Reconciling inflation with openness

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It is already understood that the increasing observational evidence for an open universe can be reconciled with inflation if our horizon is contained inside one single huge bubble nucleated during the inflationary phase transition. In this frame of ideas, we show here that the probability of living in a bubble with the right Ω_0 (≈ 0.2) can be comparable to unity, rather than infinitesimally small. For this purpose we modify both quantitatively and qualitatively an intuitive toy model of ours. Therefore, inferring from the observations that $\Omega_0 < 1$ not only does not conflict with the inflationary paradigm, but rather supports therein the occurrence of a primordial phase transition. [S0556-2821(96)00220-2]

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I. INTRODUCTION

An open universe, i.e., without enough matter to halt eventually its expansion, ($\Omega_0 \approx 0.2$), agrees with most astronomical observations (see, e.g., Ref. [1]) and with their interpretations. For example, in connection with the formation of large scale structure in the cold dark matter (CDM) scenario, it gives the best fit to the observed clustering (see, e.g., Ref. [2]) yielding also the required [3] power on the large scales; it explains the dynamics of bound objects on relatively small scales (see, e.g., Ref. [4]); it also increases the age of the Universe, alleviating the conflict with the age of globular clusters (see, e.g., [5]); finally, it is in a better agreement with direct geometrical estimates from radio source number counts (see e.g., Refs. [4,6]). A low density universe is now preferrable even for theorists (e.g., [7,8]) when they essay to explain the small scale anisotropies measured in the cosmic microwave background (CMB). Quite naturally, as the flatness prediction, $\Omega_0 = 1$, is a basic paradigm of inflation, one may resort to the addition of a vacuum energy, $\Omega_0 = \Omega_{\rm CDM} + \Omega_\Lambda = 1$ (for an early suggestion see, e.g., Ref. [9]) and build a theory for Λ, see, e.g., Ref. [8]. Nevertheless, that Ω_0 may be less than one is certainly a stimulating challenge of modern cosmology, notwithstanding the obvious caveat that nothing is certain because (i) the present small scale observations, which mostly favor a low Ω_0, may be too limited to be representative of the whole Universe and (ii) the job of interfacing theory and observations for the CMB perturbations is still in its infancy (see, e.g., Ref. [10] for another solution). On the other hand, it is also possible to choose the initial conditions in inflation so as to give $\Omega_0 \approx 0.2$ today, either by starting with an extremely small density parameter at the beginning of inflation, or by assuming that inflation lasted less than 60 e foldings or so. Both possibilities, however, enter in conflict with the very spirit of inflation because they introduce the fine tuning of the initial conditions that inflation overcame and we certainly do not want to reintroduce them; moreover, there would be also a conflict with the microwave background isotropy [11].

In this work, we move instead from the fundamental notion [12–14] that the inside of a primordial bubble nucleated for quantum tunneling from a false vacuum (FV) to a true vacuum (TV) looks like an open universe in an external de Sitter space. Recently, substantial progress (upon which we build) has been achieved along this line in two different ways. The first is the single-bubble scenario [15,16] in one field inflation, where the *identical* bubbles inflated for about 60 e foldings after nucleation: our visible universe is contained inside one of these bubbles, and appears to be locally open. The second proposal [17] is the many-bubble scenario in two-field inflation where one field drives the inflationary slow rolling and the other undergoes a quantum tunneling in a direction orthogonal to the former, generating bubblelike open universes, with all possible density parameters, from zero to unity. Then, there is no reason to expect a preferential value of Ω_0: it must then be argued that possibly quantum cosmology will explain why we live in an $\Omega_0 = 0.2$ universe.

