

## Average kinetic energy of a heavy quark in semileptonic $B$ decay

Dae Sung Hwang\*

*Department of Physics, Sejong University, Seoul 143, Korea*

C. S. Kim†

*Department of Physics, Yonsei University, Seoul 120, Korea  
and Theory Division, KEK, Tsukuba, Ibaraki 305, Japan*

Wuk Namgung‡

*Department of Physics, Dongguk University, Seoul 100, Korea*

(Received 3 April 1996)

Within the ACCMM model the average kinetic energy of the heavy quark in a heavy-light meson is calculated as  $\langle \mathbf{p}^2 \rangle = \frac{3}{2} p_F^2$  solely from the fact that the Gaussian momentum probability distribution has been taken in the ACCMM model. Therefore, the Fermi momentum parameter  $p_F$  of the ACCMM model is not a truly free parameter, but is closely related to the average kinetic energy of the heavy quark, which is theoretically calculable in principle. In this context, we determine  $p_F$  by comparing the theoretical prediction of the ACCMM model with the model-independent lepton energy spectrum of  $B \rightarrow e \nu X$  from the recent CLEO analysis, and find that  $p_F = 0.54 \pm_{0.15}^{0.16}$  GeV. We also calculate  $p_F$  in the relativistic quark model by applying the quantum mechanical variational method, and obtain  $p_F = 0.5-0.6$  GeV. We show the correspondences between the relativistic quark model and the heavy quark effective theory. We then clarify the importance of the value of  $p_F$  in the determination of  $|V_{ub}/V_{cb}|$ . [S0556-2821(96)02221-7]

PACS number(s): 12.39.Ki, 12.39.Pn, 13.20.He

### I. INTRODUCTION

In the standard  $SU(2) \times U(1)$  gauge theory of Glashow, Salam, and Weinberg the fermion masses and hadronic flavor-changing weak transitions have a somewhat less secure role, since they require a prior knowledge of the mass generation mechanism. The simplest possibility to give mass to the fermions in the theory makes use of Yukawa interactions involving the doublet Higgs field. These interactions give rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix: Quarks of different flavor are mixed in the charged weak currents by means of a unitary matrix  $V$ . However, both the electromagnetic current and the weak neutral current remain flavor diagonal. Second-order weak processes such as mixing and  $CP$  violation are even less secure theoretically, since they can be affected by both beyond the standard model virtual contributions and new physics direct contributions. Our present understanding of  $CP$  violation is based on the three-family Kobayashi-Maskawa model [1] of quarks, some of whose charged-current couplings have phases. Over the past decade, new data have allowed one to refine our knowledge about parameters of this matrix  $V$ .

In the minimal standard model  $CP$  violation is possible through the CKM mixing matrix of three families, and it is important to know whether or not the element  $V_{ub}$  is nonzero accurately. Its knowledge is also necessary to check whether

the unitarity triangle is closed or not [2]. However, its experimental value is very poorly known presently and its better experimental information is urgently required. At present, the only experimental method to measure  $V_{ub}$  is through the end-point lepton energy spectrum of the inclusive  $B$  meson semileptonic decays, e.g., CLEO [3] and ARGUS [4], and their data indicate that  $V_{ub}$  is nonzero. Recently it has also been suggested that the measurements of hadronic invariant mass spectrum [5,6] as well as hadronic energy spectrum [7] in the inclusive  $B \rightarrow X_{c(u)} l \nu$  decays can be useful in extracting  $|V_{ub}|$  with better theoretical understandings. In future asymmetric  $B$  factories with vertex detector, the hadronic invariant mass spectrum will offer alternative ways to select  $b \rightarrow u$  transitions that are much more efficient than selecting the upper end of the lepton energy spectrum, with much less theoretical uncertainties.

The simplest model for the semileptonic  $B$  decay is the spectator model which considers the decaying  $b$  quark in the  $B$  meson as a free particle. The spectator model is usually used with the inclusion of perturbative QCD radiative corrections [8]. Then the decay width of the process  $B \rightarrow X_q l \nu$  is given by

$$\begin{aligned} \Gamma_B(B \rightarrow X_q l \nu) &\equiv |V_{qb}|^2 \times \tilde{\Gamma}_B(B \rightarrow X_q l \nu) \\ &\simeq \Gamma_b(b \rightarrow q l \nu) \\ &= |V_{qb}|^2 \left( \frac{G_F^2 m_b^5}{192 \pi^3} \right) f \left( z = \frac{m_q}{m_b} \right) \\ &\quad \times \left[ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} g \left( z = \frac{m_q}{m_b} \right) \right], \quad (1) \end{aligned}$$

\*Electronic address: dshwang@phy.sejong.ac.kr

†Electronic address: kim@cskim.yonsei.ac.kr;  
cskim@kekvox.kek.jp

‡Electronic address: ngw@cakra.dongguk.ac.kr

where  $m_q$  is the mass of the final  $q$  quark decayed from the  $b$  quark. Here  $f(z)$  is the phase-space factor, and  $g(z) = (\pi^2 - 31/4)(1-z)^2 + 1.5$  is the corresponding single-gluon QCD correction [9]. As can be seen, the decay width of the spectator model depends on  $m_b^5$ , therefore a small difference of  $m_b$  would change the decay width significantly. The model of Altarelli, Cabibbo, Corbò, Maiani, and Martignelli [10] (ACCMM model) is an improvement on the naive free-quark decay spectator model, but at the cost of introducing several free parameters: the final (charm) quark mass  $m_c$ , the spectator mass  $m_{sp}$ , and the most important Fermi momentum function  $\phi(\mathbf{p}; p_F)$  that includes both binding and final state interaction effects.

In Sec. II, we determine the Fermi momentum parameter  $p_F$  by comparing the theoretical prediction of the ACCMM model with the model-independent lepton energy spectrum of  $B \rightarrow X_c l \nu$  for the whole region of electron energy, which has been recently extracted by CLEO [11]. Previously, the comparison had been hampered by the cascade decay of  $b \rightarrow c \rightarrow s l \nu$ , and only the part of lepton energy spectrum ( $E_l > 1.8$  GeV) could be compared to give  $p_F \sim 0.3$  GeV. However, we argue that the value  $p_F \sim 0.3$  GeV, which has been commonly used in experimental analyses, has no theoretical or experimental clear justification. Therefore, it is strongly recommended to determine the value of  $p_F$  more reliably and independently, when we think of the importance of its role in experimental analyses. A better determination of  $p_F$  is also interesting theoretically since it has its own physical correspondence related to the average kinetic energy ( $\langle \mathbf{p}^2 \rangle$ ) of a heavy quark inside a heavy meson. In this context we calculate theoretically the value of  $\langle \mathbf{p}^2 \rangle$  in the relativistic quark model using a quantum-mechanical variational method in Sec. III. We also compare our model with the heavy quark effective theory (HQET) in expansion of  $1/M_Q$ . The value of  $p_F$  is particularly important in the determination of the value of  $|V_{ub}/V_{cb}|$ , as we explain in Sec. IV. Section V contains our conclusions.

