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Supersymmetry, p-brane duality, and hidden spacetime dimensions

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A global superalgebra with 32 supercharges and all possible central extensions is studied in order to extract some general properties of duality and hidden dimensions in a theory that treats p-branes democratically. The maximal number of dimensions is 12, with signature (10,2), containing one space and one time dimension that are hidden from the point of view of perturbative ten-dimensional string theory or its compactifications. When the theory is compactified on $R^{d-1,1}\otimes T^{c+1,1}$ with $d+c+2=12$, there are isometry groups that relate to the hidden dimensions as well as to duality. Their combined intersecting classification schemes provide some properties of nonperturbative states and their couplings. [S0556-2821(96)00818-1]

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I. INTRODUCTION

The discovery of string dualities has led to the idea that there is a more fundamental theory than string theory that manifests itself in different forms in certain regimes of its moduli space. The several familiar string theories (type-I, type-II, heterotic) may be regarded as different starting points for perturbative expansions around some vacua of the fundamental theory, in analogy with perturbative expansions around different vacua of spontaneously broken gauge theories. A lot of evidence has accumulated by now to convince oneself that the different versions of $D=10$ superstrings and their compactifications are related to each other nonperturbatively by duality transformations [1]. Furthermore, there is evidence that the nonperturbative theory is hiding higher dimensions [2–4] (in the form of ''M theory'' [4–6]) and that it is related to various p-branes [7] and D-branes [8].

In order to explore the fundamental theory, it is desirable to find a common ground that is valid for the many versions of the theory and that is independent of the details of the language used to describe the theory. In search of such a common ground in various dimensions, I will explore a supersymmetry algebra that has 32 fermionic generators and all possible central extensions [7]. The supersymmetry is not necessarily exact; it may be broken by central extensions that are included in the algebra. The spirit is similar to charge or current algebras used in the 1960s for weak and strong interactions, avoiding complexities due to the details of the theory. The basic assumption that we make is that the superalgebra is valid in the sense of a (broken) dynamical symmetry that applies to the full theory at the level of matrix elements for (broken) supermultiplets. By studying the isometries of the superalgebra, including the central extensions, many of the features of duality may be displayed while some new features become apparent, including the following.

(1) The central extensions (as well as supercharges) have a structure consistent with 12 dimensions, with a signature of (10,2). The extra two dimensions beyond ten are hidden from the point of view of string theory. One of them is spacelike and the other is timelike.

(2) As a consequence of central extensions of the superalgebra, p-branes naturally become part of the fundamental theory, and their interaction with $p+1$ forms in supergravity are deduced. These p-branes contribute to the nonperturbative multiplets demanded by U duality and hidden higher dimensions.

(3) The structure of (broken) symmetries associated with hidden dimensions or U duality, the groups, and a classification scheme for the nonperturbative states emerge naturally from the structure of the superalgebra. The relations between the hidden symmetries may be described schematically as

$$\begin{array}{l} \underset{c\ \text{compact}+2\ \text{hidden dims}}{\mathrm{SO}(c+1,1)} \otimes \underset{\text{spacetime}}{\mathrm{SO}(d-1,1)} \rightarrow \begin{array}{c}\text{multiplets of}\\ \text{charges, states,}\end{array} \\ \qquad\downarrow \\ \left.\begin{array}{l}\left.\begin{array}{c} \mathrm{SO}(c+1)_{1\ \text{hidden dim}} \\ \mathrm{SO}(c)_L\otimes \mathrm{SO}(c)_R \\ \uparrow \\ \underset{(T\ \text{duality})}{\mathrm{SO}(c,c)}\end{array}\right\} \rightarrow \begin{array}{c}\uparrow\\ \text{maximal compact} \\ K \\ \text{(duality)}\end{array}\end{array}\right\} \rightarrow \underset{\text{(duality)}}{U} . \end{array} \tag{1.1}$$

(4) Furthermore, one may start with perturbative string states, but then add nonperturbative states that are needed in order to provide a basis for the underlying superalgebra and its isometries K and $\mathrm{SO}(c+1,1)$. This is a method of classifying *a priori* the unknown nonperturbative states.

It has been known since the 1970s that the structure of the superalgebra of type IIA in ten dimensions is intimately connected to 11 dimensions. In the context of string duality, this led to a possible ''M theory'' with signature (10,1) [4–6]. Actually, there seems to be room for (10,2) according to the general properties of the superalgebra discussed below and other more specific arguments given elsewhere,[1] and the fact that type IIB in (9,1) can be related to type IIA in (9,1) by a T-duality transformation in (10,2).

The above points will be the main topics of this paper. The last point is supported by previous work [3,13,14], in

[1]The possibility of (10,2) emerged sometime ago [9]. In connection with duality, it was discussed in a conference talk [10], where the (10,2) behavior of the central extensions and the relation to duality was emphasized. In more recent developments [11,12], other aspects of (10,2) in more detailed theories, such as ''F theory,'' have been discussed.

which consistency between nonperturbative U multiplets and $11D$ (broken) multiplets was analyzed. In this paper we point out the possibility of $12D$ (broken) multiplets.

II. 32 SUPERCHARGES AND (10,2)

It is well known that the maximum number of supercharges in a physical theory is 32. This constraint is obtained in four dimensions by requiring that supermultiplets of massless particles should not contain spins that exceed two. Assuming that the four-dimensional theory is related to a higher-dimensional one, then the higher theory can have at most 32 real supercharges. Denote the 32 supercharges by Q^a_α, where $a=1,2,\ldots,N$, and α is the spinor index in d dimensions. For example, in $d=11$, there is a single 32-component Majorana spinor ($N=1$), in $D=10$ there are two 16-component Majorana-Weyl spinors ($N=2$), etc. down to $D=4$ where there are eight 4-component Majorana spinors ($N=8$). It is important to note that 32 corresponds to counting *real* components of spinors.

