Gauge-invariant YFS exponentiation of (un)stable W^+W^- production at and beyond CERN LEP 2 energies

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We present the theoretical basis and sample Monte Carlo data for the YFS exponentiated calculation of $e^+e^-\to W^+W^-\to f_1\bar{f}'_1+\bar{f}_2f'_2$ at and beyond CERN LEP 2 energies, where the left-handed parts of f_i and f'_i are the respective upper and lower components of an $SU(2)_L$ doublet, $i=1,2$. The problem of gauge invariance of the radiation from the unstable charged spin 1 $W^\pm$ is solved in an entirely physical manner. Our formulas are illustrated in a prototypical YFS Monte Carlo event generator YFSWW2, wherein both standard model and anomalous triple gauge boson couplings are allowed. [S0556-2821(96)03021-4]

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I. INTRODUCTION

The problem of the precision calculation of the process $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions$+n(\gamma)$ at and beyond energies reached at the CERN e^+e^- collider LEP 2 is of considerable interest in connection with the verification and tests of the $SU(2)_L\times U(1)$ model of Glashow, Salam, and Weinberg [1] of the electroweak interaction. Indeed, these processes are the primary objective of the initial LEP 2 physics program, providing as they do both a window on the most precise measurement of the W rest mass and a window on the most precise test of the fundamental non-Abelian triple and quartet gauge field self-interactions in principle, for example. In this paper, we present the theoretical basis of the rigorous Yennie-Frautschi-Suura (YFS) [2] Monte Carlo (MC) approach [3] to these processes. For completeness, we shall illustrate our results with a prototypical Monte Carlo event generator, which will be exact in the infrared regime and will be of leading logarithmic accuracy through $O(\alpha^2)$ in the hard radiative regime. A more accurate realization of our results will appear elsewhere [4].

Referring now to the results in Refs. [5, 6], it is clear that there are problems of principle in carrying through a manifestly gauge-invariant realization of the production and decay of massive charged spin-1 boson pairs in e^+e^- with the presence of radiation. Indeed, some controversy did exist in the literature on just how one should proceed even in the stable particle case [5,6], where, for example, the current-splitting approach of Ref. [5] has been questioned as to accuracy and appropriateness in Ref. [6]. We note that recently it has been verified [7] that the approach in Ref. [5] is indeed accurate enough for the requirements of the LEP 2 physics program as it is currently envisioned. We will show that the YFS theory will afford us an arena in which we can resolve this controversy.

While we were preparing this manuscript, we became aware of independent and related results by Baur and Zeppenfeld (BZ) [8] for the problem of quark-pair annihilation into $W^\pm$ followed by a lepton-pair $W^\pm$ decay, a process of interest in hadron-hadron collider physics, for example. We will therefore compare our approach with that of BZ in what follows. We shall see that the two approaches are consistent with one another. Moreover, during this same period, we became aware of an independent derivation in Ref. [9] of the solution of the gauge invariance problem for radiative corrections to the processes of interest to us here. We will therefore use the results of Ref. [9] in what follows, as they are in complete agreement with our work insofar as the issue of gauge invariance is concerned.

In addition, in the original YFS paper [2], there is no explicit discussion of charged spin-1 massive radiation. Thus we will need to extend the the rigorous YFS theory to this case as well. This means that our analysis is of theoretical interest in its own right as a study of the infrared limit of massless vector radiation from massive charged vector fundamental particles.

Our work is organized as follows. In the next section, we set our notational conventions. In Sec. III, we derive the extension of YFS theory to spin-1 charged particles. In Sec. IV, we derive the corresponding YFS formula for the processes $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions$+n(\gamma)$ and show that it is *manifestly gauge invariant*. In Sec. V, we illustrate

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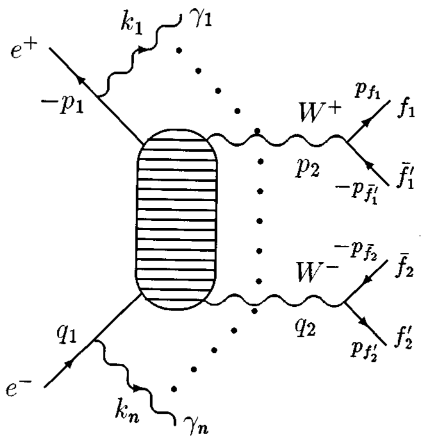

FIG. 1. Process $e^+e^-\to W^+W^-\to 4$ fermions$=f_1+\overline{f_1'}+\overline{f_2}+f_2'$, where $\binom{f_i}{f_i'}$, $i=1,2$, are $SU(2)_L$ doublets. Here p_A is the four-momentum of $A, A=f_i, f_i'$, *and* $p_1(q_1)$ and $p_2(q_2)$ are the four-momenta of $e^+(e^-)$ and $W^+(W^-)$, respectively. We use the notation $C_L\equiv P_LC\equiv\frac{1}{2}(1-\gamma_5)C$ for any C.

our YFS formula with Monte Carlo data based on the prototypical MC event generator YFSWW2 [10], which uses the Born level cross section of Ref. [11] as an input to achieve leading-logarithmic LL accuracy in the hard radiative regime and exactness in the infrared regime. Section VI contains our summary remarks, and finally some useful formulas are collected in the Appendixes.

II. PRELIMINARIES

In this section we review the relevant aspects of our YFS Monte Carlo methods as they pertain to the problem of extending our methods to the W^+W^- pair production processes of interest to us here. In this way we also set our notation and define our kinematics.