The model of this work implements also a many-bubble scenario, exploits fully the assets of two-field inflation [18] and has two useful features: the peak of the bubble nucleation (i) can be placed at any observed $\Omega_0 < 1$, and (ii) can be made narrow enough for our Universe to be regarded as typical. Furthermore, the absolute probability of residing in a bubble (whatsoever) may be made comparable to one (at least until some costraint is found), so that the use of the anthropic principle may be largely avoided. In our model bubble nucleation is made to end abruptly: thereafter, the external space (embodied in the residual fraction of false vacuum) undertakes a classical double inflationary tour exactly such as in the literature [19] that seeks a break in the canonical featureless perturbation power spectrum. In fact, this is an anomalous two-field inflation in which the one scalar field present (the other, in reality, is disguised as gravity) is exploited twice, quantistically (for the tunneling) and classically (for the second slow rolling).

Our model, already introduced [20] to produce large scale power in the CDM scenario out of the remnants of the primordial phase transition (see also [21]), contains now a result of [16] that specifies how to link the bubble's Ω_0 to the bubble's nucleation epoch N (number of e foldings between nucleation and end of inflation),

$$\Omega_0(N) = [1 + 4\exp\{2\,(N_H - N)\}]^{-1}, \tag{1}$$

where N_H corresponds to the horizon, i.e., is fixed by the request that the largest observable scale, $L_H = 2c/H_0$, crossed out the horizon N_H e foldings before the end of inflation (and is close to 60 for standard cosmological values [23]). Notice that according to Eq. (1), for $N\rightarrow\infty$, $\Omega_0\rightarrow 1$ as expected because the bubble is born with $\Omega_0=0$ and evolves toward flatness with time. Also, notice the other coincidence that bubbles born at N_H have today $\Omega_0=0.2$: this is the main difficulty in model building, because a horizon-sized bubble (or even smaller, were the need to arise) can easily be seen.

Our procedure is the following: given any value of Ω_0 we determine through Eq. (1) the corresponding $N(\Omega_0)$ and we end abruptly at that $N_E=N$ the bubble production via a feature in the potential (see below) that increases manifolds the Euclidean action. The universe contains then only bubbles that have been generated earlier, $N>N_E$, in higher number for lower N and have now attained all the $\Omega_0(N_E)<\Omega_0<1$. This break is also new with respect to our work of [20]. In Ref. [17], the lack of completion of the phase transition is dictated by the need to prevent us from seeing our bubble's walls; here, it solves that problem, but in a novel way, that may be useful in future improved work: a virtuous (or cunning) potential takes care of inflating the bubble size beyond the horizon after the bubble interior has become radiation dominated.

II. THE MODEL

To realize our scenario we need two prerequisites, [20]. First, we need two channels, a FV channel, to drive inflation in the parent universe, and a TV channel, to drive a shorter, but well appreciable, inflation inside the bubbles. Second, we need a tunneling rate tunable in time, in order to produce a nucleation peak at the right epoch. Our model has just these features: it is certainly not the unique possibility, but it is a rather simple and geometrically intuitive one.

The model works in fourth order gravity [24], and exploits two fields: one, Starobinsky's scalaron R (i.e., the Ricci scalar) drives the slow-rolling inflation; the second ψ performs the first order phase transition. The phase transition dynamics is governed both by the potential of ψ and by its coupling to R; the dynamics of the slow roll is ''built in'' in the fourth order Lagrangian: $\mathcal{L}_{\rm grav}=-R+R^2/[6M^2W(\psi)]$ ($c=\hbar=G=1$); the matter Lagrangian is instead standard and contains the usual potential $V(\psi)$. The coupling of the scalaron with ψ can be thought of as a field-dependent effective mass $M_{\rm eff}(\psi)=MW^{1/2}(\psi)\approx M$, just as the Brans-Dicke scalar-tensor coupling is a field-dependent Planck mass, and remains hidden in the Yukawa corrections [25] to Newton's gravity at 10^5–10^6 Planck lengths [24,26]. We are interested only in the last $N_T>N_H$ e foldings of inflation. This theory can be conformally transformed [27] into canonical gravity: $\tilde{g}_{\alpha\beta}=e^{2\omega}g_{\alpha\beta}, e^{2\omega}=|\partial\mathcal{L}/\partial R|=1-R/3M^2_{\rm eff}$. Then, it becomes undistinguishable from Einstein gravity with two fields ψ and ω, coupled by a potential $U(\psi,\omega)$ linear both in $V(\psi)$ and in $W(\psi)$.