## II. DETERMINATION OF $p_F$ FROM THE EXPERIMENTAL SPECTRUM

Altarelli *et al.* [10] proposed for the inclusive  $B$ -meson semileptonic decays their ACCMM model, which incorporates the bound state effect by treating the  $b$  quark as a virtual state particle, thus giving momentum dependence to the  $b$  quark mass. The virtual state  $b$  quark mass  $W$  is given by

$$W^2(\mathbf{p}) = m_B^2 + m_{sp}^2 - 2m_B \sqrt{\mathbf{p}^2 + m_{sp}^2} \quad (2)$$

in the  $B$  meson rest frame, where  $m_{sp}$  is the spectator quark mass,  $m_B$  is the  $B$  meson mass, and  $\mathbf{p}$  is the momentum of the  $b$  quark inside the  $B$  meson.

For the momentum distribution of the virtual  $b$  quark, Altarelli *et al.* considered the Fermi motion inside the  $B$  meson with the Gaussian momentum probability distribution

$$\phi(\mathbf{p}; p_F) = \frac{4}{\sqrt{\pi} p_F^3} e^{-\mathbf{p}^2/p_F^2}, \quad (3)$$

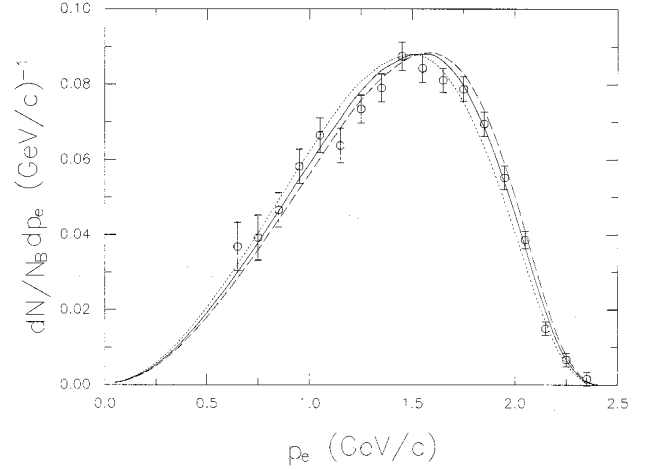


FIG. 1. The normalized lepton energy spectrum of  $B \rightarrow X_c l \nu$  for the whole region of electron energy from the recent CLEO measurement [11]. Also shown are the theoretical ACCMM model predictions, Eq. (4), using  $p_F = 0.44, 0.51, 0.59$  GeV, corresponding to the dashed, full, and dotted lines, respectively. The minimum  $\chi^2$  equals to 1.00 with  $p_F = 0.51$  GeV. We fixed  $m_{sp} = 0.0$  GeV and  $m_q = m_c = 1.5$  GeV.

where the Gaussian width,  $p_F$ , is treated as a free parameter. Then the lepton energy spectrum of the  $B$ -meson decay is given by

$$\begin{aligned} \frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q, m_B) \\ = \int_0^{p_{\max}} p^2 dp \phi(\mathbf{p}; p_F) \frac{d\Gamma_b}{dE_l}(m_b = W, m_q), \quad (4) \end{aligned}$$

where  $p_{\max}$  is the maximum kinematically allowed value of  $p = |\mathbf{p}|$ . The ACCMM model, therefore, introduces a new parameter  $p_F$  for the Gaussian momentum distribution of the  $b$  quark inside  $B$  meson, instead of the  $b$  quark mass of the spectator model. In this way the ACCMM model incorporates the bound state effects and reduces the strong dependence on  $b$  quark mass in the decay width of the spectator model.

The Fermi momentum parameter  $p_F$  is the most essential parameter of the ACCMM model, as we explained in the above. However, the experimental determination of its value from the lepton energy spectrum has been very ambiguous, because various parameters of the ACCMM model, such as  $p_F$ ,  $m_q$ , and  $m_{sp}$ , are fitted all together from the limited region of end-point lepton energy spectrum ( $E_l > 1.8$  GeV) to avoid the cascade decay of  $b \rightarrow c \rightarrow s l \nu$ , and because the perturbative QCD corrections are very sensitive in the end-point region of the spectrum. Recently, CLEO [11] extracted the model-independent lepton energy spectrum of  $B \rightarrow X_c l \nu$  for the whole region of electron energy from  $2.06 \text{ fb}^{-1}$  of  $Y(4S)$  data, which is shown in Fig. 1, with much smaller uncertainties compared to the previously measured results of ARGUS [12]. Now we compare the whole region of experimental electron energy spectrum of CLEO with the theoretical prediction of the ACCMM model, Eq. (4), to derive the

value of  $p_F$  using  $\chi^2$  analysis. With  $p_F$ ,  $m_c$ , and  $m_{sp}$  as free parameters, for one  $\sigma$  standard deviation we obtain

$$p_F = 0.54 \pm_{0.15}^{0.16} \text{ GeV}. \quad (5)$$

In Table I, we show the extracted values of  $p_F$  (in GeV) and  $\chi^2_{\min}/N_{\text{DF}}$  for the fixed input values of  $m_{sp}=0, 0.15$  GeV and  $m_q=m_c=1.4, 1.5, 1.6, 1.7$  GeV, which are the values commonly used in experimental analyses. As can be noticed, these results are strongly dependent on the input value of  $m_c$ : if we use smaller  $m_c$ , the best fit value of  $p_F$  increases, and *vice versa*. In Fig. 1, we also show the theoretical ACCMM model spectrums with  $p_F=0.44, 0.51, 0.59$  GeV (with  $m_c=1.5$  GeV,  $m_{sp}=0.0$  GeV), corresponding to dashed, full, and dotted line, respectively. The experimental data and the theoretical predictions are all normalized to the semileptonic branching ratio,  $\mathcal{B}(B \rightarrow X_c l \nu) = 10.49\%$ , following the result of CLEO [11]. Previously, we extracted similarly  $p_F$  by comparing the theoretical prediction with the experimental spectrum of ARGUS [12], and we obtained [13]  $p_F = 0.27 \pm_{0.27}^{0.22}$  GeV for the fixed input values of  $m_c=1.5$  GeV and  $m_{sp}=0.15$  GeV. As can be seen from Table I, if we fix  $m_c=1.5$  GeV and  $m_{sp}=0.15$  GeV, then we obtain from the new CLEO spectrum [11]  $p_F = 0.55 \pm_{0.07}^{0.09}$  GeV with the minimum  $\chi^2$  being about 1.0. We note that two results are apart each other within one  $\sigma$  standard deviation, but the new result from CLEO has much smaller uncertainties. In Secs. III and IV, we give in detail the related physics of this unexpected large value of the parameter  $p_F$ .

