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Black plane solutions in four-dimensional spacetimes

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The static, plane symmetric solutions and cylindrically symmetric solutions of Einstein-Maxwell equations with a negative cosmological constant are investigated. These black configurations are asymptotically anti–de Sitter–type not only in the transverse directions, but also in the membrane or string directions. Their causal structure is similar to that of Reissner-Nordström black holes, but their Hawking temperature goes with $M^{1/3}$, where M is the ADM mass density. We also discuss the static plane solutions in Einstein-Maxwell-dilaton gravity with a Liouville-type dilaton potential. The presence of the dilaton field changes drastically the structure of solutions. They are asymptotically ''anti–de Sitter–'' or ''de Sitter-type'' depending on the parameters in the theory. [S0556-2821(96)03820-9]

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I. INTRODUCTION

In general relativity, looking for the exact solutions of the Einstein field equations has been a subject of long-standing interest. Among these exact solutions, the black hole solutions take an important position because thermodynamics, gravitational theory, and quantum theory are connected in quantum black hole physics. In addition, black holes might play an important role in developing a satisfactory quantum theory of gravitation which does not exist today. With the investigation of the lower energy actions of string theories and supergravity theory, in recent years we witnessed a rapid growth of interest in the family of black configurations ranging from various lower-dimensional black holes, black strings, to higher-dimensional extended black p-branes, such as two-dimensional dilaton black holes [1–3] and black hole solutions in Jackiw-Teitelboim theory [4], three-dimensional dilaton black holes [5,6] and black strings [7,8], four-dimensional charged dilaton black holes [9–12] and black strings [13], and higher-dimensional black p-branes [10,14]. These black configurations broaden the family of black holes and manifest some new features.

In the framework of four-dimensional Einstein theory of gravitation, it is well known that generic black hole solutions to Einstein-Maxwell equations are Kerr-Newman solutions, which are characterized by only three parameters: the mass, charge, and the angular momentum. It is often referred to as the nonhair conjecture of black holes. The Kerr-Newman spacetime is asymptotically flat. When a nonzero cosmological constant is introduced, the spacetime will become asymptotically de Sitter or anti–de Sitter spacetime depending on the sign of the cosmological constant. Although the theory of general relativity in three-dimensions retains the same formal structure as the one in four-dimensions and the Einstein equations still hold, the nature of the theory in three-dimensions is very different from that of four-dimensional gravitation [15]. In the three-dimensional spacetime the number of independent components of Riemann tensor is six, the same as that of the Ricci tensor, and the Weyl tensor vanishes identically. Thus, the stress-energy tensor has only a local effect on the curvature, the curvature at a point is nonzero if and only if the sress-energy tensor does not vanish [16]. Therefore, the pure gravity is identically flat in three-dimensional theory of gravity. But, when some extended matters appear, the Einstein equations have solutions with cosmological event horizons; the existence of a positive cosmological constant does not change this conclusion [16]. However, a negative cosmological constant will change dramatically this situation. Recently, Banados, Teitelboim, and Zanelli (BTZ) [17] have found a family of black hole solutions in three-dimensional Einstein gravity. The negative cosmological constant plays a central role in the existence of BTZ black holes.

In four-dimensional spacetime, Horowitz and Strominger [14] showed that there does not exist static, cylindrically symmetric black string solutions with asymptotically flat in the transverse directions if the strong energy condition, $T_{\mu\nu}t^\mu t^\nu \geqslant \frac{1}{2}Tt^\mu t_\mu$ for all timelike vectors t^μ, is satisfied. When the energy condition is relaxed to the weak energy condition, $T_{\mu\nu}t^\mu t^\nu \geqslant 0$, the conclusion still holds. However, their proof does not rule out the existence of the asymptotically nonflat black strings. The Kaloper's black string solution [13] in the dilaton theory of gravity is a manifest example, which is basically a direct product of a spinning BTZ black hole and a real line space [18]. Therefore, the Kaloper's black strings are asymptotically anti–de Sitter in the transverse directions and flat in the string direction.

In a recent paper [19], Lemos constructed the cylindrical black hole solutions (black strings) in four-dimensional Einstein gravity with a negative cosmological constant. Huang and Liang [20] further constructed the so-called toruslike black holes (with the topology $R^2\times S^1\times S^1$). The black string solutions of Lemos are asymptotically anti–de Sitter not only in the transverse directions, but also in the string direction. One of the aims of this paper is, in Sec. II, to extend the work of Lemos to the plane symmetric solutions and cylindrically symmetric solutions in Einstein-Maxwell equations with a negative cosmological constant. Here, the negative cosmological constant plays a crucial role in these

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solutions, as in the BTZ black holes. In Sec. III, we further discuss the plane symmetric solutions in Einstein-Maxwell-dilaton gravity with a Liouville-type potential of the dilaton field. The presence of the dilaton field will change drastically the structures and quantum properties of solutions. The existence of black configurations is independent of the sign of the ''cosmological constant.'' Our conclusion and a brief discussion are included in Sec. IV.