The *ansatz* of a quartic $W(\psi)=1+8\lambda\psi^2(\psi-\psi_0)^2/\psi_0^4$, with two degenerate vacua at 0 and ψ_0 and a mass term $V(\psi)=m^2\psi^2/2$ realizes the two conditions discussed above: it carves, in fact, two parallel channels of different height, separated by a peak of height $\lambda/2$ at $\psi_{PK}=\psi_0/2$. The degeneracy of $W(\psi)$ in $\psi=0$ and $\psi=\psi_0$ is removed by $V(\psi)$; the TV channel remains exactly at $\psi_{\rm TV}=0$, while the FV channel remains approximately at $\psi_{\rm FV}=\psi_0$.

Let us now follow the evolution of a generic bubble nucleated at N. After nucleation, the bubble slow rolls down the TV channel for N e foldings, and then exits inflation, reaching the global minimum at $\omega=\psi=0$, where it reheats and enters its Friedmannian radiation-dominated era (RDE). At the same time, the external space also slow rolls for nearly N e foldings down the FV channel. After this first inflation, however, the external space undergoes a second inflation along the ψ direction, and for $\omega\approx 0$. This second inflation lasts approximately $N_2=2\pi\psi_0^2$ e foldings. Only then does the external space reach its RDE. This second inflation is crucial in our model. In fact, we know that the bubble's walls grow, as seen from inside, at the velocity of light as long as the external space is de Sitter space. During the first inflation, when both the bubble and the external space are de Sitter space, the walls expand comovingly; the bubble comoving size is then $L\approx L_He^{N-N_H}$. During the second inflation, when the bubble is in RDE, the causal horizon expands overcomovingly as a^2, where a is measured from the inside. At the end of this era the bubble size acquires, therefore, an extra factor of a becoming $L\approx L_He^{N-N_H}\times e^{N_2}$. When finally both the inside and the outside of the bubble are in the Friedmannian regime, the bubble's walls expand again comovingly (as long as they are of superhorizon size): now, however, the causal horizon is faster than comoving expansion, and the walls may become visible. The relevance of the second inflationary stage is that it allows the bubble comoving size to become as large as wanted by tuning N_2; in particular, the bubble can be yet invisible because many times larger than our present horizon. Unlike the other models, in which we will never see the walls, because the external space is always de Sitter space, in our model we will see the walls, but only in a remote future. How remote is this future depends on N_2. Since we want $N\approx N_H$, it is enough to choose N_2 larger than, say, 100, to ensure that the bubble walls are well outside our present horizon. This implies a very reasonable $\psi_0>4$.

With these general considerations in mind, we can proceed now to work out the details of the nucleation process [12]. The tunneling rate Γ can be written as $\Gamma=\mathcal{M}^4\exp(-S)$, where $\mathcal{M}$ is of the order of the energy of the false vacuum, and S is the minimal Euclidean action, i.e., the action for the so-called bounce solution of the Euclidean equation of motion. For the calculation of S we can use directly standard physics [12] provided we satisfy the thin wall limit (TWL), which is not difficult to achieve. The result is [20]

$$S=(N/N_1)^4, \quad N_1\approx\sqrt{m^3\psi_0/(M^2\lambda)}. \tag{2}$$

Thus $N\gg N_1$ to avoid spinodal decomposition [22]. In particular, we will exploit the fact that S decreases as N^4 (bubble nucleation more likely later than earlier) and increases as λ^2 (a quenching opportunity). Finally, the relevant parameter [28] $Q=4\pi\Gamma/9H^4$, which measures the number of bubbles per horizon volume per Hubble time can be written as [20]

$$Q(N)=\exp((N_0^4-N^4)/N_1^4),$$

$$(\mathcal{M}/M_{\rm eff})^4=(9/64\pi)\exp((N_0/N_1)^4). \quad (3)$$

Thus N_0 tells us when the physics is being done because it estimates the peak of the bubble spectrum.

