dirac neutrino mass these symmetries are necessary. This is very different from the original seesaw mechanism, where ν_R is unstable for the nonzero light neutrino mass. The application of the stable ν_R is to play the role of the cold dark matter. Second the light neutrino mass is suppressed also by the loop effect, so for a weak scale M_R the neutrino mass can be much smaller than the current experimental bounds. The light neutrino may be provided as a candidate for explaining the solar neutrino, atmospheric neutrino problems, and a hot dark matter component in the mixed dark matter model or still a late decaying particle in the cold dark matter model. Baryogenesis of the Universe also restricts this kind of mechanism, but the constrains are relaxed due to the loop factor. The most attractive picture is that if there is no other scale except the weak scale below the GUT, M_R is around this scale, then in this scenario with only one scale and with only neutrino particles, one may explain the observed dark matter problem and the structure formation of the Universe, and possibly other related phenomena in neutrino physics. From now on we will call the original seesaw mechanism the tree-level seesaw mechanism and the other the radiative seesaw mechanism.

Now let us implement this mechanism in a very simple model. This model is to extend the standard model by introducing a three-family of right-handed neutrinos ν_R and one more Higgs doublet Φ. We impose a Z_2 discrete symmetry on this model. Under this symmetry transformation ν_R and Φ change sign, while other fields remain the same. As a result of this symmetry, ν_R does not couple to the standard Higgs Φ_S through Yukawa coupling. Only the new Higgs doublet Φ couples to ν_R. Now we can write down all the possible interaction terms for this model. It includes the gauge interaction, Yukawa coupling, and the Higgs potential. However in this work only the Yukawa interaction for the lepton and part of the Higgs potential are relevant. The Yukawa coupling and complete Higgs potential are expressed as

$$L_Y = f_{ij}\overline{l_i}e_{Rj}\Phi_S + g_{ij}\overline{l_i}\nu_{Rj}\Phi + \text{H.c.} + M_{ij}\nu_{Ri}^T\nu_{Rj}, \tag{2}$$

$$\begin{aligned} V = & -\mu_1^2\Phi_S^+\Phi_S - \mu_2^2\Phi^+\Phi + \lambda_1(\Phi_S^+\Phi_S)^2 + \lambda_2(\Phi^+\Phi)^2 \\ & + \lambda_3(\Phi_S^+\Phi_S)(\Phi^+\Phi) + \lambda_4(\Phi_S^+\Phi)(\Phi^+\Phi_S) \\ & + \tfrac{1}{2}\lambda_5[(\Phi_S^+\Phi)^2 + (\Phi^+\Phi_S)^2]. \end{aligned} \tag{3}$$

Here l_i and e_{Ri} are the lepton doublet and the right-handed charged lepton, respectively. Since the Z_2 symmetry is exact and will not be broken, Φ will not develop a nonzero vacuum expectation value (VEV). Therefore only the L_Y lepton number is not broken, i.e., the neutrino does not obtain mass at this level. However with all the terms in L_Y and a term like $\lambda(\Phi_S^+\Phi)^2$ in the potential, it is easy to check that the lepton number symmetry is not automatically conserved anymore. In other words, the neutrino must develop a nonzero mass, but obviously this mass is generated only at loop level, see Fig. 1. If the masses of Φ and ν_R are at the same order of the magnitude M_R, the light neutrino mass can be estimated as, up to a logarithmic factor,

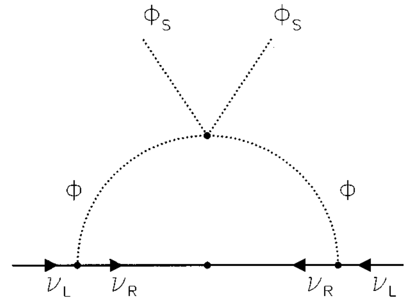

FIG. 1. The one-loop diagram for light neutrino mass generation.