In 12 dimensions, the Weyl spinor also has 32 components since $(1/2)2^{12/2}=32$, but when the signature is (11,1), the spinor is complex and has 64 real components. Therefore, as long as we consider a single time coordinate, $d=11$ is the highest allowed dimension. However, if the signature is (10,2), it is possible to impose a Majorana condition that permits a real 32-component spinor.[2] Thus, a price to pay to go beyond 11 dimensions is to consider a second timelike coordinate. It is not clear that traditional unphysical problems of two time coordinates may not be circumvented in some unknown, sufficiently constrained theory. Hence, we may entertain the possibility of (10,2) if there are some benefits for doing so, provided physical inconsistencies are eliminated. Beyond 12 dimensions, the spinor is too large, and therefore, we cannot consider $d>12$.

We need to discuss the theory and analyze its content of hidden dimensions. For example, type IIA string theory with signature (9,1) will appear to be a toroidal compactification from (10,2) on $R^{9,1}\otimes T^{1,1}$ where the extra dimensions with signature (1,1) are both considered hidden, one of them spacelike and the other timelike. More generally, we will consider toroidal compactifications on $R^{d-1,1}\otimes T^{c+1,1}$ where d is the number of ordinary Minkowski spacetime dimensions and c is the number of compactified *string* dimensions, while the two hidden dimensions are counted as extra, so that $d+c+2=12$. The 32 spinors Q^a_α may then be classified as the spinor for $SO(d-1,1)\otimes SO(c+1,1)$. The index a corresponds to the spinor of $SO(c+1,1)$. This group is not necessarily a symmetry, but it helps to keep track of the compactified dimensions, including the hidden ones. Furthermore, the same a index will be reclassified later under the maximal compact subgroup K of U duality, thus providing a bridge between duality and higher hidden dimensions. The supercharges labeled in this way are listed in Table I in various dimensions (at this stage of the discussion the K content of Table I should be ignored). The fact that the same index a is classified in *irreducible* representations of the hidden symmetries of two types is a significant point for the arguments in the rest of the paper.

Consider the maximally extended algebra of the 32 supercharges in various dimensions in the form

$$\{Q^a_\alpha,Q^b_\beta\}=\delta^{ab}\gamma^\mu_{\alpha\beta}P_\mu+\sum_{p=0,1,\ldots}\gamma^{\mu_1\cdots\mu_p}_{\alpha\beta}Z^{ab}_{\mu_1\cdots\mu_p}. \tag{2.1}$$

Since the left side is the symmetric product of 32 supercharges, the right side can have at most $\frac{1}{2}\,32\times33=528$ independent generators. The indices ab on $Z^{ab}_{\mu_1\cdots\mu_p}$ are either symmetrized or antisymmetrized and have the same permutation symmetry as $\alpha\beta$ in $\gamma^{\mu_1\cdots\mu_p}_{\alpha\beta}$. The central extensions $Z^{ab}_{\mu_1\cdots\mu_p}$ are assumed[3] to commute with Q^a_α, P_μ, but they are tensors of the Lorentz group and hence do not commute with it. According to a theorem of Haag *et al.* [15], there can be only Lorentz scalar central charges in a unitary theory in four dimensions, for interactions of *pointlike particles* ($p=0$). However, as will become clear below, in the presence of p-branes, new interactions that permit Lorentz tensors $Z^{ab}_{\mu_1\cdots\mu_p}$ are present in theories with a unitary S matrix (e.g., string theory), indicating that the theorem [15] does not apply to extended objects. In (10,2) dimensions, we will use $M=0',0,1,2,\cdots,10$ for the space index instead of μ. In the 32×32 representation (equivalent to *chirally projected* 64×64), only the 2- and 6-index gamma matrices $\gamma^{M_1M_2}_{\alpha\beta}$ and $\gamma^{M_1\cdots M_6}_{\alpha\beta}$ are symmetric in $\alpha\beta$, and furthermore, $\gamma^{M_1\cdots M_6}_{\alpha\beta}$ is self-dual (one gamma matrix index has been lowered by multiplying with the charge conjugation matrix). The remaining $\gamma^{M_1\cdots M_p}_{\alpha\beta}$ do not have definite symmetry or antisymmetry in $\alpha\beta$. Therefore, in 12 dimensions, on the right-hand side of Eq. (2.1) there can be no P_M, and the 528 generators consist of the antisymmetric tensors $Z_{M_1M_2}$ and $Z^+_{M_1\cdots M_6}$ which is self-dual. The number of components in each is

$$\frac{12\times11}{2}=66,\quad \frac{1}{2}\,\frac{12\times11\times10\times9\times8\times7}{1\times2\times3\times4\times5\times6}=462, \tag{2.2}$$

respectively. Upon compactification to (10,1), we rewrite the 12D index $M=(0',\mu)$ where $\mu=0,1,2,\cdots,10$ is an $11D$ index. Then, we have (suppressing the $0'$ index)

[2] A quick way to see this is to use Bott periodicity to relate the properties of the spinors with signatures $(2,2)\sim(10,2)$. For SO(2,2), the Weyl spinor is real since $SO(2,2)\sim SL(2,R)\times SL(2,R)$. Hence, it is also real for signature (10,2). Another quick remark is that the Lorentz group $SO(n,1)$ and the conformal group $SO(n,2)$ for n spacelike dimensions have the same spinor representations. Hence, the 32-dimensional spinor is a basis for both SO(10,1) and SO(10,2). Since it is real for 11D, it must also be real for 12D with signature (10,2).