More precisely, the problem we study herein is illustrated in Fig. 1, together with the respective kinematics: $e^++e^-\to W^++W^-+n(\gamma)\to 4$ fermions $+n(\gamma)$ at c.m. system (C.M.S.) energies $\sqrt{s}\geqslant 2\mathrm{M_W}$, so that we may neglect m_e^2/s compared to 1. This corresponds to the case of LEP 2 and of the Next linear collider (NLC) (level 0 designs of the NLC now are in progress at several laboratories [12]). Let us focus for the moment on W^+W^- pair production part of Fig. 1. In Refs. [13, 14], for the case that the W's are replaced by the fermion pair $f\bar{f}$, of rest masses m_f and of charges $\pm Q_f e$, we have realized by Monte Carlo methods the process $e^++e^-\to f+\bar{f}+n(\gamma)$ via the fundamental YFS formula

$$d\sigma=e^{2\alpha\,\mathrm{Re}B+2\alpha\tilde{B}}\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{j=1}^{n}\frac{d^3k_j}{k_j^0}\int\frac{d^4y}{(2\pi)^4}\times\exp\left[iy\left(p_1+q_1-p_2-q_2-\sum_j k_j\right)+D\right]\times\overline{\beta}_n(k_1,\ldots,k_n)\,\frac{d^3p_2d^3q_2}{p_2^0q_2^0},\tag{1}$$

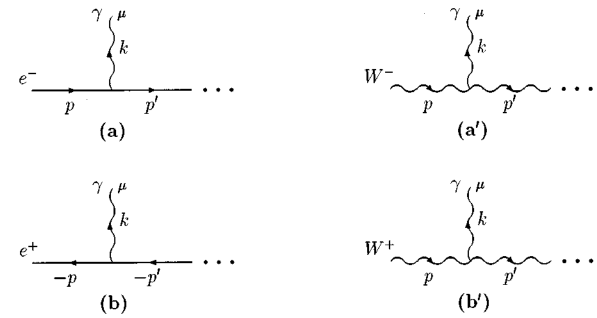

FIG. 2. Infrared emission of a photon of four-momentum k by (a) an incoming fermion of mass m_f, charge Q_fe, and four-momentum p, (a′) an incoming spin-1 charged boson of mass M_W, charge Q_We, and four-momentum p, (b) an incoming antifermion of rest mass m_f, charge $-Q_fe$, and four-momentum p, and (b′) an incoming spin-1 charged boson of rest mass M_W, charge $-Q_We$, and four-momentum p. Here the components of p, of course, must change in passing from the fermion cases to the vector boson cases if the incoming lines are on shell, for example.

where the real infrared function $\tilde{B}$ and the virtual infrared function B are given in Refs. [2, 13, 15–17], and where we note the usual connections

$$2\alpha\tilde{B}=\int^{k\leqslant K_{\max}}\frac{d^3k}{k_0}\,\tilde{S}(k),$$

$$D=\int d^3k\,\frac{\tilde{S}(k)}{k^0}\left[e^{-iyk}-\theta(K_{\max}-k)\right]\tag{2}$$

for the standard YFS infrared emission factor

$$\tilde{S}(k)=\frac{\alpha}{4\pi^2}\left[Q_fQ_{f'}\left(\frac{p_1}{p_1k}-\frac{q_1}{q_1k}\right)^2+\cdots\right]\tag{3}$$

if Q_f is the electric charge of f in units of the positron charge. Here, the "⋯" represent the remaining terms in $\tilde{S}(k)$ obtained from the one given by respective substitutions of Q_f, p_1, $Q_{f'}$, and q_1 with corresponding values for the other pairs of the respective external charged legs according to the YFS prescription in Ref. [2] (wherein due attention is taken to obtain the correct relative sign of each of the terms in $\tilde{S}(k)$ according to this latter prescription) and in Refs. [13, 14], $f\neq e$, $f'=\bar{f}$.

The YFS hard photon residuals $\overline{\beta}_i$ in Eq. (1), $i=0,1,2$, are given in Refs. [13, 14] for YFS2, YFS3 so that these latter event generators calculate the YFS exponentiated exact $O(\alpha^2)$ LL cross section for $e^+e^-\to f\bar{f}+n(\gamma)$ with multiple initial (initial+final) state radiation, respectively, using a corresponding Monte Carlo realization of Eq. (1). In the next sections, we use explicit Feynman diagrammatic methods to extend the realization of Eq. (1) in YFS2, YFS3 to the corresponding Monte Carlo realization of the respective application of Eq. (1) to $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions $+n(\gamma)$.

III. YFS THEORY FOR MASSIVE CHARGED VECTOR PARTICLES

In this section, we present the required formulas for extending the exact $O(\alpha^2)$ LL Monte Carlo realization of the hard photon residuals β_n in YFS2, YFS3 [13,14] to the corresponding Monte Carlo realization for the W^+W^--pair production processes. We begin by deriving the respective YFS real and virtual infrared functions for these latter processes.

Referring to the kinematics in Fig. 1 and to the definition of the YFS infrared functions $\tilde{S}(k)$, $S(k)$ in Ref. [2], we see that to extend the infrared YFS calculus to W's it is enough to show that in the respective infrared regime for an emitted photon of four-momentum k the amplitude for spin 1, charge Q_We, and mass M_W for the emitting particle is related to that for spin 1/2, charge Q_fe, and rest mass m_f by the substitutions of the respective Lorentz group representation factor, the charge, the corresponding four-vectors, and radiationless (Born) amplitude. From this result, it is immediate that the formulas given in Ref. [2] for for $\tilde{S}(k)$, $S(k)$ for spin 1/2 hold also for spin 1 with the corresponding substitution of charges and massive four-vectors. Let us now establish this correspondence of the infrared regimes of spin-1/2 and spin-1 massive, charged particles.