### III. AVERAGE KINETIC ENERGY OF HEAVY QUARK INSIDE HEAVY MESON

Recently considerable progresses have been achieved on the relation of the ACCMM model with QCD [14–16]. Especially Bigi *et al.* [14] derived an inequality between the expectation value of the kinetic energy of the heavy quark inside the hadron and that of the chromomagnetic operator, which gives<sup>1</sup>

$$\langle \mathbf{p}^2 \rangle \geq \frac{3}{4} (M_V^2 - M_P^2). \quad (6)$$

The experimental value of the right-hand side of Eq. (6) is  $0.36 \text{ GeV}^2$  for the  $B$  meson system [18]. This bound corresponds to  $p_F \geq 0.49$  GeV for the  $B$  meson, because in the ACCMM model the average kinetic energy,  $\langle \mathbf{p}^2 \rangle$ , can be calculated from

$$\langle \mathbf{p}^2 \rangle = \int dp p^2 \phi(\mathbf{p}; p_F) = \frac{3}{2} p_F^2. \quad (7)$$

This relation (7) was obtained solely from the fact that the Gaussian momentum probability distribution was taken in the ACCMM model, and therefore the lower bound

$p_F \geq 0.49$  GeV is independent of any other input parameter values of the ACCMM model, and is much larger than the commonly used value  $p_F \sim 0.3$  GeV. Ball *et al.* [16] also calculated  $\langle \mathbf{p}^2 \rangle$  using the QCD sum rule approach, and obtained  $\langle \mathbf{p}^2 \rangle = 0.50 \pm 0.10 \text{ GeV}^2$  for the  $B$  meson, corresponding to  $p_F = 0.58 \pm 0.06$  GeV from Eq. (7). We note that the heavy quark inside the hadron possesses more kinetic energy than the value one might expect naively from the nonrelativistic consideration. We also note that the Fermi momentum parameter  $p_F$  of the ACCMM model is not a truly free parameter, but is closely related to the average kinetic energy of heavy quark, which is theoretically calculable in principle.

We consider the relativistic potential model with the quantum-mechanical variational technique to theoretically calculate the average kinetic energy of the  $b$  quark inside the  $B$  meson, and to compare the results with the predictions of the HQET. The potential model has been successful to describe the physics of  $\psi$  and  $Y$  families with the nonrelativistic Hamiltonian [19,20]. However, for the  $B$  meson it has been difficult to apply the nonrelativistic potential model because of the relativistic motion of the light quark inside the  $B$  meson. In this work, we study the  $B$  meson system with a realistic Hamiltonian, which is relativistic for the light quark and nonrelativistic for the heavy quark, and adopt the variational method to solve it. We take the Gaussian function as the trial wave function, and obtain the ground state energy and wave function by minimizing the expectation value of the Hamiltonian.

For the  $B$  meson system we start with the Hamiltonian

$$H = M + \frac{\mathbf{p}^2}{2M} + \sqrt{\mathbf{p}^2 + m^2} + V(r), \quad (8)$$

where  $M \equiv m_b$  is the heavy quark mass and  $m \equiv m_{sp}$  is the  $u$  or  $d$  quark mass (which corresponds to the spectator light quark mass in the ACCMM model). We apply the variational method to the Hamiltonian (8) with the trial wave function

$$\psi(\mathbf{r}) = \left( \frac{\mu}{\sqrt{\pi}} \right)^{3/2} e^{-\mu^2 \mathbf{r}^2/2}, \quad (9)$$

where the parameter  $\mu$  is a variational parameter. The Fourier transform of  $\psi(\mathbf{r})$  gives the momentum space wave function  $\chi(\mathbf{p})$ , which is also Gaussian,

$$\chi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}\mu)^{3/2}} e^{-\mathbf{p}^2/2\mu^2}. \quad (10)$$

We note here that the Gaussian momentum probability distribution of the ACCMM model equals  $\phi(\mathbf{p}; p_F) = 4\pi |\chi(\mathbf{p}; \mu)|^2$ . See Eqs. (3) and (10). The ground state is given by minimizing the expectation value of  $H$ ,

$$\langle H \rangle = \langle \psi | H | \psi \rangle = E(\mu), \quad \frac{d}{d\mu} E(\mu) = 0 \quad \text{at } \mu = \bar{\mu}, \quad (11)$$

and then the value  $\bar{E} \equiv E(\bar{\mu})$  approximates the  $B$  meson mass  $M_B$ , and at the same time we get  $\bar{\mu} \equiv p_F$ , the Fermi momentum parameter in the ACCMM model. As is well known, the

<sup>1</sup>This theoretical lower bound could be significantly weakened, as shown in [17], with inclusion of the  $\alpha_s$  corrections as well as  $1/M_Q$  corrections.

TABLE I. The values of  $p_F$  (in GeV) and  $\chi_{\min}^2/N_{\text{DF}}$  for the fixed input parameter values  $m_{\text{sp}}$  and  $m_c$  (in GeV). We derived the values using  $\chi^2$  analysis by comparing the whole region of experimental electron energy spectrum of CLEO [11], which is shown in Fig. 1, with the theoretical prediction of the ACCMM model, Eq. (4), using  $p_F$  as a free parameter.

	$m_{\text{sp}}=0.00$				$m_{\text{sp}}=0.15$			
	$m_c=1.4$	1.5	1.6	1.7	$m_c=1.4$	1.5	1.6	1.7
$p_F$	$0.64 \pm 0.09$	$0.51^{+0.08}_{-0.07}$	$0.40^{+0.07}_{-0.05}$	$0.29^{+0.07}_{-0.06}$	$0.69 \pm 0.10$	$0.55^{+0.09}_{-0.07}$	$0.44^{+0.09}_{-0.06}$	$0.32^{+0.08}_{-0.03}$
$\chi_{\min}^2$	1.09	1.00	1.41	2.05	1.44	1.05	1.09	1.47

value of  $\bar{\mu}$  or  $p_F$  corresponds to the measure of the radius of the two-body bound state, as can be seen from the relation  $\langle r \rangle = 2/(\sqrt{\pi} \bar{\mu})$  or  $\langle r^2 \rangle^{1/2} = 3/(2 \bar{\mu})$ .

We now take in Eq. (8) the Cornell potential, which is composed of the Coulomb and linear potentials with a constant term:

$$V(r) = -\frac{\alpha_c}{r} + Kr + V_0 \equiv -\frac{4}{3} \frac{\alpha_s}{r} + Kr + V_0. \quad (12)$$

The additive constant  $V_0$ , which is related to the regularization concerned with the linear confining potential [21], is usually known as flavor dependent:  $V_0=0$  for the heavy-heavy meson system,  $V_0=-0.2$  GeV for the  $B$  meson system [22]. We use the value of  $K=0.19$  GeV<sup>2</sup> [23] for the string tension, and for the parameter  $\alpha_c$  ( $\equiv 4/3\alpha_s$ ) we will consider two values  $\alpha_s=0.35$  and  $0.24$  separately. The first choice  $\alpha_s=0.35$  is the value which has been determined by the best fit of  $(c\bar{c})$  and  $(b\bar{b})$  bound state spectra [23], and  $\alpha_s=0.24$  is that given by the running coupling constant for the QCD scale at  $M_B$ .