[3] For simplicity, we assume commuting central extensions. There are more involved versions of the extended superalgebra in which some of the central extensions do not commute with Q^a_α, or with each other, etc. [16]. We might expect that the noncommuting cases may arise for curved backgrounds and nontoroidal compactifications that are not discussed here.

TABLE I. Classification of Q_α^a and $Z^{ab}_{\mu_1\cdots\mu_p}$ under 11D (or 12D) and K.

$\frac{c+1.1}{d-1.1}$	32 Q_α^a SO $(c+1,1)$ (or K) $\otimes$ SO $(d-1,1)$	$p=0$ p^m,Z^{mn} X^{mnlqr}	$p=1$ p_μ,Z^n_μ X^{nlqr}_μ	$p=2$ $Z_{\mu v}$ $X^{lqr}_{\mu\nu}$	$p=3$ $X^{qr}_{\mu\nu\lambda}$	$p=4$ $X^r_{\mu_1\cdots\mu_4}$	$p=5$ $X_{\mu_1\cdots\mu_5}$	$\frac{U}{K}$
A $\frac{1,1}{9,1}$	$(\pm,16)$	1+0 +0	1+1 +0	1 +0	0	1	1^+ $+1^-$	$\frac{\mathrm{SO}(1,1)}{Z_2}$
B $\frac{1,1}{9,1}$	$\begin{pmatrix}+\\+,16\end{pmatrix}$	0+0 +0	1+2 +0	0 +0	1	0	1^+ $+2^+$	$\frac{\mathrm{SL}(2,R)}{\mathrm{SO}(2)}$
$\frac{2,1}{8,1}$	(2,16)	2+1 +0 **=3** **≈2+1**	1+2 +0 **=3** **≈2+1**	1+0 **=1** **≈1**	1	[1] +2 **=3** **≈2** +1	(1) move	$\frac{\mathrm{SL}(2)\otimes\mathrm{SO}(1,1)}{\mathrm{SO}(2)\otimes Z_2}$
$\frac{3,1}{7,1}$	$((2,0),8^+)$ $((0,2),8^-)$	3+3 +0 **=6** **≈3^+** **+3^-**	1+3 +0 **=(2,2)** **≈3+1**	1+1 **=1+1** **≈1+1**	3+[1] **=(2,2)** **≈3+1**	3^+ $+3^-$ **=6** **≈3^+** **+3^-**	(1) move	$\frac{\mathrm{SL}(3)\otimes\mathrm{SL}(2)}{SO(3)\otimes\mathrm{U}(1)}$
$\frac{4,1}{6,1}$	(4,8)	4+6 +0 =10 ≈10	1+4 +1 =5+1 ≈5+1	1+4 +[1] =5+1 ≈5+1	6 +[4] =10 ≈10	(4) move	(1) move	$\frac{\mathrm{SL}(5)}{\mathrm{SO}(5)}$
$\frac{5,1}{5,1}$	(4,4*) (4*,4)	5+10 +1 **=1+15** **≈(4,4)**	1+5+ 5+[1] **=2×6** **≈(0,5)** **+(5,0)** **+2(0,0)**	1+10 +[5] **=1+15** **=(4,4)**	10^+ $+10^-$ **=10^+** **+10^-** **≈(10,1)** **+(1,10)**	(5) move	(1) move	$\frac{\mathrm{SO}(5,5)}{\mathrm{SO}(5)\otimes\mathrm{SO}(5)}$
$\frac{6,1}{4,1}$	(8,4)	6+15 +6 +[1] **=7+21** **≈27+1**	1+6 +15+[6] **=7+21** **≈27+1**	1+20 +[15] **=1+35** **≈36**	(15) move	(6) move	(1) move	$\frac{\mathrm{E}_{6(6)}}{USp(8)}$
$\frac{7,1}{3,1}$	$(8^+,(2,0))$ $(8^-,(0,2))$	7+21 +21 +[7] **=28+28** **≈28_c**	1+7 +35 +[21] **=8+56** **≈63+1**	$1^\pm$ $+35^\pm$ **=$1^\pm$** **+$35^\pm$** **≈36_c**	(21) move	(7) move	0	$\frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)}$
$\frac{8,1}{2,1}$	(16,2)	8+28 +56 +[28] **=36+84** **≈120**	1+8+70 +[1+56] **=1+9** **+126** **≈135+1**	(1+56) move	(28) move	0	0	$\frac{\mathrm{E}_{8(8)}}{\mathrm{SO}(16)}$

$$Z_{M_1M_2}\to P_\mu\oplus Z_{\mu_1\mu_2} \quad 66=11+55,$$

$$Z^+_{M_1\cdots M_6}\to X_{\mu_1\cdots\mu_5} \quad 462=462, \qquad (2.3)$$

which are the momenta and central charges in 11 dimensions pointed out in [7].

Continuing the compactification process to lower dimensions on $R^{d-1,1}\otimes T^{c+1,1}$, each 11-dimensional index μ decomposes into $\mu\oplus m$ where μ is in d dimensions and m is in $c+1=11-d$ dimensions. Then, each 11-dimensional tensor decomposes as

$$P_\mu\to P_\mu\oplus P_m \quad 11=d+(c+1),$$

$$Z_{\mu\nu}\to Z_{\mu\nu}\oplus Z^n_\mu\oplus Z^{mn},$$

$$X_{\mu_1\cdots\mu_5}\to X_{\mu_1\cdots\mu_5}\oplus X^{m_1}_{\mu_1\cdots\mu_4}\oplus X^{m_1m_2}_{\mu_1\mu_2\mu_3}\oplus X^{m_1m_2m_3}_{\mu_1\mu_2}\oplus X^{m_1\cdots m_4}_{\mu_1}$$
$$\oplus X^{m_1\cdots m_5}. \qquad (2.4)$$

