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Time evolution and matching conditions of spinning gauge strings

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The dynamical evolution of a spinning gauge string is investigated. We find that the formation of closed timelike curves in these models is exceedingly unlikely, because they require unrealistic values of the gauge-string parameters, probably found in the spacetime surrounding a supermassive string. The junction conditions across the boundary layer of the interior and exterior string solution are investigated. The time evolution of the string core radius, in a simplified model, is numerically obtained. It turns out that the evolution of $g_{\varphi\varphi}$ cannot be made consistent with the motion of the core of the string. The behavior of the core of the cosmic string, measured in the interior time, shows that the appearance of the causality-violating regions is an expression of the helical structure of time. [S0556-2821(96)01820-6]

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I. INTRODUCTION

Spinning-bounded sources in general relativity play a crucial role in understanding the connection between the mathematics in (2+1)-dimensional gravity on the one hand and the cosmological implications on the other hand [1–5]. Quite recently, considerable attention has been given to cosmic string solutions, due to the possibility of the formation of the controversial closed timelike curves (CTC's) [6–10]. Particularly, Gott's spacetime [11], generated by two moving cosmic strings, shows the intriguing possibility of CTC's. He shows that if the relative velocity is sufficiently high, CTC's will emerge, that circle the two strings as they pass each other. To be sure, there is a Cauchy horizon separating the region with CTC's from that without them. Ori [12] shows that in a Gott spacetime the CTC's are not restricted to some interaction region when the strings are near each other, but rather CTC's exist arbitrarily far away from both strings. However, Ori's characterization of the CTC's as preexisting is misleading. Cutler [13] showed that it is possible to find complete spacelike hypersurfaces extending to infinity in the Gott spacetime prior to which there are no CTC's. So an observer in the Gott spacetime prior to the Cauchy horizon may find no CTC's in his past light cone, and yet be able to encounter CTC's in the future.

Now, it is commonly believed that Hawking's chronology protection conjecture holds, i.e., CTC's cannot arise in a realistic universe [5,14–16]. Hawking enforces the conjecture by considering quantum-mechanical instabilities leading up to the Cauchy horizon. However, the Gott spacetime has no closed null geodesics (fountains) which were the main source of instability in the wormhole solution. Thus the instabilities encountered prior to the Cauchy horizon in the Gott solution are mild compared to those encountered in the wormhole solution [17–19]. It is also unclear whether it is unphysical to have in Gott spacetime CTC's at spacelike infinity [18]. For a collapsing finite string loop, it is possible that a singularity and a black hole is formed and that any CTC is confined within the event horizon (see Headrick and Gott [18] for discussion).

The models mentioned above do not shed enough light on the situation where the spinning string has a singular boundary, separating the interior vortex solution from the exterior conical string solution. A lot of work was done in constructing an exact solution for the nonrotating situation. See, for example, Gott [20] and Hiscock [21]. Further, spinning strings are stationary. Hence, one should investigate whether or not such models could be created by a dynamical process. Soleng [7] investigated a time-dependent interior solution of a homogeneous dust planet, a cross section of a cosmic string, and found CTC's for realistic energy-momentum tensors. However, the assumptions seem to be special, by the arbitrarily chosen energy production mechanism. One can extend the model of Soleng by considering the full coupled gravity-scalar-gauge field equations on the general, axially symmetric spacetime

$$ds^2=-f(r,t)^2[dt+\omega(r,t)d\varphi]^2+f(r,t)^2dz^2+dr^2+a(r)^2d\varphi^2, \tag{1.1}$$

by treating the scalar and gauge field also time dependent. In the first instance, we will consider f time independent.

The model of a cosmic string is that of a single complex scalar field ϕ minimally coupled to a U(1)-gauge field A_μ, with the complex scalar field interacting with itself through the standard ''mexican hat'' potential. The energy-momentum tensor will be

$$T_{\mu\nu}=\tfrac{1}{2}[(\mathcal{D}_\mu\phi)^*\mathcal{D}_\nu\phi+(\mathcal{D}_\nu\phi)^*\mathcal{D}_\mu\phi]+\mathcal{F}_{\mu\lambda}\mathcal{F}_\nu^\lambda-\tfrac{1}{2}g_{\mu\nu}\mathcal{D}_\alpha\phi(\mathcal{D}^\alpha\phi)^*-\tfrac{1}{8}\lambda g_{\mu\nu}(\phi\phi^*-\eta^2)^2-\tfrac{1}{4}g_{\mu\nu}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}, \tag{1.2}$$

where $\mathcal{D}\equiv\partial_\mu+ieA_\mu$, $\phi\equiv\mathcal{Q}e^{i\theta}\equiv\mathcal{Q}e^{in\varphi}$, $A_\mu=(0,A_2,0,A_4)$, $\mathcal{F}_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu$, and η represents the energy scale of the symmetry breaking. We use the coordinate sequence (t,z,r,φ) and units for which $G=\hbar=c=1$. It is the scalar (Higgs) field which plays the role of order parameter in the Ginzburg-Landau theory of type-II superconductivity and leads to the famous Meissner effect. It prevents the magnetic

