

Enhanced photon production rate on the light cone

P. Aurenche,^{1,2,*} F. Gelis,² R. Kobes,³ and E. Petitgirard³

¹CFIF, Instituto Superior Técnico, Edifício Ciência (física), P-1096 LISBOA, Codex, Portugal

²Laboratoire de Physique Théorique ENSLAPP, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France

³Physics Department and Winnipeg Institute for Theoretical Physics, University of Winnipeg, Winnipeg, Manitoba, Canada R3B 2E9

(Received 24 April 1996)

Recent studies of the high temperature soft photon production rate on the light cone using Braaten-Pisarski resummation techniques have found the presence of collinear divergences. We show that there exists a class of terms outside the Braaten-Pisarski framework which, although also divergent, dominates over these previously considered terms. The divergences in these new terms may be alleviated by application of a recently developed resummation scheme for processes sensitive to the light cone. [S0556-2821(96)02220-5]

PACS number(s): 11.10.Wx, 11.15.Bt, 12.38.Mh

I. INTRODUCTION

The development by Braaten and Pisarski of the effective expansion of hot gauge theories [1,2], given in terms of hard thermal loops [3–6], has resolved some long-standing paradoxes in the field [7–9]. However, it is also realized that these techniques are useful down to scales of the external momenta of the order of gT , the “soft” scale; infrared problems still remain if one goes below this scale, such as in calculations involving the fast fermion damping rate [10–14], the calculation of corrections to the Debye mass [15–17], and calculations of the QCD pressure [18–20]. As well, problems also arise for processes sensitive to the behavior of the theory near the light cone, such as in the soft photon production rate [21–23], photon bremsstrahlung from a QED or QCD plasma [24,25], including the effect of Landau-Pomeranchuk suppression [26], or scalar QED and QCD dispersion relations [27–29]. All of these problems warrant extensions of the hard thermal loop resummation techniques, although it is not obvious whether or not such extensions will be perturbative in nature.

In this paper we consider the production of a real photon with momentum of $O(gT)$. Concerning the calculation of this rate the following paradox appears: a straightforward application of the hard thermal loop (HTL) effective expansion leads to a rate of $O(e^2 g^3 T^2)$ (neglecting logarithmic divergences) and the production process is dominated by diagrams involving soft fermions; on the other hand, the bremsstrahlung emission of photons by hard [momentum of $O(T)$] fermions has been estimated using semiclassical methods [24,25] and it was found to be of $O(e^2 g T^2)$ (ignoring the Landau-Pomeranchuk effect). In the framework of the hard thermal loop expansion such bremsstrahlung diagrams, involving a hard fermion loop, should be suppressed rather than enhanced.

We reexamine below the problem of the hard fermion loop contribution to the production of soft real photons in the framework of thermal field theory, going beyond the hard loop expansion. We find that the sensitive behavior of these

terms to the light cone enhances their order by a factor of $1/g^2$ relative to the soft terms. As with the soft loop contributions, these enhanced terms also exhibit a collinear divergence, which, however, is alleviated by including thermal mass effects on the hard fermion propagators as required for a consistent calculation and also in agreement with a recently developed extension to the hard thermal loop resummation scheme for processes near the light cone [28]. Although a rigorous proof that this extension is complete is still forthcoming, it is clear that these terms dominate those of the hard thermal loop effective expansion, and as such a new effective expansion in cases such as this should be investigated.

II. PRODUCTION RATE

In order to calculate the photon production rate, we must evaluate the imaginary part of the trace of the (retarded) polarization tensor:

$$q_0 \frac{d\sigma}{d^3q} = - \frac{1}{(2\pi)^3} n_B(q_0) \text{Im} \Pi^\mu{}_\mu(Q) \sim \frac{1}{g} \text{Im} \Pi^\mu{}_\mu(Q), \quad (1)$$

where the approximate equality holds for a photon of energy $q_0 \sim gT$. According to the Braaten-Pisarski theory, the four diagrams displayed in Fig. 1 could, *a priori*, contribute to soft photon production at leading order. However, the diagram of Fig. 1(b) is zero thanks to an extension of the Furry’s theorem to the effective vertex with one photon and two gluons. Moreover, in the HTL approximation, the contribution of the diagrams of Figs. 1(c) and 1(d) are known to vanish: indeed, the trace of the four-point function with two photons and two fermions vanishes while the four-gauge-boson effective vertex of Fig. 1(c) has no HTL contribution. There remains only the diagram of Fig. 1(a) which is expected to contribute a term of $O(e^2 g^2 T^2)$ to $\text{Im} \Pi^\mu{}_\mu(Q)$ since the loop momentum is soft and “effective propagators and vertices are of the same order as their bare counterparts” [1,2]. The contribution from this soft internal quark loop using the effective propagators and vertices has been calculated in Refs. [21,22], with a result that, to leading order, exhibits a collinear divergence:

*On leave of absence from ENSLAPP, B.P. 110, F-74941 Annecy-le-Vieux Cedex, France.