To summarize, our model has four free parameters: M, setting the slow-rolling inflationary rate; m, setting the energy difference between vacua; λ, setting the barrier height; and finally ψ_0, setting the separation between vacuum channels. These constants completely define the slow-rolling and the phase transition dynamics: we should fix all four of them, but for the time being we fix only the two combinations thereof we have called N_0 and N_1; we hope the remaining freedom will suffice to meet forthcoming constraints. Furthermore, there is one feature we have to insert by hand; it is the mechanism by which the bubble production is halted *ex abrupto*. We have chosen to do this by inserting *ad hoc* at the desired point, $N=N_E$, a sudden ramp in λ which then becomes effectively $\lambda(\omega)$. The detailed form of $\lambda(\omega)$ is not important as long as the increase in S is sharp and sudden enough to quench instantaneously the bubble production. In fact, given N_E as said above, we find N_0 by fixing the fraction X of the FV phase that we want to turn into the TV phase through Eq. (4) below. The visual difference with [20] consists in the fact that the FV channel here does not merge into the TV channel at some $N>0$, but plunges onto it perpendicularly only at $N\approx 0$. Therefore, as already mentioned, whatever remains in the FV phase slow rolls classically over a double inflationary path [19], an essential feature in our model (see the top panel of Fig. 1).

We now proceed to the evaluation of the tunneling probability [14]: $dn/dt=\Gamma V_{\rm FV}$, where dn is the number of bubbles per horizon nucleated in the time interval dt,

$$V_{\rm FV}=a^3(4\pi/3H^3)\exp(-I),$$

and

$$I(t)=(4\pi/3)\int_{-\infty}^{t}dt'\Gamma(t')\left(a(t')\int_{t'}^{t}dt''/a(t'')\right)^3$$

..

Incidentally, the fact that dn/dt is proportional to the FV volume left at the time t (i.e., the volume not already occupied by bubbles) implies that a turnaround is possible in the bubble production. This is because, due to the doubly exponential nature of the process, after a certain time the FV volume fraction may decrease faster than the product $a^3\Gamma$ increases. This turnaround would indicate that the transition is being completed: again, in this paper, unlike [20], we interrupt the transition sharply at N_E just before all this happens [17].

Now, bubble spectra can be obtained either through numerical integrations (see Fig. 2 in [20] where for the first time realistic bubble spectra were given) or, better, through a working analytic approximation, which is necessary to understand the complex role of our four parameters. Algebraic details will be given in future work alongside with further applications. Here, we outline the procedure: first, we change from one to the other of the four equivalent time variables at our disposal, t, N, L, and Ω_0 where L is the scale associated with N, as seen from the inflating background, $L=L_H\exp(N-N_H)$, and Ω_0 is given in Eq. (1), by writing $dn/dL=(dn/dN)/L=-(dn/dt)/HL$. Second, we get the fraction of space in bubbles of size L, $(dP/dL)=(L/L_T)^3(dn/dL)$, and third, the fraction of space in bubbles of a given Ω_0, $(dP/d\Omega_0)=(dP/dN)(d\Omega_0/dN)^{-1}$. Fourth, we evaluate the fraction of space in useful bubbles:

