Nonlinear e^+e^- pair production in a plasma by a strong electromagnetic wave

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The nonlinear process of electron-positron pair production in the field of a strong electromagnetic wave in a plasma on the basis of the Dirac model is considered. The formulas for angular distribution and total number of electron-positron pairs are obtained. The latter, in the center-of-mass frame of the produced particles, also describe the process of pair production in vacuum by a uniform periodic electric field. The obtained approximate nonlinear solution of the Dirac equation is applicable in the field when the energy of interaction with the Dirac vacuum is comparable to the electron rest mass. [S0556-2821(96)02319-3]

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I. INTRODUCTION

The multiphoton production of electron-positron pairs in a plasma by a laser radiation field is possible at ordinary densities [1], in contrast with single-photon production $\gamma\rightarrow e^+ + e^-$, which is only accessible in a superdense plasma (electron density $\rho \gtrsim 3\times 10^{34}$ cm^{-3}) [2]. However, in [1] the multiphoton pair-production probabilities were obtained in fields which are rather weak for the real perturbation of the Dirac vacuum, when $eA/\hbar\omega \ll 1$ and the theory of perturbation in the field is valid (e is the electric charge, and A and ω are the amplitude of the vector potential and the wave frequency, respectively). The minimal degree $N_{\rm min}$ of multiphoton production is determined by the process threshold $N_{\rm min} > mc^2/\hbar\omega$, and so the lowest order of perturbation theory, in which pair production by a laser field is possible, is[1] $N_{\rm min} \gg 1$.

Real e^+e^- pair production requires such high intensities that the energy of the interaction of the electron with the field over a wavelength becomes comparable to the electron rest mass: $ecE_0/\omega \gtrsim mc^2$. Let introduce a relativistic invariant parameter of the wave intensity $\xi^2 = -e^2 a_0^2/m^2c^4$ ($a_0^2 = a_{0i}a_0^i$, where a_{0i} is the amplitude of four-vector potential of the wave). For the above-mentioned fields, $\xi = eA_0/mc^2 = eE_0/mc\omega \gtrsim 1$, where A_0 and E_0 are amplitudes of the vector potential and of the electric field of the wave, respectively.

In such fields the multiphoton pair-production process goes in through nonlinear channels, for a description of which the perturbation theory is certainly not applicable. In this case the dispersion law of a plasma is nonlinear, too; i.e., the refractive index $n(\omega)$ depends on the wave intensity: $n = n(\omega,\xi^2)$. As is known [4], because of the intensity effect, the transparency range of a plasma widens and the dispersion law $n(\omega,\xi^2) < 1$, which is necessary for the production of e^+e^- pairs, holds all the more. But the intensities required for the appearance of a real nonlinearity in dispersion are far from those of the considered fields, and for these fields, as will be shown below, $\xi \lesssim 1$. As follows from the results obtained in [4], the contribution of the wave intensity in the law of dispersion becomes essential when $\xi \gg 1$. Since even for the most powerful laser fields existing, $\xi \lesssim 1$, the dispersion law of a plasma can be regarded as linear, but the pair-production process in such fields will have an essential nonlinear behavior. The multiphoton degree N is defined by the condition (reaction threshold)

$$N\hbar\omega \geqslant \frac{2mc^2}{[1-n^2(\omega)]^{1/2}}. \qquad (1)$$

This paper presents the electron-positron pair-production process through nonlinear channels by a strong laser field in a plasma. The problem is solved with the help of the Dirac model, according to which all negative-energy states are filled with electrons and the interaction of the wave occurs with that vacuum.

In Sec. II the Dirac equation in the field of the plane monochromatic electromagnetic wave propagating in a plasma is solved. The nonlinear solutions in the center-of-mass frame of the produced particles, where the field of the wave transforms into a homogeneous periodic electric field are found. The latter means that the obtained results also describe the electron-positron pair-production process by a periodic electric field in vacuum.

In Sec. III the multiphoton pair-production probabilities in a plasma by the wave field, as well as in vacuum by the periodic electric field, with the help of the obtained solutions of the Dirac equation are calculated. In the case of weak fields the results obtained in Ref. [1] are summarized in the Nth order of the perturbation theory.

II. NONLINEAR SOLUTION OF THE DIRAC EQUATION

Let a plane transverse linearly polarized electromagnetic wave with frequency ω and vector potential

$$\vec{A}(\vec{r},t) = \vec{A}_0 \cos(\omega t - \vec{\kappa}\cdot\vec{r}); \quad |\vec{\kappa}| = n\omega/c, \qquad (2)$$

propagate in a plasma with the dispersion law $n^2(\omega) = 1 - 4\pi\rho e^2/m\omega^2$.