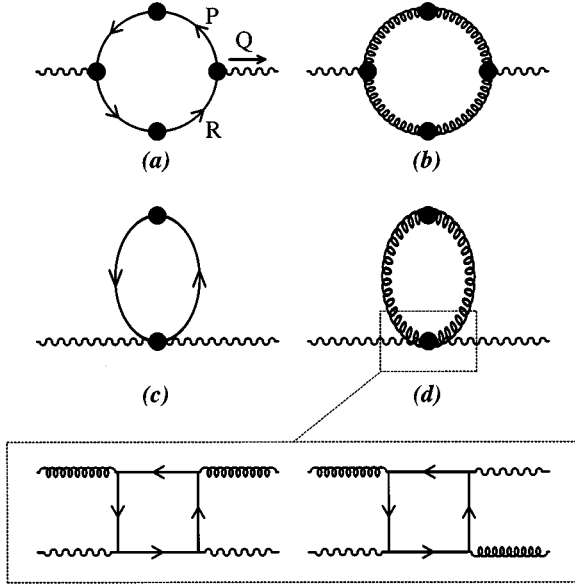


FIG. 1. Contributions to the soft photon production rate with soft internal lines.

$$\begin{aligned}
 \text{Im}\Pi^\mu{}_\mu(Q) &= -4N_c e^2 \frac{m_{\text{th}}^2}{q^2} \int \frac{d\hat{L}}{(2\pi)^{1-2\varepsilon}} \frac{q}{Q \cdot \hat{L} + i\varepsilon} \\
 &\times \int \frac{d^n P}{(2\pi)^{n-1}} 2\pi \delta(P \cdot \hat{Q}) \left(\frac{1}{2} - n_F(p_0) \right) \\
 &\times \sum_{s=\pm 1} \left[\left(1 - \frac{sp_0}{p} \right) \beta_s(P) + \left(1 - \frac{sr_0}{r} \right) \beta_s(R) \right] \\
 &= O(e^2 g^3 T^2) \frac{1}{\varepsilon}, \tag{2}
 \end{aligned}$$

where $m_{\text{th}}^2 = C_F g^2 T^2 / 8$, $P = (p_0, \vec{p})$, $p = |\vec{p}|$, ε is the regulating parameter of dimensional regularization ($n = 4 + 2\varepsilon, \varepsilon > 0$), and the effective quark propagator has been split as

$${}^*S_R(P) \equiv i \sum_{s=\pm 1} \frac{\hat{P}_s}{D_R^s(p_0 + i\varepsilon, \vec{p})}, \tag{3}$$

where \hat{P}_s is the lightlike vector $\hat{P}_s \equiv (1, s\hat{p} = s\vec{p}/p)$ and $1/D_R^s \equiv \alpha_s(P) - i\pi\beta_s(P)$.

One should note that the above result is suppressed by a factor g compared to the expected order $e^2 g^2 T^2$ [30]. This fact warrants a reexamination of the diagrams which had been found to vanish in the HTL approximation since they may well contribute at the suppressed order $e^2 g^3 T^2$. Considering Fig. 1(d), a blown-up view of the effective four-gauge-boson vertex reveals that the diagram is, in fact, equivalent to the two-loop diagrams of Fig. 2 with a hard fermion loop and a soft gluon insertion: Fig. 2 corresponds to the lowest order bremsstrahlung diagrams studied in [24,25]. The same reasoning can be made for Fig. 1(c) and leads to the diagram with a self-energy insertion of Fig. 2(b) where the gluon is now hard and the fermion of momentum $R+L$ soft. In the following we calculate only the diagrams of Fig. 1(d), beyond the hard loop approximation, because it is enhanced,