The relevant situations are illustrated in Fig. 2, where in we have the emission of a photon of four-momentum k by an incoming fermion of charge Q_fe, rest mass m_f, and four-momentum p to be compared with the analogous situation in (a′) where we have the emission of a photon of four-momentum k by an incoming spin-1 vector boson W^- of charge Q_We, rest mass M_W, and four-momentum p. In (b), we have the emission of a photon of four-momentum k by an incoming spin-1/2 antifermion of charge $-Q_fe$, rest mass m_f, and four-momentum p to be compared with (b′), wherein we have the emission of a photon of four-momentum k by an incoming spin-1 vector boson W^+ of charge $-Q_We$, rest mass M_W, and four-momentum p as well. From the standard Feynman methods, for Fig. 2(a), we have the amplitude for $f=e$ for definiteness:

$$\begin{aligned}\mathcal{M}_{2(\mathrm{a})}&=\cdots\frac{i}{\not p'-m_e+i\epsilon}(-iQ_ee\gamma^\mu)u(p)\\&=\cdots\frac{Q_ee}{-2kp+k^2+i\epsilon}(\not p-\not k+m_e)\gamma^\mu u(p)\\&=\cdots\frac{Q_ee}{k^2-2kp+i\epsilon}[2p^\mu-k^\mu+i\sigma^{\mu\alpha}k_\alpha]u(p)\\&\Rightarrow\lim_{\mathrm{IR}}\mathcal{M}_{2(\mathrm{a})}=\cdots\frac{Q_ee}{k^2-2kp+i\epsilon}(2p^\mu-k^\mu)u(p),\end{aligned}\tag{4}$$

where we define

$$\lim_{\mathrm{IR}}\mathcal{A}(k)\equiv\lim_{k\to0}\mathcal{A}(k),$$

for any function $\mathcal{A}(k)$. For Fig. 2(a′), we get the corresponding result

$$\begin{aligned}\mathcal{M}_{2(\mathrm{a}')}&=\cdots\frac{(-i)(g^{\alpha''\alpha'}-p'^{\alpha''}p'^{\alpha'}/M_W^2)}{p'^2-M_W^2+i\epsilon}(iQ_We)\\&\quad\times[g_{\alpha'\beta}(2p-k)^\mu+g^\mu_{\alpha'}(-p+2k)_\beta\\&\quad+g^\mu_\beta(-p'-2k)_{\alpha'}]\epsilon^\beta_-(p)\\&=\cdots\frac{(Q_We)(g^{\alpha''\alpha'}-p'^{\alpha''}p'^{\alpha'}/M_W^2)}{k^2-2kp+i\epsilon}[g_{\alpha'\beta}(2p-k)^\mu\\&\quad+2k_\beta g^\mu_{\alpha'}-2k_{\alpha'}g^\mu_\beta-p'_{\alpha'}g^\mu_\beta]\epsilon^\beta_-(p)\Rightarrow\lim_{\mathrm{IR}}\mathcal{M}_{2(\mathrm{a}')}\\&=\cdots\frac{(Q_We)}{k^2-2kp+i\epsilon}(2p^\mu-k^\mu)\epsilon^{\alpha''}_-(p).\end{aligned}\tag{5}$$

Similarly, for Fig. 2(b), we get the infrared limit as

$$\begin{aligned}\mathcal{M}_{2(\mathrm{b})}&=\overline{v(p)}(-Q_ee)\gamma^\mu\frac{i}{-\not p+\not k-m_e+i\epsilon}\cdots=\frac{\overline{v(p)}Q_ee\gamma^\mu(-\not p+\not k+m_e)}{k^2-2kp+i\epsilon}\cdots\\&=\frac{Q_ee\overline{v(p)}\Big[-\gamma^\mu\not p-\not p\gamma^\mu+(\not p+m_e)\gamma^\mu+\tfrac12(\not k\gamma^\mu+\gamma^\mu\not k)+\dfrac12(\gamma^\mu\not k-\not k\gamma^\mu)\Big]}{k^2-2kp+i\epsilon}\cdots\\&=\frac{Q_ee\overline{v(p)}[-2p^\mu+k^\mu-i\sigma^{\mu\alpha}k_\alpha]}{k^2-2kp+i\epsilon}\cdots\Rightarrow\lim_{\mathrm{IR}}\mathcal{M}_{2(\mathrm{b})}=\frac{\overline{v(p)}Q_ee(-2p^\mu+k^\mu)}{k^2-2kp+i\epsilon}\cdots,\end{aligned}\tag{6}$$

whereas for Fig. 2(b′) we compute the infrared limit as

$$\begin{aligned}\mathcal{M}_{2(\mathrm{b}')}&=\cdots\frac{(-i)(g^{\alpha''\beta}-p'^{\alpha''}p'^{\beta}/M_W^2)}{(p'^2-M_W^2+i\epsilon)}(iQ_We)[g_{\beta\alpha}(-2p+k)^\mu+g^\mu_\alpha(p'+2k)_\beta+g^\mu_\beta(p-2k)_\alpha]\epsilon^\alpha_+(p)\\&=\cdots\frac{(Q_We)(g^{\alpha''\beta}-p'^{\alpha''}p'^{\beta}/M_W^2)}{(k^2-2kp+i\epsilon)}[g_{\beta\alpha}(-2p+k)^\mu+2k_\beta g^\mu_\alpha-2k_\alpha g^\mu_\beta+p'_\beta g^\mu_\alpha]\epsilon^\alpha_+(p)\Rightarrow\lim_{\mathrm{IR}}\mathcal{M}_{2(\mathrm{b}')}\\&=\cdots\frac{(Q_We)}{(k^2-2kp+i\epsilon)}(-2p^\mu+k^\mu)\epsilon^{\alpha''}_+(p).\end{aligned}\tag{7}$$

This shows that the stated correspondence holds.