It is convenient to solve the problem in the center-of-mass frame of the produced pair (C frame), in which the wave

[1] In contrast with the stationary case, in a nonstationary plasma, at the sharp change of the refractive index in time, pair production by a laser field is possible already in the single-photon channel, i.e., in the first order of perturbation theory in the field [3].

vector of the photons is $\vec{\kappa}'=0$ (the refraction index of the plasma in this frame is $n'=0$). The velocity of the C frame with respect to the laboratory frame (L frame) is $V=cn$. The traveling electromagnetic wave is transformed in the C frame into a varying electric field (the magnetic field $H'=0$) with a vector potential

$$\vec{A}'(t')=\frac{\vec{A}_0}{2}\left[\exp(i\omega' t')+\exp(-i\omega' t')\right],$$

$$\omega'=\omega\sqrt{1-n^2}. \tag{3}$$

It is easily noted that with Eq. (3) taken into account the reaction threshold condition (1) is obtained from the laws of the conservation of energy $E'_-+E'_+=N\hbar\omega'$ and momentum $\vec{P}'_-+\vec{P}'_+=N\hbar\vec{\kappa}'=0$ in the C frame ($E'_-,\vec{P}'_-$, and $E'_+,\vec{P}'_+$ are the energy and momentum of the electron and positron, respectively, in the C frame).

To solve the problem of N-photon production of an e^+e^- pair in the given radiation field (2), we shall make use of the Dirac model (all vacuum negative-energy states are filled with electrons and the interaction of the external field proceeds only with this vacuum: on the other hand, the interaction with the plasma electrons reduces to a refraction of the wave only).

The Dirac equation in the field (3) has the form (here we set $\hbar=c=1$)

$$i\frac{\partial\Psi}{\partial t}=[\vec{\alpha}\cdot(\vec{P}'-e\vec{A}'(t'))+\beta m]\Psi, \tag{4}$$

where

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix},\quad \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}$$

are the Dirac matrices, with $\vec{\sigma}$ the Pauli matrices. Since in the C frame the interaction Hamiltonian does not depend on the space coordinates, the solution of Eq. (4) can be represented in the form of a linear combination of free solutions of the Dirac equation with amplitudes $a_i(t')$ depending only on time:

$$\Psi_{\vec{P}'}(\vec{r}',t')=\sum_{i=1}^{4}a_i(t')\Psi_i^{(0)}(\vec{r}',t'). \tag{5}$$

Here

$$\Psi_{1,2}^{(0)}(\vec{r}',t')=\left(\frac{E'+m}{2E'}\right)^{1/2}\begin{pmatrix}\varphi_{1,2}\\ \dfrac{\vec{\sigma}\cdot\vec{P}'}{E'+m}\varphi_{1,2}\end{pmatrix}\times\exp[i(\vec{P}'\cdot\vec{r}'-E't')],$$

$$\Psi_{3,4}^{(0)}(\vec{r}',t')=\left(\frac{E'+m}{2E'}\right)^{1/2}\begin{pmatrix}\dfrac{-\vec{\sigma}\cdot\vec{P}'}{E'+m}\chi_{3,4}\\ \chi_{3,4}\end{pmatrix}\times\exp[i(\vec{P}'\cdot\vec{r}'+E't')], \tag{6}$$

where

$$E'=(\vec{P}'^2+m^2)^{1/2},\quad \varphi_1=\chi_3=\begin{pmatrix}1\\0\end{pmatrix},\quad \varphi_2=\chi_4=\begin{pmatrix}0\\1\end{pmatrix}. \tag{7}$$

The solution (4) in the form (5) corresponds to an expansion of the wave function in a complete set of orthonormal functions of the electrons (positrons) with specified momentum [with energies $E'=\pm(\vec{P}'^2+m^2)^{1/2}$ and spin projections $S_z=\pm 1/2$]. The latter are normalized to one particle per unit volume. According to the assumed model only the Dirac vacuum is present prior to the turning on of the field: i.e.,

$$|a_3(-\infty)|^2=|a_4(-\infty)|^2=1,\quad |a_1(-\infty)|^2=|a_2(-\infty)|^2 \tag{8}$$

(the field is turned on adiabatically at $t=-\infty$). From the condition of conservation of the norm we have

$$\sum_{i=1}^{4}|a_i(t')|^2=2, \tag{9}$$

which expresses the equality of the number of created electrons and positrons, whose creation probability is, respectively, $|a_{1,2}(t')|^2$ and $1-|a_{3,4}(t')|^2$.