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field from spreading out and gives rise to the vortices. These vortices have two characteristic lengths. First, the coherence length, the length it takes the order parameter to rise from its false vacuum $\phi=0$ to its superconducting vacuum $\phi=\eta$. Second, the penetration length, i.e., how far the magnetic field will be spread out. The vortex feature has its analogy in cosmology, i.e., cosmic strings, remnants of the very early Universe of very thin tubes of false vacuum. It is known that cosmic strings can lead to structures on cosmological scales such as galaxies and clusters of galaxies. Strings can be classified according to their winding number (or topological charge) n. It is a measure of the wrapping of the scalar-field phase θ around the string. From the requirement that ϕ must be single valued, i.e., θ varies by $2\pi n$ when we make a complete turn around a closed loop, it follows that the flux of vortex lines is quantized. Then, one obtains for the energy per unit length a lower bound $\mu_n \geqslant \pi|n|\eta^2$ for $e^2 \leqslant \lambda$ (Bogomol'nyi bound). The field equations become

$$G_{\mu\nu}=-8\pi T_{\mu\nu}, \tag{1.3}$$

$$\mathcal{D}_\mu \mathcal{D}^\mu \phi - 2\frac{\partial V}{\partial \phi^*}=0, \tag{1.4}$$

$$\nabla^\nu \mathcal{F}_{\nu\mu}=\tfrac{1}{2}ie[\phi(\partial_\mu\phi^* - ieA_\mu\phi^*)-\phi^*(\partial_\mu\phi+ieA_\mu\phi)]. \tag{1.5}$$

For the static situation, where the field variables depend on r and z only, approximate solutions can be constructed [22–24]. It was found that the solutions show significant deviation from the classical vortex solution.

II. THE TIME-DEPENDENT SPINNING STRING

For the interior region $\rho<\rho_s$ we obtain from the combination $G_{14}-\omega G_{11}$ and $G_{34}-\omega G_{13}$ the equations

$$\frac{\partial_\rho^2 \omega}{\omega}+\frac{\partial_\rho \omega}{\omega}\left(4\frac{\partial_\rho f}{f}-\frac{\partial_\rho a}{a}\right)=\frac{-16\pi}{\lambda f^2}(\partial_t \mathcal{X})^2, \tag{2.1}$$

and

$$\frac{\partial_\rho \partial_t \omega}{\omega}+2\frac{\partial_t \omega}{\omega}\left(\frac{\partial_\rho f}{f}-\frac{\partial_\rho a}{a}\right)=-16\pi\eta^2 \partial_\rho \mathcal{X} \partial_t \mathcal{X}. \tag{2.2}$$

The A_2 component of the gauge field satisfies ($G_{24}=0$)

$$\frac{\partial_\rho A_2}{A_2}=-\frac{e^2}{\lambda}\mathcal{X}^2\frac{P}{\partial_\rho P}. \tag{2.3}$$

Further, from Eq. (1.5) and the complex part of Eq. (1.4),

$$\partial_t^2 P=-\lambda\eta^2 f^2\frac{\partial_\rho \omega}{\omega}\partial_\rho P, \tag{2.4}$$

$$\mathcal{X}\sim\frac{1}{\sqrt{P\omega}}, \tag{2.5}$$

where we used the redefinitions $\mathcal{Q}\equiv\eta\mathcal{X}$, $A_4\equiv(1/e)(P-n)$, and $\rho\equiv\sqrt{\lambda}\eta r$. Assume that ω, P, and $\mathcal{X}$ are separable functions of ρ and t, say $\omega(\rho,t)=\Omega(t)\mathcal{J}(\rho)$, $\mathcal{X}(\rho,t)=\Psi(t)U(\rho)$, and $P(\rho,t)=\Pi(t)\mathcal{P}(\rho)$. Substitution into Eqs. (2.1) and (2.2), yields the solutions