$$m_\nu \simeq \frac{\lambda}{16\pi^2} g^T M_R^{-1} g V^2, \tag{4}$$

where V is the VEV of the standard Higgs boson Φ_S. In the tree-level seesaw mechanism it is assumed that the couplings g and f have the same order of magnitude and similar structure, that is $gV \sim fV = m_D$. Then in this model the basic seesaw concept is realized at the loop level. Compared with the simplest tree-level seesaw model, which is the standard model plus a right-handed neutrino, our model only has one more Higgs doublet and introduces an additional Z_2 discrete symmetry. Because the Z_2 symmetry is not broken, Φ_S and Φ and ν_R and ν_l will not mix with each other, respectively. The lightest particles among ν_R and Φ are stable. In the above description the lepton number is explicitly broken. Of course the lepton number may also be broken spontaneously with the introduction of some singlet scalar fields as in the singlet majoron model [13]. In the singlet majoron model the mass term of ν_R is replaced by $h\nu_R\nu_R S$, where S is the singlet scalar field. When the S field gets a nonzero VEV, the lepton number is spontaneously broken. The difference between the tree-level seesaw model and our model is that in our model the majoron only couples to ν_R at tree level. The light neutrino couples to the majoron not through mixing but radiative correction.

Now we come to discuss the application of our model to the dark matter issue of the Universe and other issues in neutrino physics. The very interesting question is to see how the right-handed neutrino may serve as the candidate of cold dark matter. In our model in principle both ν_R and Φ can be the candidate of the cold dark matter depending on which particle is the lightest one. Here we assume that the one of ν_R is the lightest particle among ν_R and Φ, and from now on we just call it ν_R. Because Φ has direct standard gauge coupling to the Z boson, if it is the dark matter the elastic scattering cross section of Φ from the nuclei of the detector is determined by this neutral current interaction. Not having observed any signal of this reaction requires the mass of Φ to be at least a few TeV [14]. On the other hand, ν_R dark matter is not constrained much from the direct dark matter search experiments. The relic abundance of ν_R is controlled by its interaction with other light particles and the evolution of the

Universe. We are going to estimate the relic density of ν_R in our model with and without a Majoron.

First let us see the case without the Majoron. Most generally the evolution of ν_R is determined by the combined evolution equations of ν_R and Φ. The equations include the contributions from $\overline{\nu_R}\nu_R$ annihilation, $\Phi\Phi$ annihilation, and the decay from Φ to ν_R. If Φ is not almost degenerate with ν_R, i.e., the mass difference ΔM is significantly larger than the freezeout temperature of ν_R, one may neglect the presence of Φ. Then only the $\overline{\nu_R}\nu_R$ annihilation cross section $\langle\sigma_A v\rangle$ determines the relic density of ν_R. Approximately the contribution of the thermal relics of a massive cold dark matter particle to mass density of the Universe can be expressed as $\Omega h^2 \sim 10^{-37}\ \mathrm{cm}^2/\langle\sigma_A v\rangle$ [15]. To be the candidate of the cold dark matter, its annihilation cross section should be roughly as large as $10^{-37}\ \mathrm{cm}^2$. And the freezeout temperature $T_D(\nu_R)$ at which ν_R decouples from the thermal equilibrium is about $M_R/20$. For $\overline{\nu_R}\nu_R$ annihilation the dominant channel is $\overline{\nu_R}\nu_R \to \overline{\nu_l}\nu_l,\ e^+e^-\cdots$, which is related to the light neutrino mass generation. The cross section for this channel is estimated as

$$\langle\sigma_A v\rangle \sim \frac{\lambda'^2 m_D^4}{\pi M_R^6} = 10^{-39}\left(\frac{\lambda'}{1.0}\right)^2\left(\frac{m_D}{1.7\ \mathrm{GeV}}\right)^4\left(\frac{100\ \mathrm{GeV}}{M_R}\right)^6\ \mathrm{cm}^2, \tag{5}$$

where λ' represents all possible contribution from the scalar potential including the λ term. Since λ is related to the lepton number violation, λ' can be naturally much larger than λ. In fact with the parameters chosen reasonably as in the above equation, the annihilation cross section is much smaller than that needed for ν_R being the cold dark matter, the ν_R contribution overcloses the Universe. On the other hand, however, if ΔM is much smaller than $T_D(\nu_R)$, the density of Φ and ν_R are both determined by the annihilation process $\Phi\Phi\to$ light standard model particles. The cross section is estimated as