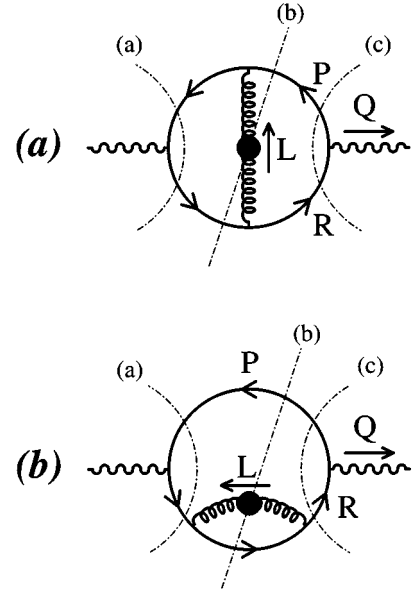


FIG. 2. Contributions to the soft photon production rate with hard internal fermion lines: (a) vertex insertion; (b) self-energy insertion.

compared to Fig. 1(c), by a factor $1/g$, due to the Bose factor associated to the soft gluon propagator.

We work in the retarded or advanced formalism of finite temperature field theory [22,31–33], which has the advantages of real time methods [34–36] but still maintains close ties with the imaginary time techniques [15,37]. For the contribution of the graph with the self-energy correction (we must keep in mind that there is another graph with a self-energy correction on the other fermionic propagator), we find the following result for the retarded self-energy up to color and group factors, which will be re-established in the final results [31,22]:

$$\begin{aligned}
 -i\Pi^\mu{}_\mu(Q)|_R &= -e^2 \int \frac{d^n P}{(2\pi)^n} \left\{ \left[\frac{1}{2} - n_F(p_0) \right] [\Delta_R(P) - \Delta_A(P)] \right. \\
 &\times [\Delta_R(R)]^2 \text{Tr} \hat{\Sigma}_R + \left[\frac{1}{2} - n_F(r_0) \right] \\
 &\times [\Delta_R^2(R) \text{Tr} \hat{\Sigma}_R - \Delta_A^2(R) \text{Tr} \hat{\Sigma}_A] \Delta_A(P) \left. \right\}, \tag{4}
 \end{aligned}$$

where the retarded and advanced propagators are defined by

$$\Delta_{R,A}(P) \equiv \frac{i}{P^2 - M^2 \pm i\varepsilon p_0} \tag{5}$$

(here we use $M=0$), and the notation $\text{Tr} \hat{\Sigma}_\alpha$ with $\alpha=R,A$ stands for

$$\text{Tr} \hat{\Sigma}_\alpha \equiv \text{Tr}(\gamma_\mu \hat{R} [-i\Sigma_\alpha(R)] \hat{R} \gamma^\mu \hat{P}). \tag{6}$$

The one-loop fermion self-energy $\Sigma_\alpha(R)$ is calculated with the following decomposition of the soft gluon propagator into its transverse, longitudinal, and gauge components [1,2]:

$$iD^{\mu\nu}(L) \equiv \frac{P_T^{\mu\nu}(L)}{L^2 - \Pi_T} + \frac{P_L^{\mu\nu}(L)}{L^2 - \Pi_L} + \text{gauge terms.} \quad (7)$$

We introduce the spectral functions

$$\rho_{T,L}(l, l_0) = \frac{i}{L^2 - \Pi_{T,L}} \Big|_R - \frac{i}{L^2 - \Pi_{T,L}} \Big|_A. \quad (8)$$

We find that the imaginary part of the photon self-energy, defined by $2i\text{Im}\Pi^\mu_\mu = \Pi^\mu_{\mu|R} - \Pi^\mu_{\mu|A}$, can be written at $Q^2=0$ as

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &= -2e^2g^2 \int \frac{d^n P}{(2\pi)^{n-1}} \epsilon(p_0) \delta(P^2) [n_F(r_0) - n_F(p_0)] \\ &\times \int \frac{d^n L}{(2\pi)^{n-1}} \epsilon(r_0 + l_0) \delta[(R+L)^2] \\ &\times [n_F(r_0 + l_0) + n_B(l_0)] \rho_{T,L}(l, l_0) \\ &\times \left[\frac{4R_\mu Q_\nu P_{T,L}^{\mu\nu}(L)}{R^2} - P_{T,L,\mu}^\mu(L) \left(1 + \frac{2Q \cdot L}{R^2} \right) \right], \quad (9) \end{aligned}$$

with appropriate spectral function $\rho_{T,L}$ and projection matrix $P_{T,L}$. This expression corresponds to cut ‘‘b’’ of Fig. 2(b). Cuts ‘‘a’’ and ‘‘c’’ vanish in dimensional regularization or, if one uses a regulating mass for the fermion, by kinematical arguments. At that point, we drop the gauge-dependent part of the gluon propagator since it is possible to verify that it does not contribute at the order we are interested in, thus leaving a gauge-independent result.

