Green-Schwarz superstring in extended configuration space and the infinitely reducible first class constraints problem

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(Received 29 March 1996)

The Green-Schwarz superstring action is modified to include some set of additional (on-shell trivial) variables. A complete constraint system of the theory turns out to be reducible both in the original and in additional variable sectors. The initial $8s$ first class constraints and $8c$ second class ones are shown to be unified with $8c$ first class and $8s$ second class constraints from the additional variables sector, resulting in SO(1,9)-covariant and linearly independent constraint sets. Residual reducibility proves to fall on second class constraints only. [S0556-2821(96)02218-7]

PACS number(s): 11.25.Hf, 04.60.Ds, 11.30.Pb

I. INTRODUCTION

The general recipe of covariant quantization of dynamical systems subject to reducible first and second class constraints was developed in Refs. [1–3]. The ''ghosts for ghosts'' mechanism [1,2] was proposed to balance the correct dynamics on the one hand and manifest covariance on the another. The application of the scheme turned out to be remarkably successful for certain cases. The antisymmetric tensor field [1], chiral superparticle [4], and high superspin theories [5] seem to be the most interesting examples.

However, in the general case there may arise an infinite tower of extra ghost variables, which makes the expression for the effective action formal. The superparticle [6] and superstring [7] models appeared to be the first (and, actually, the most important) examples of such a type. A complete constraint system of the theories in the Hamiltonian formalism includes fermionic constraints[1] that, being a mixture of eight first class and eight second class ones [$8s$ and $8c$ representations of SO(8)-little group, respectively], lie in the minimal spinor representation of the Lorentz group. The latter fact means that the covariant irreducible separation of the constraints is impossible in the original phase space [8]. However, one can realize the reducible split by making use of covariant projectors known for the superparticles [9–11] and superstring [12]. The introduction of 16 covariant primary ghosts to the (reducible) first class constraints implies 16 secondary ones, etc. There arises an infinite tower of extra ghost variables. The Lagrangian analogue of the situation is infinitely reducible Siegel symmetry [13], with spinor parameters from which only half are essential on shell. Note that within the framework of the alternative twistor-harmonic approach [14], the fermionic constraints can be separated in a covariant and irreducible manner due to the ''bridge nature'' of the harmonic variables. This formalism, however, is essentially Hamiltonian and the reparametrization invariance of the original Green-Schwarz theory turns out to be broken in the modified version [14].

Reformulation of the Batalin-Fradkin-Vilkovisky (BFV) procedure that does not involve explicit separation of constraints was presented in Refs. [15–17]. However, as was shown in Ref. [11], application of the scheme for concrete models may conflict with manifest Poincaré covariance.

In this paper we propose an alternative approach to the infinitely reducible constraints problem of $D=10$, $N=1$ Green-Schwarz superstring (GSS). The basic idea is to introduce additional pure gauge fermionic degrees of freedom subject to *reducible* constraints like those of the GSS. We choose these constraints to be a pair of Majorana-Weyl spinors with the following structure[2]: (i) The first of them is a mixture of eight first class and eight second class constraints, which are required to lie in $8c$ and $8s$ irreducible representations of SO(8) group, respectively; (ii) The second spinor contains only eight linearly-independent components that are second class constraints.

Splitting further all the fermionic constraints of the problem in covariant and reducible manner (by making use of covariant projectors [11,12]) one can combine the original fermionic first class constraints of the GSS with the first class ones from the additional variables sector into one irreducible set [which corresponds to the $8s\oplus 8c$ representation of SO(8) or Majorana-Weyl spinor of SO(1,9)]. Analogously, the second class constraints from the additional variables sector can be unified with the original second class ones resulting in covariant and irreducible constraint. For the model concerned, the resulting constraint system turns out to be completely equivalent to the initial one. Thus, the reducible fermionic first class constraints of the GSS become irreducible in the modified theory. The infinite tower of extra ghost variables, which corresponds to the first class constraints in the original formulation of the superstring, will not appear in the new version. The Lagrangian that reproduces the scheme described above is our main result.

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[1]We discuss mostly the $N=1$, $D=10$ case for which covariant quantization is the principal problem.

[2]The total number of constraints is sufficient to suppress just one canonical pair of variables.

The paper is organized as follows. In Sec. II the Green-Schwarz action is modified to include some set of additional variables. The local symmetries of the model are investigated. A complete canonical analysis of the theory is carried out in Sec. III A. Classical equivalence of the modified and original superstrings is established in Sec. III B. We do this by imposing gauge conditions for all first class constraints in the problem. Dynamics in the physical variables sector proves to coincide with that of the GSS. Note that all the gauge conditions can be imposed in covariant manner, excepting the standard light-cone gauge conditions corresponding to the super-Virasoro constraints. In Sec. IV explicitly covariant separation of the constraints is realized. The infinitely reducible first class constraints problem is resolved. Concluding remarks are presented in Sec. V. Appendix A contains our conventions and a brief description of the SO(8)-formalism used in the work. Appendix B includes essential Poisson brackets of the constraints involved.

