# How to create a two-dimensional black hole

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The interaction of a cosmic string with a four-dimensional stationary black hole is considered. If a part of an infinitely long string passes close to a black hole it can be captured. The final stationary configurations of such captured strings are investigated. It is shown that the minimal 2D surface $\Sigma$ describing a captured stationary string coincides with a *principal Killing surface*, i.e., a surface formed by Killing trajectories passing through a principal null ray of the Kerr-Newman geometry. A uniqueness theorem is proved, namely, it is shown that the principal Killing surfaces are the only stationary solutions of the string equations which enter the ergosphere and remain timelike and regular at the static limit surface. Geometrical properties of principal Killing surfaces are investigated and it is shown that the internal geometry of $\Sigma$ coincides with the geometry of a 2D black or white hole (*string hole*). The equations for propagation of string perturbations are shown to be identical with the equations for a coupled pair of scalar fields ''living'' in the spacetime of a 2D string hole. Some interesting features of the physics of 2D string holes are described. In particular, it is shown that the existence of the extra dimensions of the surrounding spacetime makes interaction possible between the interior and exterior of a string black hole; from the point of view of the 2D geometry this interaction is acausal. Possible application of this result to the information loss puzzle is briefly discussed. [S0556-2821(96)00920-4]

## I. INTRODUCTION

Black hole solutions in a spacetime of lower than four dimensions have been discussed for a long time (see, e.g., Ref. [1] and references therein). Such solutions are of interest mainly because they provide toy models which allow one to investigate unsolved problems in four-dimensional (4D) black hole physics. The interest in 2D black holes greatly increased after Witten [2] and Mandal, Sengupta, and Wadia [3] showed that 2D black hole solutions naturally arise in superstring-motivated 2D dilaton gravity. Many aspects of 2D black hole physics and its relation to 4D gravity were discussed in a number of recent publications (see, e.g., Ref. [4]). The main purpose of this paper is to show that there might exist physical objects which behave as 2D black holes. Namely, we consider a cosmic string interacting with a usual 4D stationary black hole. If an infinitely long string passes close enough to a black hole it can be captured [5,6]. Further evolution of the system (a black hole and a trapped cosmic string) can be quite complicated. One can expect that the action of the string on the black hole accelerates the latter, so that after characteristic time of order of $T=R_g/(c\mu^*)$ the black hole will have the velocity comparable with the velocity of the string. ($R_g=2GM/c^2$ is the gravitational radius of a black hole, and $\mu^*=G\mu/c^2$ is the dimensionless tension of the string.) One can also expect that during the same time $T$ the action of the string on the rotat-

ing black hole can essentially change its angular momentum. In what follows we do not discuss these interesting effects. We assume that $\mu^*\ll1$, so that a cosmic string can be considered as a test body and its gravitational field and action on the black hole can be neglected. Certainly, this approximation is valid for time intervals less than $T$.

In this paper we study stationary final states of a captured infinite test string, with end points fixed at infinity. We show that there is only a very special family of solutions describing a stationary test string which enters the ergosphere, namely, the strings lying on cones of a given angle $\theta=$ const. We demonstrate that the induced 2D geometry of a stationary string crossing the static limit surface and entering the ergosphere of a rotating black hole has the metric of a 2D black or white hole. The horizon of such a 2D string hole coincides with the intersection of the string world sheet with the static limit surface. We shall also demonstrate that the 2D string hole geometry can be tested by studying the propagation of string perturbations. The perturbations propagating along the *cone strings* ($\theta=$ const) are shown to obey the relativistic equations for a coupled system of two scalar fields. These results generalize the results of Ref. [7] where the corresponding equations were obtained and investigated for strings lying in the equatorial plane. The quantum radiation of string excitations (*stringons*) and thermodynamical properties of string holes are discussed. The remarkable property of 2D string holes as physical objects is that besides quanta (stringons) living and propagating only on the 2D world sheet there exist other field quanta (gravitons, photons, etc.), living and propagating in the surrounding physical 4D spacetime. Such quanta can enter the ergosphere as well as leave it and return back to the exterior. For this reason the

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presence of extra physical dimensions makes dynamical interaction possible between the interior and exterior of a 2D string black hole, which appears acausal from the perspective of the internal 2D geometry. The possible application of this effect to the information loss puzzle is briefly discussed.

The paper is organized as follows. In Sec. II we collect results concerning the Kerr-Newman geometry which are necessary for the following sections. In Sec. III we introduce the notion of a *principal Killing surface* and we prove that a principal Killing surface is a minimal two-surface embedded in the four-dimensional spacetime. In Sec. IV we prove the uniqueness theorem, i.e., the statement that the principal Killing surfaces are the only stationary minimal two-surfaces that are timelike and regular in the vicinity of the static limit surface of the Kerr-Newman black hole. In Sec. IV we also relate the principal Killing surfaces with the world sheets of a particular class of stationary cosmic strings, the cone strings. In Sec. V we show that the internal geometry of these world sheets is that of a two-dimensional black or white hole and we discuss the geometry of such string holes. In Section 6 we consider the propagation of perturbations along a stationary string using a covariant approach developed in Ref. [8] (see also Refs. [9–11]), and we show that the corresponding equations coincide with a system of coupled equations for a pair of scalar fields on the two-dimensional string hole background. Finally, in Sec. VII, we discuss the physics of string holes and give our conclusions.

## II. KERR-NEWMAN GEOMETRY

In Boyer-Lindquist coordinates the Kerr-Newman metric is given by [12]

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - a\,dt]^2$$

$$+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2, \tag{2.1}$$

where $\Delta = r^2 - 2Mr + Q^2 + a^2$ and $\rho^2 = r^2 + a^2\cos^2\theta$. The corresponding electromagnetic field tensor is given by

$$\mathbf{F} = \frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}\mathbf{d}r \wedge [\mathbf{d}t - a\sin^2\theta\mathbf{d}\phi]$$

$$+ \frac{2Qar}{\rho^4}\cos\theta\sin\theta\mathbf{d}\theta \wedge [(r^2+a^2)\mathbf{d}\phi - a\mathbf{d}t]. \tag{2.2}$$

The spacetime (2.1) possesses a Killing vector $\xi^\mu = (1,0,0,0)$ which is timelike at infinity. The norm of the Killing vector is

$$F \equiv -\xi^2 = 1 - \frac{2Mr - Q^2}{\rho^2}. \tag{2.3}$$

A surface $S_{\text{st}}$, where $\xi$ becomes null ($F=0$), is known as the static limit surface. It is defined by

$$r = r_{st} \equiv M + \sqrt{M^2 - Q^2 - a^2\cos^2\theta}. \tag{2.4}$$

The Kerr-Newman metric (2.1) is of type $D$ and possesses two principal null directions $l_+^\mu$ and $l_-^\mu$ Each of these null vectors obeys the relation

$$C_{\alpha\beta\gamma\delta}^{(+)}l^\beta l^\delta = Cl_\alpha l_\gamma, \tag{2.5}$$

where

$$C_{\alpha\beta\gamma\delta}^{(+)} = C_{\alpha\beta\gamma\delta} + iC_{\alpha\beta\gamma\delta}^*, \quad C_{\alpha\beta\gamma\delta}^* = \tfrac{1}{2}e_{\alpha\beta\mu\nu}C^{\mu\nu}{}_{\gamma\delta}. \tag{2.6}$$