The contribution of the vertex diagram to the retarded self-energy can be written as [31,22]

$$\begin{aligned} -i\Pi^\mu_\mu(Q)|_R &= -e^2 \int \frac{d^n P}{(2\pi)^n} \text{TR}_{T,L} \left\{ \left[\frac{1}{2} - n_F(p_0) \right] \right. \\ &\times \{ [V_{RRA}(P, Q, -R) \Delta_R(P) \\ &- V_{ARA}(P, Q, -R) \Delta_A(P)] \Delta_R(R) \} \\ &\times \left[\frac{1}{2} - n_F(r_0) \right] \{ [V_{ARA}(P, Q, -R) \Delta_R(R) \\ &- V_{ARR}(P, Q, -R) \Delta_A(R)] \Delta_A(P) \} \left. \right\}, \quad (10) \end{aligned}$$

where all the Dirac algebra factors are included in

$$\text{TR}_{T,L} \equiv [P_{T,L}^{\rho\sigma}(L)] \text{Tr}(\gamma_\mu \not{R} \gamma_\rho (\not{R} + L) \gamma^\mu (\not{P} + L) \gamma_\sigma \not{P}) \quad (11)$$

and the functions $V_{\alpha\beta\gamma}(P, Q, -R)$ contain the scalar part of the diagram of Fig. 3. They are defined by [31,22]:

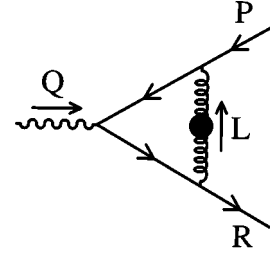


FIG. 3. The three-point vertex function.

$$\begin{aligned} V_{\alpha\beta\delta}(P, Q, -R) &\equiv -g^2 \int \frac{d^n L}{(2\pi)^n} \left\{ \left(\frac{1}{2} + n_B(l_0) \right) \right. \\ &\times \left[\frac{i}{L^2 - \Pi} \Big|_R - \frac{i}{L^2 - \Pi} \Big|_A \right] \Delta_\alpha(P+L) \Delta_\beta(R+L) \\ &+ \left(\frac{1}{2} - n_F(r_0 + l_0) \right) [\Delta_R(R+L) \\ &- \Delta_A(R+L)] \Delta_\beta(P+L) \frac{i}{L^2 - \Pi} \Big|_\delta \\ &+ \left(\frac{1}{2} - n_F(p_0 + l_0) \right) [\Delta_R(P+L) \\ &- \Delta_A(P+L)] \Delta_\beta(R+L) \frac{i}{L^2 - \Pi} \Big|_{\bar{\alpha}} \left. \right\}. \quad (12) \end{aligned}$$

Plugging this expression in Eq. (10), one obtains the imaginary part of the self-energy:

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &= -\frac{1}{2} e^2 g^2 \int \frac{d^n P}{(2\pi)^{n-1}} [n_F(r_0) - n_F(p_0)] \\ &\times \int \frac{d^n L}{(2\pi)^{n-1}} \rho_{T,L}(l, l_0) \text{TR}_{T,L} \\ &\times \left\{ \epsilon(p_0) \delta(P^2) \epsilon(r_0 + l_0) \delta[(R+L)^2] \right. \\ &\times \frac{n_F(r_0 + l_0) + n_B(l_0)}{R^2(P+L)^2} + \epsilon(r_0) \delta(R^2) \epsilon(p_0 + l_0) \\ &\times \delta[(P+L)^2] \frac{n_F(p_0 + l_0) + n_B(l_0)}{P^2(R+L)^2} \left. \right\}. \quad (13) \end{aligned}$$

Two classes of terms appear which can be interpreted in terms of cut diagrams: the first term in the curly brackets above corresponds to cut ‘‘b’’ in Fig. 2(a) and it is to be combined with Eq. (9) while the second term is associated with the other self-energy correction mentioned above. Both classes of terms give an identical contribution to the photon production rate. Keeping only the dominant terms for P, R hard and Q, L soft, we find the total contribution to the imaginary part of the self-energy to be