It follows that we obtain the YFS infrared functions [2] $\tilde{S}(k)$ and $S(k)$ for real and virtual soft photon emission from $W^{+,-}$ lines by substituting the respective mass M_W into the corresponding expressions for these functions for emission from $e^{+,-}$ lines:

$$\overset{(\sim)}{S}(k)_{e\bar{e}}\big|_{m=M_W} = \overset{(\sim)}{S}(k)_{W^-W^+},$$

$$\overset{(\sim)}{S}(k)_{ee}\big|_{m=M_W} = \overset{(\sim)}{S}(k)_{W^-W^-},$$

$$\overset{(\sim)}{S}(k)_{\bar{e}\bar{e}}\big|_{m=M_W} = \overset{(\sim)}{S}(k)_{W^+W^+}, \tag{8}$$

where the subscripts indicate the respective YFS infrared functions for $e\bar{e}$, ee, $\bar{e}\bar{e}$, W^-W^+, W^-W^-, and W^+W^+ in the obvious manner.

One important point needs to be discussed before we turn to the application of Eqs. (8) to $W^{\pm}$ pair production. This concerns the so-called Coulomb effect [18]—the enhanced $1/\beta$ behavior of the $O(\alpha)$ virtual correction to the Born cross section due to the exchange of $k\to 0$ virtual photons, where β is the c.m.s velocity of one of the W's. Since the YFS virtual infrared function $2\alpha B_{W^-W^+}\equiv\int[d^4k/(k^2-m_\gamma^2+i\epsilon)]S(k)_{W^-W^+}$, where m_γ is our photon infrared regulator mass, describes precisely this regime of the virtual photon phase space as well, we need to remove this Coulomb effect from $B_{W^-W^+}$ so that it can be treated via the methods of Ref. [18] as accurately as one desires without double counting it. This we do by defining the analytic subtraction

$$B'_{W^-W^+}(\beta)=B_{W^-W^+}(\beta)-\frac{\theta(\beta_t-\beta)}{\beta}\lim_{\beta\to 0}\beta B_{W^-W^+}(\beta)$$
$$=B_{W^-W^+}(\beta)-\frac{\pi}{4\beta}\,\theta(\beta_t-\beta), \tag{9}$$

where θ is the usual step function and here we determine β_t, the transition velocity which separates the nonrelativistic regime from the relativistic one insofar as the Coulomb corrections are concerned, by the requirement that the $O(\alpha^2)$ Coulomb correction $(\pi\alpha/\beta)^2/12$, be less than 0.03% for $\beta\geqslant\beta_t$—this gives $\beta_t\cong 0.382$. For $\beta\geqslant\beta_t$, the Coulomb correction series is a well-behaved part of the usual perturbation series and does not need special treatment in our analysis. Our requirement that the $O(\alpha^2)$ Coulomb correction stay below 0.03% for $\beta\geqslant\beta_t$ is determined with an eye toward an ultimate goal of 0.1% total precision on our calculations—this goal is, however, beyond the scope of the current paper. It is apparent that $B'_{W^-W^+}$ and $B_{W^-W^+}$ contain the same infrared divergences so that we may use $B'_{W^-W^+}$ in our YFS exponentiation algebra without any change in the cancellation of infrared singularities to all orders in α so that such a use of $B'_{W^-W^+}$ is fully justified by the original YFS arguments [2]—the resulting cross sections are fully independent of the substitution of $B'_{W^-W^+}$ for $B_{W^-W^+}$ when the theory is summed to all orders in α as is proved in the original YFS paper. Thus we will make this substitution here and treat the Coulomb effect entirely according to the methods in Ref. [18]. It can therefore be seen that our procedure for arriving at a smooth transition, in our complete cross section, within the limits of its physical precision tag, between the Coulomb-dominated $\beta\to 0$ regime and the relativistic $\beta\to 1$ regime of the $W^{+,-}$ charge form factor is entirely consistent with the smooth interpolation of Schwinger in Ref. [19] between the analogous regimes of the respective charge form factors for spin-0 and spin-1/2 massive charged particles.

We now turn to the application of the results in this section to the realization of our YFS Monte Carlo approach to multiple photon radiative effects in $e^+e^-\to W^+W^-+n(\gamma)$ at LEP 2 and NLC energies.

IV. GAUGE-INVARIANT YFS MONTE CARLO APPROACH FOR $e^+e^-\to W^+W^-+n(\gamma)$

In this section we apply the results of the preceding section to develop a Monte Carlo event generator YFSWW2 [10], which realizes the YFS exponentiated multiple photon radiation in the process $e^+e^-\to W^+W^-+n(\gamma)$, where we will also allow the W's to decay to four-fermion final states. In the development presented here, we shall work to the leading logarithmic $\bar{\beta}_0$ level in the respective YFS hard photon residuals $\bar{\beta}_n$ in Eq. (1) as it is applied to W-pair production via the substitutions in Eqs. (8).