$$\Psi(t)=\pm\sqrt{\frac{-\lambda a_1}{16\pi}t+b},$$

$$\Pi(t)=\alpha_1\cos kt+\alpha_2\sin kt,$$

$$\Omega(t)=\exp\left[\frac{-a_1a_2\lambda}{32\pi}t^2+a_2b\sqrt{\frac{-\lambda a_1}{16\pi}t+c}\right],$$

$$\mathcal{J}(\rho)=\frac{a(\rho)^2}{f(\rho)^2}\exp\left[\frac{-8\pi\eta^2}{a_2}U(\rho)^2+a_3\right], \tag{2.6}$$

where a_i, α_i, b, c, and k are constants. The radii of the core false vacuum and magnetic field tube are then at $\rho_\phi\approx 1$ and $\rho_A\approx 1/\sqrt{\alpha}$, respectively, where $\alpha\equiv e^2/\lambda$ the ratio of the scalar- to gauge-field masses. Further, $a(\rho)$ satisfies the equation (in the case of $f=1$)

$$\partial_\rho^2 a+3\gamma U\partial_\rho U\partial_\rho a+\left[\gamma(\partial_\rho U)^2(1+2\gamma U^2)+\gamma U\partial_\rho^2 U - \frac{1}{2}a_1U^2\right]a=0, \tag{2.7}$$

where $\gamma\equiv-8\pi\eta^2/a_2$.

If we substitute for the the ρ-dependent part of the scalar field the Nielsen-Olesen behavior $U=(1-e^{-\rho})$, we can investigate the metric component $g_{\varphi\varphi}=a(\rho)^2-\omega^2$ for some values of the constant a_i, b, c, and γ. $a(\rho)$ can be solved from Eq. (2.7) as a power series

$$a(\rho)=\rho-\frac{2}{3}\gamma\rho^3+\frac{5}{8}\gamma\rho^4+\left(-\frac{7}{20}\gamma+\frac{7}{30}\gamma^2+\frac{1}{40}a_1\right)\rho^5+\cdots. \tag{2.8}$$

In Fig. 1 we plotted $g_{\varphi\varphi}$ for large symmetry-breaking scale η [in comparison with the grand unified theory (GUT) scale]. We see that $g_{\varphi\varphi}$ can become negative, an unpleasant feature: there is a causality-violating region. The Killing vector ∂_φ, which has closed orbits, becomes timelike in that region. The moment upon which $g_{\varphi\varphi}$ becomes negative depends on several parameters [see Eq. (2.6)]. For example, the smaller the η, the later the negative region will be entered. Moreover, this moment depends on the behavior of the space-part U of the Higgs field, as can be seen from Eq. (2.6). Further, as we shall see in Sec. IV, the matching conditions between the interior and exterior string solution will fix some constants. For example, the sign of a_2, which makes the distinction between the appearance of the breakdown of $g_{\varphi\varphi}$ at finite time or at time infinity.

III. MATCHING CONDITIONS AND THE BOUNDARY LAYER

Consider the exterior metric

$$ds_+^2=-f^2[dt+md\varphi]^2+f^2dz^2+d\rho^2+\beta(\rho)^2d\varphi^2, \tag{3.1}$$