II. SOLUTIONS OF EINSTEIN-MAXWELL EQUATIONS WITH A NEGATIVE COSMOLOGICAL CONSTANT

In this section, we discuss the static, plane symmetric solutions and cylindrically symmetric solutions of the Einstein-Maxwell equations with a negative cosmological constant, respectively. Let us first consider the case of plane symmetry.

A. Black plane solutions

The Einstein-Maxwell equations have a well-known solution possessing the plane symmetry. Its line element is given by [21]

$$ds^2=-\left(\frac{m}{z}+\frac{e^2}{z^2}\right)dt^2+\left(\frac{m}{z}+\frac{e^2}{z^2}\right)^{-1}dr^2+z^2(dx^2+dy^2), \tag{1}$$

where m and e are two integration constants, and $-\infty<t,x,y,z<\infty$. When $m/z+e^2/z^2<0$, the solution describes a spatially homogeneous spacetime. It is static as $m/z+e^2/z^2>0$. In the latter case, the solution (1) has four Killing vectors: a timelike $\partial/\partial t$ and three spacelike $\partial/\partial x$, $\partial/\partial y$, and $x\partial/\partial y-y\partial/\partial x$, indicating the static, plane symmetry of spacetime (1). But the solution is of a naked singularity at $z=0$ and its physical meanings are unclear. On the other hand, Einstein equations with a negative cosmological constant ($3\alpha^2$) admit the plane symmetric anti–de Sitter solution

$$ds^2=-\alpha^2z^2dt^2+(\alpha^2z^2)^{-1}dz^2+\alpha^2z^2(dx^2+dy^2). \tag{2}$$

If one redefines $Z=-1/(\alpha^2z^2)$, Eq. (2) then becomes

$$ds^2=(\alpha Z)^{-2}(-dt^2+dZ^2+dx^2+dy^2), \tag{3}$$

which is just the half of the spacetime of supergravity domain walls [22,23], because the gravitational field of the supergravity domain walls can be interpreted in terms of the domain wall interpolating between the Minkowski spacetime and the plane anti–de Sitter spacetime. The spacetime (3) is geodesically incomplete and has a Cauchy horizon at $Z\to-\infty$. This Cauchy horizon is also unstable [24], as the Cauchy horizon in the Reissner-Nordström black holes.

Combining Eqs. (1) and (2), we find that a static plane symmetric solution possessing event horizons will appear. The singularity at the $z=0$ plane in Eq. (1) will be enclosed by these event horizons. We start with the action

$$S=\frac{1}{16\pi}\int_V d^4x\sqrt{-g}(R+6\alpha^2-F^{\mu\nu}F_{\mu\nu})-\frac{1}{8\pi}\int_{\partial V}d^3x\sqrt{-h}K, \tag{4}$$

where R is the scalar curvature, $F_{\mu\nu}$ is Maxwell field, and $3\alpha^2=-\Lambda>0$ denotes the negative cosmological constant. The quantity h is the induced metric on ∂V, and K its extrinsic curvature. Varying the action (4) yields the equations of motion

$$G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=3\alpha^2g_{\mu\nu}+8\pi T^{EM}_{\mu\nu}, \tag{5}$$

$$0=\partial_\mu(\sqrt{-g}F^{\mu\nu}),\quad F_{\mu\nu,\rho}+F_{\nu\rho,\mu}+F_{\rho\mu,\nu}=0, \tag{6}$$

where

$$T^{EM}_{\mu\nu}=\frac{1}{4\pi}\left(F_{\mu\lambda}F^{\lambda}_{\nu}-\frac{1}{4}g_{\mu\nu}F^2\right) \tag{7}$$

is the stress-energy tensor of the Maxwell field. The general metric of static plane symmetry can be written as

$$ds^2=-A(r)dt^2+B(r)dr^2+C(r)(dx^2+dy^2), \tag{8}$$

where we have taken $r=|z|$ because of the reflection symmetry with respect to the $z=0$ plane. In the metric (8) solving Eqs. (5) and (6), we find

$$A(r)=B^{-1}(r)=\alpha^2r^2-\frac{m}{r}+\frac{q^2}{r^2}, \tag{9}$$

$$C(r)=\alpha^2r^2, \tag{10}$$

$$F_{tr}=\frac{q}{\alpha^2r^2}, \tag{11}$$

where m and q are two integration constants related to the Arnowitt-Deser-Misner (ADM) mass and electric charge of the solutions, respectively. Because of the noncompactibility of the coordinates x and y, for simplicity, we only consider the mass and charge per unit area in the x-y plane. The electric charge density Q can be obtained by the Gauss theorem

$$Q=\frac{1}{4\pi}\int F_{tr}C(r)dxdy=\frac{q}{2\pi}, \tag{12}$$

where we have considered the two integral surfaces at $z=\pm r$. With the help of the Euclidean action method of black membranes [25], the ADM mass density M is found to be $M=\alpha^2m/4\pi$. Thus, we obtain the static plane symmetric solutions of Eqs. (5) and (6);

$$ds^2=-\left(\alpha^2r^2-\frac{4\pi M}{\alpha^2r}+\frac{(2\pi Q)^2}{\alpha^4r^2}\right)dt^2+\left(\alpha^2r^2-\frac{4\pi M}{\alpha^2r}+\frac{(2\pi Q)^2}{\alpha^4r^2}\right)^{-1}dr^2+\alpha^2r^2(dx^2+dy^2). \tag{13}$$