Here, $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor, $e_{\alpha\beta\mu\nu}$ is the totally antisymmetric tensor, and $C_\pm$ are nonvanishing complex numbers. The Goldberg-Sachs theorem [13] implies that the integral lines $x^\mu(\lambda)$ of principal null directions

$$\frac{dx^\mu}{d\lambda_\pm} = \mp l_\pm^\mu \tag{2.7}$$

are null geodesics ($l^\mu l_\mu = 0$, $l^\mu l_{;\mu}^\nu = 0$) and their congruence is shear free. We denote by $\gamma_+$ and $\gamma_-$ ingoing and outgoing principal null geodesics, respectively, and choose the parameter $\lambda_\pm$ to be an affine parameter along the geodesic. The explicit form of $l_\pm$ is given by

$$l_\pm^\mu = ((r^2+a^2)/\Delta, \mp 1, 0, a/\Delta),$$

$$l_{\pm\mu} = (-1, \mp\rho^2/\Delta, 0, a\sin^2\theta). \tag{2.8}$$

The normalization has been chosen so that $l_\pm$ are future directed and such that

$$l_+^\mu l_{-\mu} = -2\rho^2/\Delta. \tag{2.9}$$

The Killing equation implies that the tensor $\xi_{\mu;\nu}$ is antisymmetric and its eigenvectors with nonvanishing eigenvalues are null. In the Kerr-Newman geometry $\xi_{\mu;\nu}$ is of the form

$$\xi_{\mu;\nu} = (\Delta F'/2\rho^2)l_{+[\mu}l_{-\nu]} + [2ia(1-F)\cos\theta/\rho^2]m_{[\mu}\bar{m}_{\nu]}, \tag{2.10}$$

where we have made use of the complex null vectors $m$ and $\bar{m}$, that complete the Kinnersley null tetrad. In the normalization where $m^\mu\bar{m}_\mu = 1$, they take the form

$$m^\mu = \frac{1}{\sqrt{2}\rho}(ia\sin\theta, 0, 1, i/\sin\theta),$$

$$m_\mu = \frac{1}{\sqrt{2}\rho}(-ia\sin\theta, 0, \rho^2, i(a^2+r^2)\sin\theta). \tag{2.11}$$

The remarkable property of the Kerr-Newman geometry is that the principal null vectors $l_\pm$ are eigenvectors of $\xi_{\mu;\nu}$. Namely, one has

$$\xi_{\mu;\nu}l_\pm^\nu = \mp\kappa l_{\pm\mu},$$

$$\kappa = \pm\frac{1}{2}l_\pm^\nu(\xi^2)_{;\nu} = \frac{1}{2}F_{,r} = \frac{Mr^2 - rQ^2 - Ma^2\cos^2\theta}{\rho^4}. \tag{2.12}$$

These equations [(2.10) and (2.12)], will play an important role later in our analysis.

Notice also that the electromagnetic field tensor **F** has the form:

$$F_{\mu\nu} = -\frac{\Delta}{\rho^2}\left(\frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}\right)l_{+[\mu}l_{-\nu]}$$
$$+\frac{4iQar\cos\theta}{\rho^4}m_{[\mu}\bar{m}_{\nu]}, \qquad (2.13)$$

so that

$$F_{\mu\nu}l^{\nu}_{\pm} = \mp\frac{Q(r^2 - a^2\cos^2\theta)}{\rho^4}l_{\mu\pm}. \qquad (2.14)$$

## III. PRINCIPAL KILLING SURFACES

Our aim is to consider stationary configurations of cosmic strings in the gravitational field of a charged rotating black hole. In particular, we are interested in the situation when a string is trapped by a black hole; that is when the string crosses the black holes static limit surface and enters the ergosphere. We neglect the thickness of the string and its own gravitational field. In this approximation the string evolution is described by a timelike 2D world sheet (for general properties of cosmic strings, see, for instance, Refs. [14,15]). The dynamical equations obtained by variation of the Nambu-Goto action for a string imply that this world sheet is a minimal surface. So, the mathematical problem we are trying to solve is to find stationary timelike minimal surfaces which intersect the static limit surface of a rotating black hole. For this purpose we begin by considering the general properties of stationary timelike surfaces.

Let $\Sigma$ be a two-dimensional timelike surface embedded in a stationary spacetime, and let $\xi$ be the corresponding Killing vector which is timelike at infinity. $\Sigma$ is said to be *stationary* if it is everywhere tangent to the Killing vector field $\xi$. For any such surface $\Sigma$ there exists two linearly independent null vector fields $l$, tangent to $\Sigma$. We assume that the integral curves of $l$ form a congruence and cover $\Sigma$ (i.e., each point $p \in \Sigma$ lies on exactly one of these integral curves).

Thus, we can construct a stationary timelike surface $\Sigma$ in the following way: consider a null ray $\gamma$ with tangent vector field $l$ such that $\xi \cdot l$ is nonvanishing everywhere along $\gamma$. There is precisely one Killing trajectory with tangent vector $\xi$ that passes through each point $p \in \gamma$. This set of Killing trajectories passing through $\gamma$ forms a stationary 2D surface $\Sigma$. We define $l$ over $\Sigma$ by Lie propagation along each Killing trajectory. We call $\gamma$ a basic ray of $\Sigma$. It is easily verified that $l$ remains null when defined in this manner over $\Sigma$.

We can use the Killing time parameter $u$ and the affine parameter $\lambda$ along $\gamma$ as coordinates on $\Sigma$. In these coordinates $\zeta^A = (u,\lambda)$ one has $x^{\mu}_{,0} = \xi^{\mu}$ and $x^{\mu}_{,1} = l^{\mu}$ and the induced metric $G_{AB} = g_{\mu\nu}x^{\mu}_{,A}x^{\nu}_{,B}(A,B,\ldots = 0,1)$ is of the form

$$dS^2 = G_{AB}d\zeta^A d\zeta^B = -Fdu^2 + 2(\xi \cdot l)du\,d\lambda. \qquad (3.1)$$

In the case of a black hole the Killing vector $\xi$ becomes null at the static limit surface $S_{\text{st}}$. In what follows we always choose $l$ to be that of the two possible null vector fields on $\Sigma$

which does not coincide with $\xi$ on the static limit surface $S_{\text{st}}$. In this case the metric (3.1) is regular at $S_{\text{st}}$. Now, introduce two vectors $n^{\mu}_R$ ($R=2,3$) normal to the 2D surface $\Sigma$:

$$g_{\mu\nu}n^{\mu}_R n^{\nu}_S = \delta_{RS}, \qquad g_{\mu\nu}x^{\mu}_{,A}n^{\nu}_R = 0, \qquad (3.2)$$

which satisfy the completeness relation

$$g^{\mu\nu} = G^{AB}x^{\mu}_{,A}x^{\nu}_{,B} + \delta^{RS}n^{\mu}_R n^{\nu}_S. \qquad (3.3)$$

These two normal vectors span the vector space normal to the surface at a given point, and they are uniquely defined up to local rotations in the $(n_2, n_3)$ plane.