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &= -2e^2g^2 \int \frac{d^n P}{(2\pi)^{n-1}} \int \frac{d^n L}{(2\pi)^{n-1}} q n'_F(p_0) \\ &\quad \times n_B(l_0) \rho_{T,L}(l, l_0) \epsilon(p_0) \epsilon(r_0 + l_0) \delta(P^2) \\ &\quad \times \delta[(R+L)^2] 4R_\rho P_\sigma P_{T,L}^{\rho\sigma} \frac{L^2}{R^2(P+L)^2}. \end{aligned} \quad (14)$$

The presence of the L^2 factor in the numerator clearly indicates that our calculation is carried out beyond the HTL approximation where such terms are neglected. Estimating naively the order of magnitude of our result [using $L^2 \sim g^2 T^2$ and $R^2 \sim (P+L)^2 \sim gT^2$] we find it to be $O(e^2 g^3 T^2)$, i.e., of the same order as the supposedly dominant soft fermion loop contribution of Eq. (2). However, the denominator $R^2(P+L)^2$ is responsible for collinear divergences which drastically modify this naive estimate. Using the δ function constraints one easily rewrites

$$\begin{aligned} \frac{-4}{R^2(P+L)^2} &= \frac{1}{P \cdot Q} \frac{1}{P \cdot Q + Q \cdot L} \\ &= \frac{1}{Q \cdot L} \left(\frac{1}{P \cdot Q} - \frac{1}{P \cdot Q + Q \cdot L} \right) \approx \frac{2}{Q \cdot L} \frac{1}{P \cdot Q}. \end{aligned} \quad (15)$$

The first equality shows the presence of two very close collinear singularities ($P \cdot Q = 0$) since the two poles differ only by the soft $Q \cdot L$ term. The last equality holds true to leading order only after the integration over the whole phase space in Eq. (14) is performed. Introducing the angular variable $u = 1 - \cos\theta$ between the lightlike momenta P and Q the above expression becomes, near $u = 0$,

$$\frac{1}{R^2(P+L)^2} \sim \frac{p}{qL^2} \frac{1}{pqu}. \quad (16)$$

This form shows the presence of a logarithmic collinear divergence and the order of the residue at the pole in u is $1/g^4 T^4$ instead of the naively expected $1/g^2 T^4$. Concentrating then on this collinear limit, and leaving details of the calculations to a future paper, we find the dominant divergent term to be

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &\approx (-1)_L 4e^2g^2 N_C C_f \frac{1}{(2\pi)^4} \frac{1}{q} \\ &\quad \times \int_{p^*}^\infty dp p^2 n_F(p) [1 - n_F(p)] \int_0^{l^*} \frac{dl}{l} \\ &\quad \times \int_{-l}^l \frac{dl_0}{l_0} L^2 \rho_{L,T}(l_0, l) \int_0^1 \frac{du}{u}, \end{aligned} \quad (17)$$

where p^* and l^* are some intermediate momenta between the hard and soft scale. We have reintroduced at that point the color factor N_C and group factor C_f in the result. The symbol $(-1)_L$ is $+1$ for the transverse gluon mode and -1 for the longitudinal one. We find in the l integration that the region between l^* and ∞ gives a negligible contribution, so we can take $l^* \rightarrow \infty$. Similarly, we can take $p^* \rightarrow 0$. We notice a nice factorization of Eq. (17) into a ‘‘hard thermal

loop’’ integral over p , a soft gluonic integral over l and l_0 , and an integral over the angular variable u leading to the logarithmic collinear divergence. It is possible to show by kinematical considerations that only the Landau damping part of the spectral function $\rho_{L,T}$ contributes to the divergent piece; this is the reason why the integration domain has been limited to $L^2 < 0$. The occurrence of a collinear divergence, as in Eq. (17), was noticed in the dispersion relations for scalar QED near the light cone [27].