II. ACTION AND LOCAL SYMMETRIES

The action functional to be examined is of the form

$$S=S_{\rm GS}+S_{\rm add}, \tag{1}$$

where

$$S_{\rm GS}=\int d\tau d\sigma\left\{-\frac{1}{4\pi\alpha'}\sqrt{-g}g^{\alpha\beta}\Pi^m{}_\alpha\Pi_{m\beta}-\frac{1}{2\pi\alpha'}\epsilon^{\alpha\beta}\partial_\alpha X^m i\Theta\Gamma_m\partial_\beta\Theta\right\},$$

$$S_{\rm add}=\int d\tau d\sigma\left\{-\frac{1}{2}\epsilon^{\alpha\beta}\Lambda_m(\partial_\alpha A^m{}_\beta-\partial_\beta A^m{}_\alpha-\partial_\alpha\Theta\Gamma^m\chi_\beta+i\partial_\beta\Theta\Gamma^m\chi_\alpha+i\chi_\alpha\Gamma^m\chi_\beta)-\Phi\Lambda^2\right\},$$

and $\Pi^m{}_\alpha\equiv\partial_\alpha X^m-i\Theta\Gamma^m\partial_\alpha\Theta$, $\sqrt{-g}\equiv\sqrt{-\det g_{\alpha\beta}}$. The first term in Eq. (1) is the Green-Schwarz action [7], while the second term is the action of additional variables. All the variables are treated on equal footing. The latin indices are designed for target manifold tensors, the greek ones are set for worldsheet tensors (for instance, $\chi^A{}_\alpha$ is $D=10$ Lorentz spinor and $D=2$ worldsheet vector). Statistics of the fields corresponds to their tensor structure, i.e., X^m, $g^{\alpha\beta}$, $A^m{}_\alpha$, Λ^m, Φ are bosons, while Θ^A, $\chi^A{}_\alpha$ are fermions. The matrix $\epsilon^{\alpha\beta}$ is chosen in the form $\epsilon^{\alpha\beta}=-\epsilon^{\beta\alpha}$, $\epsilon^{01}=-1$.

Since the $S_{\rm add}$ contains only derivatives of the Θ, the modified superstring is invariant under standard global supersymmetry transformations.

Local symmetries of the theory, except the standard reparametrizations of worldsheet and the Weyl transformations, include a modification of the Siegel transformations[3]

$$\delta_k\Theta=2i\Pi_{m\alpha}\tilde{\Gamma}^m k^{-\alpha},$$
$$\delta_k X^m=i\Theta\Gamma^m\delta_k\Theta,$$
$$\delta_k(\sqrt{-g}g^{\alpha\beta})=16\sqrt{-g}P^{-\alpha\gamma}(\partial_\gamma\Theta k^{-\beta}),$$
$$\delta_k\chi_\alpha=\partial_\alpha(\delta_k\Theta),$$
$$\delta_k A^m{}_\alpha=i\Theta\Gamma^m\partial_\alpha(\delta_k\Theta), \tag{2}$$

where

$$P^{\pm\alpha\beta}\equiv\frac{1}{2}\left(g^{\alpha\beta}\pm\frac{\epsilon^{\alpha\beta}}{\sqrt{-g}}\right),\quad k^-\equiv P^-k,$$

and a set of new symmetries acting on the additional variables subspace. Here we list them with brief comments.

There is a pair of bosonic symmetries with $D=10$ vector ξ^m and $D=2$ vector μ_α parameters

$$\delta_\xi A^m{}_\alpha=\partial_\alpha\xi^m, \tag{3}$$

$$\delta_\mu A^m{}_\alpha=\Lambda^m\mu_\alpha,\qquad \delta_\mu\Phi=-\frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha\mu_\beta, \tag{4}$$

which mean that the fields $A^m{}_\alpha$ and Φ may be gauged away. Note that the system (3), (4) is reducible. This can easily be seen by taking $\xi^m=\Lambda^m\nu$, $\mu_\beta=\partial_\beta\nu$, where ν is an arbitrary function. With such a choice $(\delta_\xi-\delta_\mu)|_{\rm on\ shell}\equiv0$, which means functional dependence of generators of the transformations. In addition to transformations (3) and (4), the action (1) possesses the fermionic symmetries

$$\delta_{s^+}\chi_\alpha=\Lambda_n\tilde{\Gamma}^n s^+{}_\alpha,$$
$$\delta_{s^+}\Phi=\epsilon^{\alpha\beta}i(\partial_\alpha\Theta-\chi_\alpha)s^+{}_\beta, \tag{5}$$
$$\delta_{s^-}\chi_\alpha=\Lambda_n\tilde{\Gamma}^n s^-{}_\alpha,$$
$$\delta_{s^-}\Phi=i\epsilon^{\alpha\beta}(\partial_\alpha\Theta-\chi_\alpha)s^-{}_\beta. \tag{6}$$

The symmetries (5) and (6) are reducible. The transformation of parameters, under which Eqs. (5) and (6) are invariant (modulo equations of motion), is of the form

$$s'_\alpha=s_\alpha+\Lambda_n\Gamma^n\varkappa_\alpha$$

[3]To check the k invariance of the action it is necessary to use to the Fierz identity $\Gamma^m_{A(B}\Gamma^m_{CD)}=0$ and the property of the $P^\pm$ projectors: $P^{\pm\alpha\gamma}P^{\pm\beta\sigma}=P^{\pm\beta\gamma}P^{\pm\alpha\sigma}$. Note as well that the k symmetry is reducible. The following transformation of parameters $k'_\beta=k_\beta+\Pi_{n\gamma}\Gamma^n P^{-\gamma\sigma}\varkappa_{\sigma\beta}$, with the $\varkappa_{\sigma\beta}$ being an arbitrary function, does not change Eq. (2) (modulo equations of motion), which means linear dependence of generators of the transformations.

with an arbitrary $\varkappa_\alpha$. It is interesting to note that the reducible symmetries (2) and (6) can be replaced by one irreducible symmetry if one supposes that $(\Lambda\Pi_\alpha)^2\neq 0$. Actually, consider the transformation

$$\delta_\omega\Theta=2i\Pi_{m\alpha}\tilde{\Gamma}^m\omega^{-\alpha},$$

$$\delta_\omega X^m=i\Theta\Gamma^m\delta_\omega\Theta,$$

$$\delta_\omega(\sqrt{-g}g^{\alpha\beta})=16\sqrt{-g}P^{-\alpha\gamma}(\partial_\gamma\Theta\,\omega^{-\beta}),$$

$$\delta_\omega\chi_\alpha=\partial_\alpha(\delta_\omega\Theta)+\Lambda_m\tilde{\Gamma}^m\omega^-{}_\alpha,$$