This solution is asymptotically anti–de Sitter not only in the transverse directions, but also in the membrane directions. By calculating the scalar curvature invariants in the spacetime (13), the solution (13) is singular only at $r=0$ plane. The vacuum background ($M=Q=0$) corresponding to Eq. (13) is

$$ds^2=-\alpha^2r^2dt^2+(\alpha^2r^2)^{-1}dr^2+\alpha^2r^2(dx^2+dy^2), \tag{14}$$

which is just the plane anti–de Sitter spacetime (2). In addition, if the condition

$$Q^2\leqslant\frac{3\alpha^6}{4\pi^2}\left(\frac{\pi M}{\alpha^4}\right)^{4/3} \tag{15}$$

is satisfied, the Eq. $A(r)=0$, i.e.,

$$\alpha^2r^2-\frac{4\pi M}{\alpha^2 r}+\frac{(2\pi Q)^2}{\alpha^4r^2}=0 \tag{16}$$

has two positive real roots:

$$r_\pm=\frac{1}{2}\left[\sqrt{2R}\pm\left(-2R+\frac{8\pi M}{\alpha^4\sqrt{2R}}\right)^{1/2}\right], \tag{17}$$

where

$$R=\left\{\frac{\pi^2M^2}{\alpha^8}+\left[\left(\frac{\pi^2M^2}{\alpha^8}\right)^2-\left(\frac{4\pi^2Q^2}{3\alpha^6}\right)^3\right]^{1/2}\right\}^{1/3}+\left\{\frac{\pi^2M^2}{\alpha^8}-\left[\left(\frac{\pi^2M^2}{\alpha^8}\right)^2-\left(\frac{4\pi^2Q^2}{3\alpha^6}\right)^3\right]^{1/2}\right\}^{1/3}. \tag{18}$$

The other two roots of Eq. (16) are imaginary numbers and have no physical meanings. Because $A(r)\geqslant 0$ when $0\leqslant r\leqslant r_-$ and $r\geqslant r_+$, $A(r)\leqslant 0$ when $r_-\leqslant r\leqslant r_+$. Therefore, the two positive roots can be interpreted as the outer horizon and inner horizon of the plane symmetric solutions, respectively. The causal structure of solution (13) is similar to that of Reissner-Nordström black holes. The singularity at $r=0$ is enclosed by event horizons. Unlike the spherically symmetric black holes, here the singularity is in the plane at $r=0$. In addition, it is worth noting that, in fact, the solution (13) has four-event horizons, two outer horizons at $z=\pm r_+$, two inner horizons $z=\pm r_-$. The singularity at the plane $z=0$ is enclosed by these horizons. When the equality in Eq. (15) holds, the two horizons coincide and the horizon becomes

$$r_{\rm ext}=\left(\frac{\pi M}{\alpha^4}\right)^{1/3}. \tag{19}$$

This case corresponds to the extremal black plane solutions. Here, we point out that if the negative cosmological constant is replaced by a positive one, the solution (13) will become an asymptotically de Sitter solution, and has a single cosmological horizon. The singularity at $r=0$ becomes a cosmological singularity. We have no interest for this situation and do not discuss it in detail.

We now turn to thermodynamics of the black plane solutions. To do this, it is convenient to employ the Euclidean action method [25–27]. Analytically, extending the solution (13) to its Euclidean section, we obtain

$$ds^2=\left(\alpha^2r^2-\frac{4\pi M}{\alpha^2r}+\frac{(2\pi Q)^2}{\alpha^4r^2}\right)d\tau^2+\left(\alpha^2r^2-\frac{4\pi M}{\alpha^2r}+\frac{(2\pi Q)^2}{\alpha^4r^2}\right)^{-1}dr^2+\alpha^2r^2(dx^2+dy^2), \tag{20}$$

where τ is the Euclidean time. The requirement of the absence of the conical singularity in the Euclidean spacetime (20) causes the Euclidean time τ to have a period β_H, which satisfies

$$\beta_H^{-1}=\frac{A'}{4\pi\sqrt{AB}}\bigg|_{r_+}=\frac{1}{2\pi}\left(\alpha^2r_++\frac{2\pi M}{\alpha^2r_+^2}-\frac{(2\pi Q)^2}{\alpha^4r_+^3}\right), \tag{21}$$