The second fundamental form is defined as

$$\Omega_{RAB} = g_{\mu\nu}n^{\mu}_R x^{\rho}_{,A}\nabla_{\rho}x^{\nu}_{,B}. \qquad (3.4)$$

The condition that a surface $\Sigma$ is minimal can be written in terms of the trace of the second fundamental form as

$$\Omega^A_{RA} \equiv G^{AB}\Omega_{RAB} = 0. \qquad (3.5)$$

We find that in the metric (3.1) the second fundamental form is given by

$$\Omega^A_{RA} = g_{\mu\nu}G^{AB}n^{\mu}_R x^{\gamma}_{,A}\nabla_{\gamma}x^{\nu}_{,B}$$
$$= g_{\mu\nu}n^{\mu}_R\left(\frac{2}{(\xi \cdot l)}l^{\gamma}\nabla_{\gamma}\xi^{\nu} + \frac{F}{(\xi \cdot l)^2}l^{\gamma}\nabla_{\gamma}l^{\nu}\right). \qquad (3.6)$$

Consider a special type of a stationary timelike two-surface in the Kerr-Newman geometry. Namely, a surface for which the null vector $l$ coincides with one of the principal null geodesics $l_{\pm}$ of the Kerr-Newman geometry. We call such surface $\Sigma_{\pm}$ a *principal Killing surface* and $\gamma_{\pm}$ its *basic ray*. We shall use indices $\pm$ to distinguish between quantities connected with $\Sigma_{\pm}$. The fact that $l_{\pm}$ are geodesics ensures that $l^{\gamma}_{\pm}\nabla_{\gamma}l^{\mu}_{\pm} \propto l^{\mu}_{\pm}$. In addition, from Eq. (2.12), $l^{\gamma}_{\pm}\nabla_{\gamma}\xi^{\mu} \propto l^{\mu}_{\pm}$ which, because of the contraction with $n^{\nu}_R$, guarantees that $\Omega_{RA}{}^A$ vanishes for a principal Killing surface, i.e., every principal Killing surface is minimal. Thus, $\Sigma_{\pm}$ are stationary solutions of the Nambu-Goto equations.

It should be stressed that the principal Killing surfaces are only very special stationary minimal surfaces. A principal Killing surface is uniquely determined by indicating two coordinates (angles) of a point where it crosses the static limit surface. Because of the axial symmetry only one of these two parameters is nontrivial. A general stationary string solution in the Kerr-Newman spacetime can be obtained by separation of variables (Ref. [6], see also Sec. IV) and it depends on three parameters (two of which are nontrivial).

## IV. UNIQUENESS THEOREM

We prove now that the only stationary timelike minimal two-surfaces that cross the static limit surface $S_{\text{st}}$ and are regular in its vicinity are the principal Killing surfaces.

Consider a stationary timelike surface $\Sigma$ described by the line element (3.1). By using the completeness relation (3.3) and the metric (3.1), we obtain

$$\Omega^2 \equiv \Omega_{RA}{}^A \Omega^R{}_B{}^B = z^\mu z_\mu,$$

$$z^\mu \equiv \frac{2}{(\xi \cdot l)} l^\gamma \nabla_\gamma \xi^\mu + \frac{F}{(\xi \cdot l)^2} l^\gamma \nabla_\gamma l^\mu. \quad (4.1)$$

In other words $\Sigma$ is minimal if and only if $z^\mu$ is null so that $\Omega^2 = 0$ (clearly, if $\Sigma$ is a principal Killing surface then $z^\mu \propto l^\mu_\pm$ and this condition is satisfied). In general, we observe that $l \cdot z$ vanishes as $l^\mu$ is null and as $\xi_{\mu;\nu}$ is antisymmetric. Thus, if $z^\mu$ is null then it must be proportional with $l^\mu$. The condition that $\Omega^2 = 0$ in the line element (3.1) then becomes

$$2(\xi \cdot l) l^\rho \nabla_\rho \xi^\mu + F l^\rho \nabla_\rho l^\mu + (\xi \cdot l) l^\rho \frac{d}{dx^\rho} \left( \frac{F}{\xi \cdot l} \right) l^\mu = 0. \quad (4.2)$$

It is easily verifed that Eq. (4.2) is invariant under reparametrizations of $l^\mu$, i.e., if $l^\mu$ satisfies Eq. (4.2) then so does $g(x) l^\mu$. Thus, without loss of generality we may normalize $l^\mu$ so that $l \cdot \xi = -1$. Then, Eq. (4.2) becomes:

$$-2 l^\rho \nabla_\rho \xi^\mu + F l^\rho \nabla_\rho l^\mu + l^\rho \frac{dF}{dx^\rho} l^\mu = 0. \quad (4.3)$$

Since $l^\rho \nabla_\rho l^\mu$ is regular on $\Sigma$, this equation at the static limit surface ($F = 0$) reduces to:

$$\left( \xi_{\mu;\rho} - \frac{1}{2} \frac{dF}{dx^\rho} l_\mu \right) l^\rho = 0, \quad (4.4)$$

that is, $l^\rho$ is a real eigenvector of $\xi_{\mu;\rho}$. From Eq. (2.10) follows that the only real eigenvector of $\xi_{\mu;\rho}$ must be either $l_+$ or $l_-$. Thus, we have $l \propto l_\pm$ at the static limit surface.

Now, suppose there exists a timelike minimal surface $\Sigma$ different from $\Sigma_\pm$. At the static limit surface $\Sigma$ must have $l \propto l_+$ (or $l \propto l_-$). In the vicinity of the static limit surface, $l$ can have only small deviations from $l_+$. From the conditions $l \cdot l = 0$ and $l \cdot \xi = -1$, we then get the following general form of $l$ in the vicinity of the static limit surface:

$$l = \left[ 1 + \frac{i a \sin\theta}{\sqrt{2} \rho} (B - \bar{B}) \right] l_+ + \bar{B} m + B \bar{m} + O(B^2), \quad (4.5)$$

up to first order in $(B, \bar{B})$. We then insert this expression into Eq. (4.2), contract by $\bar{m}_\mu$, and keep only terms linear in $(B, \bar{B})$:

$$-2 \bar{m}_\mu l^\rho \nabla_\rho \xi^\mu = \frac{-2 i a (1 - F) \cos\theta}{\rho^2} \bar{B} + O(B^2), \quad (4.6)$$

$$\bar{m}_\mu l^\rho \frac{dF}{dx^\rho} l^\mu = l^\rho_+ \frac{dF}{dx^\rho} \bar{B} + O(B^2) = -F' \bar{B} + O(B^2), \quad (4.7)$$

$$\bar{m}_\mu F l^\rho \nabla_\rho l^\mu$$

$$= F l^\rho_+ \frac{d\bar{B}}{dx^\rho} - 2 F l^\mu_+ m^\rho \bar{m}_{(\rho;\mu)} \bar{B} + F \bar{m}^\rho \bar{m}_\mu l^\mu_{+;\rho} B + O(B^2)$$

$$= -F \frac{d\bar{B}}{dr} - \frac{(\rho + 2 i a \cos\theta) F}{\rho^2} \bar{B} + O(B^2), \quad (4.8)$$

where the last equality was obtained by direct calculation using Eqs. (2.8) and (2.11). Thus, altogether:

$$F \frac{d\bar{B}}{dr} = -B \left[ \frac{dF}{dr} + \frac{2 i a \cos\theta}{\rho^2} + \frac{F}{\rho} \right] + O(B^2). \quad (4.9)$$