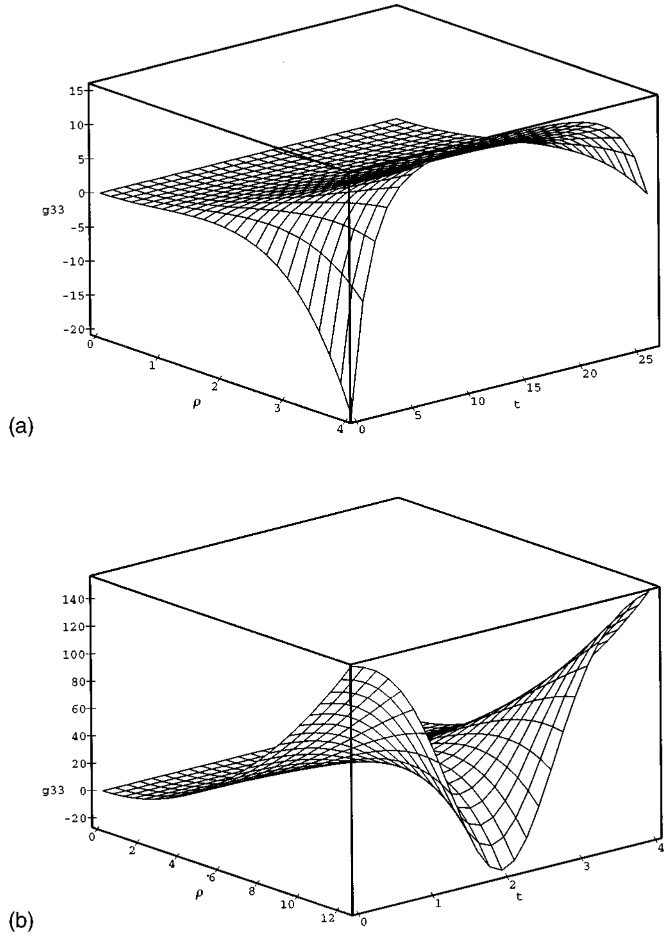

FIG. 1. Plot of the metric component $g_{\varphi\varphi}$ for $\eta=0.14$, $\lambda=1$, and $a_2>0$ (a). It turns out, the smaller the η, the later the breakdown of $g_{\varphi\varphi}$ occurs. The figure (b) represents the situation $a_2<0$.

where $m=4\mathcal{J}$ represents the angular momentum. This metric can be transformed to

$$ds_+^2=f^2[-dt^{*2}+dz^2]+d\rho^2+\beta(\rho)^2 d\varphi^2, \qquad (3.2)$$

by $t=t^*-m\varphi$, leaving a helical structure of time [4]. This jump property can also be formulated in context with torsion in Riemann-Cartan geometry [8,9,25] or to quantum-mechanical concepts [10]. Well known is the peculiar gravitational time delay effect [26]. For isolated GUT strings, $\beta(\rho)\rightarrow(1-4\mu)(\rho+\rho_0)$, where the angle deficit $\Delta\varphi=8\pi\mu$, μ the linear mass density of the string and ρ_0 a constant determining the origin of the exterior coordinate, so that the radial coordinates coincide in the interior and exterior coordinate system. It was argued [26], in the static limit with all the matter concentrated in the core, that the radius ρ_s of core of the string will always remain smaller than $4\mathcal{J}/(1-4\mu)$. So, when one takes for the maximum possible angular momentum per unit length, $\mathcal{J}=\mu\rho_s(1-4\mu)$, then ρ_s will always be larger then $4\mathcal{J}/(1-4\mu)$ as long as $\mu<\frac{1}{4}$, which is a realistic condition. In Sec. IV we compare this result with our time evolution of ρ_s.

Now the interior solution must be matched onto the exterior solution. Consider the hypersurface Σ, the boundary between the exterior and interior spacetime

$$ds_\Sigma^2\equiv\gamma_{ab}d\xi^a d\xi^b=-\epsilon d\tau^2+dz^2+\rho_s(\tau)^2 d\varphi^2, \qquad (3.3)$$

where $\rho_s(\tau)$ is the width of the scalar-gauge field core, of order 1 for $\alpha\approx1$ and τ the proper time on the boundary layer (latin indices take the values 0, 1, and 3). The condition of metric continuity on the hypersurface results in

$$f(\rho_s(\tau))=1,\;\; \omega(\rho_s(\tau))=4J(\rho_s(\tau)),\;\; a(\rho_s(\tau))=\beta(\rho_s(\tau)). \qquad (3.4)$$

Further, we have the discontinuity of the extrinsic curvature $\mathcal{K}_{ij}$, determined by the shell's stress tensor S_{ij} [27,28]

$$\epsilon([\mathcal{K}_j^i]-\delta_j^i[\mathcal{K}])=-8\pi S_j^i, \qquad (3.5)$$

where $[\mathcal{K}_{ij}]\equiv\mathcal{K}_{ij}^+-\mathcal{K}_{ij}^-$ represents the jump in the extrinsic curvature, the so-called Lanczos tensor, and $\epsilon=1$ for time-like hypersurfaces.[1]

Further, from the Einstein equations, we obtain

$$S_{i|j}^j+[T_i^n]=0, \qquad (3.6)$$

$$\{\mathcal{K}_j^i\}S_i^j+[T_n^n]=0, \qquad (3.7)$$

where $\{\mathcal{K}_{ij}\}\equiv\frac{1}{2}(\mathcal{K}_{ij}^++\mathcal{K}_{ij}^-)$, n denotes the coordinate in the direction of an outgoing normal to the shell and the vertical bar denotes covariant differentiation with respect to the metric on Σ. Following Berezin *et al.* [28], we obtain, from Eqs. (3.5) and (3.7)

$$S_i^j\mathcal{K}_j^{i+}+4\pi\epsilon S_i^j(S_j^i-\tfrac{1}{2}\delta_j^i S)+[T_n^n]=0. \qquad (3.8)$$