For example, for $(d=10,\ c=0)$, the type 11A superalgebra is recovered, with the 528 operators $(P_\mu, P_{10}, Z_{\mu\nu}, Z_\mu, X_{\mu_1\cdots\mu_4}, X^\pm_{\mu_1\cdots\mu_5})$ where the $\pm$ indicate self-antiself dual, respectively. In Table I in each row labeled by $(d-1,1)/(c+1,1)$, the numbers of each central extension of P,Z,X type with p Lorentz indices is indicated (these are the numbers that are not in bold characters). Since each of the P,Z,X are antisymmetric tensors in $c+1$ dimensions, these numbers correspond to representations of SO$(c+1)$ (which includes rotations into one of the extra dimensions). As we go to lower dimensions, we must use the duality between p indices and $d-p$ indices to reclassify and count the central extensions $Z^{ab}_{\mu_1\cdots\mu_p}\sim Z^{ab}_{\mu_1\cdots\mu_{d-p}}$. In the table a number in parentheses means that it should be omitted from there and instead moved in the same row to the location where the same number appears in brackets. This corresponds to the equivalence of p indices and $d-p$ indices. When $p=d-p$, there are self-dual or antiself-dual tensors. Their numbers are indicated with additional superscripts $\pm$ in the form $1^{\pm}$, $2^{\pm}$, $3^{\pm}$, $10^{\pm}$, $35^{\pm}$ wherever they occur.

The total number of central extensions P,Z,X found according to this compactification procedure for each value of p are indicated in Table I in bold characters. These totals are the same numbers found by counting the number of possibilities ab on $Z^{ab}_{\mu_1\cdots\mu_p}$. The bold numbers following the $=$sign correspond to representations of SO$(c+1,1)$ (making a connection to 12D) and those following the $\approx$ sign correspond to representations of K (to be discussed later in connection to duality).

III. CENTRAL CHARGES AND p-BRANES

What is the meaning of the p-form central extension $Z^{ab}_{\mu_1\cdots\mu_p}$? Since this is a charge in a global algebra there ought to exist a $(p+1)$-form local current $J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)$ whose integral over a spacelike surface embedded in d dimensions gives

$$Z^{ab}_{\mu_1\cdots\mu_p}=\int d^{d-1}\Sigma^{\mu_0}J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x). \tag{3.1}$$

The current couples to the fields of low energy physics (i.e., supergravity). In the case of usual central charges that are Lorentz singlets Z^{ab} (i.e., $p=0$), the current is associated with charged particles. Such a current may be constructed as usual from world lines (or equivalently, from local fields) as

$$J^{ab}_{\mu_0}(x)=\int d\tau\sum_i z_i^{ab}\delta^d(x-X^i(\tau))\partial_\tau X^i_{\mu_0}(\tau). \tag{3.2}$$

The z_i^{ab} are the charges of the particles labeled by i. This current couples in the action to a gauge field $A^{\mu_0}_{ab}$, and it appears as the source in the equation of motion of the Abelian[4] gauge field

$$S\sim\sum_i\int d\tau A^{\mu}_{ab}(X^i(\tau))\partial_\tau X^i_{\mu}(\tau)z_i^{ab}=\int d^dx A^{\mu}_{ab}(x)J^{ab}_{\mu}(x),$$

$$\partial_\lambda\partial^{[\lambda}A^{\mu]}_{ab}(x)=J^{\mu}_{ab}(x). \tag{3.3}$$

Therefore, there are as many gauge fields as there are central extensions of type $p=0$. These gauge fields occur as massless particles in the Neveu-Schwarz–Neveu-Schwarz (NS-NS) and Ramond-Ramond (R-R) sectors of the superstring. The charges Z^{ab} associated with the NS-NS sector occur perturbatively in string theory (Kaluza-Klein momenta and winding numbers), but the charges associated with the R-R sector are nonperturbative from the point of view of string theory (topological solitonic charges). On the other hand, from the point of view of the superalgebra, they occur at an equal footing, and will be treated on an equal basis from the point of view of the (broken) symmetries that we discuss later.

Central charges with $p\geqslant 1$ have been usually omitted in past discussions due to the theorem in [15]. The theorem allows only $p=0$ central extensions. This was derived under the assumption of a unitary S matrix based on pointlike interactions in four dimensions. However, let us now discuss the implications of central charges in the presence of extended objects and in any dimension. For $p=1$, the central extension is a vector $Z^{ab}_{\mu_1}$, which requires a local current that is an antisymmetric tensor $J^{ab}_{\mu_0\mu_1}(x)$ in the Lorentz indices.[5] An antisymmetric current cannot be constructed from particles but it can be constructed from strings as

$$J^{ab}_{\mu_0\mu_1}(x)=\int d\tau d\sigma\sum_i z_i^{ab}\delta^d(x_\mu-X^i_\mu(\tau,\sigma))\times\partial_\tau X^i_{[\mu_0}(\tau,\sigma)\partial_\sigma X^i_{\mu_1]}(\tau,\sigma). \tag{3.4}$$

z_i^{ab} is the charge of the ith string. Just like the particles discussed above, the charged strings also are expected to form a multiplet of the (broken) symmetries, and they interact with the low energy supergravity fields through antisymmetric gauge potentials $B^{\mu_0\mu_1}_{ab}(x)$, with an action

$$S\sim\sum_i\int d\tau d\sigma B^{\nu\mu}_{ab}(X^i(\tau,\sigma))\partial_\tau X^i_{[\nu}(\tau,\sigma)\partial_\sigma X^i_{\mu]}(\tau,\sigma)z_i^{ab}=\int d^dx B^{\nu\mu}_{ab}(x)J^{ab}_{\nu\mu}(x). \tag{3.5}$$

In this expression one can recognize the familiar string coupling to an antisymmetric tensor in the world sheet formulation. The equation of motion for $B^{\nu\mu}_{ab}(x)$ involves the (Abelian) gauge-invariant field strength $H^{\lambda\nu\mu}_{ab}=\partial^{[\lambda}B^{\nu\mu]}_{ab}(x)$ and the above current as a source

[4]The gauge fields are Abelian since we assumed commuting central charges. As noted in a previous footnote, a non-Abelian version is expected if the background is curved rather than flat.