It is convenient to rewrite this equation in the form

$$\frac{d\bar{B}}{dr*} = -\Omega \bar{B}, \quad \Omega \equiv \frac{dF}{dr} + \frac{2 i a \cos\theta}{\rho^2} + \frac{F}{\rho}, \quad (4.10)$$

and we have introduced the tortoise-coordinate $r*$ defined by

$$\frac{dr}{dr*} = F(r). \quad (4.11)$$

Near the static limit surface the complex frequency $\Omega$ is given by:

$$\Omega = \frac{2(r_{st} - M + i a \cos\theta)}{r_{st}^2 + a^2 \cos^2\theta} + O(r - r_{st}) \equiv \Omega_{st} + O(r - r_{st}). \quad (4.12)$$

The solution of equation (4.10) near the static limit surface is then given by:

$$\bar{B} = c e^{-\Omega_{st} r*}, \quad c = \text{const.} \quad (4.13)$$

Notice that $Re(\Omega_{st}) > 0$, thus $\bar{B}$ is oscillating with infinitely growing amplitude near the static limit surface. A solution regular near the static limit surface ($r* \to -\infty$) can, therefore, only be obtained for $c = 0$, which implies that $B = \bar{B} = 0$, thus we have shown that $\Sigma$ is minimal if and only if $l \propto l_\pm$. This proves the uniqueness theorem: The only stationary timelike minimal two-surfaces that cross the static limit surface $S_{st}$ and are regular in its vicinity are the principal Killing surfaces.

We now discuss the physical meaning of this result. For that purpose it is convenient to introduce the ingoing $(+)$ and outgoing $(-)$ Eddington-Finkelstein coordinates $(u_\pm, \varphi_\pm)$:

$$du_\pm = dt \pm \Delta^{-1}(r^2 + a^2) dr, \quad d\varphi_\pm = d\phi \pm \Delta^{-1} a \, dr, \quad (4.14)$$

and to rewrite the Boyer-Lindquist metric (2.1) as

$$ds^2 = -\frac{\Delta}{\rho^2} [du_\pm - a \sin^2\theta \, d\varphi_\pm]^2 + \frac{\sin^2\theta}{\rho^2} [(r^2 + a^2) d\varphi_\pm$$
$$- a \, du_\pm]^2 + \rho^2 d\theta^2 \pm 2 \, dr [du_\pm - a \sin^2\theta \, d\varphi_\pm]. \quad (4.15)$$

The electromagnetic field tensor (2.2) is

$$\mathbf{F} = \frac{Q(r^2 - a^2 \cos^2\theta)}{\rho^4} \mathbf{d}r \wedge [\mathbf{d}u_\pm - a \sin^2\theta \mathbf{d}\varphi_\pm]$$

$$+ \frac{2 a r Q \cos\theta \sin\theta}{\rho^4} \mathbf{d}\theta \wedge [(r^2 + a^2) \mathbf{d}\varphi_\pm - a \mathbf{d}u_\pm]. \quad (4.16)$$

We have shown that any stationary minimal two-surface that crosses the static limit must have $x^\mu_{,1}=l^\mu_\pm$ (up to a constant factor). Using the explcit form of $l_\pm$ in Boyer-Lindquist coordinates (2.8), we can choose the affine parameter along $\gamma_\pm$ to coincide with $r$ such that $x'=\mp l_\pm$, where the prime denotes derivative with respect to $r$. We can then read off $\theta'$ and $\phi'$ for these surfaces $\Sigma_\pm$:

$$\theta'=0, \quad \phi'\pm a/\Delta=0. \quad (4.17)$$

In the Eddington-Finkelstein coordinates the induced metric on $\Sigma_\pm$ is then

$$dS^2=-Fdu^2_\pm\pm 2\,dr\,du_\pm, \quad F=1-\frac{2Mr-Q^2}{\rho^2}. \quad (4.18)$$

The induced electromagnetic field tensor is

$$\mathbf{F}=\frac{Q}{\rho^4}(r^2-a^2\cos^2\theta)\mathbf{d}r\wedge\mathbf{d}u_\pm, \quad (4.19)$$

that is, the induced electric field is

$$E_r=\frac{Q}{\rho^4}(r^2-a^2\cos^2\theta). \quad (4.20)$$

Equations (4.17) imply that a principal Killing string is located at the cone surface $\theta=$const. These so-called cone strings are thus the only stationary world sheets that can cross the static limit surface and are timelike and regular in its vicinity. It was shown, on the other hand, that the general stationary string solution in the Kerr-Newman spacetime can be obtained by separation of variables [6]:

$$\left(H_{rr}\frac{dr}{d\lambda}\right)^2=\frac{a^2b^2}{\Delta^2}-\frac{q^2}{\Delta}+1,$$

$$\left(H_{\theta\theta}\frac{d\theta}{d\lambda}\right)^2=q^2-\frac{b^2}{\sin^2\theta}-a^2\sin^2\theta, \quad (4.21)$$

$$\left(H_{\phi\phi}\frac{d\phi}{d\lambda}\right)^2=b^2,$$

where $b$ and $q$ are arbitrary constants, while

$$H_{rr}=\frac{\Delta-a^2\sin^2\theta}{\Delta}, \quad H_{\theta\theta}=\Delta-a^2\sin^2\theta, \quad H_{\phi\phi}=\Delta\sin^2\theta. \quad (4.22)$$

In this general three-parameter family of solutions, parametrized by $b,q$ and some initial angle $\phi_0$, the stationary strings crossing the static limit surface are determined by Eqs. (4.17), that is

$$\varphi_\pm=\text{const}, \quad q^2=2ab, \quad \sin^2\theta=\text{const}=b/a, \quad (4.23)$$

i.e., a two-parameter family of solutions (notice, however, that due to the axial symmetry only one of these parameters $b$ is nontrivial). Physically, it means that a stationary cosmic string can only enter the ergosphere in very special ways, corresponding to the angles (4.23).

## V. GEOMETRY OF 2D STRING HOLES

The metric (4.18) for $\Sigma_+$ describes a black hole, while for $\Sigma_-$ it describes a white hole. For $a=0$, $\Sigma_\pm$ are geodesic surfaces in the 4D spacetime and they describe two branches of a geodesically complete 2D manifold. However, it should be stressed that for the generic Kerr-Newman geometry ($a\neq 0$), only one of two null basic lines of the principal Killing surface, namely, the ray $\gamma_\pm$ with tangent vector $l\pm$, is geodesic in the four-dimensional embedding space. The other basic null ray is geodesic in $\Sigma_\pm$ but not in the embedding space. This implies that, in general (when $a\neq 0$), the principal Killing surface is not geodesic. Furthermore, it can be shown that $\Sigma_\pm$ considered as a 2D manifold is geodesically incomplete with respect to its null geodesic $\gamma'$.

As a consequence of $\Sigma_\pm$ not being geodesic (when $a\neq 0$), it is possible, as we shall now demonstrate, to send causal signals from the inside of the 2D black hole to the outside of the 2D black hole by exploiting the two extra dimensions of the 4D spacetime.