It is not difficult to calculate $\mathcal{K}_{ij}^+$ for the exterior metric (3.2) (see Appendix). The terms $\partial_\rho\beta$ and $\partial_\rho f$ in $\mathcal{K}_{ij}^+$ are determined, in the case of the supermassive string, by [29]

$$\partial_\rho\beta=\frac{-4\pi\eta}{\sqrt{\lambda}}(\mathcal{X}_{\rm out}^2-1)P_{\rm out}+\frac{1-4\pi|n|\eta}{\sqrt{\lambda}}, \quad \partial_\rho f=0, \qquad (3.9)$$

where $\mathcal{X}_{\rm out}$ and $P_{\rm out}$ are the exterior string fields. Further, for the surface stress tensor S_{ij}, we can apply the thin-wall approximation

$$S_{ij}=\lim_{\delta\to0}\int_{\rho_s-(1/2)\delta}^{\rho_s+(1/2)\delta}T_{ij}d\rho,$$

where δ represents the thickness of the shell and use an averaging procedure [30,28], or write out Eq. (3.6) in components:

[1]In the most simple case, when there would be no jump in the derivatives of g_{ij}, we would obtain, for γ [see Eqs. (2.6) and (2.7)],

$$\gamma=\frac{-1}{(\rho_s+\rho_0)U\partial_\rho U}.$$

γ heavily affects the behavior of $g_{\varphi\varphi}$. It is negative because $U(\rho_s)\partial_r U(\rho_s)$ is positive for the Nielsen-Olesen vortex solution. So, a_2 must be positive. This means that Fig. 1(a) is physically more likely.

$$\dot{S}_0^0+\frac{\dot{\rho}_s}{\rho_s}(S_0^0-S_3^3)=\frac{1}{4}\sqrt{\lambda}\,\eta^3\partial_\rho U^2\partial_t\Psi^2+\frac{\sqrt{\lambda}\,\eta}{4e^2a^2}\partial_\rho\mathcal{P}^2\partial_t\Pi^2, \tag{3.10}$$

and

$$\dot{S}_3^0+\frac{\dot{\rho}_s}{\rho_s}S_3^0=\frac{\sqrt{\lambda}\,\eta\omega}{e^2a^2}\partial_\rho\mathcal{P}^2\partial_t\Pi^2. \tag{3.11}$$

Further, an equation of state must be specified. Following Laguna-Castillo *et al.* [31,30], we take $S_0^0=-S_1^1$. We also consider the simplified model with $S_3^3=S_0^0+C$ (C a constant).

Substituting into Eq. (3.8) the results of the Appendix, we obtain the following differential equations for ρ_s, S_0^0, and S_3^0 [$T_{nn}^{\mathrm{out}}=0$ and $f=1$].

$$\begin{aligned}\ddot{\rho}_s=&-\frac{S_3^3}{S_0^0}\frac{\sqrt{\lambda}\,\eta\beta\partial_\rho\beta}{\rho_s^2}(\dot{\rho}_s^2+1)+\frac{\sqrt{\dot{\rho}_s^2+1}}{\sigma_{\mathrm{out}}S_0^0}\\&\times\left[8\pi\epsilon\left(S_0^{02}-\frac{1}{\rho_s^2}S_3^{02}+\frac{1}{4}S_3^{32}\right)+\frac{\eta^2U^2(\partial_t\Psi)^2}{2}\right.\\&\times\left(\Omega^2e^{(-16\pi\eta^2U^2+2a_3)/a_2}-1\right)-\frac{(\partial_t\Pi)^2\mathcal{P}^2}{2e^2a^2}\\&-\frac{\lambda\eta^4}{2}(\partial_\rho U)^2\Psi^2-\lambda\eta^2\frac{(\partial_\rho\mathcal{P})^2\Pi^2}{2a^2e^2}\\&\left.+\frac{\eta^2\Psi^2U^2\Pi^2\mathcal{P}^2}{2a^2}+V\right],\end{aligned} \tag{3.12}$$

$$\dot{S}_0^0=C\frac{\dot{\rho}_s}{\rho_s}+\sqrt{\lambda}\,\eta^3U\partial_\rho U\Psi\partial_t\Psi+\frac{\sqrt{\lambda}\,\eta}{e^2a^2}\mathcal{P}\partial_\rho\mathcal{P}\Pi\partial_t\Pi, \tag{3.13}$$

and

$$\dot{S}_3^0+\frac{\dot{\rho}_s}{\rho_s}S_3^0=\frac{\sqrt{\lambda}\,\eta}{a^2e^2}\Omega\mathcal{J}\Pi\partial_t\Pi\mathcal{P}\partial_\rho\mathcal{P}. \tag{3.14}$$