$$\langle\sigma v\rangle \simeq \frac{\pi\alpha^2}{M_R^2} \simeq 10^{-35}\left(\frac{100\ \mathrm{GeV}}{M_R}\right)^2\ \mathrm{cm}^2, \tag{6}$$

where α is the fine structure constant. At this extreme situation with the parameters chosen as in Eq. (5), the annihilation process is too strong, it contributes only a small portion of needed dark matter density. Although for ΔM between these two extreme situations one needs to solve the combined evolution equations, one can certainly expect a certain range of ΔM from $T_D(\nu_R)$ to M_R, the relic ν_R is able to contribute a closure density to the Universe. A similar case is investigated quantitatively but in a different model [16]. Its numerical calculation supports this expectation in our model.

As we already mentioned the light neutrino mass depends on the parameter λ. With $\lambda \leqslant 1$, one obtains $m_{\nu_\tau} \leqslant 100$ keV, $m_{\nu_\mu} \leqslant 300$ eV, $m_{\nu_e} \leqslant 10^{-2}$ eV. We investigate three possible choices for neutrino mass, which are interesting in neutrino physics. The first is $m_{\nu_\tau} \simeq 5$ eV, then ν_τ can be the hot dark matter component needed for the large scale structure formation in the mixed cold dark matter model. In this case the ν_μ mass is close to 10^{-2} eV and ν_e is very light as expected from the seesaw mechanism. If the mass of ν_μ is a few times smaller than 10^{-2} eV, ν_μ, ν_e oscillation may offer a solution to the solar neutrino problem through the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism. If it is a few times larger, the mass square difference for these two neutrino species is just what is needed for the atmospheric neutrino problem. Because all three light neutrino masses are very small, the constrain from the baryogenesis of the Universe, which requires that the primordial baryon asymmetry not be washed out by the coexistence of the $B-L$ violation process for neutrino mass generation and gauged $B+L$ violation process, is satisfied. The second choice is to have the mass of ν_τ around 0.1 eV, and the mass of ν_μ around 3×10^{-3} eV and ν_e much lighter. In this case three neutrino oscillations can possibly explain both solar and atmospheric neutrino problems, but no candidate of hot dark matter is provided. The third choice is with three neutrinos as heavy as about 1 keV, 5 eV, and 10^{-4} eV. With these neutrino masses, ν_μ may serve as the candidate of hot dark matter and the oscillation between ν_μ and ν_e can explain the Liquid Scintillation Neutrino Detector (LSND) neutrino oscillation experimental data [17]. However the keV τ neutrino must decay fast enough in order not to delay the beginning of the matter dominated epoch of the Universe too much. This demands the lifetime $\tau(\nu_\tau) \leqslant 2\times10^2(1\,\mathrm{keV}/m_{\nu_\tau})^2$ yr [18]. In our model the dominant decay modes for ν_τ are $\nu_\tau \to \nu_{(\mu,e)} + (\mu,e)^\pm$. Its lifetime is therefore estimated as $\tau(\nu_\tau) \geqslant 10^{12}(\mathrm{keV}/m_{\nu_\tau})^5$ yr. We see that this constrain rules out the third choice.

Now we proceed to discuss the Majoron model. In the Majoron model the relic density of ν_R is not only determined by the annihilation processes $\overline{\nu_R}\nu_R \to \overline{\nu_l}\nu_l,\ e^+e^-,\ldots$ but also by the process $\nu_R\nu_R \to \phi_R\phi_R$, here ϕ_R is the Majoron associated with spontaneous lepton number breaking. The coupling between ϕ_R and other standard model particles is only induced by the loop effect and proportional to some power of Yukawa coupling, so it is negligible in considering of the relic density of ν_R. We estimate the cross section for the second annihilation process $\nu_R\nu_R \to \phi_R\phi_R$ as

$$\langle\sigma_A v\rangle \sim \frac{h^4}{3\pi M_R^2}\left(\frac{P}{E}\right)^2 \simeq \frac{h^4 T}{\pi M_R^3} \tag{7}$$