It is evident that there exist causal lines leaving the ergosphere and entering the black hole exterior. It means that "interior" and "exterior" of a 2D black hole can be connected by 4D causal lines. We now show that (at least for the points lying close to the static limit surface), the causal line can be chosen as a null geodesic. Consider for simplicity the stationary string corresponding to ($\theta=\pi/2$, $\varphi_+=0$) and crossing the static limit surface in the equatorial plane of a Kerr black hole. We will demonstrate that there exists an outgoing null geodesic in the 4D spacetime connecting the point $(r,\varphi_+)=(2M-\epsilon,0)$ of the cosmic string inside the ergosphere with the point $(r,\varphi_+)=(2M+\epsilon,0)$ of the cosmic string outside the ergosphere, for small $\epsilon$. An outgoing null geodesic, corresponding to positive energy at infinity $E$ and angular momentum at infinity $L_z$ in the equatorial plane of the Kerr black hole background, is determined by [16]

$$r^2\frac{dr}{d\lambda}=\mathcal{P}, \quad (5.1)$$

$$r^2\frac{du_+}{d\lambda}=-a\mathcal{U}+\frac{r^2+a^2}{\Delta}[\mathcal{P}+\mathcal{Q}], \quad (5.2)$$

$$r^2\frac{d\varphi_+}{d\lambda}=-\mathcal{U}+\frac{a}{\Delta}[\mathcal{P}+\mathcal{Q}], \quad (5.3)$$

where

$$\mathcal{U}\equiv aE-L_z, \quad \mathcal{Q}\equiv Er^2+a\mathcal{U}, \quad \mathcal{P}^2\equiv\mathcal{Q}^2-\Delta\mathcal{U}^2, \quad (5.4)$$

and we consider the case where $dr/d\lambda>0$. Inside the ergosphere the 4D geodesic must follow the rotation of the black hole because of the dragging effect, that is, $d\varphi_+/d\lambda>0$ (for $a>0$). However, after leaving the ergosphere the geodesic can reach a turning point in $\varphi_+$ and then return ($d\varphi_+/d\lambda<0$) towards the cosmic string outside the static limit surface. To be more precise: provided $-L_z>aE$, there will be a turning point in $\varphi_+$ outside the static limit surface at $r=r_0$:

$$r_0 = \frac{2M(aE - L_z)}{-L_z - aE} > 2M. \tag{5.5}$$

Obviously the turning point in $\varphi_+$ can be put at any value of $r$ outside the static limit surface. If we choose $E$ and $L_z$ such that:

$$r_0 = 2M + \epsilon - \frac{M}{2a^2}\epsilon^2, \tag{5.6}$$

then, after reaching the turning point in $\varphi_+$, the geodesic will continue in the direction opposite to the rotation of the 4D black hole with constant $r = 2M + \epsilon$ (to first order in $\epsilon$) and eventually reach the point $(r, \varphi_+) = (2M + \epsilon, 0)$ of the cosmic string outside the ergosphere.

We close this section with the following remarks.

Notice that the (outer) horizon of the 2D black hole coincides with the static limit of the 4D rotating black hole. The 2D surface gravity, which is proportional to the 2D temperature, is given by

$$\kappa^{(2)} = \frac{1}{2} \frac{dF}{dr}\bigg|_{r = r_{\mathrm{st}}} = \frac{\sqrt{M^2 - Q^2 - a^2\cos^2\theta}}{2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2 - a^2\cos^2\theta}}. \tag{5.7}$$

The surface gravity of the 4D Kerr-Newman black hole is

$$\kappa^{(4)} = \frac{\sqrt{M^2 - Q^2 - a^2}}{2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2 - a^2}}, \tag{5.8}$$

and then it can be easily shown that

$$\kappa^{(2)} \geqslant \kappa^{(4)}. \tag{5.9}$$

That is to say, the 2D temperature is higher than the 4D temperature (except at the poles where they coincide) and it is always positive. Even if the 4D black hole is extreme, the 2D temperature is nonzero.

As we show in Appendix A, the solutions of the form (4.18) can also be obtained in 2D dilaton gravity:

$$\mathcal{S} = \frac{1}{2\pi}\int dt\, dx\, \sqrt{-g}\, e^{-2\phi}[R + 2(\nabla\phi)^2 + V(\phi)], \tag{5.10}$$

with the dilaton potential

$$V(\phi) = \left[\frac{2}{r^2}(rF)_{,r}\right]_{\big|r = e^{-\phi}/\lambda}, \tag{5.11}$$

if the dilaton field has the form

$$\phi = -\ln(\lambda r), \quad \lambda = \mathrm{const}. \tag{5.12}$$

It should be stressed that this observation does not mean that we can use the dilaton-gravity equations in order to describe the dynamics of 2D string holes, or to determine the back reaction of the string excitations on the geometry of string holes.

## VI. STRING PERTURBATION PROPAGATION

A general transverse perturbation about a background Nambu-Goto string world sheet can be written as (summing over the $R$ indices)

$$\delta x^\mu = \Phi^R n_R^\mu, \tag{6.1}$$

where the normal vectors are defined by Eqs. (3.2). The equations of motion for the perturbations, $\Phi_R$ follow from the following effective action for stringons [8,9]:

$$\mathcal{S}_{\mathrm{eff}} = \int d^2\zeta\sqrt{-G}\,\Phi^R\{G^{AB}(\delta_R^T\nabla_A + \mu_R{}^T{}_A)(\delta_{TS}\nabla_B + \mu_{TSB}) + \mathcal{V}_{RS}\}\Phi^S, \tag{6.2}$$

where $\mathcal{V}_{RS} = \mathcal{V}_{(RS)}$ are scalar potentials and $\mu_{RSA} = \mu_{[RS]A}$ are vector potentials which coincide with the normal fundamental form

$$\mu_{RSA} = g_{\mu\nu}n_R^\mu x_{,A}^\rho\nabla_\rho n_S^\nu. \tag{6.3}$$

The scalar potentials are defined as

$$\mathcal{V}_{RS} \equiv \Omega_{RAB}\Omega_S{}^{AB} - G^{AB}x_{,A}^\mu x_{,B}^\nu R_{\mu\rho\sigma\nu}n_R^\rho n_S^\sigma. \tag{6.4}$$

The equations describing the propagation of perturbations on the world-sheet background are then found to be

$$\{\delta_{RS}\Box + 2\mu_{RS}{}^A\partial_A + \nabla_A\mu_{RS}{}^A - \mu_R{}^{TA}\mu_{STA} + \mathcal{V}_{RS}\}\Phi^S = 0. \tag{6.5}$$

We note that the perturbations (6.1) and the effective action (6.2) are invariant under rotations of the normal vectors, i.e., invariant under the transformations $n_R \mapsto \tilde{n}_R = \Lambda_R{}^S n_S$, $\Phi^R \to \tilde{\Phi}^R = \Lambda^R{}_S\Phi^S$, where

$$[\Lambda]_R{}^S = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix}, \tag{6.6}$$

for some arbitrary real function $\Psi$. Thus, we have a ''gauge'' freedom in our choice of normal vectors.