which is just the Hawking temperature of the black plane solutions. For extremal black planes (19) the temperature vanishes. When $Q=0$, that is, for neutral black plane solutions, the temperature, $\beta_H^{-1}=3M^{1/3}(\alpha/4\pi)^{2/3}$, goes with $M^{1/3}$, which is very different from that of Schwarzschild black holes. It implies that the difference in topology structures will change greatly the quantum properties of black configurations. Following Ref. [27], consider the black membrane and its surroundings contained by two infinite parallel plates at $z_B=\pm r_B$. We regard the interior of the plates as the thermodynamical system and the two plates as the boundaries. In a grand canonical ensemble, we must fix the boundary conditions of the system. The Euclidean manifold (20) is regular with a product topology $R^2\times R^2$, and boundary $S^1\times R^2$. On the boundary, the inverse temperature β_B has the Toman's relation,

$$\beta_B=\beta_H A^{1/2}(r_B), \tag{22}$$

and the electric potential ϕ_B is fixed as

$$\phi_B=\frac{2\pi Q}{\alpha^2}\left(\frac{1}{r_+}-\frac{1}{r_B}\right)A^{-1/2}(r_B). \tag{23}$$

The Euclidean action can be obtained by Euclideanizing the action (4):

$$S_E=-\frac{1}{16\pi}\int_V d^4x\sqrt{-g}(R+6\alpha^2-F^{\mu\nu}F_{\mu\nu})+\frac{1}{8\pi}\int_{\partial V}d^3x\sqrt{-h}K. \tag{24}$$

In the Euclidean spacetime of Eq. (8), the scalar curvature and extrinsic curvature are, respectively,

$$R=-g^{-1/2}(g^{1/2}A'/AB)'-2G_0^0 \tag{25}$$

and

$$K=-g^{-1/2}(g^{1/2}B^{-1/2})', \tag{26}$$

where G_0^0 is the 0-0 component of the Einstein tensor. Similar to the mass and charge, we only calculate the Euclidean action per unit area in the following discussions. Substituting Eqs. (25) and (26) into Eq. (23), with the help of Eqs. (9)–(11), we have

$$S_E=\beta_B\left(-\frac{\alpha^2 r}{2\pi}A^{1/2}(r)\right)\bigg|_{r_B}-\frac{\alpha^2 r_+^2}{2}-\beta_B\frac{2\pi Q^2}{\alpha^2}\left(\frac{1}{r_+}-\frac{1}{r_B}\right)A^{-1/2}(r_B). \tag{27}$$

In order to obtain the Euclidean action of black plane solutions, we must eliminate the contribution of the vacuum background (14) with the same boundary conditions from the action (27). The Euclidean action of the vacuum background is easy to get

$$S_{VE}=-\beta_B\frac{\alpha^3 r_B^2}{2\pi}. \tag{28}$$

Thus, we obtain the Euclidean action of black plane solutions:

$$S_{ME}=\frac{\beta_B\alpha^2 r}{2\pi}(\alpha r-A^{1/2}(r))\bigg|_{r_B}-\frac{\alpha^2 r_+^2}{2}-\beta_B\frac{2\pi Q^2}{\alpha^2}\left(\frac{1}{r_+}-\frac{1}{r_B}\right)A^{-1/2}(r_B). \tag{29}$$

Comparing the Euclidean action (29) with the formula of thermodynamic potential, we get the internal E, entropy S, and the chemical potential μ corresponding to the electric charge Q, respectively:

$$E=\frac{\alpha^2}{2\pi}(\alpha r^2-rA^{1/2}(r))|_{r_B}, \tag{30}$$

$$S=\frac{\alpha^2 r_+^2}{2}=\frac{\sigma}{4}, \tag{31}$$

$$\mu=\frac{2\pi Q}{\alpha^2}\left(\frac{1}{r_+}-\frac{1}{r_B}\right)A^{-1/2}(r_B)\equiv\phi_B, \tag{32}$$

where $\sigma=2\alpha^2 r_+^2$ denotes the area of horizon of black membranes and the prefactor 2 is because of two outer horizon surfaces. From Eq. (31) we see that the entropy S also satisfies $\frac{1}{4}$ area formula of black hole entropy. In terms of the relativistic thermodynamics, the proper energy E^* and proper chemical potential μ^* are, respectively,

$$E^*=EA^{1/2}(r_B)\xrightarrow{r_B\to\infty}=M$$
$$=\text{ADM mass density of the black membranes}, \tag{33}$$

$$\mu^*=\mu A^{1/2}(r_B)\xrightarrow{r_B\to\infty}=\phi_H$$
$$=\text{electric potential at the horizon}, \tag{34}$$

where $\phi_H=(2\pi Q/\alpha^2)(1/r_+)$. With the help of Eqs. (30)–(32), we easily obtain the first law of thermodynamics for the system:

$$dE\equiv\left(\frac{\partial E}{\partial S}\right)_{Q,\sigma}dS+\left(\frac{\partial E}{\partial Q}\right)_{S,\sigma}dQ+\left(\frac{\partial E}{\partial\sigma}\right)_{S,Q}d\sigma,$$
$$=\beta_B^{-1}dS+\phi_B dQ-Pd\sigma, \tag{35}$$

where $p\equiv-(\partial E/\partial\sigma)_{S,Q}$ is the surface pressure of the system and $\sigma=2\alpha^2 r_B^2$ the surface area of the system. If one rewrites Eq. (35) by using proper quantities at $r_B\to\infty$, it then reduces to

$$dM=\beta_H^{-1}dS+\phi_H dQ, \tag{36}$$

which is just the first law of black hole thermodynamics.