In Sec. IV we solve these equations numerically, by substituting the solutions (2.6), bearing in mind the relations $t=t^*-\omega\varphi$ and $\dot{t}^*=\sqrt{1+\dot{\rho}_s^2}$. The equation for $\ddot{\rho}$, [Eq. (3.12)] is comparable with the evolution equation of the spherical symmetric phase separation boundary studied by Berezin *et al.* [28]:

$$\ddot{\rho}=-\frac{2S_2^2}{S_0^0}\frac{(1+\dot{\rho}^2)}{\rho}+\frac{\sqrt{1+\dot{\rho}^2}}{S_0^0}(\epsilon_{\mathrm{out}}-\epsilon_{\mathrm{in}}), \tag{3.15}$$

where $\epsilon_{\mathrm{out}},\epsilon_{\mathrm{in}}$ represent the energy densities. For simple perfect fluids, the evolution behavior is comparable with the well-known equations for detonation waves [28]. In our case, the coefficient of the first term in the right-hand side of Eq. (3.12) contains the exterior solution $\beta(\rho_s)$, which, in turn, determines the mass density of the string. Further, the second term contains the time-dependent string-field components. If we substitute for the moment for β the well-known approximate solution, $(1-4\mu)\rho_s+\rho_0$, we obtain

$$\ddot{\rho}_s=-\sqrt{\lambda}\,\eta\frac{S_3^3(1-4\mu)[(1-4\mu)\rho_s+\rho_0]}{S_0^0\rho_s^2}(\dot{\rho}_s^2+1)+SF(t^*), \tag{3.16}$$

where $SF(t^*)$ represents the time-dependent string-field components of Eq. (3.12). Without this term, Eq. (3.16) can be integrated to yield

$$\begin{aligned}\sqrt{\dot{\rho}_s^2+1}\sim&\frac{1}{(1-4\mu)^2+1}\left[\rho_s-\frac{\rho_0}{1-4\mu}+\frac{\rho_0^2}{(1-4\mu)^2-1}\rho_s^{-1}\right.\\&\left.+\cdots+D\rho_s^{-(1-4\mu)^2}e^{(1-4\mu)\rho_0/\rho_s}\right],\end{aligned} \tag{3.17}$$

with D an integration constant. Solution (3.17) already shows the critical behavior of $\dot{\rho}_s$ with respect to μ and ρ_0.

IV. THE EVOLUTION OF THE SHELL

We solved the Eqs. (3.12)–(3.14) numerically in the most simple situation, where $a=\rho_s$ and $f=1$. We used the most realistic situation of Fig. 1(a), where the constant a_2 is positive (the breakdown of $g_{\phi\phi}$ will eventually take place in the far future). For β we will take for the moment, $(1-4\mu)(\rho_s+\rho_0)$, with μ the linear mass density. In Fig. 2 we plotted ρ_s as function of interior time and proper time ($\sigma_{\mathrm{out}}=1$ and $\epsilon=1$) for $\eta=0.01$ and compared the evolution with $\rho_m\equiv\omega/(1-4\mu)$. We took for μ the lower bound $\pi\eta^2$. In order to be able to compare the evolution of ρ_s with the evolution of $g_{\varphi\varphi}$, we used the same values for the several constants as in Fig. 1. Further, we took $e^2=\lambda=1$. We see that merely for large interior time ρ_s becomes smaller than ρ_m.

In Fig. 3 we plotted ρ_s for two different values of C, representing the field fluctuations [see Eqs. (3.12) and (3.13)]. It is evident that the evolution depends critically on the fine tuning of C and η. Further, it appears that by fine tuning the parameter, the evolution of ρ_s can be halted, a relatively long period of proper time. In all runs where ρ_s starts to blow up, the proper time stops flowing and the moment the interior time is comparable with the moment where $g_{\varphi\varphi}$ tends to become negative. In Fig. 4 we plotted ρ_s for $\eta=0.1$. We see that ρ_s decreases rapidly and the causality-violating region will never be encountered.