$$\delta_\omega A^m{}_\alpha=i\Theta\Gamma^m\partial_\alpha(\delta_\omega\Theta),$$

$$\delta_\omega\Phi=i\epsilon^{\alpha\beta}(\partial_\alpha\Theta-\chi_\alpha)\omega^-{}_\beta, \tag{7}$$

which is a formal sum of Eqs. (2) and (6) with $k^-=s^-\equiv\omega^-$. Two remarks concerning this symmetry are relevant. First, it is straightforward to check that there is no transformation of parameters that leaves Eq. (7) invariant, i.e., all 16 parameters are effective on shell. Secondly, the original k^- and s^- transformations (each of them has eight essential parameters on shell) can be extracted from Eq. (7) by taking

$$\omega_{1\beta}=\frac{1}{(\Lambda\Pi)^2}(\Lambda\Pi_\sigma)P^{+\sigma\gamma}\Lambda_n\Gamma^n\tilde{\Gamma}^m\Pi_{m\gamma}k_\beta,$$

$$\omega_{2\beta}=\frac{1}{(\Lambda\Pi)^2}(\Lambda\Pi_\sigma)P^{+\sigma\gamma}\Pi_{n\gamma}\Gamma^n\tilde{\Gamma}^m\Lambda_m s_\beta. \tag{8}$$

Equations of motion for the theory (1) are of the form

$$\Pi^m{}_\alpha\Pi_{m\beta}=\frac{1}{2}g_{\alpha\beta}g^{\gamma\delta}\Pi^m{}_\gamma\Pi_{m\delta},$$

$$\partial_\beta(\sqrt{-g}g^{\beta\alpha}\partial_\alpha X^m+2\sqrt{-g}P^{-\beta\alpha}i\Theta\Gamma^m\partial_\alpha\Theta)=0,$$

$$\Pi_{m\alpha}\Gamma^m P^{-\alpha\beta}\partial_\beta\Theta=0, \tag{9a}$$

$$\Lambda^2=0,$$

$$\epsilon^{\alpha\beta}(\partial_\alpha A^m{}_\beta-\partial_\beta A^m{}_\alpha-i\partial_\alpha\Theta\Gamma^m\chi_\beta+i\partial_\beta\Theta\Gamma^m\chi_\alpha$$
$$+i\chi_\alpha\Gamma^m\chi_\beta)+4\Phi\Lambda^m=0,$$

$$\partial_\alpha\Lambda_m=0,$$

$$i(\chi_\alpha-\partial_\alpha\Theta)\Gamma^m\Lambda_m=0. \tag{9b}$$

Note that Eqs. (9a) are just the Green-Schwarz superstring equations. In the light-cone gauge this system reduces to $\Box X^i=0$, $\partial_-\Theta^a=0$, where i and a are, respectively, vector and spinor indices of SO(8) group. It turns out that there are no more dynamical degrees of freedom in the question. We will prove this fact in the next section by moving to the Hamiltonian formalism and imposing all gauge conditions.

III. CANONICAL FORMALISM

A. Dirac procedure

Denoting momenta conjugate to the variables[4] (X^m, Θ^B, N, N_1, g^{11}, Λ^m, $A^m{}_\alpha$, $\chi^B{}_\alpha$, Φ) as (π_m, $P_{\theta B}$, P_N, P_{N_1}, P_g, $\pi_{\Lambda m}$, $\pi_{Am}{}^\alpha$, $P^\alpha_{\chi B}$ π_Φ) one gets

$$\pi_m=\frac{1}{2\pi\alpha'}\left(\frac{1}{N}\Pi_{m0}-\frac{N_1}{N}\Pi_{m1}+i\Theta\Gamma_m\partial_1\Theta\right),$$

$$\tilde{L}\equiv P_\theta+i\Theta\Gamma^m\left(\pi_m+\frac{1}{2\pi\alpha'}\Pi_{m1}\right)-i\chi_1\Gamma^m\Lambda_m\approx 0,$$

$$\pi_{\Lambda m}\approx 0,\quad \pi_{Am}{}^0\approx 0,\quad \pi_{Am}{}^1-\Lambda_m\approx 0,$$

$$P_\chi{}^0\approx 0,\quad P_\chi{}^1\approx 0,\quad \pi_\Phi\approx 0,$$

$$P_N\approx 0,\quad P_{N_1}\approx 0,\quad P_g\approx 0, \tag{10}$$

where $\partial_1\equiv\partial/\partial\sigma$. The first equation in Eq. (10) determines $\partial_0 X^m$ as a function of the other canonical variables. The remaining equations are primary constraints.

The canonical Hamiltonian is given by

$$H=\int d\sigma\left\{N_1(\hat{\pi}\Pi_1)+N\frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)\right.$$
$$-A_0\partial_1\Lambda+i(\chi_1-\partial_1\Theta)\Gamma^m\Lambda_m\chi_0+\Phi\Lambda^2+\tilde{L}\lambda_\theta+P_N\lambda_N$$
$$+P_{N_1}\lambda_{N_1}+P_g\lambda_g+\pi_\Lambda\lambda_1+\pi_A{}^0\lambda_2$$
$$\left.+(\pi_A{}^1-\Lambda)\lambda_3+P_\chi{}^0\lambda_4+P_\chi{}^1\lambda_5+\pi_\Phi\lambda_6\right\}, \tag{11}$$

where

$$\hat{\pi}^n\equiv\pi^n-\frac{1}{2\pi\alpha'}i\Theta\Gamma^n\partial_1\Theta$$

and $\lambda_\theta,\lambda_N,\lambda_{N_1},\lambda_g,\lambda_1-\lambda_6$ are Lagrange multipliers corresponding to the primary constraints. The preservation in time of the primary constraints implies the secondary ones

$$\hat{\pi}_m\Pi^m{}_1\approx 0,\quad \frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)\approx 0,$$