[5]New symmetric tensors, other than the symmetric stress tensor $\delta^{ab}T_{\mu_0\mu_1}(x)$ associated with the momentum $\delta^{ab}P_{\mu_1}(\sim Z^{ab}_{\mu_1})$, are not allowed in the superalgebra, since they would couple to new ''gravitons.''

$$\partial_\lambda \partial^{[\lambda} B_{ab}^{\nu\mu]}(x) = J_{ab}^{\nu\mu}(x). \tag{3.6}$$

A well-known example is type IIB superstring with its two antisymmetric tensors. In this case the ab indices on $B_{ab}^{\nu\mu}(x)$ correspond to a symmetric traceless 2×2 matrix. More antisymmetric tensors are found in compactifications to lower-dimensional string theories.

This example also shows that central extensions that are not Lorentz singlets are present in a unitary theory with nontrivial scattering. Therefore, the theorem in [15], while valid for point-particle interactions, should not be applicable in the presence of p-branes and their interactions.

The generalization to the higher values of p is straightforward: In order to have a charge that is a p-form we need a current $J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)$ that is a $(p+1)$-form. This in turn requires a p-brane to construct the current

$$J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x) = \int d\tau d\sigma_1 \ldots d\sigma_p \sum_i z_i^{ab} \delta^d(x - X^i(\tau,\vec{\sigma})) \times \partial_r X^i_{[\mu_0} \cdots \partial_{\sigma_p} X^i_{\mu_p]}(\tau,\sigma_1,\ldots,\sigma_p), \tag{3.7}$$

and its coupling to supergravity fields requires a $(p+1)$-form gauge potential $A^{ab}_{\mu_0\mu_1\cdots\mu_p}(x)$ such that

$$S \sim \int d^dx A_{ab}^{\mu_0\mu_1\cdots\mu_p}(x) J^{ab}_{\mu_0\mu_1\cdots\mu_p}(x) = \sum_i \int d\tau d\sigma_1 \ldots d\sigma_p A_{ab}^{\mu_0\mu_1\cdots\mu_p}(X^i)\partial_\tau X^i_{[\mu_0} \cdots \partial_{\sigma_p} X^i_{\mu_p]} z_i^{ab} \tag{3.8}$$

and

$$\partial_\lambda \partial^{[\lambda} A_{ab}^{\mu_0\mu_1\cdots\mu_p]}(x) = J_{ab}^{\mu_0\mu_1\cdots\mu_p}(x). \tag{3.9}$$

As is well known by now, there are perturbative as well as nonperturbative couplings of p-branes to supergravity in various dimensions. Hence, the $Z^{ab}_{\mu_1\cdots\mu_p}$ are present in the superalgebra and they correspond simply to the charges of p-branes. The classification of their ab indices under duality groups is the subject of the next section, but here we already see that there is a one-to-one correspondence between the p-forms $Z^{ab}_{\mu_1\cdots\mu_p}$ and the $(p+1)$-form gauge potentials $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$ that appear as massless states in string theory in the NS-NS or R-R sectors.

The main message is that from the point of view of the superalgebra, all p-branes appear to be at an equal footing. Isometries of the superalgebra that will be discussed below treat them equally and may mix them with each other in various compactifications. The theory in d dimensions has $(p+1)$-forms $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$ which appear as massless particles in the string version of the fundamental theory. These act as gauge potentials and couple at low energies to charged p-branes. This generates a nontrivial central extension $Z^{ab}_{\mu_1\cdots\mu_p}$ in the superalgebra. The number of such central extensions (ab indices) is in one-to-one correspondence with the number of the $(p+1)$-forms $A_{ab}^{\mu_0\mu_1\cdots\mu_p}$, and these numbers can be obtained by counting the possible combination of (symmetric-antisymmetric) indices ab associated with the supercharges.

IV. RECLASSIFICATION AND DUALITY

In the discussion above we concentrated on the 11D (or 12D) content of the supercharges and the central extensions. We now turn to duality. In string theory the T-duality group is directly related to the number of compactified left-right string dimensions. In our notation, the number of compactified string dimensions is c. Therefore, for a string of type II, it is

$$T = \mathrm{SO}(c,c). \tag{4.1}$$

Its maximal compact subgroup is

$$k = \mathrm{SO}(c)_L \otimes \mathrm{SO}(c)_R, \tag{4.2}$$

where L,R denote left-right movers respectively.[6] The supercharges $Q_\alpha^{\,a}$ naturally know about this group, since they too can be split into left-right movers in even d dimensions: then the index a on left-right chiral charges $Q_\alpha^{\,a}$ corresponds precisely to the spinor index of $\mathrm{SO}(c)_L \otimes \mathrm{SO}(c)_R$. For odd d dimensions the same is true, but the L/R split is defined by going to the next smaller value of c.