With the Gaussian trial wave functions, Eqs. (9) and (10), the expectation value of each term of the Hamiltonian (8) is given as follows:

$$\begin{aligned} \left\langle \frac{\mathbf{p}^2}{2M} \right\rangle &= \left\langle \chi(\mathbf{p}) \left| \frac{\mathbf{p}^2}{2M} \right| \chi(\mathbf{p}) \right\rangle = \frac{3}{4M} \mu^2, \\ \langle \sqrt{\mathbf{p}^2 + m^2} \rangle &= \langle \chi(\mathbf{p}) | \sqrt{\mathbf{p}^2 + m^2} | \chi(\mathbf{p}) \rangle \\ &= \frac{4\mu}{\sqrt{\pi}} \int_0^\infty e^{-x^2} \sqrt{x^2 + (m/\mu)^2} x^2 dx, \\ \langle V(r) \rangle &= \left\langle \psi(\mathbf{r}) \left| -\frac{\alpha_c}{r} + Kr + V_0 \right| \psi(\mathbf{r}) \right\rangle \\ &= \frac{2}{\sqrt{\pi}} (-\alpha_c \mu + K/\mu) + V_0. \end{aligned} \quad (13)$$

Then we have

$$\begin{aligned} E(\mu) = \langle H \rangle &= M + \frac{1}{2M} \left( \frac{3}{2} \mu^2 \right) + \frac{2}{\sqrt{\pi}} (-\alpha_c \mu + K/\mu) + V_0 \\ &+ \frac{4\mu}{\sqrt{\pi}} \int_0^\infty e^{-x^2} \sqrt{x^2 + (m/\mu)^2} x^2 dx. \end{aligned} \quad (14)$$

In our previous study [24], we obtained the last integral in Eq. (14) as a power series of  $(m/\mu)^2$ . And when we write up to the order of  $(m/\mu)^4$ , we now get

$$\begin{aligned} E(\mu) &= M + \frac{3}{4M} \mu^2 + \frac{2}{\sqrt{\pi}} (-\alpha_c \mu + K/\mu) + V_0 \\ &+ \frac{2\mu}{\sqrt{\pi}} \left[ 1 + \frac{1}{2} (m/\mu)^2 + \left( \frac{5}{32} - 2c_1 \right) (m/\mu)^4 \right. \\ &\left. + \frac{1}{4} (m/\mu)^4 \ln(m/\mu) \right] + O((m/\mu)^6), \end{aligned} \quad (15)$$

where  $c_1 \approx -0.0975$ . Up to the order of  $(m/\mu)^2$ ,  $E(\mu)$  becomes

$$\begin{aligned} E(\mu) &= M + \frac{3}{4M} \mu^2 \\ &+ \frac{2}{\sqrt{\pi}} [(1 - \alpha_c) \mu + (K + \frac{1}{2} m^2)/\mu] + V_0, \end{aligned} \quad (16)$$

and the next-order terms  $[O((m/\mu)^4)]$  contribute only less than 1%. Then, we find the minimum value of  $E(\mu)$  in Eq. (16) by the variational method, and the minimum point is given by

$$\frac{\partial}{\partial \mu} E(\mu) = \frac{3}{2M} \mu + \frac{2}{\sqrt{\pi}} (\beta - \gamma/\mu^2) = 0, \quad (17)$$

where

$$\beta \equiv 1 - \alpha_c = 1 - \frac{4}{3} \alpha_s \quad \text{and} \quad \gamma \equiv K + \frac{1}{2} m^2. \quad (18)$$

We rewrite Eq. (17) as

$$(\beta \mu^2 - \gamma) + \frac{b}{M} \mu^3 = 0, \quad (19)$$

where  $b = 3\sqrt{\pi}/4$  is a constant. Then, we expand  $\bar{\mu}$ , which satisfies Eq. (19), as a power series of  $1/M$ ,

$$\bar{\mu} = a_0 + a_1 \frac{1}{M} + a_2 \frac{1}{M^2} + \dots, \quad (20)$$

and by matching the order by the order in Eq. (19), we get

TABLE II. The numerical values of the coefficients  $a_0, a_1, a_2$  in the  $1/M$  expansion of  $\bar{\mu}$ , Eq. (20), and the values of  $\bar{\mu}$  which minimizes  $E(\mu)$  in Eq. (16). We varied  $\alpha_s=0.35, 0.24$  and the light quark mass  $m(\equiv m_{\text{sp}})=0.00, 0.15, 0.30$  GeV.

		$a_0$	$a_1$	$a_2$	$\bar{\mu}$
$\alpha_s=0.35$	$m_{\text{sp}}=0.00$	0.60	-0.60	1.50	0.54
	$m_{\text{sp}}=0.15$	0.61	-0.63	1.62	0.54
	$m_{\text{sp}}=0.30$	0.67	-0.76	2.13	0.61
$\alpha_s=0.24$	$m_{\text{sp}}=0.00$	0.53	-0.36	0.63	0.49
	$m_{\text{sp}}=0.15$	0.54	-0.38	0.68	0.49
	$m_{\text{sp}}=0.30$	0.59	-0.46	0.89	0.54

$$a_0 = \sqrt{\frac{\gamma}{\beta}}, \quad a_1 = -\frac{b}{2} \left( \frac{\gamma}{\beta^2} \right), \quad (21)$$

$$a_2 = \frac{5b^2}{8} \sqrt{\frac{\gamma}{\beta}} \left( \frac{\gamma}{\beta^3} \right), \quad \dots$$

As can be easily seen, since  $b/M \ll 1$ , Eq. (19) has an approximate solution  $\bar{\mu} \approx \sqrt{\gamma/\beta} = a_0$ .

Using Eqs. (20) and (21), we can obtain the numerical values of the coefficients  $a_0, a_1, a_2$ , and that of  $\bar{\mu}$  which minimizes  $E(\mu)$  in Eq. (16), for  $\alpha_s=0.35$  and  $0.24$  separately. We also considered three different values of the light quark mass  $m(\equiv m_{\text{sp}})=0.00, 0.15, 0.30$  GeV, in order to see the dependence of the results on the light quark mass  $m$ . As we can see from Eqs. (17) and (18), the effect of  $m$  comes in only through the little modification of  $\gamma$ , because  $\gamma \approx K + m^2/2 \approx K$ . The results of this calculation for  $a_0, a_1, a_2$ , and  $\bar{\mu}$  with the input values of  $\alpha_s$  and the light quark mass  $m(\equiv m_{\text{sp}})$  are presented in Table II. As previously explained, we fixed  $K=0.19$  GeV<sup>2</sup> and  $V_0=-0.2$  GeV. However, the exact value of  $V_0$  is irrelevant in our calculations of  $\bar{\mu}$ , Eqs. (20) and (21), but it is necessary for the calculation of  $\bar{\Lambda}$  in Eq. (23) below.