[4]Within the framework of canonical formalism it is useful to make an invertible change of variables [23]

$$g^{00},g^{01},g^{11}\rightarrow N=-\frac{1}{\sqrt{-g}g^{00}},\quad N_1=-\frac{g^{01}}{g^{00}},g^{11},$$

where $\sqrt{-g}=\sqrt{-\det g_{\alpha\beta}}$. In terms of the new variables the discovery of secondary constraints becomes evident.

$$\Lambda^2\approx 0,\quad (\chi_1-\partial_1\Theta)\Gamma^n\Lambda_n\approx 0, \partial_1\Lambda^m\approx 0, \tag{12}$$

and conditions on the Lagrange multipliers

$$\lambda_1{}^m=0, \tag{13a}$$

$$\lambda_{3\,m}=-i\chi_1\Gamma_m\lambda_\theta+\partial_1 A_{m\,0}+i(\chi_1-\partial_1\Theta)\Gamma_m\chi_0+2\Phi\pi_{Am}{}^1, \tag{13b}$$

$$\pi_{Am}{}^1\Gamma^m(\lambda_\theta-\chi_0)=0, \tag{13c}$$

$$2i\Gamma^m\left(\hat{\pi}_m+\frac{1}{2\pi\alpha'}\Pi_{m\,1}\right)(\lambda_\theta-(N+N_1)\partial_1\theta) - i\Gamma^m\pi_{Am}{}^1(\lambda_5-\partial_1\chi_0)=0. \tag{13d}$$

Equations (13c) and (13d) are sufficient to determine λ_θ. Actually, multiplying Eq. (13c) by $\tilde{\Gamma}^n(\hat{\pi}_n+1/(2\pi\alpha')\Pi_{n\,1})$ and Eq. (13d) by $\tilde{\Gamma}^n\pi_{An}{}^1$ and then taking the sum one gets

$$\lambda_\theta=\frac{1}{2\pi_A{}^1[\hat{\pi}+1/(2\pi\alpha')\Pi_1]}\left\{\tilde{\Gamma}^n\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n\,1}\right)\times\Gamma^m\pi_{Am}{}^1\chi_0+\tilde{\Gamma}^m\pi_{Am}{}^1\Gamma^n\times\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)(N+N_1)\partial_1\Theta\right\}, \tag{14}$$

provided that $\pi_A{}^1(\hat{\pi}+1/(2\pi\alpha')\Pi_1)\neq 0$ on shell. The latter condition can always be realized by choosing appropriate gauge-fixing conditions and initial data to the equations of motion.[5] Inserting further Eq. (14) into Eq. (13d) one finds

$$\pi_{An}{}^1\Gamma^n(\lambda_5-\partial_1\chi_0)=0. \tag{15}$$

Equation (15) determines half of the λ_5, that can easily be seen by moving to the SO(8)-formalism. In SO(8) notation the condition (15) reads (see Appendix A)

$$(\lambda_{5\dot{a}}-\partial_1\chi_{0\dot{a}})-\frac{1}{\sqrt{2}\pi_A{}^{+1}}\gamma^i_{\dot{a}a}\pi_{Ai}{}^1(\lambda_{5a}-\partial_1\chi_{0a})=0.$$

It is straightforward to check further that the secondary constraints are identically conserved in time if Eqs. (12) and (13) hold. Thus, there are no more constraints in the problem.

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[5]The constraints $\partial_1\pi_{An}{}^1\approx 0$, $(\pi_A{}^1)^2\approx 0$ together with the equation of motion $\partial_0\pi_{An}{}^1=0$ imply $\pi_{Am}{}^1=n_m$, where n_m is a constant null vector (initial data). Choosing the initial data to be $n^m=(n^0,0,\ldots,0,n^0)$, $n^0\neq 0$, and imposing standard light-cone gauge conditions (see Sec. III B) $X^-=\alpha'\tau p^-$, $\pi^-=1/(2\pi)p^-$, where $p^-\neq 0$ is a complete momentum of the superstring, one gets

$$\pi_A{}^1[\hat{\pi}+1/(2\pi\alpha')\Pi_1]=-\frac{1}{2\pi}n^+\left(p^- - \frac{2}{\alpha'}i\theta\Gamma^-\partial_1\theta\right)\neq 0.$$

To separate the original constraints of the theory into first and second class, consider the equivalent constraints system

$$\pi_{\Lambda n}\approx 0,\quad \Lambda_n-\pi_{An}{}^1\approx 0, \tag{16a}$$

$$\begin{gathered}\pi_{An}{}^0\approx 0,\quad \pi_\Phi\approx 0,\\ P_\chi{}^0\approx 0,\quad P_N\approx 0,\\ P_{N_1}\approx 0,\quad P_g\approx 0,\\ (\pi_A{}^1)^2\approx 0,\quad \partial_1\pi_{An}{}^1\approx 0,\end{gathered} \tag{16b}$$

$$\hat{\pi}_n\Pi^n{}_1+L\partial_1\Theta\approx 0, \frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)+L\partial_1\Theta\approx 0, \tag{16c}$$

$$L\equiv P_\theta+i\Theta\Gamma^m\left(\pi_m+\frac{1}{2\pi\alpha'}\Pi_{m1}\right)-i\partial_1\Theta\Gamma^m\pi_{Am}{}^1-\partial_1 P_\chi{}^1 \approx 0, \tag{16d}$$

$$P_\chi{}^1\approx 0,\quad (\chi_1-\partial_1\Theta)\Gamma^n\pi_{An}{}^1\approx 0. \tag{16e}$$

The constraints (16a) are second class. The constraints system (16b), (16c) is first class. Among 16 fermionic constraints (16d) half are first class and another half are second class (see Sec. IV). Analogously, the first equation in Eq. (16e) contains eight first class and eight second class constraints while the latter implies eight linearly independent second class constraints (see Sec. IV). The essential Poisson brackets of the constraints (16) are gathered in the Appendix B.