Specifically, on using the results in Eqs. (8), we arrive at the representation, for the process $e^+e^-\to W^+W^-+n(\gamma)\to f_1+\bar{f}'_1+f'_2+\bar{f}_2+n(\gamma)$, of the fundamental YFS cross section formula

$$d\sigma=e^{2\alpha\,\mathrm{Re}B+2\alpha\tilde{B}}\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{j=1}^{n}\frac{d^3k_j}{k_j^0}\int\frac{d^4y}{(2\pi)^4}$$
$$\times\exp\left[iy\left(p_1+q_1-p_2-q_2-\sum_j k_j\right)+D\right]\bar{\beta}_n(k_1,\ldots,k_n)$$
$$\times\frac{d^3p_{f_1}d^3p_{\bar{f}'_1}d^3p_{f'_2}d^3p_{\bar{f}_2}}{p^0_{f_1}p^0_{\bar{f}'_1}p^0_{f'_2}p^0_{\bar{f}_2}}, \tag{10}$$

where, referring to the kinematics in Fig. 1, we have the identifications $p_2=p_{f_1}+p_{\overline{f_1'}}$, $q_2=p_{f_2'}+p_{\overline{f_2}}$, for the W^+,W^- four-momenta, respectively. The YFS infrared functions $\widetilde{B}(K_{\rm max})$, D, and B, and by Eqs. (8), are obtained from the the corresponding ones for the process $e^+e^-\to f+f+n(\gamma)$ as given in Eq. (3) and in Refs. [13, 17] via the substitutions $(Q_f,p_f,m_f)\to(Q_{W^-},q_2,M_W)$, $(Q_{\bar f},p_{\bar f},m_f)\to(Q_{W^+},p_2,M_W)$. Their analytical representations are given in Appendix A. The hard photon residuals β_n now contain both the production and decay of the W's, which may, of course, occur either on or off the W mass shell. We will work to the β_0 level, and we have the identification

$$\frac{1}{2}\bar{\beta}_0=\frac{d\sigma_{\rm Born}}{d\Omega_+ d\Omega_-}, \tag{11}$$

where $d\Omega_{+(-)}$ is the differential decay solid angle of $f_1(f_2')$ in the $W^{+(-)}$ rest frame, for example. Here we take the respective Born cross section $d\sigma_{\rm Born}$ from Ref. [11] for definiteness. The result (10) has been realized via the YFS MC methods of two of us (S.J. and B.F.L.W.), see, for example, Refs. [3, 13], in the program YFSWW2 [10].

Before we illustrate the type of predictions which we make with YFSWW2 for the LEP 2 or NLC physics scenarios, let us address one further important theoretical point concerning the fundamental result (10). Specifically, the reader may note that the $W^\pm$ decay width Γ_W does not appear in the YFS infrared functions B and $\widetilde{B}$ as they are given via the result (8). Yet, evidently, when a $W^\pm$ of four-momentum $p'=p+k$ emits radiation of four-momentum k, the propagator denominator that would most naively be present immediately preceding the emission vertex would be $D_W(p')=p'^2-M_W^2+ip'^2\Gamma_W/M_W$, and for $p^2=M_W^2$ this does not reduce to the YFS [2] infrared algebraic form k^2+2kp. The attendant problems with electromagnetic gauge invariance are now well known [5,6,8,9]. The solution to this apparent dilemma is already presented in Refs. [8, 9]. Whenever the $W^\pm$ radiate, one can include their decay width in their propagation provided one also allows the radiated photon to couple to charges in the graphs which generate the nonzero width to its respective order as well; see Refs. [8, 9] for a detailed discussion of these graphs, which are illustrated in Fig. 3 for the case that Γ_W is computed to $O(\alpha)$, for example. The net effect, as explained in detail in Refs. [8, 9] and as we have independently verified, is that the emission vertex for the photon in question is multiplied by a factor which replaces the preemission denominator $D_W(p')=p'^2-M_W^2+ip'^2\Gamma_W/M_W$ for $p^2=M_W^2$ with the desired YFS infrared algebraic form k^2+2kp, thereby maintaining electromagnetic gauge invariance.

We turn now to sample Monte Carlo data from YFSWW2 in the LEP 2- and NLC-type energy regimes. This we do in the next section.

V. SAMPLE MONTE CARLO DATA FOR $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions$+n(\gamma)$

In this section, we present sample Monte Carlo data for the process $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions$+n(\gamma)$ in the LEP 2- or NLC-type energy regimes. We have used our Monte Carlo event generator YFSWW2 as presented above,

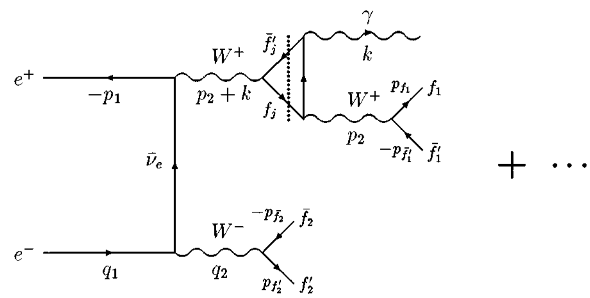

FIG. 3. Imaginary parts necessary to include in the amplitude for $e^+e^-\to W^+W^-\to 4$ fermions$=f_1+\overline{f_1'}+\overline{f_2}+f_2'$ in order to maintain gauge invariance in the presence of radiation by the W's themselves. Here the notation is identical to that in Fig. 1 and the vertical dotted line indicates the standard Bjorken-Landau-Cutkosky isolation of the respective imaginary part.

and we have had in mind in particular the illustration, among other things, of the effect of the $W^\pm$ contribution to the YFS real and virtual infrared functions in Eq. (10), as this effect has not been presented elsewhere.