To end this subsection, we write down the metric of charged black plane solutions with a pressureless null radiation in the advanced time coordinates:

$$ds^2=-\left(\alpha^2 r^2-\frac{4\pi M(v)}{\alpha^2 r}+\frac{(2\pi Q(v))^2}{\alpha^4 r^2}\right)dv^2+2dvdr+\alpha^2 r^2(dx^2+dy^2), \tag{37}$$

which is the Vaidya-like metric of black membranes. Equation (37) implies that the stress-energy tensor of the radiation is

$$T^R_{\mu\nu}=\rho(v,r)l_\mu l_\nu, \tag{38}$$

where $l_\mu=-\partial_\mu v$ is the four-velocity of the null radiation, and the energy density $\rho(v,r)$ satisfies

$$\rho(v,r)=\frac{\dot M(v)}{4\alpha^2 r^2}-\frac{\pi Q\dot Q(v)}{\alpha^4 r^3}, \tag{39}$$

where an overdot stands for derivative with respect to v. Following Refs. [28–30], we can easily show that the inner horizon r_- is unstable. When the ingoing radiation has a power-law tail, a nonscalar curvature singularity will be developed at the inner horizon. When an outgoing null flux is added to the metric (37), the mass inflation will take place inside the black plane solutions, as in the Reissner-Nordström black holes.

B. Black string solutions

The black string solutions to Einstein equations with a negative cosmological constant have been constructed by Lemos [19]:

$$ds^2=-\left(\alpha^2 r^2-\frac{m}{r}\right)dt^2+\left(\alpha^2 r^2-\frac{m}{r}\right)^{-1}dr^2+r^2d\theta^2+\alpha^2 r^2dz^2, \tag{40}$$

where $-\infty<t,\ z<\infty$, $0\leqslant r<\infty$, and $0\leqslant\theta\leqslant 2\pi$, and the integration constant m is related to the ADM mass density of the black strings. Huang [31] has recently discussed the generalization of the solution (40) to include the electric charge. However, some expressions in Ref. [31] are incorrect. For completeness, here we reexamine the charged black string solutions to Einstein-Maxwell equations with a negative cosmological constant. To construct the cylindrically symmetric solution, by identifying the coordinate x in Eq. (6) with a period 2π, and replacing the variable y by z, we obtain the static, cylindrically symmetric solution of Eqs. (5) and (6):

$$ds^2=-\left(\alpha^2r^2-\frac{4M}{\alpha r}+\frac{4Q^2}{\alpha^2r^2}\right)dt^2+\left(\alpha^2r^2-\frac{4M}{\alpha r}+\frac{4Q^2}{\alpha^2r^2}\right)^{-1}dr^2+r^2d\theta^2+\alpha^2r^2dz^2, \tag{41}$$

$$F_{tr}=\frac{2Q}{\alpha r^2}, \tag{42}$$

where the two constants M and Q are the ADM mass and charge per unit length in the z direction. The spacetime (41) is asymptotically anti–de Sitter in the transverse directions and string directions, unlike the Kaloper's black strings in the dilaton gravity [13]. The singularity at $r=0$ is enclosed by the horizons $r_\pm$ if the condition

$$Q^2\leqslant\frac{3}{4}M^{4/3} \tag{43}$$

holds. Same as the black plane solutions, the black strings (41) have two horizons:

$$r_\pm=\frac{1}{2}\left[\sqrt{2R}\pm\left(-2R+\frac{8M}{\alpha^3\sqrt{2R}}\right)^{1/2}\right], \tag{44}$$

where

$$R=\left\{\frac{M^2}{\alpha^6}+\left[\left(\frac{M^2}{\alpha^6}\right)^2-\left(\frac{4Q^2}{3\alpha^4}\right)^3\right]^{1/2}\right\}^{1/3}+\left\{\frac{M^2}{\alpha^6}-\left[\left(\frac{M^2}{\alpha^6}\right)^2-\left(\frac{4Q^2}{3\alpha^4}\right)^3\right]^{1/2}\right\}^{1/3}. \tag{45}$$

Euclideanizing the metric (41), we can get the Hawking temperature of the black strings:

$$\beta_H^{-1}=\frac{1}{2\pi}\left(\alpha^2r_++\frac{2M}{\alpha r_+^2}-\frac{4Q^2}{\alpha^2r_+^3}\right). \tag{46}$$

Similar to the previous subsection, we have the entropy per unit length and the first law of thermodynamics for charged black strings:

$$S=\frac{\pi\alpha r_+^2}{2}=\frac{1}{4}\sigma, \tag{47}$$

$$dM=\beta_H^{-1}dS+\phi_H dQ, \tag{48}$$

where $\sigma=2\pi\alpha r_+^2$ is the area of horizon per unit length and $\phi_H=2Q/\alpha r_+$ the electric potential at the horizon r_+. When $Q=0$, i.e., for neutral black strings, the inner horizon disappears and $r_+=\alpha^{-1}(4M)^{1/3}$. Thus, the Hawking temperature (46) becomes $\beta_H^{-1}=(3\alpha/2\pi)(M/2)^{1/3}$. Therefore, the temperature of black strings also goes with $M^{1/3}$, as the case of black membranes. When the equality in Eq. (43) holds, the two horizons coincide and Hawking temperature vanishes. This corresponds to the extremal black strings. As the case of black membranes, the causal structure of charged black strings is similar to that of Reissner-Nordström black holes.