Note that among 1+10 constraints $(\pi_A{}^1)^2\approx 0$, $\partial_1\pi_{An}{}^1\approx 0$ only 10 are functionally independent as a consequence of the identity $\partial_1(\pi_A{}^1)^2-2\pi_A{}^{n1}\partial_1\pi_{An}{}^1\equiv 0$. Independent constraints can be extracted in the light-cone basis as follows:

$$\pi_A{}^{-1}-\frac{1}{2\pi_A{}^{+1}}\pi_A{}^{i1}\pi_{Ai}{}^1\approx 0,\quad \partial_1\pi_A{}^{+1}\approx 0,\quad \partial_1\pi_{Ai}{}^1\approx 0.$$

As was mentioned above, it is impossible to separate eight first class and eight second class constraints, being combined in the L, in a covariant and irreducible manner. For the model concerned, covariant projectors into first and second class constraints are constructed in Sec. IV.

An explicit counting of the degrees of freedom shows that there are 16 bosonic and eight fermionic phase-space degrees of freedom in the model, which coincides with the number of degrees of freedom in the Green-Schwarz theory. Note as well that, after use of the Dirac algorithm, there remained 1+2+10+1 bosonic and 16+8 fermionic undefined Lagrange multipliers. Since the local symmetries, considered in Sec. II, have just this number of parameters being independent on shell, we conclude that they exhaust all the essential Lagrangian symmetries of the model.

B. Gauge-fixing and physical dynamics

In imposing gauge-fixing conditions two criteria should be satisfied [18]. First, the Poisson bracket of original first class constraints and gauges must be an invertible matrix when restricted to constraints and gauges surface. Second, gauge conditions are to be consistent with equations of motion; i.e., there must not appear new constraints from the condition of preservation in time of the gauges.[6] With this remark, consider first gauge conditions fixing all the undefined Lagrange multipliers in the theory. The equations

$$N\approx 1,\quad N_1\approx 0,\quad g^{11}\approx 1,\quad A^m{}_0\approx 0,\quad \Phi\approx 1/2, \tag{17a}$$

$$\chi_0\approx 0,\quad (\chi_1-\partial_1\Theta)\Gamma^n\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)\approx 0 \tag{17b}$$

prove to be suitable for this goal. Preservation in time of the gauges (17) yields

$$\begin{aligned}&\lambda_N=0,\quad \lambda_{N_1}=0,\quad \lambda_g=0,\\ &\lambda_2{}^m=0,\quad \lambda_6=0,\quad \lambda_4=0,\end{aligned} \tag{18}$$

and

$$\begin{aligned}&(\lambda_5-\partial_1\lambda_\theta)\Gamma^n\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)+(\chi_1-\partial_1\Theta)\Gamma^n\\ &\quad\times\left(\frac{2}{\pi\alpha'}i\partial_1\Theta\Gamma_n\lambda_\theta+\partial_1\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)\right)\\ &=0.\end{aligned} \tag{19}$$

Taking into account that the constraint $(\chi_1-\partial_1\Theta)\Gamma^n\pi_{An}{}^1\approx 0$ together with the gauge $(\chi_1-\partial_1\Theta)\Gamma^n[\hat{\pi}_n+1/(2\pi\alpha')\Pi_{n1}]\approx 0$ imply

$$\chi_1-\partial_1\Theta\approx 0, \tag{20}$$

while Eqs. (13c), (15), (16b), and (19) mean

$$\lambda_5-\partial_1\lambda_\theta=0, \tag{21}$$

one concludes that all the Lagrange multipliers have been fixed. In the gauge chosen, the canonical variables (N,P_N), (N_1,P_{N_1}), (g^{11},P_g), $(A^m{}_0,\pi_{Am}{}^0)$, $(\chi_0,P_\chi{}^0)$, $(\chi_1,P_\chi{}^1)$, (Φ,π_Φ) are unphysical and may be dropped after introducing the corresponding Dirac brackets.

We consider now gauge conditions to the remaining first class constraints.

(i) (Θ,P_θ) *sector*. There are eight first class constraints being nontrivially combined with eight second class ones in Eq. (16d). In this case one can adopt the condition

$$\Gamma^n\pi_{An}{}^1\Theta\approx 0, \tag{22}$$

[6] In the general case one can admit new constraints if they will later be treated as gauge conditions for some of the original first class constraints.

or in the SO(8)-formalism [we write out only the linearly independent part of Eq. (22)]

$$\Theta_{\dot{a}}-\frac{1}{\sqrt{2}\pi_A{}^{+1}}\gamma^i_{\dot{a}a}\Theta_a\pi_{Ai}{}^1\approx 0. \tag{23}$$

Because $\Gamma^n\pi_{An}{}^1\lambda_\theta\approx 0$ [see Eq. (14)] the gauge (22) is consistent with the equations of motion. It is straightforward to check as well that the Poisson bracket of the first class constraints being contained in the L with the gauge (23) is an invertible matrix.

After gauge fixing, the only dynamical variables in the sector are Θ_a. Taking into account that in the gauge chosen Eq. (14) takes the form

$$\lambda_\theta=\partial_1\Theta, \tag{24}$$

one concludes that physical dynamics of the superstring (1) in the odd-variables sector is described by

$$\partial_0\Theta_a=\partial_1\Theta_a$$

or

$$\partial_-\Theta_a=0,\quad a=1,\ldots,8, \tag{25}$$

just as in the Green-Schwarz model.

(ii) $(A^m{}_1,\pi_{Am}{}^1)$ *sector*. The constraints to be discussed are of the form

$$(\pi_A{}^1)^2\approx 0,\quad \partial_1\pi_A{}^{m1}\approx 0. \tag{26}$$

The equations of motion in the sector read

$$\partial_0\tilde{A}^m{}_1=0,\quad \partial_0\tilde{\pi}_A{}^{m1}=0, \tag{27}$$

where canonical transformation (see as well Ref. [24])

$$A^m{}_1\rightarrow\tilde{A}^m{}_1=A^m{}_1-\tau\pi_A{}^{m1},\quad \pi_A{}^{m1}\rightarrow\tilde{\pi}_A{}^{m1}=\pi_A{}^{m1}$$

has been made. Imposing then the following gauge condition to Eq. (26),

$$\tilde{A}^m{}_1\approx 0,$$

one concludes that there are no physical degrees of freedom in the sector.