Specifically, we will use the four-fermion final state $f_1=c$, $f_1'=s$, $f_2=\nu_e$, $f_2'=e$ for definiteness. With an eye toward studies of the triple gauge boson couplings in the standard model SU(2)×U(1) theory of Glashow, Salam, and Weinberg [1], we further follow the notation of Ref. [11] and compute our results for no anomalous three-gauge-boson couplings and for the values $\delta\kappa=0.1,\delta\lambda=0.1$ for the deviations of the coupling parameters κ, λ away from their tree-level standard model expectations of 1.0, 0.0, respectively, in the notation of Ref. [11]. This we do for three LEP 2-type c.m.s. energies $\sqrt{s}=175$, 190, and 205 GeV and for the NLC-type energy 500 GeV. The cross sections which we obtain from our simulation with YFSWW2 were each determined from 10^5 events (except for 175 GeV, where we had 10^6 events) and for the full solid angle acceptance for each of the four-fermions in the final state. The results were computed for both the pure initial state YFS exponentiated β_0-level case, as in Ref. [20], referred to as the ISR (initial state radiation) case in the following, and in the case when the $W^\pm$ contribution to the soft YFS exponentiated radiative effects is treated exactly, which we refer to as the ISR$+Y'$ case in the following. Furthermore, for definiteness, and clarity, the effect of the Coulomb correction, as given in Ref. [18], is also illustrated in our results, both for the simple YFS form factor ISR case and for the full YFS form factor ISR$+Y'$ case—the presence of the Coulomb correction is indicated by "Coul." in the following. The corresponding results are illustrated in Table I.

What we see is that the effect of the full YFS form factor is at the level of 0.14%, 0.39%, and 0.52%, respectively, beyond the usual Coulomb effect at the LEP 2 energies 175, 190, and 205 GeV and is at the level of 0.81% beyond the usual Coulomb effect at NLC energies for the case of the SM couplings, giving a total effect beyond initial state radiation of 3.00%, 2.59%, 2.44%, and 2.06% for the c.m.s. energies 175, 190, 205, and 500 GeV, respectively. For the case of the anomalous couplings of $\delta\kappa=\delta\lambda=0.1$, the corresponding results are 0.22%, 0.47%, 0.62%, and 0.25%, respectively, beyond the usual Coulomb effect for total corrections of

TABLE I. Results of the 10^5 (except for $E_{c.m.}=175$ GeV, where it is 10^6) statistics sample from YFSWW2 for the total cross section σ [pb]. The upper results at each value of energy are for standard model coupling constants, while the lower ones are for anomalous coupling constants ($\delta\kappa=\delta\lambda=0.1$). See the text for more details.

$E_{c.m.}$ [GeV]	ISR	ISR+Coul. corr	ISR+Coul. corr. +Y' corr.
175	0.4906±0.0002	0.5046±0.0002	0.5053±0.0002
	0.4898±0.0002	0.5037±0.0002	0.5048±0.0002
190	0.6060±0.0007	0.6193±0.0007	0.6217±0.0007
	0.6034±0.0007	0.6166±0.0007	0.6195±0.0007
205	0.6359±0.0008	0.6480±0.0008	0.6514±0.0008
	0.6315±0.0008	0.6436±0.0008	0.6476±0.0008
500	0.2910±0.0003	0.2946±0.0003	0.2970±0.0004
	0.3538±0.0004	0.3582±0.0004	0.3591±0.0004

3.06%, 2.67%, 2.55%, and 1.50%, respectively, for the same c.m.s. energies. Thus we see that at LEP 2 and NLC energies, the full form factor does indeed modulate the effect of the anomalous couplings; this enhances its importance at both LEP 2 and NLC energies. Since the targeted accuracy of the theoretical precision for the LEP 2 WW-pair production cross section is 0.5% [7], evidently, the full form factor effect is very important for LEP 2 physics scenarios.

Indeed, we have looked into the manifestation of these effects in the $W^{\pm}$ production angular distributions. We show in Figs. 4 and 5 four respective differential distributions for the total cross sections in Table I for the LEP 2 energy of $\sqrt{s}=190$ GeV and for the NLC energy of $\sqrt{s}=500$ GeV, where we feature the W^- production angle distribution in the c.m. system for all three SM coupling scenarios in Table I and with the final ''ISR+Coul. corr.+Y$'$-corr.'' scenario for the anomalous coupling case. We see in these figures that the

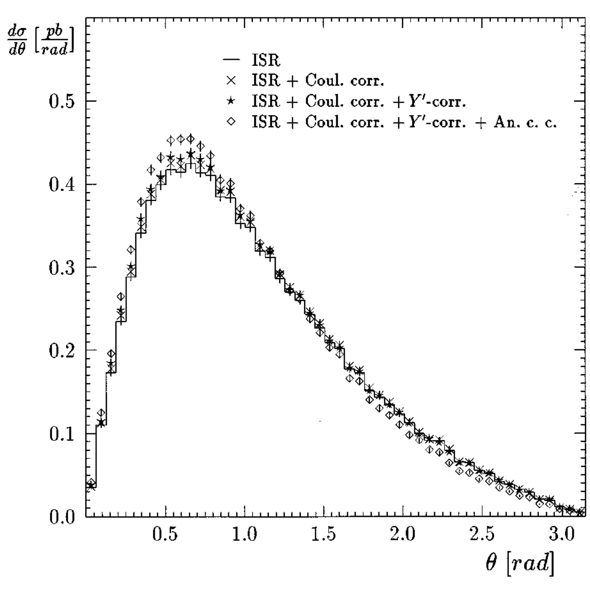

FIG. 4. W^- angular distributions for $E_{c.m.}=190$ GeV. See the text for more details.