Finally, we write down here the Vaidya-like metric of black strings (41),

$$ds^2=-\left(\alpha^2r^2-\frac{4M(v)}{\alpha r}+\frac{4Q^2(v)}{\alpha^2r^2}\right)dv^2+2dv\,dr+r^2d\theta^2+\alpha^2r^2dz^2, \tag{49}$$

where the energy density of the null radiation is

$$\rho(v)=\frac{\dot{M}(v)}{2\pi\alpha r^2}-\frac{Q\dot{Q}(v)}{\pi\alpha^2r^3}. \tag{50}$$

Similarly, by using the metric (49) we can show that the inner horizon is also unstable and a scalar curvature singularity will replace the inner horizon when the charged black strings are perturbed by ingoing and outgoing null fluxes.

So far, we have investigated the static, plane symmetric solutions and cylindrically symmetric solutions in the Einstein-Maxwell equations with a negative cosmological constant. The causal structure of these solutions is similar to that of Reissner-Nordström black holes. Therefore, they can be interpreted as the black membranes and black strings, respectively. These black configurations are asymptotically anti–de Sitter-type not only in the transverse directions, but also in the membrane or string directions. In these solutions, the negative cosmological constant plays an important role. In the following section, we will see that when the dilaton field is present, the structure of the plane solutions will be changed greatly. The role of the negative cosmological constant seems to be lowered.

III. BLACK PLANE SOLUTIONS IN EINSTEIN-MAXWELL-DILATON GRAVITY

In recent years, many black hole solutions have been found in the dilaton gravity. Due to the dilaton field, the usual black hole structure and quantum properties are changed drastically. In this section, we would like to look for the plane symmetric solution in the Einstein-Maxwell-dilaton gravity with a Liouville-type dilaton potential, whose action is

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}[R-2(\nabla\phi)^2-6\alpha^2\eta e^{2b\phi}-e^{-2a\phi}F^2], \tag{51}$$

where ϕ is the dilaton field, the Liouville-type potential represents the ‘‘cosmological constant term,’’ a and b are two constants, and $\eta=\pm1$, representing the sign of the ‘‘cosmological constant.’’ This action (51) has been considerably investigated in the context of three- and four-dimensional dilaton black holes [5,12]. Varying the action (51), we obtain the equations of motion

$$R_{\mu\nu}=2\partial_\mu\phi\partial_\nu\phi+3\alpha^2\eta e^{2b\phi}g_{\mu\nu}+2e^{-2a\phi}(F_{\mu\lambda}F_\nu^{\lambda}-\tfrac{1}{4}g_{\mu\nu}F^2), \tag{52}$$

$$0=\partial_\mu(\sqrt{-g}e^{-2a\phi}F^{\mu\nu}), \tag{53}$$

$$\nabla^2\phi=3\alpha^2 b\eta e^{2b\phi}-\frac{a}{2}e^{-2a\phi}F^2. \tag{54}$$

We consider again the static plane solutions of Eqs. (52)–(54) in the metric

$$ds^2=-A(r)dt^2+B(r)dr^2+C(r)(dx^2+dy^2). \tag{55}$$

From Eq. (53) we have

$$F_{tr}=\frac{Q}{\sqrt{ABC}}e^{2a\phi}, \tag{56}$$

where Q is an integration constant. Thus, Eqs. (52) and (54) reduce to

$$\frac{A'C'}{2ABC}-\frac{C''}{BC}+\frac{C'^2}{2BC^2}+\frac{B'C'}{2B^2C}=\frac{2}{B}\phi'^2, \tag{57}$$

$$\frac{A''}{2AB}-\frac{A'B'}{4AB^2}-\frac{A'^2}{4A^2B}+\frac{A'C'}{2ABC}=-3\alpha^2\eta e^{2b\phi}+\frac{Q^2}{A^2B^2C^2}e^{2a\phi}, \tag{58}$$

$$-\frac{C''}{2BC}+\frac{B'C'}{4B^2C}-\frac{A'C'}{4ABC}=3\alpha^2\eta e^{2b\phi}+\frac{Q^2}{A^2B^2C^2}e^{2a\phi}, \tag{59}$$

$$\frac{1}{\sqrt{ABC}}\left[\sqrt{AB}\left(\frac{C}{B}\right)\phi'\right]'=3\alpha^2 b\eta e^{2b\phi}+\frac{aQ^2}{A^2B^2C^2}e^{2a\phi}, \tag{60}$$