For example, in four dimensions the $N=8$ real Majorana spinors are rewritten as 8 pseudo-real Weyl spinors of left or right type that are each other's complex conjugates. In Table I these were classified as pseudo-real representations

$$\begin{gathered}(8^+,(2,0)), \quad (8^-,(0,2)) \\ \mathrm{SO}(7,1)_{\text{hidden}} \otimes \mathrm{SO}(3,1)_{\text{space}}.\end{gathered} \tag{4.3}$$

Now we reclassify them as

$$\begin{gathered}([(4,0)+(0,4^*)],(2,0)), \quad ([(4^*,0)+(0,4)],(0,2)) \\ (\mathrm{SO}(6)_L \times \mathrm{SO}(6)_R)_{k\subset T} \otimes \mathrm{SO}(3,1)_{\text{space}}.\end{gathered} \tag{4.4}$$

[6]The notation for duality groups, such as $\mathrm{SO}(c,c)$, is used somewhat loosely in this paper, for brevity. The T,U duality groups mentioned in this paper are supposed to be interpreted as discrete groups, such as $\mathrm{SO}(c,c,Z)$, etc. This is not apparent from the superalgebra point of view, but is true in string theory. Under T-duality transformations the quantized Kaluza-Klein and winding numbers of string states transform into each other under $\mathrm{SO}(c,c,Z)$. In addition there is an induced transformation on the oscillators in the internal dimensions under the subgroup $k = \mathrm{SO}(c)_L \otimes \mathrm{SO}(c)_R$, where the effective parameters of the induced transformation depend on the discrete $\mathrm{SO}(c,c,Z)$ as well as the torus parameters G_{ij}, B_{ij}, and hence, it is equivalent to being continuous. Therefore, because of T duality, all perturbative string states must fall into linear representations of $k = \mathrm{SO}(c)_L \otimes \mathrm{SO}(c)_R$, which is larger than the $\mathrm{SO}(c)$ expected naively. In a similar sense, the transformations under K are also equivalent to being continuous, even though those of U are discrete. For a clarification of these points see [13].

The common internal group in $\mathrm{SO}(7,1)_{\mathrm{hidden}}$ and $\mathrm{SO}(6)_L\times\mathrm{SO}(6)_R$ is SO(6), but besides this common subgroup these two groups are not related to each other by group-subgroup relationships. Thus, their transformations on the physical states of the theory must act on rather different modules that have intersections with each other.

More generally, for any dimension, investigating the supercharges listed in Table I shows that the index a that was classified there under the hidden noncompact group $\mathrm{SO}(c+1,1)_{\mathrm{hidden}}$ can be reclassified under the perturbatively explicit maximal compact subgroup $k\subset T$ of T duality, $k=\mathrm{SO}(c)_L\otimes\mathrm{SO}(c)_R$ (see Table III in Ref. [13]). These two groups are not subgroups of each other, but they do have a common subgroup $\mathrm{SO}(c)$. Recall that c is the number of compactified string dimensions (other than the two hidden dimensions), and $\mathrm{SO}(c)$ is the (broken) rotation group in these internal dimensions.

In each case one may notice that the N supercharges $Q^{\,a}_{\alpha}$ transform irreducibly under $\mathrm{SO}(c+1,1)_{\mathrm{hidden}}$, but reducibly under $k=\mathrm{SO}(c)_L\times\mathrm{SO}(c)_R$. However, we can obtain an irreducible representation $Q^{\,a}_{\alpha}$ by defining a larger *compact* group K that contains k, as well as the maximal compact part of $\mathrm{SO}(c+1,1)_{\mathrm{hidden}}$. That is

$$K\supset\mathrm{SO}(c)_L\otimes\mathrm{SO}(c)_R \quad\text{and}\quad K\supset\mathrm{SO}(c+1). \tag{4.5}$$

Thus, we look for the *compact* group K that contains $\mathrm{SO}(c)_L\times\mathrm{SO}(c)_R$, $\mathrm{SO}(c+1)$ and that has an *irreducible* representation for the index a (total dimension N). Note that the group K must mix one extra dimension with others. Furthermore, the central extensions of type P,Z,X, that already display the extra dimension, have to fall into representations of K that contain them. The *minimal* compact K that we find through this reasoning is listed in the last column of Table I. By virtue of containing $k\subset T$, the group $K\supset k$ must be related to a larger group of duality U that contains T. After finding K as described, U is determined uniquely by looking for the smallest noncompact group that contains $\mathrm{SO}(c,c)$ and for which K is the maximal compact subgroup. The subgroup hierarchy that emerges is given in Eq. (1.1). For example, in four dimensions (with $d=4$, $c=6$), it is

$$\begin{array}{ll}\mathrm{SO}(7,1)_{\mathrm{hidden}}\otimes\mathrm{SO}(3,1)\to a: & \begin{array}{c}(8^+,(2,0))\\+(8^-,(0,2))\end{array}\\ \quad\downarrow & \qquad\uparrow\\ \left.\begin{array}{l}\mathrm{SO}(7)_{\mathrm{hidden}}\\ \mathrm{SO}(6)_L\otimes\mathrm{SO}(6)_R\end{array}\right\}\to\mathrm{SU}(8) & \\ \quad\uparrow & \\ T=\mathrm{SO}(6,6) & \end{array}\left.\vphantom{\begin{array}{c}a\\a\\a\\a\\a\end{array}}\right\}\to\mathrm{E}_{7(7)}. \tag{4.6}$$

The a index which was classified as the spinors $8^{\pm}$ under SO(7,1) or as $[(4,0)+(0,4^*)]$ or $[(4^*,0)+(0,4)]$ under $k=\mathrm{SO}(6)_L\otimes\mathrm{SO}(6)_R$ is now reclassified as the 8 or 8^* of SU(8). This group is the minimal compact group containing both SO(7) and $\mathrm{SO}(6)_L\otimes\mathrm{SO}(6)_R=\mathrm{SU}(4)_L\otimes\mathrm{SU}(4)_R$. Furthermore, the smallest noncompact group containing both SU(8) and SO(6,6) is $\mathrm{E}_{7(7)}$. This way of describing K or U does not use the details of supergravity or string theory. It merely hinges on the number of supercharges and their reclassifications in maximally irreducible representations as described above. We emphasize that the scheme takes advantage of the hidden dimensions.