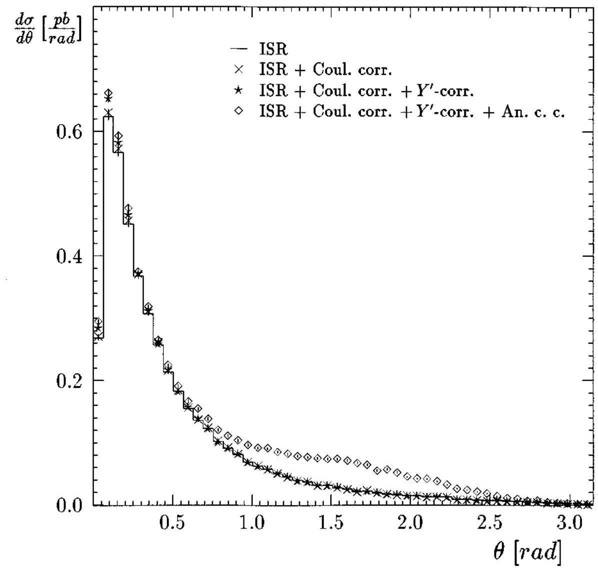

FIG. 5. W^- angular distributions for $E_{c.m.}=500$ GeV. See the text for more details.

full form factor effect modulates the distributions most strongly near their peaks, near the forward direction for the W^- when the incoming electron direction is used as the reference direction. As the anomalous couplings modulate these distributions over a large range of the respective production angles, we see that the smaller (larger) values of the full form factor effects in Table I for the anomalous cases relative to the SM cases for NLC (LEP 2) energies is consistent with the shapes of the distributions in Figs. 4 and 5. Also evident in the figures is the more pronounced anomalous coupling effect at NLC energies, as expected.

We end this section by noting that we have also implemented the full YFS form factor effect and the anomalous couplings as well in an unpublished version of the program KORALW [20] of three of us (M.S., S.J., and W.P.) and we have checked that the results from YFSWW2 and from this new version of KORALW [21] are in agreement within the statistical errors of the simulations. This is an important cross-check on the results in this paper it will be presented in detail elsewhere [21].

VI. CONCLUSIONS

In this paper we have developed the YFS theory of charged spin-1 bosons in the presence of possible nonzero widths. We have applied our theory to the LEP 2 or NLC process $e^+e^-\rightarrow W^+W^-+n(\gamma)$, allowing for the W pair to decay to four-fermions. The result is the description, via the Monte Carlo event generator YFSWW2, of the respective $n(\gamma)$ effects on an event-by-event basis, in which the infrared singularities are cancelled to all orders in α.

Specifically, in our realization of the YFS theory for charged spin-1 bosons, we have maintained the electromagnetic gauge invariance of the $SU(2)_L\times U(1)$ theory following the works in Refs. [8, 9]. In addition, we have also avoided any doubling counting of the so-called Coulomb effect by removing it analytically from the YFS virtual infrared function B. This resulted in the definition of a new YFS virtual infrared function B'. We have illustrated our calculations with explicit Monte Carlo data at the LEP 2 c.m.s. energies $\sqrt{s}=175$, 190, and 205 GeV and at the NLC energy $\sqrt{s}=500$ GeV, for both standard model and anomalous VWW vertices $V=\gamma,Z$. We find in all cases that the effect of the radiation by the $W^{\pm}$ themselves is important, both in the production angular distributions and in the over all normalization. In our YFS Monte Carlo realization of this effect, we have worked to the leading β_0 level in the current analysis. The higher order hard photon residuals β_n, $n\geqslant 1$, will be considered elsewhere [4] in this connection.

In summary, our Monte Carlo event generator YFSWW2 now calculates, on an event-by-event basis, the multiple photon effects in the process $e^+e^-\to W^+W^-+n(\gamma)\to 4$ fermions$+n(\gamma)$ and includes, for the first time ever, the effects of the radiation by the $W^{\pm}$ themselves in the respective YFS exponentiated soft photons, without doubling counting the so-called Coulomb effect and without spoiling the electromagnetic gauge invariance of the $SU(2)_L\times U(1)$ theory.[1]

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APPENDIX: YFS INFRARED FUNCTIONS

In this appendix we present some analytical representations of the YFS infrared (IR) functions corresponding to emission of virtual and real photons for the W^+W^--pair production in the e^+e^- annihilation. An important feature of these representations is that they are stable and fast in numerical evaluation. Thus they are particularly suited for Monte Carlo implementations.

1. Virtual photon IR function for the s channel

The s-channel virtual photon YFS IR function $\mathrm{Re}B(s)$ for any two charged particles with four-momenta p_1,p_2 and masses m_1,m_2 reads

$$2\alpha\ \mathrm{Re}B(s,m_1,m_2)=\frac{\alpha}{\pi}\Bigg\{\left(\frac{1}{\rho}\ln\frac{\mu(1+\rho)}{m_1m_2}-1\right)\ln\frac{m_\gamma^2}{m_1m_2}+\frac{\mu\rho}{s}\ln\frac{\mu(1+\rho)}{m_1m_2}+\frac{m_1^2-m_2^2}{2s}\ln\frac{m_1}{m_2}$$
$$+\frac{1}{\rho}\left[\pi^2-\frac{1}{2}\ln\frac{\mu(1+\rho)}{m_1^2}\ln\frac{\mu(1+\rho)}{m_2^2}-\frac{1}{2}\ln^2\frac{m_1^2+\mu(1+\rho)}{m_2^2+\mu(1+\rho)}-\mathrm{Li}_2\left(\frac{2\mu\rho}{m_1^2+\mu(1+\rho)}\right)\right.$$
$$\left.-\mathrm{Li}_2\left(\frac{2\mu\rho}{m_2^2+\mu(1+\rho)}\right)\right]-1\Bigg\},\qquad\text{(A1)}$$

where

$$\mu=p_1p_2,\quad s=2\mu+m_1^2+m_2^2,$$
$$\rho=\sqrt{1-\left(\frac{m_1m_2}{\mu}\right)^2},\qquad\text{(A2)}$$

and m_γ is a fictitious photon mass used to regularize the IR singularity.