(iii) (X^m,π_m) *sector*. There are super-Virasoro constraints (16c) and equations of motion

$$\partial_0 X^m=2\pi\alpha'\pi^m,\quad \partial_0\pi^m=\frac{1}{2\pi\alpha'}\partial_1\partial_1 X^m. \tag{28}$$

In this case one can impose the standard light-cone gauges [19]

$$X^-=\alpha'\tau p^-,\quad \pi^-=\frac{1}{2\pi}p^-, \tag{29}$$

where p^- is the complete momentum of the superstring. It is easy to check that the conditions (29) are consistent with Eq. (28). Making use of Eq. (16c) to express the variables X^+ and π^+ as functions of other variables and taking into account the gauges (29) one can conclude that the physical dynamics in the sector is described by

$$\partial_0 X^i = 2\pi\alpha' \pi^i, \quad \partial_0 \pi^i = \frac{1}{2\pi\alpha'} \partial_1\partial_1 X^i, \tag{30}$$

or, eliminating the π^i,

$$\Box X^i = 0, \quad i = 1, \ldots, 8. \tag{31}$$

Thus, physical dynamics of the superstring (1) is determined by Eqs. (25) and (31), which coincides with the Green-Schwarz superstring dynamics.

IV. COVARIANT SEPARATION OF CONSTRAINTS. RESOLVING THE INFINITELY REDUCIBLE FIRST CLASS CONSTRAINTS PROBLEM

In the previous sections we have modified the GSS so as to include a set of additional pure gauge variables. In the extended phase space covariant separation of constraints present no special problem. Actually, consider the constraint system

$$\pi_{An}{}^1 \tilde{\Gamma}^n P_\chi{}^1 \approx 0, \tag{32a}$$

$$b_n \tilde{\Gamma}^n P_\chi{}^1 \approx 0, \quad (\chi_1 - \partial_1 \Theta)\Gamma^n \pi_{An}{}^1 \approx 0, \tag{32b}$$

$$b_n \tilde{\Gamma}^n L \approx 0, \tag{32c}$$

$$\pi_{An}{}^1 \tilde{\Gamma}^n L \approx 0, \tag{32d}$$

where $b^n \equiv \hat{\pi}^n + 1/(2\pi\alpha')\,\Pi^n{}_1$, which is completely equivalent to Eqs. (16e) and (16d) due to the condition $(b\pi_A{}^1) \neq 0$ (see Sec. III A). Moving to the SO(8)-formalism it is straightforward to check now that Eq. (32a) includes eight linearly independent first class constraints; Eq. (32b) contains 8+8 independent second class ones; Eqs. (32c) and (32d) imply eight first class and eight second class constraints, respectively. For instance, rewriting Eqs. (32a) and (32b) into SO(8) formalism one gets (see Appendix A)

$$\nu_a \equiv \sqrt{2}\pi_A{}^{+1} P_{\chi a}{}^1 + \pi_{Ai}{}^1 \gamma^i{}_{a\dot{a}} P_{\chi\dot{a}}{}^1 \approx 0, \tag{33a}$$

$$\varphi_{\dot{a}} \equiv \sqrt{2} b^- P_{\chi\dot{a}}{}^1 + b_i \gamma^i{}_{\dot{a}a} P_{\chi a}{}^1 \approx 0, \tag{33b}$$

$$\psi_{\dot{a}} \equiv \sqrt{2}\pi_A{}^{+1}(\chi_{1\dot{a}} - \partial_1\Theta_{\dot{a}}) - (\chi_{1a} - \partial_1 \Theta_a)\gamma^i{}_{a\dot{a}}\pi_{Ai}{}^1 \approx 0. \tag{33c}$$

Evaluating then the Poisson brackets of the constraints

$$\{\varphi_{\dot{a}}, \psi_{\dot{b}}\} \approx 2b^- \pi_A{}^{+1}\left(\delta_{\dot{a}\dot{b}} - \frac{1}{2b^-\pi_A{}^{+1}} b_i \gamma^i_{\dot{a}c}\gamma^j_{c\dot{b}}\pi_{Aj}{}^1\right) \equiv \Delta_{\dot{a}\dot{b}}, \tag{34}$$

(all other brackets vanish[7]), and taking into account that the matrix in the right-hand side of Eq. (34) is invertible,

$$\Delta\tilde{\Delta} = 1,$$

$$\tilde{\Delta}_{\dot{a}\dot{b}} = -\frac{1}{2(b\pi_A{}^1)}\left(\delta_{\dot{a}\dot{b}} - \frac{1}{2b^-\pi_A{}^{+1}}\pi_{Ai}{}^1\gamma^i_{\dot{a}c}\gamma^j_{c\dot{b}} b_j\right), \tag{35}$$

one concludes that the constraint ν_a is first class, while $\varphi_{\dot{a}}$ and $\psi_{\dot{b}}$ are second class. Analogous calculations can be performed for the constraints (32c) and (32d). Thus, the constraint system (16a)-(16e) can be covariantly split into first and second class.