With  $\bar{\mu}$  of Eqs. (20) and (21), we can get the following expectation values of the terms in the Hamiltonian (8):

$$\begin{aligned} \frac{T}{2M} &\equiv \frac{\langle \mathbf{p}^2 \rangle (\bar{\mu})}{2M} = \frac{3\bar{\mu}^2}{4M} \\ &= \frac{1}{2M} \left[ \frac{3\gamma}{2\beta} - \frac{3b}{2} \sqrt{\frac{\gamma}{\beta}} \left( \frac{\gamma}{\beta^2} \right) \frac{1}{M} + \frac{9b^2}{4} \left( \frac{\gamma^2}{\beta^4} \right) \frac{1}{M^2} \right] \\ &\quad + O\left( \frac{1}{M^3} \right), \end{aligned} \quad (22)$$

<sup>2</sup>The numerical value of  $\bar{\mu}$  is fairly insensitive to the potential we choose. In Ref. [25],  $\bar{\mu}$  has been calculated numerically from six different potential models, and found to be  $\bar{\mu}=0.56 \pm 0.02$  GeV, where the error is only the statistical error of the six different results.

$$\begin{aligned} \bar{\Lambda} &\equiv \langle \sqrt{\mathbf{p}^2 + m^2} + V(r) \rangle (\bar{\mu}) = \frac{2}{\sqrt{\pi}} \left( \beta \bar{\mu} + \frac{\gamma}{\bar{\mu}} \right) + V_0 \\ &= (V_0 + 2\sqrt{\gamma\beta}) + 0 \times \frac{1}{M} \\ &\quad + \frac{b^2}{4} \sqrt{\frac{\gamma}{\beta}} \left( \frac{\gamma}{\beta^2} \right) \frac{1}{M^2} + O\left( \frac{1}{M^3} \right), \end{aligned} \quad (23)$$

Finally,  $E(\bar{\mu})$  in Eq. (16) is expressed as a power series in  $1/M$ :

$$\begin{aligned} E(\bar{\mu}) &= M + \bar{\Lambda} + \frac{T}{2M} \\ &\equiv M + (V_0 + 2\sqrt{\gamma\beta}) + \frac{1}{2M} \left( \frac{3}{2} \frac{\gamma}{\beta} \right) \\ &\quad - \left[ \frac{b(3-b)}{4} \sqrt{\frac{\gamma}{\beta}} \left( \frac{\gamma}{\beta^2} \right) \right] \frac{1}{M^2} + O\left( \frac{1}{M^3} \right). \end{aligned} \quad (24)$$

In Eq. (24), the  $M$ -independent terms come from  $\langle \sqrt{\mathbf{p}^2 + m^2} + V(r) \rangle$ , which can be considered as the contributions of the light degrees of freedom. The term of the order of  $1/M$  is from the heavy quark momentum squared  $\langle \mathbf{p}^2 \rangle$ , that is, from the average kinetic energy of the heavy quark inside the heavy-light meson. Both  $\langle \sqrt{\mathbf{p}^2 + m^2} + V(r) \rangle$  and  $\langle \mathbf{p}^2 \rangle$  contribute to the term of the order of  $1/M^2$ . In the HQET, the mass of a heavy-light meson is represented [26] by

$$M_M = M + \bar{\Lambda} + \frac{1}{2M} (T + \nu_M \Omega) + O\left( \frac{1}{M^2} \right), \quad (25)$$

where  $\bar{\Lambda} \equiv \lim_{M \rightarrow \infty} (M_M - M)$  is the contribution from the light degrees of freedom, for which Neubert obtained [26]  $\bar{\Lambda} = 0.57 \pm 0.07$  GeV.  $T \equiv \langle \mathbf{p}^2 \rangle$  is the expectation value of the kinetic energy of the heavy quark (up to  $2M$ ) inside a heavy-light meson, and  $\Omega$  is the expectation value of the energy due to the chromomagnetic hyperfine interaction with  $\nu_V = 1/4$  and  $\nu_P = -3/4$ . In this paper we do not consider the chromomagnetic hyperfine interaction term. We will present a detailed study on the correspondences between the relativistic quark model and the heavy quark effective theory in another forthcoming paper [27]. Here we calculated only  $T$  and  $\bar{\Lambda}$  up to the order of  $1/M^2$  by using Eqs. (22) and (23), and obtained the values shown in Table III. In Table III, we also show the values of the Fermi momentum parameter  $p_F(\equiv \bar{\mu}$ , shown in Table II) of the ACCMM model using the relation (7).

Gremm *et al.* [28] recently extracted the average kinetic energy,  $T \equiv \langle \mathbf{p}^2 \rangle$ , by comparing the prediction of the HQET [29] with the shape of the inclusive  $B \rightarrow X l \nu$  lepton energy spectrum [30] for  $E_l \geq 1.5$  GeV, in order to avoid the contamination from the secondary leptons of cascade decays of  $b \rightarrow c \rightarrow s l \nu$ . They obtained  $\lambda_1 (\equiv -T) = -0.35 \pm 0.05$  GeV<sup>2</sup> for  $|V_{ub}/V_{cb}| = 0.08$  and  $\lambda_1 (\equiv -T) = -0.37 \pm 0.05$  GeV<sup>2</sup> for  $|V_{ub}/V_{cb}| = 0.1$ , which correspond to  $p_F = 0.48 \pm 0.03$  GeV and  $p_F = 0.50 \pm 0.03$  GeV, respectively. Their results are remarkably close to our value in Eq. (5)

TABLE III. The average kinetic energy  $T$  (up to  $2M$ ) of the heavy quark, the contribution of the light degrees of freedom  $\bar{\Lambda}$ , and the Fermi momentum parameter  $p_F$  of the  $B$ -meson system, for  $\alpha_s=0.35, 0.24$  and  $m(\equiv m_{\text{sp}})=0.00, 0.15, 0.30$  GeV. The results obtained by the  $\chi^2$  analysis of the recent CLEO lepton energy spectrum, and those from the HQET and the QCD sum rule approaches are also presented.