where a prime denotes derivative with respect to r. From Eqs. (57)–(60), we obtain a set of solutions

$$A(r)=\frac{1}{B(r)}=-\frac{4\pi M}{N\alpha^N}r^{1-N}-\frac{6\alpha^2\eta}{N(2N-1)}r^N+\frac{2Q^2}{N\alpha^{2N}}r^{-N}, \tag{61}$$

$$C(r)=(\alpha r)^N, \tag{62}$$

$$\phi(r)=-\frac{\beta}{2}\ln r, \tag{63}$$

where M is the quasilocal mass density [32], and

$$\beta=\sqrt{2N-N^2}, \tag{64}$$

$$a=b=\beta/N. \tag{65}$$

In the spacetime described by Eqs. (61) and (62), the scalar curvature is

$$R=\frac{\beta^2}{2r^2}A(r)+12\alpha^2\eta r^{N-2}. \tag{66}$$

Obviously, the curvature diverges at $r=0$. Therefore, the $r=0$ plane is a singularity plane in solutions (61). The plane symmetric solutions (61) manifest some interesting properties because of the parameter N. We will separately discuss the cases of $\eta=-1$ and $\eta=1$.

(1) $\eta=-1$. That is, the Liouville-type potential corresponds to a ''negative cosmological constant.'' The solution (61) is of different asymptotic properties as the N takes different values. From the solution Equations (61)–(65), we have $0<N<2$, but $N\neq 1/2$. When $N=2$, the solution can be reduced to the one of Einstein-Maxwell equations with a negative cosmological constant (13).

(i) When $1/2<N<2$, the second term (r^N) in Eq. (61) is dominant as $r\to\infty$. In that case, the solution (61) is an asymptotically ''anti–de Sitter'' solution, where the word ''anti–de Sitter'' means that the solution has no cosmological horizon. The other horizons are given by the Eq. $A(r)=0$: i.e.,

$$\frac{3\alpha^2}{(2N-1)}r^{2N}-\frac{2\pi M}{\alpha^N}r+\frac{Q^2}{\alpha^{2N}}=0. \tag{67}$$

Because of the higher order of r in Eq. (67), in general, the solution (61) will have the multihorizon structures. A simpler case is $N=1$, in this case we have

$$r_\pm=\frac{1}{3\alpha^3}(\pi M\pm\sqrt{\pi^2M^2-3\alpha^2Q^2}). \tag{68}$$

When $M^2>3\alpha^2Q^2/\pi^2$, the solution (61) has two horizons, outer horizon r_+ and inner horizon r_-; when $M^2=3\alpha^2Q^2/\pi^2$, the solution (61) has a single horizon $r_+=\pi M/(3\alpha^3)$; this corresponds to the extremal plane solution; when $M^2<3\alpha^2Q^2/\pi^2$, the solution (61) will have no horizon and the singularity at $r=0$ becomes naked. Evidently, the causal structure of this case is similar to that of Reissner-Nordström black holes. The Hawking temperature is

$$\beta_H^{-1}=\frac{1}{2\pi}\left(3\alpha^2-\frac{Q^2}{\alpha^2r_+^2}\right). \tag{69}$$

From Eq. (69) we can see that if $Q=0$, the temperature is a constant. This is very different from the case in the absence of the dilaton field (21). For a generic N, the Hawking temperature is

$$\beta_H^{-1}=\frac{1}{2\pi}\left(-\frac{2\pi M(1-N)}{N\alpha^N}r_+^{-N}+\frac{3\alpha^2}{(2N-1)}r_+^{N-1}-\frac{Q^2}{\alpha^{2N}}r_+^{-N-1}\right). \tag{70}$$

It should be noted that for some special N, the solution (61) will have no horizon. For example, when $N=3/2$, the solution has no horizon, and the singularity at $r=0$ is naked.

(ii) When $0<N<1/2$, the first term (r^{1-N}) in Eq. (61) is dominant as $r\to\infty$. In that case, the solution (61) is an asymptotically ''de Sitter'' solution, where the ''de Sitter'' means that the solution (61) has the cosmological horizon. In general, the plane solution will be of the inner horizons, outer horizon, and the cosmological horizon. These horizons are determined by the equation

$$\frac{3\alpha^2}{(1-2N)}r^{2N}+\frac{2\pi M}{\alpha^N}r-\frac{Q^2}{\alpha^{2N}}=0. \tag{71}$$

In particular, we find that, in some special cases, although there exists the cosmological horizon, the inner and outer horizons are absent, the singularity at $r=0$ is a cosmological singularity. A manifest example is $N=1/4$; the solution has only the cosmological horizon:

$$r_{\rm coh}=\left[\frac{\alpha^{1/4}}{4\pi M}(-6\alpha^2+(36\alpha^4+8\pi MQ^2\alpha^{-3/4})^{1/2})\right]^{1/2}. \tag{72}$$

This situation is very like the Reissner–Nordström–de Sitter spacetime when the charge Q exceeds a critical value. But there exists an essential difference in the causes. The former is purely because of the parameter N; the latter is due to the relation of black hole hairs (mass, charge, and cosmological constant).