V. CONCLUSIONS AND OUTLOOK

The metric of a spinning string can have causality-violating regions. However, this phenomenon seems to be artificial. In the stationary situation, an arresting proof can, at some rate, be furnished [15]. The results of the time-dependent model, presented here in a simplified form, confirms merely the conjecture that the formation of CTC's model will be exceedingly unlikely for physically realistic parameters. One conjectures that when a complete loop is taken around the string (so φ acquires a phase shift of $e^{2\pi in}$), the interior time jumps by a factor $8\pi\omega$ and will be equal to the period that $g_{\varphi\varphi}$ remains positive. When the string core radius approaches the causality-violating regions (see Fig. 2), measured in interior time, the proper time on the string core stops flowing and the required parameters are

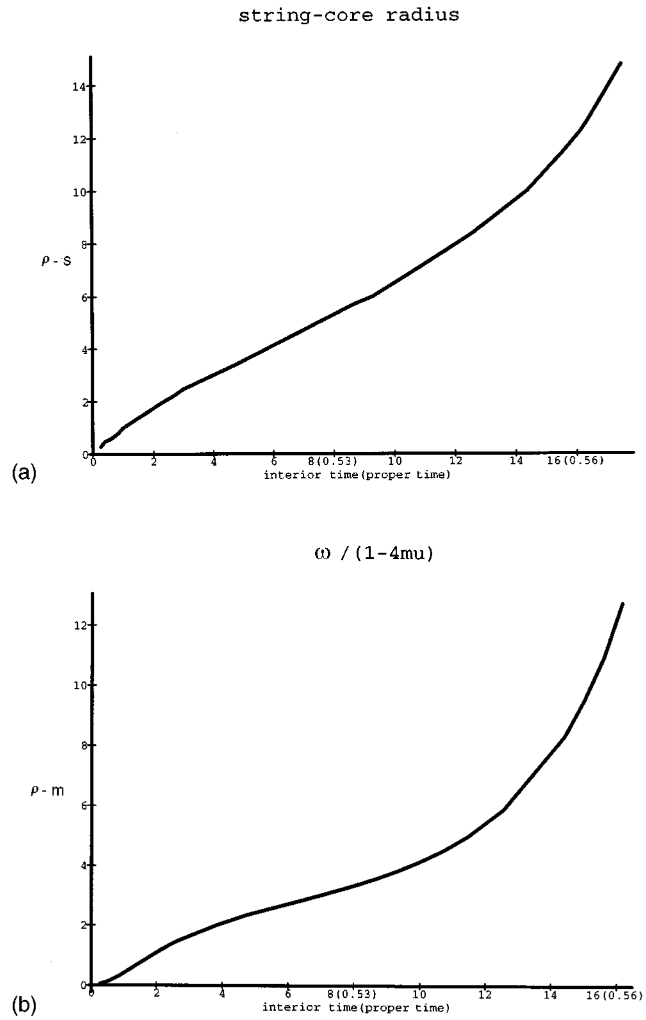

FIG. 2. Typical example of the time evolution in interior time of the string core radius for $\eta=0.01$ ($\sigma_{\rm out}=1$ and $\epsilon=1$). Further, the same values for the several parameters as for Fig. 1 were used. Between brackets the proper time is marked. Notice that at the moment ρ_s blows up, $g_{\varphi\varphi}$ tends to become negative (see Fig. 1) and the proper time stops flowing. In (b) we plotted $\rho_m \equiv \omega/(1-4\mu)$.

such that $g_{\varphi\varphi}$ will never enter the negative region.

Our model is far from being complete. First, one should handle the surface energy-momentum tensor S_i^j more carefully [28]. Second, one should not use a constant value for the mass density μ, but instead a mass density which follows from the exterior string field equations (3.9). Third, a thorough investigation of the differential equations for the several parameters is necessary, specially for the signs of $\sigma_{\rm out}$ and ϵ. From Eq. (3.12) we see that the time-dependent term in the right-hand side has an essential effect on the evolution of the core of the string and complicates the conditions on σ and ϵ with respect to the energy density. These items are currently under consideration by the author.