Consider the scalar potential $\mathcal{V}_{RS} \equiv \Omega_{RAB}\Omega_S{}^{AB} - G^{AB}x_{,A}^\mu x_{,B}^\nu R_{\mu\rho\sigma\nu}n_R^\rho n_S^\sigma$. It is easily verified that the first term $\Omega_{RAB}\Omega_S{}^{AB}$ vanishes for the principal Killing surface $\Sigma_\pm$ independently of any choice of normal vectors $n_R^\rho$. It is also possible to show that the second term on the right-hand side is invariant under rotations of the vectors $n_R$, i.e., gauge invariant, in the Kerr-Newman spacetime (see Appendix B). The symmetry and gauge invariance of $\mathcal{V}_{RS}$ show that it must be proportional to $\delta_{RS}$, i.e., $\mathcal{V}_{RS} = \mathcal{V}\delta_{RS}$. Now, using the completeness relation (3.3) we find

$$\mathcal{V} = \tfrac{1}{2}\delta^{RS}\mathcal{V}_{RS}$$

$$= -\tfrac{1}{2}\,G^{AB}x_{,A}^\mu x_{,B}^\nu R_{\mu\rho\sigma\nu}\delta^{RS}n_R^\rho n_S^\sigma$$

$$= \tfrac{1}{2}G^{AB}x_{,A}^\mu x_{,B}^\nu(R_{\mu\nu} + G^{CD}x_{,C}^\rho x_{,D}^\sigma R_{\mu\rho\sigma\nu}). \tag{6.7}$$

Making use of a representation of the Ricci tensor $R_{\mu\nu}$ in terms of the Kinnersley null tetrad, namely,

$$R_{\mu\nu} = \frac{2Q^2}{\rho^4}[m_{(\mu}\bar{m}_{\nu)} + (\Delta/2\rho^2)l_{+(\mu}l_{-\nu)}], \qquad (6.8)$$

we are able to calculate the first term of Eq. (6.7) as

$$G^{AB}x^{\mu}_{,A}x^{\nu}_{,B}R_{\mu\nu} = \pm 2(\mp R_{\mu\nu}l^{\mu}_{\pm}\xi^{\nu}) = \frac{2Q^2}{\rho^4}l^{\mu}_{\pm}\xi_{\mu} = -\frac{2Q^2}{\rho^4}. \qquad (6.9)$$

To calculate the second term of Eq. (6.7) we use the Gauss-Codazzi equations [17] for a two-surface $\Sigma$ embedded in a four-dimensional spacetime: namely,

$$R^{(2)}_{ABCD} = (\Omega_{RAC}\Omega_{BD}{}^R - \Omega_{RAD}\Omega_{BC}{}^R) + R_{\mu\rho\sigma\nu}x^{\mu}_{,A}x^{\rho}_{,B}x^{\sigma}_{,C}x^{\nu}_{,D}. \qquad (6.10)$$

Contracting Eq. (6.10) over $A$ and $C$ and then over $B$ and $D$, one finds that the scalar curvature on $\Sigma$ is just the sectional curvature in the tangent plane of $\Sigma$: i.e.,

$$R^{(2)} = G^{AC}G^{BD}R_{\mu\rho\sigma\nu}x^{\mu}_{,A}x^{\rho}_{,B}x^{\sigma}_{,C}x^{\nu}_{,D} \qquad (6.11)$$

which is identically the second term in Eq. (6.7), except for the sign. Finally,

$$\mathcal{V} = -\frac{1}{2}\left(R^{(2)} + 2\frac{Q^2}{\rho^4}\right)$$
$$= 2\left(\frac{Q^2(r^2 - a^2\cos^2\theta) - Mr(r^2 - 3a^2\cos^2\theta)}{\rho^6}\right), \qquad (6.12)$$

where we have used the fact that $R^{(2)} = -F''$.

It remains to determine the normal fundamental form $\mu_{RSA}$. Now, as $\mu_{RSA} = \mu_{[RS]A}$, we can write $\mu_{RSA} = \mu_A\epsilon_{RS}$. It is then straightforward to verify that under the gauge transformation (6.6), $\mu_{RSA}$ transforms as

$$\mu_{RSA} \mapsto \tilde{\mu}_{RSA} = \mu_{RSA} + \epsilon_{RS}x^{\mu}_{,A}\partial_{\mu}\Psi, \qquad (6.13)$$

or, in light of the previous definition,

$$\mu_A \mapsto \tilde{\mu}_A = \mu_A + x^{\mu}_{,A}\partial_{\mu}\Psi. \qquad (6.14)$$

We define $n_R$ over $\Sigma_{\pm}$ by parallel transport along a principal null trajectory and then by Lie transport along trajectories of the Killing vector, effectively fixing a gauge. That is, on $\Sigma_{\pm}$:

$$l^{\mu}_{\pm}n^{\nu}_{R;\mu} = 0, \quad \xi^{\mu}_{\pm}n^{\nu}_{R;\mu} = n^{\mu}_R\xi_{\nu;\mu}. \qquad (6.15)$$

With this covariantly constant definition of $n_R$, using Eq. (B1) of Appendix B, we find that

$$\mu_{RS1} = n^{\mu}_R l^{\nu}_{\pm}n_{S\mu;\nu} = 0, \qquad (6.16)$$

$$\mu_{RS0} = n^{\mu}_R n^{\nu}_S \xi_{\mu;\nu} = \tfrac{1}{2}\epsilon_{RS}(n^{\mu}_2 n^{\nu}_3 - n^{\mu}_3 n^{\nu}_2)\xi_{\mu;\nu}$$
$$= i\epsilon_{RS}M^{\mu}\bar{M}^{\nu}\xi_{\mu;\nu}.$$

In order to take advantage of the decomposition of $\xi_{\mu;\nu}$ in terms of the Kinnersley null tetrad (2.10), we note that $M_{\pm}$ and $m$ are related by the null rotation

$$M_{\pm} = m + El_{\pm}, \qquad (6.17)$$

where $E = \xi \cdot m$. Thus,

$$\mu_{RS0} = -\mu\epsilon_{RS}, \qquad (6.18)$$

where $\mu = -a(1-F)\cos\theta/\rho^2$. If we let $\ell_{\pm A} = x^{\mu}_{,A}l_{\pm\mu}$ then we can write the normal fundamental form in this gauge as

$$\mu_{RSA} = \mu\ell_{\pm A}\epsilon_{RS}, \qquad (6.19)$$

so that here $\mu_A = \mu\ell_{\pm A}$.

However, a more convenient choice of gauge has $\mu_{RSA} \propto \epsilon_{RS}\eta_A$ where $\eta_A = x^{\mu}_{,A}\xi_{\mu}$ is a Killing vector on $\Sigma_{\pm}$, see Ref. [7]. This corresponds to a choice of the function $\Psi$ on $\Sigma$ such that $\eta_A \propto \tilde{\mu}_A = \mu\ell_{\pm A} + x^{\mu}_{,A}\partial_{\mu}\Psi$. If we let $\Psi = \Psi(r)$, then it follows that, on $\Sigma$,

$$x^{\mu}_{,A}\partial_{\mu}\Psi = \mp\Psi'(F\ell_{\pm A} - \eta_A). \qquad (6.20)$$

Clearly, if $\Psi' = \pm\mu/F$, then $\tilde{\mu}_A = (\mu/F)\eta_A$. With this choice of gauge we find that the equations of motion reduce to

$$(\Box + \mathcal{V} + \mu^2/F)\tilde{\Phi}_R + 2\frac{\mu}{F}\epsilon_{RS}\eta^A\partial_A\tilde{\Phi}^S = 0, \qquad (6.21)$$

where

$$\mu = -\frac{a(1-F)\cos\theta}{\rho^2}, \qquad (6.22)$$

$$\mathcal{V} = 2\left(\frac{Q^2(r^2 - a^2\cos^2\theta) - Mr(r^2 - 3a^2\cos^2\theta)}{\rho^6}\right). \qquad (6.23)$$