(2) $\eta=1$. Namely, the Liouville-type potential corresponds to a ''positive cosmological constant.'' In that case, $A(r)\to+\infty$ as $r\to0$, and $A(r)\to-\infty$ as $r\to+\infty$, therefore, the Eq. $A(r)=0$ determining the horizons of solutions has at least a positive root between $0<r<\infty$. For $1/2<N<2$ and $0<N<1/2$, the solutions (61) are all asymptotically ''de Sitter'' solutions, that is, these solutions have the cosmological horizons. Of course, for generic parameter N, these solutions could have the inner horizons and outer horizon, indicating the multihorizon feature. These horizons are given by

$$\frac{3\alpha^2}{(2N-1)}r^{2N}+\frac{2\pi M}{\alpha^N}r-\frac{Q^2}{\alpha^{2N}}=0. \tag{73}$$

However, unlike the case $\eta=-1$, when $N=1$, 1/4, or 3/2, the solution (61) has only a cosmological horizon, which is

$$r_{\rm coh}=\frac{1}{3\alpha^3}(-\pi M+\sqrt{\pi^2M^2+3\alpha^2Q^2}), \tag{74}$$

for $N=1$,

$$r_{\rm coh}=\left[\frac{\alpha^{1/4}}{4\pi M}(6\alpha^2+(36\alpha^4+8\pi MQ^2\alpha^{-3/4})^{1/2})\right]^{1/2}, \tag{75}$$

for $N=1/4$,

$$r_{\rm coh}=\left[\frac{Q^2}{3\alpha^5}+\left(\left(\frac{Q^2}{3\alpha^5}\right)^2+\left(\frac{4\pi M}{9\alpha^{7/2}}\right)^3\right)^{1/2}\right]^{1/3}+\left[\frac{Q^2}{3\alpha^5}-\left(\left(\frac{Q^2}{3\alpha^5}\right)^2+\left(\frac{4\pi M}{9\alpha^{7/2}}\right)^3\right)^{1/2}\right]^{1/3}, \tag{76}$$

for $N=3/2$. The Hawking temperature for these cosmological horizons is

$$\beta_H^{-1}=\frac{1}{2\pi}\left|\left(-\frac{2\pi M(1-N)}{N\alpha^N}r^{-N}-\frac{3\alpha^2}{(2N-1)}r^{N-1}-\frac{Q^2}{\alpha^{2N}}r^{-N-1}\right)\right|_{r=r_{\rm coh}}. \tag{77}$$

Similarly, the cylindrically symmetric solution in the action (51) can also be obtained. The causal structures of them are similar to those of the plane symmetric solutions of equations (61)–(65). For simplicity, here we do not present them.

IV. CONCLUSION AND DISCUSSIONS

In this work we have discussed the static, plane symmetrically solutions and cylindrically symmetric solutions in Einstein-Maxwell equations with a negative cosmological constant. The singularity at $r=0$ can be enclosed by event horizons. Their causal structure is very similar to the one of Reissner-Nordström black holes, but the Hawking temperature goes with $M^{1/3}$. These black configurations are asymptotically anti–de Sitter–type, not only in the transverse directions, but also in the membrane or string directions. In these solutions with horizons, the negative cosmological constant plays a crucial role, as in the three-dimensional BTZ black holes. We have also investigated the plane symmetric solutions in Einstein-Maxwell-dilaton gravity with a Liouville-type dilatonic potential. The presence of the dilaton field changes drastically the structure of the solutions to Einstein-Maxwell equations with a cosmological constant. In particular, there exist the black plane solutions for the ''positive cosmological constant'' and ''negative cosmological constant.'' These solutions are asymptotically ''anti–de Sitter–type'' or ''de Sitter–type,'' depending on the parameters N and η.

In the plane symmetric solutions, an interesting phenomenon is that, if one removes the reflection symmetry with respect to the $z=0$ plane, the black plane solution becomes that the singularity at $z=0$ plane is enclosed by event horizon in one direction and naked in the another direction. For example, for neutral plane solutions,

$$ds^2=-\left(\alpha^2z^2-\frac{4\pi M}{\alpha^2 z}\right)dt^2+\left(\alpha^2z^2-\frac{4\pi M}{\alpha^2 z}\right)dz^2+(\alpha z)^2(dx^2+dy^2). \tag{78}$$

The solution has a singularity at $z=0$ plane. When $M>0$, obviously, it has a horizon at $z=(4\pi M/\alpha^4)^{1/3}$ in the positive z direction. But, the singularity is naked in the negative z direction. When $M<0$, the situation is opposite. The property is the new feature of these black plane solutions. Of course, the problems of physics might have the reflection symmetry. Finally, we would like to point out that the black plane solutions have been also discussed by Cvetic [23] in the context of supergravity domain walls.

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