To proceed, sum rules may then be used to reduce the integrations of Eq. (17) down to a single one; for example, for the transverse contribution we can use

$$\begin{aligned} \frac{1}{\pi} \int_{-\infty}^{+\infty} dz z \rho_T(zl, l) &= \frac{2}{l^2}, \\ \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{dz}{z} \rho_T(zl, l) &= \frac{2}{l^2}, \end{aligned} \quad (18)$$

$$\frac{1}{\pi} \int_{-1}^{+1} dz z \rho_T(zl, l) = \frac{2}{l^2} [1 - Z_T(l)],$$

$$\frac{1}{\pi} \int_{-1}^{+1} \frac{dz}{z} \rho_T(zl, l) = 2 \left[\frac{1}{l^2 + m_{\text{mag}}^2} - \frac{Z_T(l)}{\omega_T^2(l)} \right],$$

where

$$Z_T \equiv \frac{2\omega_T^2(\omega_T^2 - l^2)}{3m_{\text{mag}}^2\omega_T^2 - (\omega_T^2 - l^2)^2}, \quad (19)$$

where $\omega_T(l)$ is the energy of the solution to the dispersion relation and m_g is the gluon Debye mass. Note that we have introduced a phenomenological ‘‘magnetic mass’’ $m_{\text{mag}} \sim g^2 T$ by hand to regulate potential infrared divergence for the transverse contribution. We find that the divergent piece of the transverse term can be written as

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &\approx \frac{e^2g^2 N_C C_f}{12\pi} \frac{T^3}{q} \frac{1}{\epsilon} \\ &\quad \times \int_0^\infty \frac{dl}{l} \left\{ \frac{m_{\text{mag}}^2}{l^2 + m_{\text{mag}}^2} - Z_T \frac{\omega_T^2 - l^2}{\omega_T^2} \right\} \\ &\quad \sim O(e^2gT^2) \frac{1}{\epsilon}. \end{aligned} \quad (20)$$

Similar sum rules can be used to evaluate the longitudinal contribution which is found to be

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &\approx - \frac{e^2g^2 N_C C_f}{12\pi} \frac{T^3}{q} \frac{1}{\epsilon} \\ &\quad \times \int_0^\infty \frac{dl}{l} \left\{ \frac{3m_g^2}{l^2 + 3m_g^2} - Z_L \frac{\omega_L^2 - l^2}{\omega_L^2} \right\} \\ &\quad = O(e^2gT^2) \frac{1}{\epsilon}. \end{aligned} \quad (21)$$

We note that the transverse contribution requires the ‘‘magnetic mass’’ introduced earlier in the sum rules in order to be infrared safe but both contributions display a divergence which is seen to be enhanced by a factor of $1/g^2$ relative to the soft fermion loop contribution of Eq. (2).

In our discussion of the collinear divergences care has been taken to keep the exact kinematics in the evaluation of the denominators: thus, in Eq. (15), the ‘‘soft’’ term $Q \cdot L$ was not neglected compared to the ‘‘hard’’ term $P \cdot Q$. But there are other soft corrections to hard propagators which should also be included, namely, those associated to thermal mass effects. Taking these into account amounts to applying a further resummation of hard internal lines [27,28]. This resummation is in addition to the HTL resummation of Braaten and Pisarski for soft lines, and is important for processes that are sensitive to the behavior near the light cone. It also has the virtue of being a gauge-invariant resummation summarized by a compact effective action. In the present case this involves using the dressed fermion propagator given in the limit $p_0, p \gg gT$ by

$$\frac{P_0 \gamma^0 - \omega_+(p) \hat{P} \cdot \vec{\gamma}}{p^2 - M_\infty^2 + O(M_\infty^4/p^2)} \approx \frac{\mathbf{P}}{p^2 - M_\infty^2}, \quad (22)$$

where M_∞ is the fermionic thermal mass in the hard regime and $\omega_+(p) \approx \sqrt{p^2 + M_\infty^2}$ is the energy of the ‘‘particle mode’’ of the fermionic dispersion relation. Furthermore, this asymptotic mass is insensitive to a soft modification of the hard propagator. Carrying through the calculations with such a propagator, one finds the analogous relation to the final result of Eq. (17) as

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &\approx (-1)_L 4e^2 g^2 N_c C_f \frac{1}{(2\pi)^4} \frac{1}{q} \\ &\times \int_{p^*}^\infty dp p^2 n_F(p) [1 - n_F(p)] \int_0^{l^*} \frac{dl}{l} \\ &\times \int_{-l}^l \frac{dl_0}{l_0} L^4 \rho_{L,T}(l_0, l) \\ &\times \int_0^1 \frac{du}{\sqrt{1-u}} \frac{1}{L^2 u - 4M_\infty^2}. \end{aligned} \quad (23)$$