Equation (6.21) can also be written in the form:

$$[G^{AB}(\delta_{RT}\nabla_A + \epsilon_{RT}\mathcal{A}_A)(\delta_{TS}\nabla_B + \epsilon_{TS}\mathcal{A}_B) + \delta_{RS}\mathcal{V}]\tilde{\Phi}^S = 0, \qquad (6.24)$$

where $\mathcal{A}_A \equiv \mu\eta_A/F = (-\mu, \pm\mu/F)$ and we used the identity $G^{AB}\nabla_A(\mu\eta_B/F) = 0$. Here, $\mathcal{A}_A$ plays the role of a vector potential while $\mathcal{V}$ is the scalar potential. Notice that the time component of $\mathcal{A}_A$ as well as $\mathcal{V}$ are finite everywhere, while the space component of $\mathcal{A}_A$ diverges at the static limit surface. But this divergence can be removed by a simple world-sheet coordinate transformation:

$$d\tilde{t} = du_{\pm} \mp F^{-1}(r)dr, \quad d\tilde{r} = dr. \qquad (6.25)$$

The perturbation equation still takes the form (6.24) but now the potentials are given by

$$\tilde{\mathcal{A}}_A = (-\mu, 0), \quad \tilde{\mathcal{V}} = \mathcal{V}, \qquad (6.26)$$

that is, the potentials $(\tilde{\mathcal{A}}_A, \mathcal{V})$ are finite everywhere. There is, however, a divergence at the static limit surface in the time component of $\tilde{\mathcal{A}}^A$, but such situations are well known from ordinary electromagnetism; this divergence does not destroy the regularity of the solution.

## VII. STRING-HOLE PHYSICS

In conclusion, we discuss some problems connected with the proposed string-hole model of two-dimensional black and white holes. The basic observation made in this paper is that the interaction of a cosmic string with a 4D black hole in which the string is trapped by the 4D black hole opens new channels for the interaction of the black hole with the surrounding matter. The corresponding new degrees of freedom are related to excitations of the cosmic string (stringons). These degrees of freedom can be identified with physical fields propagating in the geometry of the 2D string hole. There are two types of string holes corresponding to two types of the principal Killing surfaces $\Sigma_+$ and $\Sigma_-$. The first of them has the geometry of a 2D black hole while the second has the geometry of a 2-D white hole. The physical properties of "black" and "white" string holes are different. For a regular initial state a "black" string hole at late time is a source of a steady flux of thermal "stringons." This effect is an analogue of the Hawking radiation [18]. In the simplest case when a stationary cosmic string is trapped by a Schwarzschild black hole, so that the string hole has 2D Schwarzschild metric, the Hawking radiation of stringons was investigated in Ref. [19]. For such string holes their event horizon coincides with the event horizon of the 4D black hole, and the temperature of the "stringon" radiation coincides with the Hawking temperature of the 4D black hole. For this reason the thermal excitations of the cosmic string will be in the state of thermal equilibrium with the thermal radiation of the 4D black hole.

The situation is different in the general case when a stationary string is trapped by a rotating charged black hole. For the Kerr-Newman black hole the static limit surface is located outside the event horizon. The event horizon of the 2D string hole does not coincide with the Kerr-Newman black hole horizon, except for the case where the cosmic string goes along the symmetry axis. For this reason the surface gravity, and hence the temperature of the 2D black hole differ from the corresponding quantities calculated for the Kerr-Newman black hole. The surface gravity of the 2D black hole is

$$\kappa^{(2)} = \frac{1}{2} \frac{dF}{dr}\bigg|_{r=r_{st}} = \frac{\sqrt{M^2 - Q^2 - a^2 \cos^2\theta}}{2M^2 - Q^2 + 2M\sqrt{M^2 - Q^2 - a^2\cos^2\theta}}, \quad (7.1)$$

and it is always larger than the surface gravity of the four-dimensional Kerr-Newman black hole, Eq. (5.7). The reason why the temperature of a 2D black hole differs from the temperature of the four-dimensional Kerr-Newman black hole can be qualitatively explained if we note that for quanta located on the string surface (stringons) the angular momentum and energy are related.

In the general case ($a \neq 0$), a principal Killing surface in the Kerr-Newman spacetime is not geodesic. This property might have some interesting physical applications. Consider a black string hole and choose a point $p$ inside its event horizon but outside the event horizon $r = r_+$ of the four-dimensional Kerr-Newman black hole. Consider a timelike line $\gamma_0$ representing a static observer located outside the horizon of the 2D black hole at $r = r_0$. There evidently exists an ingoing principal null ray crossing $\gamma_0$ and passing through $p$. It was shown that there exists a future-directed 4D null geodesic which begins at $p$ and crosses $\gamma_0$. In other words, a causal signal from $p$ propagating in the 4D embedding spacetime can connect points of the 2D string hole interior with its exterior. For this reason stringons propagating inside the 2D string hole can interact via extra dimensions with the stringons in the 2D string hole exterior. Such an interaction from the 2D point of view is acausal. (Certainly, this interaction becomes impossible in case when the inner stringon is located so deep inside the 2D hole that it is under the Kerr-Newman 4D horizon.)

This interaction of Hawking stringons with their quantum correlated partners, created inside the string hole horizon, might change the spectrum of the Hawking radiation, as well as its higher correlation functions. This effect might have an interesting application for study of the information loss puzzle.

In conclusion, we have shown that in the case of interaction of a cosmic string with a black hole a 2D string hole can be formed. It opens an interesting possibility of testing some of the predictions of 2D gravity. We do not know at the moment whether it is also possible to "destroy" a 2D string hole by applying physical forces which change its motion and allow the cosmic string to be extracted back from the ergosphere. We hope to return to this and other questions connected with the unusual physics of string holes elsewhere.

## APPENDIX A: STRING BLACK HOLES AND DILATON GRAVITY

In this appendix we show that the 2D string holes can also be obtained as solutions of 2D dilaton gravity with a suitably chosen dilaton potential. To be more specific, we consider the action of 2D dilaton gravity

$$\mathcal{S} = \frac{1}{2\pi} \int dt\, dx\, \sqrt{-g}\, e^{-2\phi}[R + 2(\nabla\phi)^2 + V(\phi)], \quad (A1)$$

where the dilaton potential $V(\phi)$ will be specified later. In two dimensions we can choose the conformal gauge

$$g_{\mu\nu} = e^{2\rho} \times \text{diag.}(-1, 1), \quad \rho = \rho(t, x), \quad (A2)$$

so that

$$R = 2e^{-2\rho}(\rho_{,tt} - \rho_{,xx}). \quad (A3)$$

The action (A1) then takes the form

$$\mathcal{S} = \frac{1}{\pi} \int dt \quad dx\, e^{-2\phi}[\rho_{,tt} - \rho_{,xx} + \phi_{,x}^2 - \phi_{,t}^2 + \tfrac{1}{2}e^{2\rho}V(\phi)]. \quad (A4)$$

The corresponding field equations read

$$\rho_{,xx} - \rho_{,tt} + \phi_{,tt} - \phi_{,xx} + \phi_{,x}^2 - \phi_{,t}^2 + \frac{1}{4} e^{2\rho}(V' - 2V) = 0,$$

$$\phi_{,xx} - \phi_{,tt} + 2(\phi_{,t}^2 - \phi_{,x}^2) + \frac{1}{2} e^{2\rho}V = 0, \qquad \text{(A5)}$$

where $V' \equiv dV/d\phi$. Now, consider the special solutions

$$\rho = \rho(x), \quad \phi = \phi(x), \qquad \text{(A6)}$$

and introduce the coordinate $r$:

$$\frac{dr}{F(r)} = dx, \quad e^{2\rho} = F. \qquad \text{(A7)}$$

Then the metric (A2) leads to

$$dS^2 = -F(r)dt^2 + F^{-1}(r)dr^2 \qquad \text{(A8)}$$

which is precisely the form of our 2D string holes (4.18), in the coordinates defined by

$$d\tilde{t} = du_\pm \mp F^{-1}(r)dr, \quad d\tilde{r} = dr. \qquad \text{(A9)}$$