Since the same N-dimensional basis of supercharges labeled by a knows about both duality and the hidden dimensions, this must provide a bridge for relating properties of the states of the theory under both qualities. The first consequence of this is the reclassification of the central extensions $Z^{ab}_{\mu_1\cdots\mu_p}$. Previously, they were classified under 11D (or 12D) as in Table I (the numbers following the = sign). But now the combination ab corresponds to the symmetric or antisymmetric product of the N-dimensional representation of K. Therefore, *the central extensions are now also classified under* K. The result is the total dimension listed in Table I (the numbers following the ≈ sign). These numbers are indeed dimensions of irreducible multiplets under K.

For example, in four dimensions the central extensions whose (real) numbers are 56, 63, 72 for $p=0,1,2$, respectively, are reclassified as the complex $\mathbf{28}_c$, real **63**, and complex $\mathbf{36}_c$ of $K=\mathrm{SU}(8)$. These correspond to the following combinations of the SU(8) ab indices on $Z^{ab}_{\mu_1\cdots\mu_p}$, recalling that $a\to 8$ or 8^*:

$$\begin{aligned} p=0:&\quad (8\times8)_{\mathrm{antisymm}}=28_c,\\ p=1:&\quad (8\times8^*)=63+1,\\ p=2:&\quad (8\times8)_{\mathrm{symmetric}}=36_c. \end{aligned} \tag{4.7}$$

The $p=1$ singlet **1** corresponds to the momentum P^{μ}. The complex conjugates $28_c^*,36_c^*$ contain the same real components as $28_c,36_c$. On the other hand, these same total dimensions correspond to the irreducible representations of $\mathrm{SO}(7,1)_{\mathrm{hidden}}$ as follows. Using the fact that the supercharges can be viewed as the spinors $8^+\oplus8^-$, their products give the following $\mathrm{SO}(7,1)_{\mathrm{hidden}}$ representations for the indices ab on $Z^{ab}_{\mu_1\cdots\mu_p}$:

$$\begin{aligned} p=0:&\quad (8^{\pm}\times8^{\pm})_{\mathrm{antisymm}}=28^{\pm},\\ p=1:&\quad (8^+\times8^-)=8_v+56_v,\\ p=2:&\quad (8^{\pm}\times8^{\pm})_{\mathrm{symmetric}}=1^{\pm}+35^{\pm}. \end{aligned} \tag{4.8}$$

Note that the momentum P^{μ} is now part of the 8_v. By decomposing the representations for each p with respect to the common subgroup,

$$\mathrm{SU}(8)\supset\mathrm{SO}(7)\subset\mathrm{SO}(7,1)_{\mathrm{hidden}}, \tag{4.9}$$

the same sets of SO(7) representations are recovered from either Eq. (4.7) or Eq. (4.8). This SO(7) already contains one of the hidden dimensions and classifies the central extensions of types P,Z,X separately as listed in Table I.

The main point is that the supercharges as well as the central extensions are now classified under hidden (broken) symmetries of two different types. The first one $\mathrm{SO}(c+1,1)_{\mathrm{hidden}}$ relates to 11 or perhaps 12 hidden dimensions, and the second one $K\subset U$ relates to U duality. The common compact subgroup $\mathrm{SO}(c+1)$ already contains nonperturbative information about the spacelike hidden dimension, but more information about the hidden timelike dimension and about U duality is contained in the larger group structures K, SO($c+1$,1).

V. NONPERTURBATIVE STATES

Under the assumption that the superalgebra is valid as a dynamical (broken) symmetry in the entire theory, all states would belong to multiplets of the (broken) superalgebra, including the central extensions and the p-branes associated with them. One would then expect to be able to classify the physical states of the theory according to the (broken) isometries K, SO($c+1$,1). However, since these groups are not contained in each other, we should have different modules of $\mathrm{SO}(c+1,1)_{\mathrm{hidden}}$ and $K \subset U$ that have intersections with each other in the form of (broken) SO($c+1$) multiplets, since this is the largest common subgroup:

$$K \supset \mathrm{SO}(c+1) \subset \mathrm{SO}(c+1,1)_{\mathrm{hidden}}. \tag{5.1}$$

It seems reasonable to make the hypothesis that the complete set of states of the theory could be classified with either group, but that each such classification would contain the same set of SO($c+1$) representations. One of our aims is to test this hypothesis. Each one of these classifications contains nonperturbative states related to either duality or hidden dimensions. By finding them and studying their couplings consistent with the superalgebra, one would be able to learn certain global properties of the underlying theory.

Some of the couplings described by $Z^{ab}_{u_1 \cdots \mu_p}$ are perturbative while others are nonperturbative in the string language, but all couplings or states are on an equal footing from the point of view of the superalgebra and its isometries. One must include open p-branes in the form of D-branes since they couple to closed p-branes. Therefore, we expect that various excitations of open or closed charged p-branes $X^{i}_{\mu}(\tau,\sigma_1,\cdots,\sigma_p)$ (and their supersymmetric partners) occur on an equal footing in supermultiplets that contain the (broken) group structures revealed above. String theory states at various excitation levels by themselves may not necessarily form the needed multiplets in higher dimensions (10,1) or (10,2) or in U duality. However, some combination of open or closed p-brane states are expected to fill complete multiplets of the isometries or broken symmetries of the global superalgebra. By starting from the known superstring states, the supermultiplets connected to them can be found, and the nonperturbative states can be identified.