		$T$	$\bar{\Lambda}$	$p_F (\equiv \bar{\mu})$
$\alpha_s=0.35$	$m_{\text{sp}}=0.00$	0.45	0.44	0.54
	$m_{\text{sp}}=0.15$	0.47	0.46	0.54
	$m_{\text{sp}}=0.30$	0.57	0.52	0.61
$\alpha_s=0.24$	$m_{\text{sp}}=0.00$	0.36	0.52	0.49
	$m_{\text{sp}}=0.15$	0.38	0.54	0.49
	$m_{\text{sp}}=0.30$	0.45	0.61	0.54
From CLEO data [11]		—	—	$0.54 \pm_{0.15}^{0.16}$
Bigi <i>et al.</i> [14]		$\geq 0.36$	—	$\geq 0.49$
Ball <i>et al.</i> [16]		$0.50 \pm 0.10$	—	$0.58 \pm 0.06$
Neubert [26]		—	$0.57 \pm 0.07$	—
Gremm <i>et al.</i> [28] $ V_{ub}/V_{cb} =0.08$		$0.35 \pm 0.05$	—	$0.48 \pm 0.03$
Gremm <i>et al.</i> [28] $ V_{ub}/V_{cb} =0.10$		$0.37 \pm 0.05$	—	$0.50 \pm 0.03$

extracted from the recent model-independent lepton energy spectrum of  $B \rightarrow X_c l \nu$  [11], as explained in Sec. II.

We summarize Sec. III by noting that the value of the Fermi momentum parameter of the ACCMM model is  $p_F=0.5-0.6$  GeV and is much larger than  $\sim 0.3$  GeV, as can be seen from Table III, and the heavy quark inside the hadron possesses much more kinetic energy than the value one might expect naively from the nonrelativistic consideration.

#### IV. DEPENDENCE OF $|V_{cb}|$ AND $|V_{ub}/V_{cb}|$ ON THE AVERAGE KINETIC ENERGY OF HEAVY QUARK INSIDE $B$ MESON

Now we consider the dependence on the average kinetic energy of the  $b$  quark (or equivalently Fermi momentum parameter  $p_F$  of the ACCMM model) in the  $B$  meson semileptonic decay,  $\langle \mathbf{p}^2 \rangle$ , of the measurements of  $|V_{cb}|$  and  $|V_{ub}/V_{cb}|$ . The  $B$  meson inclusive branching fraction is related to the CKM matrix  $V_{cb}$  and  $V_{ub}$  by

$$\mathcal{B}(B \rightarrow X l \nu) / \tau_B = \tilde{\Gamma}_c |V_{cb}|^2 + \tilde{\Gamma}_u |V_{ub}|^2 \approx \tilde{\Gamma}_c |V_{cb}|^2, \quad (26)$$

where the factors  $\tilde{\Gamma}_q \equiv \tilde{\Gamma}_B(B \rightarrow X_q l \nu)(p_F)$  must be calculated from theory. [See Eq. (1).] CLEO has extracted  $|V_{cb}|=0.040 \pm 0.001 \pm 0.004$  from their measurements [11] of

$$\begin{aligned} \mathcal{B}(B \rightarrow X l \nu) &= (10.49 \pm 0.17 \pm 0.43)\%, \\ \tau_B &= (1.61 \pm 0.04) \text{ psec}, \end{aligned} \quad (27)$$

and by assuming  $\tilde{\Gamma}_c = (39 \pm 8) \text{ psec}^{-1}$ . If we instead theoretically calculate  $\tilde{\Gamma}_c$  in the ACCMM model by using  $p_F=0.5-0.6$  GeV, the result of the ACCMM model becomes

$$\begin{aligned} |V_{cb}| &= |V_{cb}|_{\text{CLEO}} \sqrt{\frac{\tilde{\Gamma}_c^{(\text{CLEO})}}{\tilde{\Gamma}_c^{(p_F=0.5-0.6)}}} \\ &\approx |V_{cb}|_{\text{CLEO}} \times 1.1 = 0.044 \pm 0.001 \pm 0.004. \end{aligned} \quad (28)$$

We can easily understand this large correction ( $\sim 10\%$ ) in  $|V_{cb}|$  due to the change in  $p_F$ , because within ACCMM model from Eqs. (1) and (2),

$$\tilde{\Gamma}_c \propto m_b^5 = W^5 \approx (m_B^2 - 2m_B p_F)^{5/2},$$

and, therefore,

$$\tilde{\Gamma}_c^{(p_F=0.3)} / \tilde{\Gamma}_c^{(p_F=0.5)} \approx 1.25. \quad (29)$$

The ACCMM model also provides an inclusive lepton energy spectrum of the  $B$  meson semileptonic decay to obtain the value of  $|V_{ub}/V_{cb}|$ . The lepton energy spectrum is useful in separating  $b \rightarrow u$  transitions from  $b \rightarrow c$ , since the end-point region of the spectrum is completely composed of  $b \rightarrow u$  decays. In applying this method one integrates Eq. (4) in the range  $2.3 \text{ GeV} < E_l$  at the  $B$  meson rest-frame, where only  $b \rightarrow u$  transitions exist [31]. So we theoretically calculate<sup>3</sup>

$$\tilde{\Gamma}(p_F) \equiv \int_{2.3} dE_l \frac{d\tilde{\Gamma}_B}{dE_l}(p_F, m_{\text{sp}}, m_q, m_B). \quad (30)$$

In Eq. (30) we specified only the  $p_F$  dependence explicitly on the left-hand side. Then one compares the theoretically calculated  $\tilde{\Gamma}(p_F)$  with the experimentally measured width  $\tilde{\Gamma}_{\text{expt}}$  in the region  $2.3 \text{ GeV} < E_l$ , to extract the value of  $|V_{ub}|$  from the relation

$$\tilde{\Gamma}_{\text{expt}} = |V_{ub}|^2 \tilde{\Gamma}(p_F). \quad (31)$$

<sup>3</sup>We note that the dependences of the lepton energy spectrum on perturbative and non-perturbative QCD corrections [8,29] as well as on the unavoidable specific model parameters (e.g., the parameter  $p_F$  of the ACCMM model [10]) are strongest at the end-point region of the inclusive  $d\Gamma/dE_l$  distribution. Therefore, Eq. (30) may have very limited validity for the determination of  $|V_{ub}/V_{cb}|$ , as shown in [32].

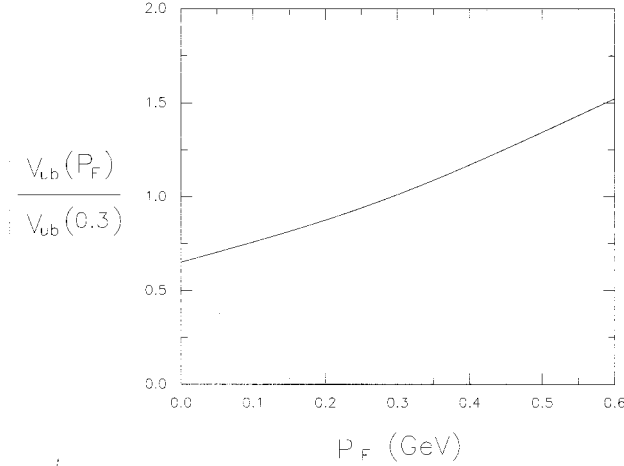


FIG. 2. The ratio  $|V_{ub}(p_F)/V_{ub}(p_F=0.3)|$  as a function of  $p_F$ .