APPENDIX: THE LANCZOS TENSOR

In order to be able to calculate the extrinsic curvature

$$\mathcal{K}_{ij}^{\pm}=-\frac{\partial x_{\pm}^{\alpha}}{\partial \xi^i}\frac{\partial x_{\pm}^{\beta}}{\partial \xi^j}\nabla_{\beta}\mathcal{N}_{\alpha}, \tag{A1}$$

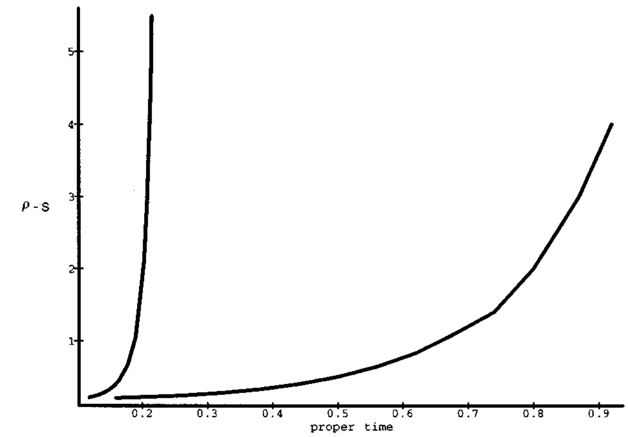

FIG. 3. Time evolution in proper time of the string core radius for $\eta=0.01$ and two different values of the field-fluctuation C, $C=0.1$ and $C=0.01$.

we need the unit normal vector $\mathcal{N}_{\mu}=\partial_{\mu} n_{|\Sigma}$, where $n(x^{\mu})$ is given by [27,28]

$$n(x^{\mu})=\sigma\frac{F(x^{\mu})}{\sqrt{|\partial_{\nu}F\partial^{\nu}F|}}, \tag{A2}$$

and where $F(x^{\mu})=0$ represents the equation for Σ (in the x_+^{μ} or x_-^{μ} coordinates). Further, $\mathcal{N}^{\mu}\mathcal{N}_{\mu}=\epsilon$, with $\epsilon=1$ for timelike hypersurfaces and $\epsilon=-1$ for spacelike hypersurfaces. σ determines the global geometry, i.e., how the inner geometry is stuck together with the outer one. $\sigma=1$ corresponds to increasing radius in the outward direction. In our situation we have for $F(x_+^{\mu})$

$$F(x_+^{\mu})=\beta(\rho)-R(t^*)$$

(for the moment β, the physical radius of the string, independent of time), $R(t^*)$ unknown. For the normal vector we then obtain

$$\mathcal{N}_0=-\sigma f\dot{\rho}_s, \quad \mathcal{N}_2=\sigma\sqrt{\dot{\rho}_s+1}, \tag{A3}$$

where the overdot means $d/d\tau$. The relation between the exterior time t^* and the proper time τ is

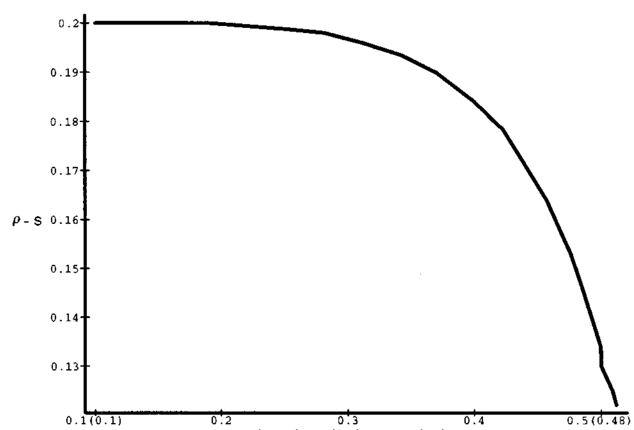

FIG. 4. Example of the evolution of ρ_s for $\eta=0.1$.

$$\dot{t}^* = \sqrt{1+\dot{\rho}_s^2}. \tag{A4}$$

Further, for the shell's ''radius,'' we have

$$\frac{dR}{dt^*} = \frac{f\dot{\rho}_s}{\sqrt{\dot{\rho}_s^2+1}}. \tag{A5}$$

For the several components of $\mathcal{K}_{ij}^+$, we obtain

$$\mathcal{K}_{00}^+ = \sigma \frac{1}{\sqrt{\dot{\rho}_s^2+1}}\left[\ddot{\rho}_s + \frac{\partial_\rho f}{f}(\dot{\rho}_s^2+1)\right], \tag{A6}$$

$$\mathcal{K}_{11}^+ = -\sigma f \partial_\rho f \sqrt{\dot{\rho}_s^2+1}, \tag{A7}$$

and

$$\mathcal{K}_{33}^+ = -\sigma\beta\partial_\rho\beta\sqrt{\dot{\rho}_s^2+1}. \tag{A8}$$

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