Substituting Eq. (5) into Eq. (4), multiplying by the Hermitian conjugate functions $\Psi_i^{(0)^+}(\vec{r}',t')$, and taking into account orthogonality of the eigenfunctions (6) and (7), we obtain a system of differential equations for the unknown functions $a_i(t')$. Since in the C frame there is symmetry with respect to the direction $\vec{A}_0$ (the $O\vec{Y}$ axis), we can take, without loss of generality, the vector $\vec{P}'$ to lie in the $X'Y'$ plane ($P'_z=0$). Further, having introduced, to simplify the notation, the new symbols

$$a_1(t')\equiv b_1(t'),$$

$$a_4(t')\equiv b_4(t')\left[1-\frac{P_y'^2}{E'^2}\right]^{-1/2}\left[\frac{P'_xP'_y}{E'(E'+m)}+i\left(1-\frac{P_y'^2}{E'(E'+m)}\right)\right], \tag{10}$$

we obtain for the amplitudes $b_1(t')$ and $b_4(t')(|b_4(t')|=|a_4(t')|)$ the following system of equations:

$$\frac{db_1(t')}{dt'}=i\frac{eP'_yA'_y(t')}{E'}b_1(t')+ieA'_y(t')\left(1-\frac{P_y'^2}{E'^2}\right)^{1/2}b_4(t')\exp(2iE't'),$$

$$\frac{db_4(t')}{dt'}=-i\frac{eP'_yA'_y(t')}{E'}b_4(t')+ieA'_y(t')\times\left(1-\frac{P_y'^2}{E'^2}\right)^{1/2}b_1(t')\exp(-2iE't'). \tag{11}$$

A similar system of equations is also obtained for the amplitudes $b_2(t')$ and $b_3(t')$

To solve the system (11), we make the transformations

$$b_1(t')=c_1(t')\exp\left[i\,\frac{P'_y}{E'}\int_{-\infty}^{t'}A'_y(\eta)d\eta\right],$$

$$b_4(t')=c_4(t')\exp\left[-i\,\frac{P'_y}{E'}\int_{-\infty}^{t'}A_y(\eta)d\eta\right],\qquad(12)$$

where the $c_1(t')$ and the $c_4(t')$ satisfy the initial conditions, according to Eqs. (8) and (10), $|c_1(-\infty)|=0$, and $|c_4(-\infty)|=0$.

For the new amplitudes $c_1(t')$ and $c_4(t')$ from Eqs. (11), we obtain the system of equations

$$\frac{dc_1(t')}{dt'}=f(t')c_4(t'),$$
$$\frac{dc_4(t')}{dt'}=-f^*(t')c_1(t'),\qquad(13)$$

where

$$f(t')=ieA'_y(t')\left(1-\frac{P'^2_y}{E'^2}\right)^{1/2}\times\exp\left[2iE't'-\frac{2ieP'_y}{E'}\int_{-\infty}^{t'}A'_y(\eta)d\eta\right].\qquad(14)$$

We can obtain the solution (11), which satisfies the initial conditions of the problem (8), with the help of successive approximations, if

$$\left|\int_{-\infty}^{t'}f(\tau)d\tau\right|\ll 1.\qquad(15)$$

Then, for the transition amplitude $c_1(t')$, we have

$$c_1(t')=\sum_{j=0}^{\infty}B_{2j+1}(t'),$$

where

$$B_{2j+1}(t')=(-1)^j\int_{-\infty}^{t'}f(\tau_1)d\tau_1\int_{-\infty}^{\tau_1}f^*(\tau_2)d\tau_2\times\int_{-\infty}^{\tau_2}f(\tau_3)d\tau_3\cdots\int_{-\infty}^{\tau_{2j-1}}f^*(\tau_{2j})d\tau_{2j}\times\int_{-\infty}^{\tau_{2j}}f(\tau_{2j+1})d\tau_{2j+1}.\qquad(16)$$