In the real experimental situations [3,4,12,31], the only measured quantity is the number of events in this region of high  $E_l$  compared to the total semileptonic events number, i.e., the branching fraction  $\tilde{\Gamma}_{\text{expt}}/\tilde{\Gamma}_{\text{sl}}^{\text{total}}$ . Since the value  $\tilde{\Gamma}_{\text{sl}}^{\text{total}}$  is proportional to  $|V_{cb}|^2$ , only the combination  $|V_{ub}/V_{cb}|^2$  is extracted.

We now consider the possible dependence of  $|V_{ub}/V_{cb}|^2$  as a function of the parameter  $p_F$  from the following relation:

$$\begin{aligned} \frac{\tilde{\Gamma}_{\text{expt}}}{\tilde{\Gamma}_{\text{sl}}^{\text{total}}} &\propto \left| \frac{V_{ub}}{V_{cb}} \right|_{p_F=p_F}^2 \tilde{\Gamma}(p_F) \\ &= \left| \frac{V_{ub}}{V_{cb}} \right|_{p_F=0.3}^2 \tilde{\Gamma}(p_F=0.3), \end{aligned} \quad (32)$$

where  $|V_{ub}/V_{cb}|_{p_F=p_F}^2$  is determined with an arbitrary value of the Fermi momentum parameter  $p_F$ . On the right-hand side we used  $p_F=0.3$  GeV because this value is commonly used in the experimental determination of  $|V_{ub}/V_{cb}|$ . Then one can get a relation

$$\left| \frac{V_{ub}}{V_{cb}} \right|_{p_F=p_F} = \left| \frac{V_{ub}}{V_{cb}} \right|_{p_F=0.3} \sqrt{\frac{\tilde{\Gamma}(0.3)}{\tilde{\Gamma}(p_F)}}. \quad (33)$$

We numerically calculated the theoretical ratio  $\tilde{\Gamma}(0.3)/\tilde{\Gamma}(p_F)$  by using Eqs. (4) and (30) with  $m_{\text{sp}}=0.15$  GeV,  $m_q=m_u=0.15$  GeV, which are the values commonly used by experimentalists, and  $m_B=5.28$  GeV. We show the values of  $|V_{ub}(p_F)/V_{ub}(p_F=0.3)|$  as a function of  $p_F$  in Fig. 2. If we use  $p_F=0.5-0.6$  GeV, instead of  $p_F=0.3$  GeV, in the experimental analysis of the end-point region of lepton energy spectrum, the value of  $|V_{ub}/V_{cb}|$  becomes significantly changed.

Previously the CLEO [31] analyzed with  $p_F=0.3$  GeV the end-point lepton energy spectrum to get

$$10 \times |V_{ub}/V_{cb}| = 0.76 \pm 0.08$$

(ACCMM with  $p_F=0.3$  [31]),

$$= 1.01 \pm 0.10$$

[Isgur, Scora, Grinstein, and Wise (ISGW) [33]]. (34)

As can be seen, those values differ by two standard deviations<sup>4</sup>. However, if we use  $p_F=0.5-0.6$  GeV, the result of the ACCMM model becomes

$$10 \times |V_{ub}/V_{cb}| \approx 1.07 \pm 0.11$$

(ACCMM with  $p_F=0.5-0.6$ ), (35)

and these two models are in a good agreement for the value of  $|V_{ub}/V_{cb}|$ .

We note here that the dependence of  $|V_{ub}/V_{cb}|$  on the parameter  $p_F$  is much stronger compared to that of  $|V_{cb}|$ . This is because the  $p_F$  dependence of the inclusive distribution  $d\Gamma/dE_l$  is particularly sensitive if we restrict ourselves only in the limited region of the end point, as shown in Eq. (30). We would like to emphasize again that the measurements of the hadronic invariant mass spectrum [5,6] in the inclusive  $B \rightarrow X_{c(u)} l \nu$  decays can be much more useful in extracting  $|V_{ub}|$  with better theoretical understandings, where we can use almost the whole region of the decay spectrum: i.e., in the forthcoming asymmetric  $B$  experiments with microvertex detectors, BABAR and BELLE, the total separation of  $b \rightarrow u$  semileptonic decays from the dominant  $b \rightarrow c$  semileptonic decays would be experimentally viable using the measurement of inclusive hadronic invariant mass distributions. And we could determine the ratio of CKM matrix elements  $|V_{ub}/V_{cb}|$  from the ratio of those measured total integrated decay rates [6], which is theoretically described by the phase-space factor and the well-known perturbative QCD correction only.

## V. CONCLUSIONS

The value of the Fermi momentum parameter of the ACCMM model  $p_F \sim 0.3$  GeV, which has been commonly used in experimental analyses, has no theoretical or experimental clear justification. Therefore, it is strongly recommended to determine the value of  $p_F$  more reliably and independently, when we think of the importance of its role in experimental analyses. It is particularly important in the determination of the value of  $|V_{ub}/V_{cb}|$ . We note that the dependence of  $|V_{ub}/V_{cb}|$  on the parameter  $p_F$  is very strong, because the inclusive lepton energy distribution is particularly sensitive to the variation of  $p_F$  if we restrict ourselves only in the limited region of the end point. A better determination of  $p_F$  is also interesting theoretically since it has its

<sup>4</sup>There now exists an improved version of the ISGW model, so-called ISGW2 [34], which gives a considerably harder end-point spectrum than that of ISGW. Therefore, it seems clear that the prediction of ISGW on  $|V_{ub}/V_{cb}|$ , Eq. (34), will decrease when reanalyzed by experimentalists, even though the changes would be small [34].

own physical correspondence related to the average kinetic energy  $\langle \mathbf{p}^2 \rangle$  of the heavy quark inside the  $B$  meson. Within the ACCMM model the average kinetic energy is calculated as  $\langle \mathbf{p}^2 \rangle = \frac{3}{2} p_F^2$ , solely from the fact that the Gaussian momentum probability distribution has been taken in the ACCMM model. Therefore, the Fermi momentum parameter  $p_F$  of the ACCMM model is not a truly free parameter, but is closely related to the average kinetic energy of heavy quark, which is theoretically calculable in principle.

In this context we theoretically calculated the value of  $p_F$  in the relativistic quark model using the quantum-mechanical variational method. It turns out that  $p_F = 0.5\text{--}0.6$  GeV, which is consistent with the value of  $p_F$  determined by comparing the ACCMM model prediction and the model-independent lepton energy spectrum of the CLEO measurement,  $p_F = 0.54 \pm_{0.15}^{0.16}$  GeV. We note that the value of the Fermi momentum parameter of the ACCMM model is much larger than  $\sim 0.3$  GeV, and the heavy quark inside the hadron possesses much more kinetic energy than the value one might expect naively from the nonrelativistic consideration. We also found the correspondences between the relativistic quark model and the heavy quark effective theory by the  $1/M_Q$  expansion, and the result shows that they are consistent with each other.