Let us now discuss Eqs. (32). The remarkable observation is that the reducible first class constraints (32a), (32c) [reducible second class constraints (32d) and the first equation in Eq. (32b)] can be combined to form an irreducible constraints set. Actually, consider the constraint system

$$\pi_{An}{}^1 \tilde{\Gamma}^n P_\chi{}^1 + b_n \tilde{\Gamma}^n L \approx 0, \tag{36a}$$

$$b_n \tilde{\Gamma}^n P_\chi{}^1 + \pi_{An}{}^1 \tilde{\Gamma}^n L \approx 0, \tag{36b}$$

$$(\chi_1 - \partial_1\Theta)\Gamma^n \pi_{An}{}^1 \approx 0, \tag{36c}$$

which is completely equivalent to the original one [Eqs. (16d) and (16e)] due to the constraints $b^2 \approx 0$, $(\pi_A{}^1)^2 \approx 0$. The constraints (36a) are first class and linearly independent. Analogously, the constraints (36b) are second class and irreducible. The remaining constraints (36c) are second class and reducible.

Thus, in the modified version (1) of the superstring, the fermionic first class constraints form an irreducible set. They will require only 16 covariant ghost variables (and 16 conjugate momenta) in constructing the BRST charge. The infinite tower of extra ghost variables, which appeared for the GSS, will not arise in the modified model [the remaining reducible first class constraints $(\pi_A{}^1)^2 \approx 0$, $\partial_1 \pi_{An}{}^1 \approx 0$ are of first stage of reducibility and can be taken into account along the standard lines [1–3]].

Note that the operators extracting the first and second class constraints from the initial mixed constraint system are not strict projectors. For instance, $\Gamma^n b_n \tilde{\Gamma}^m b_m = b^2 \approx 0$. In certain cases [11], however, it is more convenient to deal with the first and second class constraints that were extracted by means of strict projectors. For the case at hand the suitable projectors are of the form

$$P^{\pm} = \frac{1}{2}(1 \pm K),$$

$$K = \frac{1}{2\sqrt{(b\pi_A{}^1)^2 - b^2(\pi_A{}^1)^2}}\tilde{\Gamma}^{[n}\Gamma^{m]} b_n \pi_{Am}{}^1. \tag{37}$$

In terms of the operators covariant (redundant) split of the constraints looks like

$$L^- \approx 0, \quad P_\chi{}^{+1} \approx 0 \text{ first class,}$$

$$L^+ \approx 0, \quad P_\chi{}^{-1} \approx 0 \text{ second class,}$$

where $L^{\pm} \equiv LP^{\pm}$, $P_\chi{}^{\pm 1} \equiv P_\chi{}^1 P^{\pm}$. Generalization of Eqs. (36a)–(36c) reads

[7] Note that the constraints $\nu_a \approx 0$ and $\varphi_{\dot{a}} \approx 0$ are equivalent to $P_{\chi a}{}^1 \approx 0$, $P_{\chi\dot{a}}{}^1 \approx 0$ due to Eq. (35).

$$L^{-}+P_{\chi}{}^{+1}\approx 0,\quad L^{+}+P_{\chi}{}^{-1}\approx 0,$$
$$(\chi_1-\partial_1\Theta)\Gamma^n\pi_{An}{}^1\approx 0. \tag{38}$$

V. FINAL REMARKS

In this work the infinitely reducible first class constraints problem of the original GSS has been resolved. However, there still remain (infinitely) reducible second class constraints in the question. As is known, within the framework of the standard BFV formalism first and second class constraints are treated in a different manner. First class constraints contribute to the BRST charge while second class ones appear in the path integral measure [3,20]. In this sense, the problem of covariant quantization of the GSS reduces to constructing a correct integral measure for the theory (1). The weak Dirac bracket construction [11] appears to be suitable for this goal and this work is in progress now.

Note as well that the proposed techniques can be directly applied to modification of the superparticle (and superstring) due to Siegel [21,22]. In that case, there are only eight linearly independent fermionic first class constraints in the initial formulation, and use of the scheme will lead to the system with all fermionic constraints being irreducible. After this, covariant quantization is straightforward and the results will be presented elsewhere.

APPENDIX A

In this paper we use generalized notations in which two inequivalent minimal spinor representations of the Lorentz group (right-handed and left-handed Majorana-Weyl spinors) are distinguished by the position of its indices. We set lower index for the right-handed spinor ψ_A, $(A=1,\ldots,16)$ and upper index for the left-handed one ψ^A. The generalized 16×16 Dirac matrices are real and symmetric, obeying the standard algebra

$$\Gamma^m\tilde{\Gamma}^n+\Gamma^n\tilde{\Gamma}^m=2\eta^{mn},\quad \eta^{nm}=\mathrm{diag}(-,+,\ldots). \tag{A1}$$

In analyzing the constraint systems of the superparticle, superstring models it is useful to represent a Majorana-Weyl spinor of SO(1,9) group as a Majorana one of SO(8) group

$$\Psi_A=(\psi_a,\overline{\psi_{\dot a}}),\quad a,\dot a=1,\ldots,8, \tag{A2}$$

where indices $a,\dot a$ label two inequivalent minimal spinor representations of SO(8) group ($8c$ and $8s$ representations, respectively). This correspondence becomes evident in the basis of the Γ matrices,

$$\Gamma^0=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & \mathbf{1}_8\end{pmatrix},\quad \tilde{\Gamma}^0=\begin{pmatrix}-\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix},$$
$$\Gamma^i=\begin{pmatrix}0 & \gamma^i_{a\dot a}\\ \gamma^i_{\dot a a} & 0\end{pmatrix},\quad \tilde{\Gamma}^i=\begin{pmatrix}0 & \gamma^i_{a\dot a}\\ \gamma^i_{\dot a a} & 0\end{pmatrix}, \tag{A3}$$
$$\Gamma^9=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix},\quad \tilde{\Gamma}^9=\begin{pmatrix}\mathbf{1}_8 & 0\\ 0 & -\mathbf{1}_8\end{pmatrix},$$

where the $\gamma^i_{a\dot a},\gamma^i_{\dot a a}\equiv(\gamma^i_{a\dot a})^T$ are SO(8) γ matrices [19]:

$$\gamma^i_{a\dot a}\gamma^j_{\dot a b}+\gamma^j_{a\dot a}\gamma^i_{\dot a b}=2\delta^{ij}\delta_{ab},\quad i=1,\ldots,8. \tag{A4}$$

Let now b^n be a lightlike vector

$$-2b^+b^-+b^ib_i=b^2=0. \tag{A5}$$

The useful observation is that under the assumption (A5) the equation

$$b_n(\tilde{\Gamma}^n\psi)^A=0 \tag{A6}$$

determines only eight linearly independent conditions.