in terms of the energy E and three-momentum P of ν_R in the center-of-mass frame, and T is the temperature of the Universe. It is noticed that for this process it is p-wave dominated. The s-wave contribution is forbidden as a result of momentum and CP conservation as well as the statistics. This is roughly a weak interaction cross section if h is around the order of 1 and M_R at weak scale. Since the second process dominates over the first annihilation process, it is the second annihilation cross section that determines the relic density of ν_R at present. To get a feeling of the numbers, $\langle\sigma_A v\rangle \sim 10^{-37}\ \mathrm{cm}^2$ with $h\sim0.1$ and $M_R \sim 100$ GeV. Since ϕ_R decouples from the standard model particles at a high energy scale $\sim M_R/20$ larger than a few GeV, the Majoron contributes to the effective number of light neutrino species N_ν less than 0.1 when primordial nucleosynthesis commences. Hence the condition $N_\nu \leqslant 3.3$ at the time of nucleosynthesis [19] is satisfied. Other restrictions from cosmology and astrophysics can also be easily obeyed. The strongest one is due to the cooling of red giants. It requires that the coupling between the electron pair and the Majoron is weaker than 10^{-11} [20]. In our model this coupling is safely smaller than this number because this coupling is induced through a one-loop diagram and is proportional to the square of the electron mass.

The distinguished feature of the Majoron model is that it offers new decay channels for the heavier light neutrino. In our model ν_τ can decay to other two lighter neutrinos plus a Majoron. We consider one interesting situation here, the mass of ν_τ is about the order of 10 keV, ν_μ is a few eV. In a previous model without a Majoron this possibility is ruled out. However due to the new decay channel to Majoron the lifetime of ν_τ can be much shorter. The dominant decay channel is to ν_μ plus a Majoron. We estimate the lifetime of ν_τ as

$$\tau(\nu_\tau) \simeq 16\pi \left(\frac{m_{\nu_\tau}}{M_R} \right)^{-4} m_{\nu_\tau}^{-1}$$

$$= 10^3 \left(\frac{10 \ \text{keV}}{m_{\nu_\tau}} \right)^5 \left(\frac{M_R}{100 \ \text{GeV}} \right)^4 \ \text{yr}. \qquad (8)$$

Its dependence on the light neutrino mass is similar to that in the original singlet majoron model, though the decay mechanism is different, in our model the nonvanishing contribution to this decay occurs at a two-loop level. To see what kind role the τ neutrino can play in the cosmology, we need to be more specific. We take $M_R = 50$ GeV and $m_{\nu_\tau} = 30$ keV and find the lifetime $\tau \simeq 0.2$ yr. A neutrino with this mass and lifetime can just be a late decaying particle which is required for the large scale structure formation of the Universe in the cold dark matter model [18]. Nevertheless at the same time we have to require the ν_μ to be lighter than a few eV in order not to violate the same requirement. The mass hierarchy between ν_τ and ν_μ is about 1 order of magnitude larger than that expected from the above-mentioned seesaw relation m_τ^2/m_μ^2, though we think it is still reasonable. Here again we see the advantage of the radiative seesaw model. Even if the mass of ν_τ is as small as 10 keV, M_R can be around 100 GeV due to the loop factor, so that ν_τ can decay fast.

In conclusion, we discussed a version of the seesaw mechanism which is realized radiatively and gave a concrete model to exhibit its interesting new features. Generally speaking, in this mechanism the constrains from cosmology and astrophysics are relaxed compared with that in the tree-level seesaw model. We emphasize and focus on the weak scale seesaw mechanism in our model. The most interesting application of this new mechanism is on cosmology and neutrino physics. We pointed out that the lightest right-handed neutrino ν_R can be a good candidate of cold dark matter. And at the same time the light neutrino may provide a hot dark matter or late decaying particle for large scale structure formation, or offer solutions to other problems in neutrino physics. Finally we would like to point out that if ν_R is the dark matter of the Universe, there are two possible ways to find out its signal. The first is through the high energy collider experiment. The ν_R pair can be produced through a process like $e^+e^- \to \overline{\nu_R}\nu_R$. Since ν_R is invisible and a Majorana particle, the best signal is to search for a like sign charged lepton pair. Another way is to look for the annihilation products $\mu^+\mu^-$ of a dark matter ν_R pair in the indirect dark matter search experiments.

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