We are interested in nonlinear pair production in the strong wave field. For that let us calculate the first term of the sum (16):

$$B_1(t')=\int_{-\infty}^{t'}f(\tau_1)d\tau_1,$$

substituting the concrete form of the wave vector potential $A'_y(\eta)$ from expression (3) into expression (14) and carrying out the integration. Then for $B_1(t')$ we obtain

$$B_1(t')=\frac{E'}{2P'_y}\left(1-\frac{P'^2_y}{E'^2}\right)^{1/2}\times\sum_{s=-\infty}^{+\infty}\frac{s\omega'}{2E'-s\omega'}\,\mathcal{J}_s(z)e^{2iE't'-is\omega't'},\quad(17)$$

where $\mathcal{J}_s(z)$ is the Bessel function,

$$z\equiv 2\xi\,\frac{m}{E'}\,\frac{P'_y}{\omega'},\quad \xi=\frac{eE'_0}{mc\omega'},\quad E'_0=\frac{\omega'}{c}A_0.$$

Since ξ, is an relativistic invariant parameter, in formulas (17) $\xi=eE_0/mc\omega$, where ω and E_0 are the frequency and amplitude of the electric field of the wave in the L frame.

For existing fields, when $\xi\lesssim 1$, the condition (15) always satisfies $|B_1(t')|\ll 1$, but the latter is not enough, yet, in order to be confined to that term in determination of the amplitude $c_1(t')$. Because the resonant term $S=N=2E'/\omega$ $(N\gg 1)$ gives a real contribution in the multiphoton pair production process and in expression (17), the maximal value of the Bessel function can be shifted from the resonant value. Since $N\gg 1$, that shift will be as small and negligible as possible when the argument of the Bessel function is $Z\sim N\gg 1$. So the condition, when the pair-production process will have an essential nonlinear character, is

$$Z=2\xi\,\frac{m}{E'}\,\frac{P'_y}{\omega'}\gg 1.\qquad(18)$$

If the condition (18) is satisfied, we can be restricted to the first term of the sum (16) for the amplitude $c_1(t')$:

$$c_1(t')=B_1(t').\qquad(19)$$

So the obtained approximate solution of Dirac equation is applicable in such intensities of the electromagnetic wave, when the conditions (15) and (18) are satisfied simultaneously:

$$\frac{1}{N}\ll\xi\lesssim 1.\qquad(20)$$

III. MULTIPHOTON PROBABILITIES OF NONLINEAR PAIR PRODUCTION

According to Eqs. (10) and (12), for the transition amplitude of the electron from the Dirac vacuum to the state with positive energy (in a definite spinor state) in the wave field we have:

$$|a_1(t')|^2=|b_1(t')|^2=|c_1(t')|^2.$$

To obtain the probability amplitude for the production of electrons and positrons after the wave has been turned off we introduce a small detuning of the resonance in Eq. (17), corresponding to an N-photon transition: $2E'=N\omega'+\Gamma$ $(\Gamma\ll\omega')$.

The production probability of the e^+e^- pair, summed over the spin states, is determined by the quantity

$$|a_1(t')|^2+|a_2(t')|^2=2|a_1(t')|^2\equiv 2|C_1(t')|^2.$$

The differential probability of the N-photon process per unit time and phase-space volume $d^3p'/(2\pi)^3$ (the normalization volume $V=1$) in the center-of-mass frame of the produced particles is given by

$$dw_N^C=\frac{dW_N^C(t')}{t'}=2\lim_{t'\to\infty}\frac{|c_1(t')|^2}{t'}\frac{d^3p'}{(2\pi)^3}. \tag{21}$$

Substituting expression (17) into Eq. (21) and making use of the definition of the δ function in the form

$$\lim_{t'\to\infty}\frac{\sin^2\Gamma t'}{\pi\Gamma^2 t'}=\delta(\Gamma)=\delta(2E'-N\omega'),$$

we obtain

$$dw_N^C=\frac{N^2\omega'^2(E'^2-P_y'^2)}{16\pi^2P_y'^2}\mathcal{J}_N^2\left(\frac{2eA_0P_y'}{E'\omega'}\right)\times\delta\left(E'-\frac{N\omega'}{2}\right)d^3p'. \tag{22}$$

Integrating the expression (22) over d^3p', we obtain the total probability of the N-photon e^+e^- pair production in a plasma by the strong electromagnetic wave:

$$w_N^C=\frac{N^5\omega'^5}{32\pi p'}\left\{\left[\frac{2Z_0^2}{4N^2-1}-1\right]\mathcal{J}_N^2(Z_0)+\frac{Z_0^2}{2N(2N-1)}\mathcal{J}_{N-1}^2(Z_0)+\frac{Z_0^2}{2N(2N+1)}\mathcal{J}_{N+1}^2(Z_0)-\frac{4p'^2}{N^2\omega'^2}\frac{Z_0^{2N}}{2^{2N}(2N+1)(N!)^2}\times{}_2F_3(N+\tfrac{1}{2},N+\tfrac{1}{2},N+1,2N+1,N+\tfrac{3}{2},-Z_0^2)\right\}, \tag{23}$$