We note that taking the $M_\infty \rightarrow 0$ limit of Eq. (23) at the collinear point results in Eq. (17), reproducing the collinear divergence as well as the dependence on the magnetic mass. The angular integration in Eq. (23) is easily performed and one arrives at

$$\begin{aligned} \text{Im}\Pi^\mu_\mu(Q) &\approx (-1)_L \frac{e^2 g^2 N_c C_f T^3}{12\pi} \frac{2}{q} \frac{1}{\pi} \\ &\times \int_0^1 \frac{dx}{x} \tilde{I}_{T,L}(x) \int_0^{+\infty} dw \\ &\times \frac{\sqrt{w/(w+4)} \tanh^{-1} \sqrt{w/(w+4)}}{[w + \tilde{R}_{T,L}(x)]^2 + [\tilde{I}_{T,L}(x)]^2}, \end{aligned} \quad (24)$$

with $w = -L^2/M_\infty^2$, $\tilde{R}_{T,L}(x) \equiv \text{Re}\Pi_{T,L}(x)/M_\infty^2$, and $\tilde{I}_{T,L}(x) \equiv \text{Im}\Pi_{T,L}(x)/M_\infty^2$. Obviously, the remaining integrals are finite and dimensionless. We note, though, that as in the massless case of Eq. (17), sum rules can be used to reduce Eq. (23) down to a one-dimensional integral. Details will be given in a future work; we simply describe here some general features about the result which are evident from this form and from Eq. (23). The first is that the order of the contributions is not changed by inclusion of the asymptotic mass, and as such, it is also enhanced by a factor of $1/g^2$ relative to the soft contribution of Eq. (2). The second one is that the presence of M_∞ regulates the collinear divergence associated with the lower limit on the u integration, since $L^2 < 0$. The final one is that the former sensitivity on the magnetic mass scale disappears when one considers specifically $M_\infty \sim O(gT)$, but, as noted after Eq. (23) in taking $M_\infty \rightarrow 0$, would reappear at scales below this. This result is thus of the same order as that of Ref. [25], neglecting the Landau-Pomeranchuk effect, but we differ from it since we find that both the transverse and the longitudinal modes contribute to the same order while in Ref. [25] the interaction in the medium is assumed to be static.

III. CONCLUSIONS

Although the resummation of the hard fermion line by inclusion of the asymptotic mass regulates the collinear divergence, we should note that it is not rigorously known if such a mass is the only term present at this order in general. In particular, terms in an effective propagator which might arise from higher loop diagrams than the hard thermal one-loop terms giving rise to this asymptotic mass may contribute. Thus, we cannot say with absolute certainty that the terms discussed here are the only ones which contribute at this order, although arguments exist that this may in fact be the case. Indeed, in this regard cancellations may occur: it is known in some examples that a constant damping term inserted in an effective propagator will cancel against the corresponding vertex corrections [10,28,38,39]. What does seem clear, though, is that there is an enhancement mechanism present in processes near the light cone which falls outside of the usual Braaten-Pisarski resummation of soft internal lines. It is also possible through a similar mechanism that other processes sensitive to the behavior of the theory near the light cone, such as the photon production rate for slightly virtual photons [40], may also get contributions from similar terms with hard internal momenta. Work along these lines, as well as details of the calculations reported here, will be presented elsewhere.

ACKNOWLEDGMENTS

We thank R. Baier and A. Rebhan for valuable discussions. The work of P.A. and F.G. was supported in part by the EEC program ‘‘Human Capital and Mobility,’’ Network ‘‘Physics at High Energy Colliders,’’ Contract No. CHR-CT93-0357 (DG 12 COMA). The work of R.K. and E.P. was supported by the Natural Sciences and Engineering Research Council of Canada.