If we use  $p_F = 0.5\text{--}0.6$  GeV, instead of  $p_F = 0.3$  GeV, in the experimental analysis of the end-point region of lepton energy spectrum, the value of  $|V_{ub}/V_{cb}|$  is increased by the factor of 1.3–1.5 compared with the case of  $p_F = 0.3$  GeV. Here we would like to emphasize that the measurements of the hadronic invariant mass spectrum in the inclusive  $B \rightarrow X_{c(u)} l \nu$  decays can be much more useful in extracting  $|V_{ub}|$  with better theoretical understandings. In future asymmetric  $B$  factories with vertex detector, the hadronic invariant mass spectrum will offer alternative ways [5,6] to select  $b \rightarrow u$  transitions that are much more efficient than selecting the upper end of the lepton energy spectrum, with much less theoretical uncertainties.

#### ACKNOWLEDGMENTS

The work was supported in part by the Korean Science and Engineering Foundation, Project No. 951-0207-008-2, in part by Non-Directed-Research-Fund, Korea Research Foundation 1993, in part by the CTP, Seoul National University, in part by Yonsei University Faculty Research Grant, in part by Daeyang Foundation at Sejong University, in part by the Basic Science Research Institute Program, Ministry of Education 1997, Project No. BSRI-97-2425, and in part by the Japanese Ministry of Education, Science and Culture.

- 
- [1] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [2] For example, see C. S. Kim, J. L. Rosner, and C. P. Yuan, *Phys. Rev. D* **42**, 96 (1990); H. Quinn, in *CP Violation, Its Implications to Particle Physics and Cosmology*, Proceedings of the Topical Conference, Tsukuba, Japan, 1993, edited by Y. Kuno and Y. Okada [*Nucl. Phys. B (Proc. Suppl.)* **37A**, 21 (1994)].
- [3] CLEO Collaboration, R. Fulton *et al.*, *Phys. Rev. Lett.* **64**, 16 (1990).
- [4] ARGUS Collaboration, H. Albrecht *et al.*, *Phys. Lett. B* **234**, 409 (1990); **241**, 278 (1990); **255**, 297 (1991).
- [5] V. Barger, C. S. Kim, and R. J. N. Phillips, *Phys. Lett. B* **235**, 187 (1990); **251**, 629 (1990); C. S. Kim, D. S. Hwang, P. Ko, and W. Namgung, *CP Violation, Its Implications to Particle Physics and Cosmology* [2], p. 69; *Phys. Rev. D* **50**, 5762 (1994).
- [6] C. S. Kim, Report No. hep-ph/9605201, 1996 (unpublished).
- [7] A. O. Bouzas and D. Zappalà, *Phys. Lett. B* **333**, 215 (1994).
- [8] N. Cabibbo, G. Corbò, and L. Maiani, *Nucl. Phys.* **B155**, 93 (1979); G. Corbò, *ibid.* **B212**, 99 (1983); M. Jezabek and J. H. Kühn, *ibid.* **B320**, 20 (1989).
- [9] C. S. Kim and A. D. Martin, *Phys. Lett. B* **225**, 186 (1989).
- [10] G. Altarelli, N. Cabibbo, G. Corbò, L. Maiani, and G. Martinelli, *Nucl. Phys.* **B208**, 365 (1982).
- [11] CLEO Collaboration, B. Barish *et al.*, *Phys. Rev. Lett.* **76**, 1570 (1996).
- [12] ARGUS Collaboration, H. Albrecht *et al.*, *Phys. Lett. B* **318**, 397 (1993).
- [13] Dae Sung Hwang, C. S. Kim, and Wuk Namgung, *Phys. Rev. D* **53**, 4951 (1996).
- [14] I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, *Int. J. Mod. Phys. A* **9**, 2467 (1994); *Phys. Lett. B* **328**, 431 (1994); *Phys. Rev. D* **52**, 196 (1995).
- [15] C. Csáki and L. Randall, *Phys. Lett. B* **324**, 451 (1994).
- [16] P. Ball and V. M. Braun, *Phys. Rev. D* **49**, 2472 (1994); E. Bagan, P. Ball, V. M. Braun, and P. Gosdzinsky, *Phys. Lett. B* **342**, 362 (1995).
- [17] A. Kapustin, Z. Ligeti, M. B. Wise, and B. Grinstein, *Phys. Lett. B* **375**, 327 (1996).
- [18] Particle Data Group, L. Montanet *et al.*, *Phys. Rev. D* **50**, 1173 (1994).
- [19] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, *Phys. Rev. D* **17**, 3090 (1978); **21**, 313(E) (1980); **21**, 203 (1980).
- [20] C. Quigg and J. L. Rosner, *Phys. Rep.* **56**, 167 (1979); E. J. Eichten and C. Quigg, *Phys. Rev. D* **49**, 5845 (1994); **52**, 1726 (1995).
- [21] W. Lucha, F. F. Schöberl, and D. Gromes, *Phys. Rep.* **200**, 127 (1991).
- [22] L. P. Fulcher, *Phys. Rev. D* **50**, 447 (1994).
- [23] K. Hagiwara, A. D. Martin, and A. W. Peacock, *Z. Phys. C* **33**, 135 (1986).
- [24] D. S. Hwang, C. S. Kim, and W. Namgung, *Z. Phys. C* **69**, 107 (1995).
- [25] D. S. Hwang and G.-H. Kim, *Phys. Rev. D* **53**, 3659 (1996).
- [26] M. Neubert, *Phys. Rev. D* **46**, 1076 (1992); *Phys. Rep.* **245**, 259 (1994).
- [27] D. S. Hwang, C. S. Kim, and W. Namgung (in preparation).
- [28] M. Gremm, A. Kapustin, Z. Ligeti, and M. B. Wise, *Phys. Rev. Lett* **77**, 20 (1996).
- [29] I. I. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, *Phys. Rev. Lett.* **71**, 496 (1993); A. V. Manohar and M. B. Wise, *Phys. Rev. D* **49**, 1310 (1994); B. Blok, L. Koyrakh, M.



- Shifman, and A. I. Vainshtein, *ibid.* **49**, 3356 (1994); A. Falk, M. Luke, and M. Savage, *ibid.* **49**, 3367 (1994); T. Mannel, Nucl. Phys. B **423**, 396 (1994); M. Neubert, Phys. Rev. D **49**, 3392 (1994).
- [30] CLEO Collaboration, J. Bartelt *et al.*, Report No. CLEO/CONF 93-19, 1993 (unpublished).
- [31] CLEO Collaboration, J. Bartelt *et al.*, Phys. Rev. Lett. **71**, 4111 (1993).
- [32] R. Akhoury and I. Z. Rothstein, Phys. Rev. D **54**, 2349 (1996).
- [33] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989).
- [34] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).