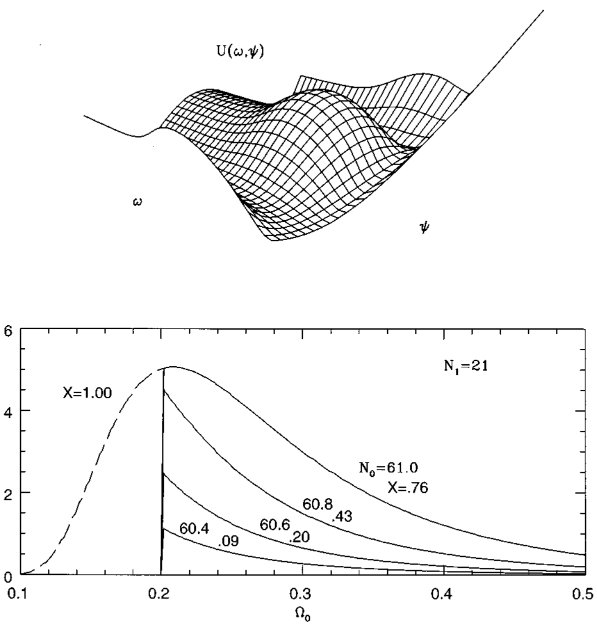

FIG. 1. Upper panel: plot of the conformal potential $U(\omega,\psi)$ in arbitrary units. The front slice is the plane $\omega=0$ where U becomes the parabola $V(\psi)$. The communication for the classical slow roll between the FV channel (right) and the TV channel (left) is established only at $\omega\approx 0$: this allows a second inflation (of the background) of tunable length toward the absolute vacuum $\omega=\psi=0$. Lower panel: the four full lines show spectra of universe bubbles vs density parameter Ω_0 sharply peaked at $\Omega_0=0.2$: each curve is labeled by the value of N_0 and by the value of $X<1$ which apply; in each case the phase transition is ended at $N_E=60$ by the ramp in the barrier shown above. The broken line achieves instead $X=1$.

$$X(N_0,N_E)=\int_{\infty}^{N_E}(dP/dN)dN. \quad (4)$$

In the bottom panel of Fig. 1 we give five examples of spectra obtained with $N_1=21$ from four values of N_0 decreasing from top to bottom, as shown. The leftmost broken curve, $N_0=61$, achieves the completion of the phase transition, $N_E=0$, and is hence labeled with $X=1$. This curve is important because it achieves turnaround approximately at $\Omega_0=0.2$ or $N=60$: hence, when the break in the bubble production is introduced at $N_E=60$, the same curve, now shown as the uppermost solid line with a vertical cut, yields a truncated spectrum with $X=0.76$ and the peak where needed. *A fortiori*, lower values of N_0, which would achieve later turnarounds, attain lower values of X as shown, provided the break is kept in place, but continue to peak at $\Omega_0=0.2$.

III. CONCLUSIONS

We contributed one special toy model to the lore of the flat, inflationary universe filled to a non-negligible fraction by superhorizon-sized underdense bubbles, which approximate open universes. This, of course, reconciles the astronomical observations in favor of $\Omega_0 \approx 0.2$ with inflation. Our own bubble universe is one of an infinite number of similar bubbles. Contrary to the single-bubble scenario [14–16] and the many-bubble model of Ref. [17], in our model the external space also ends inflation a tunable number of *e* foldings after the bubbles enter their RDE's; the bubbles themselves reenter the horizon in the distant future. The interesting features of our model are that (i) we can tune the parameters to achieve maximal probability for the nucleation of TWL bubbles around any observed $\Omega_0 < 1$ without assuming special initial conditions and without destroying the CMB isotropy, and that (ii) this probability is not infinitesimally small.

It is worth remarking again that the measure of Ω_0 along with the assumption that the Universe had an inflationary epoch, and that our position is generic, puts strong constraints on the shape and on the fundamental parameters of the primordial potential and eventually will fix them, although for the time being we are limited to such combinations thereof like N_0 and N_1.

Inside each bubble one has the usual mechanism of generation of inflationary perturbations [6,15,16]. It is then possible that reducing the local Ω_0 to 0.2 is sufficient to reconcile canonical CDM with large scale structure. However, evidences are increasing toward the presence of huge voids in the distribution of matter in the present Universe, and for velocity fields that are difficult to explain without a new source of strong inhomogeneities. If this were the case, the need may arise for an additional primordial phase transition occurring 50 or so *e* foldings before the end of inflation, exactly as in Ref. [20].

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