- [1] R. Pisarski, *Physica A* **158**, 146 (1989), **158** 246 246 (1989); *Phys. Rev. Lett.* **63**, 1129 (1989).
- [2] E. Braaten and R. D. Pisarski, *Nucl. Phys.* **B337**, 569 (1990); **B339**, 310 (1990).
- [3] V. V. Klimov, *Sov. Phys. JETP* **55**, 199 (1982).
- [4] H. A. Weldon, *Phys. Rev. D* **26**, 1394 (1982); **26**, 2789 (1982).
- [5] J. C. Taylor and S. M. H. Wong, *Nucl. Phys.* **B346**, 115 (1990).
- [6] J. Frenkel and J. C. Taylor, *Nucl. Phys.* **B334**, 199 (1990); *Z. Phys. C* **49**, 515 (1991).
- [7] R. Baier, in *Banff/CAP Workshop on Thermal Field Theory*, edited by F. Khanna, R. Kobes, G. Kunstatter, and H. Umezawa (World Scientific, Singapore, 1994).
- [8] M. Thoma, Report No. hep-ph/9503400, 1995 (unpublished).
- [9] R. Kobes, in *Thermal Field Theories and Their Applications*, Proceedings of the Workshop, Dalian, China, 1995, edited by Y. K. Gui (World Scientific, Singapore, 1996), Report No. hep-ph/9511208 (unpublished).
- [10] V. V. Lebedev and A. V. Smilga, *Ann. Phys. (N.Y.)* **202**, 229 (1990).
- [11] C. P. Burgess and A. L. Marini, *Phys. Rev. D* **45**, R17 (1992).
- [12] A. Rebhan, *Phys. Rev. D* **46**, 482 (1992).
- [13] T. Altherr, E. Petitgirard, and T. del Rio Gaztelurrutia, *Phys. Rev. D* **47**, 703 (1993).
- [14] R. D. Pisarski, *Phys. Rev. D* **47**, 5589 (1993).
- [15] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989).
- [16] A. K. Rebhan, *Phys. Rev. D* **48**, 3967 (1993); *Nucl. Phys.* **B430**, 319 (1994).
- [17] E. Braaten and A. Nieto, *Phys. Rev. Lett.* **73**, 2402 (1994).
- [18] A. P. de Almeida and J. Frenkel, *Phys. Rev. D* **47**, 640 (1993).
- [19] E. Braaten, *Phys. Rev. Lett.* **74**, 2164 (1995).
- [20] E. Braaten and A. Nieto, *Phys. Rev. D* **51**, 6990 (1995).
- [21] R. Baier, S. Peigné, and D. Schiff, *Z. Phys. C* **62**, 337 (1994).
- [22] P. Aurenche, T. Becherawy, and E. Petitgirard, Report No. hep-ph/9403320, 1993 (unpublished).
- [23] E. Quack and P. Henning, *Phys. Rev. D* **54**, 3125 (1996).
- [24] H. A. Weldon, *Phys. Rev. D* **44**, 3955 (1991).
- [25] J. Cleymans, V. V. Goloviznin, and K. Redlich, *Phys. Rev. D* **47**, 989 (1993).
- [26] L. D. Landau and I. Ya. Pomeranchuk, *Dokl. Akad. Nauk SSSR* **92**, 535 (1953); **92**, 735 (1953).
- [27] U. Kraemmer, A. K. Rebhan, and H. Schulz, *Ann. Phys. (N.Y.)* **238**, 286 (1995); *Phys. Rev. D* **52**, 2994 (1995); Report No. hep-ph/9505307, 1995 (unpublished).
- [28] F. Flechsig and A. K. Rebhan, *Nucl. Phys.* **B464**, 279 (1996).
- [29] F. Flechsig and H. Schulz, *Phys. Lett. B* **349**, 504 (1995).
- [30] It is worth recalling, in this respect, that the trace of the polarization tensor has no hard thermal loop (HTL) contribution while the transverse and the longitudinal polarization tensors each have a HTL contribution.
- [31] P. Aurenche and T. Becherawy, *Nucl. Phys.* **B379**, 259 (1992).
- [32] C. M. A. van Eijck and Ch. G. van Weert, *Phys. Lett. B* **278**, 305 (1992).
- [33] C. M. A. van Eijck, Ch. G. van Weert, and R. Kobes, *Phys. Rev. D* **50**, 4097 (1994).
- [34] A. Fetter and J. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971).
- [35] A. Niemi and G. W. Semenoff, *Ann. Phys. (N.Y.)* **152**, 105 (1984); *Nucl. Phys.* **230**, 181 (1984).
- [36] R. Kobes and G. W. Semenoff, *Nucl. Phys.* **260**, 714 (1985); **272**, 329 (1986).
- [37] N. P. Landsman and Ch. G. van Weert, *Phys. Rep.* **145**, 141 (1987).
- [38] M. Le Bellac and P. Reynaud, in *Banff/CAP Workshop on Thermal Field Theory* [7].
- [39] M. Carrington, *Phys. Rev. D* **48**, 3836 (1993).
- [40] E. Braaten, R. D. Pisarski, and T. C. Yuan, *Phys. Rev. Lett.* **64**, 2242 (1990).