2. Virtual photon IR function for the t and u channels

The t-channel virtual photon YFS IR function $\mathrm{Re}B(t)$ for two charged particles with four-momenta p_1,p_2 and masses m,M, where $m\ll M,|t|$ reads

$$2\alpha\ \mathrm{Re}B(t,m,M)=\frac{\alpha}{\pi}\Bigg\{\left(\ln\frac{|t|}{mM}+\ln\zeta-1\right)\ln\frac{m_\gamma^2}{m^2}$$
$$+\frac{\zeta}{2}\left(\ln\frac{|t|}{mM}+\ln\zeta\right)-\frac{1}{2}\ln\frac{|t|}{m^2}\ln\frac{|t|}{M^2}$$
$$-\ln\frac{M}{m}\left(\ln\frac{|t|}{mM}+\ln\zeta+\frac{\zeta-3}{2}\right)$$
$$-\ln\zeta\left(\ln\frac{|t|}{mM}+\frac{1}{2}\ln\zeta\right)+\mathrm{Li}_2\left(\frac{1}{\zeta}\right)-1\Bigg\},\qquad\text{(A3)}$$

where

$$\zeta=1+\frac{M^2}{|t|},\quad t=(p_1-p_2)^2.\qquad\text{(A4)}$$

The u-channel IR function $\mathrm{Re}B(u)$ can be obtained simply by replacing $t\to u$ in the above formula.

[1] This program is available from the authors at the WWW URL http://enigma.phys.utk.edu/pub/YFSWW/ and we look forward with excitement to its application to imminent LEP 2 data.

3. Real photon IR functions

The YFS IR function $\widetilde{B}$ corresponding to the emission of real photons with energy $E_\gamma \leqslant K_{\max}$ in a process involving any two unit charged particles with four-momenta p_1, p_2 (both outgoing) and masses m_1, m_2 can be expressed as

$$2\alpha\widetilde{B}(p_1,p_2;K_{\max}) = \frac{\alpha}{\pi}\left\{\left(\frac{1}{\rho}\ln\frac{\mu(1+\rho)}{m_1 m_2}-1\right)\ln\frac{4K_{\max}^2}{m_\gamma^2} + \frac{1}{2\beta_1}\ln\frac{1+\beta_1}{1-\beta_1}+\frac{1}{2\beta_2}\ln\frac{1+\beta_2}{1-\beta_2} + \mu A_4(p_1,p_2)\right\}, \quad \text{(A5)}$$

where $\beta_i = \sqrt{1-m_i^2/E_i^2}$, and μ and ρ are defined in Eq. (A2). The most complicated part of the above expression is the function $A_4(p_1,p_2)$. It can be expressed as a combination of some number of logarithms and dilogarithms[2] (see also Ref. [22] for a similar calculation)

$$A_4(p_1,p_2) = \frac{1}{\sqrt{(Q^2+\omega^2)(Q^2+\delta^2)}}\left\{\ln\frac{\sqrt{\Delta^2+Q^2}-\Delta}{\sqrt{\Delta^2+Q^2}+\Delta} \times [X_{23}^{14}(\eta_1)-X_{23}^{14}(\eta_0)]+Y(\eta_1)-Y(\eta_0)\right\}, \quad \text{(A6)}$$

where

$$X_{kl}^{ij}(\eta) = \ln\left|\frac{(\eta-y_i)(\eta-y_j)}{(\eta-y_k)(\eta-y_l)}\right|,$$

$$Y(\eta) = Z_{14}(\eta)+Z_{21}(\eta)+Z_{32}(\eta)-Z_{34}(\eta) + \tfrac{1}{2}X_{34}^{12}(\eta)X_{14}^{23}(\eta),$$

$$Z_{ij}(\eta) = 2\mathrm{Re}\ \mathrm{Li}_2\left(\frac{y_j-y_i}{\eta-y_i}\right)+\frac{1}{2}\ln^2\left|\frac{\eta-y_i}{\eta-y_j}\right|, \quad \text{(A7)}$$

and

$$\eta_0 = \sqrt{E_2^2-m_2^2}, \quad \eta_1 = \sqrt{E_1^2-m_1^2}+\sqrt{\Delta^2+Q^2},$$

$$y_{1,2} = \frac{1}{2}\left[\sqrt{\Delta^2+Q^2}-\Omega+\frac{\omega\delta\pm\sqrt{(Q^2+\omega^2)(Q^2+\delta^2)}}{\sqrt{\Delta^2+Q^2}+\Delta}\right],$$

$$y_{3,4} = \frac{1}{2}\left[\sqrt{\Delta^2+Q^2}+\Omega+\frac{\omega\delta\pm\sqrt{(Q^2+\omega^2)(Q^2+\delta^2)}}{\sqrt{\Delta^2+Q^2}-\Delta}\right], \quad \text{(A8)}$$

where we used the notation

$$\Delta = E_1-E_2, \quad \Omega = E_1+E_2,$$

$$\delta = m_1-m_2, \quad \omega = m_1+m_2,$$

$$Q^2 = -(p_1-p_2)^2. \quad \text{(A9)}$$

The only approximation used in deriving the above formulas is $m_\gamma \ll K_{\max}$.

As one can easily check the dependence of the above functions on the IR regulator m_γ cancels out in the sum $2\alpha\mathrm{Re}B+2\alpha\widetilde{B}$, which is used to construct the YFS form factor.

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