It still needs to be shown that Eqs. (A6) and (A7) are actually solutions to Eqs. (A5). The equations reduce to

$$\phi_{,rr} - \phi_{,r}^2 + \frac{F_{,r}}{F}\phi_{,r} - \frac{1}{4F}(V' - 2V) = \frac{F_{,rr}}{2F},$$

$$\phi_{,rr} + \frac{F_{,r}}{F}\phi_{,r} - 2\phi_{,r}^2 + \frac{1}{2F}V = 0. \qquad \text{(A10)}$$

It can now be easily verified that both equations are solved by a ''logaritmic dilaton'' provided the dilaton potential takes the form ($b = a \sin^2\theta$)

$$V(\phi) = \left[\frac{2}{r^2}(rF)_{,r}\right]_{|r=e^{-\phi/\lambda}}, \qquad \text{(A11)}$$

$$\phi = -\ln(\lambda r), \quad \lambda = \text{const}, \qquad \text{(A12)}$$

for an arbitrary function $F(r)$. For our 2D string holes, $F(r)$ is given by Eq. (2.3). The dilaton potential (A11) then takes the explicit form

$$V(\phi) = 2\lambda^2 e^{2\phi}\left[1 - \frac{4Me^{-\phi}/\lambda - Q^2}{e^{-2\phi}/\lambda^2 + a^2 - ab}\right.$$

$$\left. + \frac{2e^{-\phi}(2Me^{-2\phi}/\lambda^2 - Q^2 e^{-\phi}/\lambda)}{\lambda(e^{-2\phi}/\lambda^2 + a^2 - ab)^2}\right]. \qquad \text{(A13)}$$

This result holds for the general cone strings. A somewhat simpler expression is obtained for strings in the equatorial plane:

$$V(\phi) = 2\lambda^2 e^{2\phi}[1 - Q^2\lambda^2 e^{2\phi}], \quad \theta = \pi/2. \qquad \text{(A14)}$$

## APPENDIX B: GAUGE INVARIANCE OF THE SCALAR POTENTIAL

In this appendix we show that $\mathcal{V}_{RS}$, as defined in Eq. (6.4), is gauge invariant, i.e., invariant under the transforma-

tion (6.6) in the Kerr-Newman spacetime. Let $M = (n_2 + in_3)/\sqrt{2}$ where $\{n_2, n_3\}$ span the two-dimensional vector space normal to the cone string world sheet. Then, under the transformation specified by Eq. (6.6) $M^\mu \mapsto \widetilde{M}^\mu = e^{i\Psi}M^\mu$. We note that the combination $M^\mu \bar{M}^\nu$ is invariant under this transformation.

We will make use of the equalities

$$M^\mu \bar{M}^\nu = \frac{1}{2}(n_2^\mu n_2^\nu + n_3^\mu n_3^\nu) - \frac{i}{2}(n_2^\mu n_3^\nu - n_3^\mu n_2^\nu), \quad \text{(B1)}$$

$$M^\mu M^\nu = \frac{1}{2}(n_2^\mu n_2^\nu - n_3^\mu n_3^\nu) + \frac{i}{2}(n_2^\mu n_3^\nu + n_3^\mu n_2^\nu). \quad \text{(B2)}$$

Now consider

$$G^{AB} x_{,A}^\mu x_{,B}^\nu R_{\mu\rho\sigma\nu} n_R^\rho n_S^\sigma = (g^{\mu\nu} - \delta^{TQ} n_T^\mu n_Q^\nu)R_{\mu\rho\sigma\nu}n_R^\rho n_S^\sigma$$

$$= -R_{\rho\sigma}n_R^\rho n_S^\sigma - \delta^{TQ}R_{\mu\rho\sigma\nu}n_T^\mu n_Q^\nu n_R^\rho n_S^\sigma. \quad \text{(B3)}$$

The second term on the right-hand side can be written as

$$\delta^{TQ}R_{\mu\rho\sigma\nu}n_T^\mu n_Q^\nu n_R^\rho n_S^\sigma = (n_2^\mu n_2^\nu + n_3^\mu n_3^\nu)R_{\mu\rho\sigma\nu}n_R^\rho n_S^\sigma$$

$$= \delta^{RS}R_{\mu\rho\sigma\nu}n_2^\mu n_2^\nu n_3^\rho n_3^\sigma$$

$$= -\delta^{RS}R_{\mu\rho\sigma\nu}M^\mu M^\nu \bar{M}^\rho \bar{M}^\sigma, \quad \text{(B4)}$$

making use of Eq. (B1) and the symmetries of the Riemann tensor only. This form is explicitly gauge invariant in any spacetime geometry.

It remains to verify that the term $R_{\rho\sigma}n_R^\rho n_S^\sigma$ is also gauge invariant. We note that $M$ and the complex null vector $m$ of the Kinnersley tetrad are related by the null rotation $M = m + El$. We may then use the fact that $m$ and $l_\pm$ are eigenvectors of $R_{\rho\sigma}$ [see Eq. (6.8)] to show

$$R_{\rho\sigma}M^\rho M^\sigma = R_{\rho\sigma}(m^\rho m^\sigma + 2Em^\rho l_\pm^\sigma + E^2 l_\pm^\rho l_\pm^\sigma) = 0. \quad \text{(B5)}$$

Notice that this holds in any gauge as $M^\rho M^\sigma \mapsto \widetilde{M}^\rho \widetilde{M}^\sigma = e^{2i\Psi}M^\rho M^\sigma$. Thus, equating real and imaginary parts of $R_{\rho\sigma}M^\rho M^\sigma$ to zero one finds

$$R_{\rho\sigma}n_2^\rho n_2^\sigma = R_{\rho\sigma}n_3^\rho n_3^\sigma, \quad R_{\rho\sigma}n_2^\rho n_3^\sigma = -R_{\rho\sigma}n_3^\rho n_2^\sigma = 0. \quad \text{(B6)}$$

Thus, under a gauge transformation, we find that

$$R_{\rho\sigma}\widetilde{n}_2^\rho \widetilde{n}_3^\sigma = R_{\rho\sigma}(\cos\Psi n_2^\rho - \sin\Psi n_3^\rho)(\sin\Psi n_2^\sigma + \cos\Psi n_3^\sigma) = 0. \quad \text{(B7)}$$

It then follows that

$$R_{\rho\sigma}\widetilde{n}_2^\rho \widetilde{n}_2^\sigma = R_{\rho\sigma}(\cos\Psi n_2^\rho - \sin\Psi n_3^\rho)(\cos\Psi n_2^\sigma - \sin\Psi n_3^\sigma)$$

$$= R_{\rho\sigma}n_2^\rho n_2^\sigma. \quad \text{(B8)}$$

Similarly, $R_{\rho\sigma}n_3^\rho n_3^\sigma$ remains unchanged under rotation. Thus, we conclude that $\mathcal{V}_{RS}$ is gauge invariant as $\Omega_{RAB}\Omega_S^{AB}$ vanishes independently of gauge in the Kerr-Newman spacetime.

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