Actually, rewriting Eq. (A6) in the SO(8) formalism one gets

$$\sqrt{2}b^+\psi_a+b_i\gamma^i_{a\dot a}\overline{\psi_{\dot a}}=0, \tag{A7a}$$

$$\sqrt{2}b^-\overline{\psi_{\dot a}}+b_i\gamma^i_{\dot a a}\psi_a=0. \tag{A7b}$$

By virtue of Eqs. (A4) and (A5), Eq. (A7b) is a consequence of Eq. (A7a), provided that the standard light-cone assumption $b^+\neq 0$ has been made.

APPENDIX B

In this Appendix we list the essential Poisson brackets of the constraints (16a)–(16e):

$$\{L_A,L_B\}=2i\Gamma^n_{AB}\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)\delta(\sigma-\sigma'),$$

$$\{L_A,\hat{\pi}\Pi_1\}=-2i(\Gamma^n\partial_1\Theta)_A\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)\delta(\sigma-\sigma'),$$

$$\left\{L_A,\frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)\right\}$$
$$=-2i(\Gamma^n\partial_1\Theta)_A\left(\hat{\pi}_n+\frac{1}{2\pi\alpha'}\Pi_{n1}\right)\delta(\sigma-\sigma'),$$

$$\{\hat{\pi}\Pi_1,\hat{\pi}\Pi_1\}=2\hat{\pi}\Pi_1(\sigma)\partial_\sigma\delta+\partial_1(\hat{\pi}\Pi_1)\delta,$$

$$\left\{\hat{\pi}\Pi_1,\frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)\right\}$$
$$=\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2(\sigma)\partial_\sigma\delta\right)$$
$$+\partial_1\left[\frac{1}{2}\left(2\pi\alpha'\hat{\pi}^2+\frac{1}{2\pi\alpha'}\Pi_1{}^2\right)\right]\delta,$$

$$\left\{ \frac{1}{2}\left(2\pi\alpha' \hat{\pi}^2 + \frac{1}{2\pi\alpha'}\Pi_1{}^2 \right), \frac{1}{2}\left(2\pi\alpha' \hat{\pi}^2 + \frac{1}{2\pi\alpha'}\Pi_1{}^2 \right) \right\} = 2(\hat{\pi}\Pi_1)(\sigma)\partial_\sigma\delta + \partial_1(\hat{\pi}\Pi_1)\delta. \tag{B1}$$

In obtaining Eq. (B1) the Fierz identity

$$(\Gamma^n\psi)_A(\Gamma_n\varphi)_B + (\Gamma^n\psi)_B(\Gamma_n\varphi)_A = \Gamma^n_{AB}(\varphi\Gamma_n\psi) \tag{B2}$$

and the standard properties of the δ function

$$\partial_\sigma\delta = -\partial_{\sigma'}\delta, \quad F(\sigma')\partial_\sigma\delta = \partial_\sigma F(\sigma)\delta + F(\sigma)\partial_\sigma\delta \tag{B3}$$

have been used.

[1] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. **120B**, 166 (1983).

[2] I.A. Batalin and E.S. Fradkin, Phys. Lett. **122B**, 157 (1983); P.N. Lebedev Institute Report No. 259, 1982 (unpublished).

[3] I.A. Batalin and E.S. Fradkin, P.N. Lebedev Institute Report No. 165, 1983 (unpublished).

[4] A.A. Deriglazov, Int. J. Mod. Phys. A **8**, 1093 (1993).

[5] I.L. Buchbinder, S.M. Kuzenko, and A.G. Sibiryakov, Phys. Lett. B **352**, 29 (1995).

[6] L. Brink and J. Schwarz, Phys. Lett. **100B**, 310 (1981).

[7] M. Green and J. Schwarz, Phys. Lett. **136B**, 367 (1984).

[8] I. Bengtsson and M. Cederwall, Göteborg Report No. 84-21 1984 (unpublished).

[9] J.M. Evans, Nucl. Phys. **B331**, 711 (1990).

[10] L. Brink, M. Henneaux, and C. Teitelboim, Nucl. Phys. **B293**, 505 (1987).

[11] A.A. Deriglazov, A.V. Galajinsky, and S.L. Lyakhovich, Nucl. Phys. B (to be published).

[12] J.M. Evans, Phys. Lett. B **233**, 307 (1989).

[13] W. Siegel, Phys. Lett. **128B**, 397 (1983).

[14] E. Nissimov, S. Pacheva, and S. Solomon, Nucl. Phys. **B297**, 349 (1988); **B317**, 344 (1989).

[15] I.A. Batalin and I.V. Tyutin, Nucl. Phys. **B381**, 619 (1992).

[16] I.A. Batalin and I.V. Tyutin, Phys. Lett. B **317**, 354 (1993).

[17] I.A. Batalin and I.V. Tyutin, Mod. Phys. Lett. A **8**, 3757 (1993).

[18] P.A.M. Dirac, *Lectures on Quantum Mechanics*, Yeshiva University, Belfer Graduate School of Science (Academic Press, New York, 1964).

[19] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, 1987).

[20] E.S. Fradkin and T.E. Fradkina, Phys. Lett. **72B**, 343 (1978).

[21] W. Siegel, Class. Quantum Grav. **2**, 95 (1985).

[22] W. Siegel, Nucl. Phys. **B263**, 93 (1985).

[23] L. Brink and M. Henneaux, *Principles of String Theory* (Plenum Press, New York and London, 1988).

[24] D.M. Gitman and I.V. Tyutin, Class. Quantum Grav. **7**, 2131 (1990).