where ${}_2F_3(N+\tfrac{1}{2},N+\tfrac{1}{2};N+1,2N+1,N+\tfrac{3}{2};-Z_0^2)$ is the generalized hypergeometric function and

$$Z_0=\frac{2m\xi}{\omega'}\left(1-\frac{4m^2}{N^2\omega'^2}\right)^{1/2}.$$

As is seen from Eq. (22), the pair-production probability decreases highly at the directions perpendicular to the field ($P_y'=0$), and the obtained approximate nonlinear solution describes the process behavior well at the angles not too close to $\pi/2$. And so expression (23), which is a result of integration over all angles, does not contain a large error.

The quantity W_N is a relativistic invariant, and so formula (23) defines the pair-production probability in the L frame as well. As for the angular distribution of the probability of N-photon pair production in the L frame, it can be obtained from the expression $dW_N^C(t')$ for the differential probability in the C frame by a Lorentz transformation. Here the quantity multiplying d^3p' is the expression of $dW_N^C(t')$ [see Eq. (21)] transforms like the time component of the current density four-vector of the electrons in the Dirac vacuum ($E'<0$). One must here take into account that the momentum of real electrons coincides with the momentum of the vacuum electron $\vec{P}'$, while the momentum of a positron equals $-\vec{P}'$ and the vacuum phase-space volume element $d^3p'/(2\pi)^3$ (in unit volume) goes over correspondingly into the volume element in momentum space of electrons and positrons. Further, transforming the quantities in Eq. (22) from the C frame to the L frame, we obtain for the differential probability of N-photon pair production per unit time in the L frame:

$$dw_N^L=\frac{dW_n^L(t)}{t}=\frac{N^2\omega^2(1-n^2)(E-nP_x)}{16\pi^2P_y^2E}\times\left[\frac{(E-nP_x)^2}{1-n^2}-P_y^2\right]\mathcal{J}_N^2\left[\frac{2eA_0P_y}{\omega(E-nP_x)}\right]\times\delta\left[E-nP_x-\frac{N\omega(1-n^2)}{2}\right]d^3p, \tag{24}$$

where E and $\vec{P}$ are the energy and momentum of the produced electron or positron. Integrating Eq. (24) over the electron (positron) energy, we obtain the angular distribution of the probability of the N-photon production of electrons (positrons) per solid angle element, $do=\sin\vartheta d\vartheta d\varphi$ (the azimuthal asymmetry of the probability in the L frame is due to the linear polarization of the wave: in the case of circular polarization the probability distribution has azimuthal symmetry):

$$dw_N^L=\sum_{\nu=1}^{2}\frac{N^3\omega^3(1-n^2)^2}{32\pi^2(P_\nu-nE_\nu\cos\vartheta)\sin\vartheta\cos^2\varphi}\left[\frac{N^2\omega^2(1-n^2)}{4}-P_\nu^2\sin^2\vartheta\cos^2\varphi\right]\mathcal{J}_N^2\left[\frac{4m\xi P_\nu\sin\vartheta\cos\varphi}{N\omega^2(1-n^2)}\right]d\vartheta d\varphi, \tag{25}$$

where

$$P_{1,2}=\frac{Nn\omega(1-n^2)\cos\vartheta\pm[N^2\omega^2(1-n^2)^2-4m^2(1-n^2\cos^2\vartheta)]^{1/2}}{2(1-n^2\cos^2\vartheta)},$$

$$E_{1,2}=\frac{N\omega(1-n^2)\pm n\cos\vartheta[N^2\omega^2(1-n^2)^2-4m^2(1-n^2\cos^2\vartheta)]^{1/2}}{2(1-n^2\cos^2\vartheta)}. \tag{26}$$