Following the arguments in [13], the states of the full theory may be classified as

$$\phi_{\mathrm{indices}}(\mathrm{base}), \tag{5.2}$$

where the base consists of the commuting 528 bosonic generators of the superalgebra. These include the continuous momentum and the quantized central extensions $Z^{ab}_{\mu_1 \cdots \mu_p}$ that are at an equal footing. These quantum numbers are classified in linear representations of the (broken) isometries K or SO($c+1$,1) as given in Table I.[7] If the superalgebra is valid in the full theory then the *indices* must also fall into linear representations of K or SO($c+1$,1) in order to provide a basis for its (broken) isometries. Thus, both the indices as well as the base contain information about nonperturbative states through duality transformations or rotations into the hidden dimensions. Classifying the states under these groups relates the properties of nonperturbative states to those of the perturbative string states.

A possible scheme for finding the nonperturbative states is as follows. First, identify the perturbative string states, classify them under supermultiplets, and identify their classification under the perturbatively explicit $\mathrm{SO}(c)_L \otimes \mathrm{SO}(c)_R$. Then, try to reclassify them under the bigger (broken symmetry) group K. If additional states are needed to make complete K multiplets add them (these extra states are presumably p-branes, D-branes). There may be nonunique ways of completing K multiplets. If so, then try to make it consistent with the presence of the hidden dimensions by making sure that the SO($c+1$) representations embedded in K multiplets are consistent with the structure of the central charges listed in the table. When this is achieved, one should also check that it is all consistent with a compactification of a collection of states that starts in 11 dimensions, i.e., consistency with 11-dimensional (broken) multiplets with signature (10,1). One may need to add at this stage more nonperturbative states that are not in the same K multiplet with some perturbative string state (presumably, more p- or D-brane states). So far, one should expect consistency with ''M theory.'' Finally, check if the structure of the representations that emerge in this way can also be made consistent with 12 dimensions, with signature (10,2) (perhaps by adding more states). In this way, many properties of nonperturbative states could be deduced. Such a program was initiated in previous papers [3,13,14]. The results obtained there (involving string states at many excited levels) are in agreement with the presence of many of the structures outlined here as far as (10,1) and K structures are concerned. It would be interesting to extend these ideas to explore (10,2).

It would also be of interest to analyze ''M theory'' and ''F theory'' from the point of view of the general properties of the superalgebra, and discriminate between general properties based on the superalgebra versus the properties of the theory that depend on more detailed features. As mentioned in the footnotes, non-Abelian versions of the superalgebra are possible, and in fact, expected when the p-branes propagate on curved backgrounds. It would be of interest to relate them to properties of various compactifications of ''M theory'' and ''F theory'' in order to learn about some of their general global properties.

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[7]According to the dimensions of representations in Table I, the 0-brane Z^{ab} central extensions seem to correspond to complete linear representations of U for all dimensions except for $d=3$ (120 is not a representation $E_{8(8)}$). Similarly, higher p-branes $Z^{ab}_{\mu_1 \cdots \mu_p}$ do not generally form linear representations of U. Furthermore, the $Z^{ab}_{\mu_1 \cdots \mu_p}$ seem to form complete representations of SO(c,c) for all cases except for ($d=5$, $p=3$), ($d=3$,4, $p=2$). We interpret these observations to mean that the base is not generally a bunch of *linear* representation of either T- or U-duality groups, but it is a bunch of *linear* representation of K or SO($c+1$,1).

[1] C. Hull and P. Townsend, Nucl. Phys. **B438**, 109 (1995).
[2] E. Witten, Nucl. Phys. **B443**, 85 (1995), and in Strings '95, Future Perspectives in String Theory, Proceedings of the Conference, Los Angeles, California, edited by I. Bars *et al.* (unpublished), Report No. hep-th/9507121 (unpublished).
[3] I. Bars, Phys. Rev. D **52**, 3567 (1995).
[4] P. K. Townsend, Phys. Lett. B **350**, 184 (1995).
[5] J. Schwarz, Phys. Lett. B **367**, 97 (1996); in *Theory of Elementary Particles*, Proceedings of the 29th International Ahrenshoop Symposium, Buckaw, Germany, 1995, edited by D. Lust and G. Weight [Nucl. Phys. B (Proc. Suppl.) **49**, (1996)], Report No. hep-th/9509148 (unpublished); ''*M*-theory extensions of *T* duality,'' Report No. hep-th/9601077 (unpublished).
[6] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996).
[7] P. Townsend, ''*p*-brane democracy,'' Report No. hep-th/9507048 (unpublished).
[8] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); see also, J. Polchinski, S. Chaudhuri, and C. V. Johnson, ''Notes on *D*-branes,'' Report No. hep-th/9602052 (unpublished).
[9] M. Blencowe and M. Duff, Nucl. Phys. **B310**, 387 (1988).
[10] I. Bars, in Proceedings of Conference on Frontiers in Quantum Field Theory, Toyonaka, Japan, 1995, Report No. hep-th/9604200 (unpublished).
[11] C. Vafa, Nucl. Phys. **B469**, 403 (1996); D. R. Morrison and C. Vafa, ''Compactifications of *F*-theory on Calabi-Yau threefolds,'' Report No. hep-th/9602114 (unpublished); Report No. hep-th/9603161 (unpublished); E. Witten, Nucl. Phys. **B471**, 195 (1996).
[12] D. Kutasov, E. Martinec, and M. O'Loughlin, ''Vacua of *M* theory and $N=2$ strings,'' Report No. hep-th/9603116 (unpublished).
[13] I. Bars and S. Yankielowicz, Phys. Rev. D **53**, 4489 (1996); I. Bars, in *Theory of Elementary Particles* [5], p. 115, Report No. hep-th/9601164 (unpublished).
[14] A. Sen, Nucl. Phys. **B450**, 103 (1995).
[15] R. Haag, J. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88**, 257 (1975).
[16] For some examples, see E. Bergshoeff and E. Sezgin, Phys. Lett. B **232**, 96 (1989); **354**, 256 (1995).