The angle φ varies from 0 to 2π, while ϑ varies from 0 to $\vartheta_{\max}$, which is determined from the energy and momentum conservation laws (26). Further, depending on the value of the plasma refractive index n, the electron (positron) production at the given angle is possible for a particular momentum or for one of two momenta with different magnitude. For values

$$n \leqslant \left(1-\frac{2m}{N\omega}\right)^{1/2}$$

[in this case the threshold condition (1) for the process is certainly satisfied], we should take in Eqs. (26) only the upper sign, corresponding to the fact that in the probability (25) only $\nu=1$ (P_1) remains and $\vartheta_{\max}=\pi$; i.e., particles are produced in all directions for the given angle ϑ with definite momentum. In the opposite case we must also take into account the reaction threshold condition in the region of values of the index of refraction,

$$\left(1-\frac{2m}{N\omega}\right)^{1/2} < n < \left(1-\frac{4m^2}{N^2\omega^2}\right)^{1/2},$$

and an electron (positron) is produced in a given direction with one of two different values of momentum P_1 and P_2 in a cone, opened forward, whose opening angle is

$$\vartheta_{\max}=\mathrm{Arcsin}\{[(1-n^2)(N^2\omega^2(1-n^2)-4m^2)]^{1/2}/2mn\}.$$

The problem of e^+e^- pair production by the photon field is solved in the C frame and the probability expressions (21)–(23) in that frame are adduced with express purpose. This is of independent physical interest, since Eqs. (21)–(23) describe the process of pair production in vacuum by a uniform periodic electric field [see Eq. (4)],

$$\vec{E}(t)=\vec{E}_0 \cos \omega t, \tag{27}$$

with the reaction threshold [see Eq. (1) when $n'=0$]

$$N\hbar\omega \geqslant 2mc^2. \tag{28}$$

This process has been studied by various methods in a number of papers [5–12]; however, the multiphoton probabilities were obtained by using the perturbation theory in the field: $eA/\omega \ll 1$ (for the special case $\vartheta=90°$ as well) [11]. Expressions (22) and (23) ($\omega'=\omega$, $P'=P$, $E'=E$, $A_0=E_0/\omega$) obtained here define the multiphoton production probabilities of an e^+e^- pair by the field (27) in vacuum through the nonlinear channels.

By integrating over the electron (positron) energy, we obtain the angular distribution of the nonlinear production of electrons (positrons) in the periodic electric field [in contrast to the pair production by the photon field (25), here there is azimuthal symmetry]:

$$dw_N=\frac{N^3\omega^3}{32\pi}\frac{N^2\omega^2\sin^2\vartheta+4m^2\cos^2\vartheta}{(N^2\omega^2-4m^2)^{1/2}\cos^2\vartheta} \times \mathcal{J}_N^2\left[\frac{2eE_0(N^2\omega^2-4m^2)^{1/2}\cos\vartheta}{N\omega^3}\right]\sin\vartheta\, d\vartheta, \tag{29}$$

where ϑ is the angle between the directions of the momentum of produced electrons (positrons) and the electric field.

Finally, we consider the case of weak fields, $eA/\omega \ll 1$ ($\xi \ll 1/N$), when the perturbation theory is applicable. In this case, as was noted above, we cannot be confined to the first term of the sum (16), since every term $B_{2S+1}(t')$ of the sum at $Z \ll 1$ [see Eq. (17) for the expression of Z] includes a resonant multiplier $\sim \xi^N$ (at $2S+1 \leqslant N$) in the lowest order of perturbation theory. Then from Eq. (16) we obtain the formula of perturbation theory for the pair production probability in the C frame, which coincides with the result of Ref. [1], and has a more compact analytical form (here we could get free of the sum of unwieldy products):

$$dw_N^c=2\pi\Phi^2\delta(2E'-Nw')\frac{d^3P'}{(2\pi)^3}, \tag{30}$$

where

$$\Phi=\beta\left(\frac{Z}{2}\right)^N\omega'\left[\frac{1}{(N-1)!}+\sum_{K=1}^{[(N-1)/2]}\sum_{S_1=1}^{N-2K}\cdots\sum_{S_j=1}^{N-1-(S_1+\cdots+S_{j-1})-2K+j}\cdots\sum_{S_{2K}=1}^{N-1-(S_1+\cdots+S_{2K-1})} \times\frac{(-1)^{S_2+S_4+\cdots+S_{2K}}}{(N-S_1)(S_1+S_2)\cdots[N-(S_1+S_2+\cdots+S_{2K-1})](S_1+S_2+\cdots+S_{2K})} \times\frac{\beta^{2K}}{(S_1-1)!(S_2-1)!\cdots(S_{2K}-1)![N-1-(S_1+S_2+\cdots+S_{2K})]!}\right]. \tag{31}$$

Here $N \geqslant 3$, and parameters

$$\beta=\frac{E'}{2P'_y}\left(1-\frac{P'^2_y}{E'^2}\right)^{1/2}, \quad Z=N\xi\frac{mP'_y}{E'^2}; \quad \xi \ll \frac{1}{N}.$$

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