

Speed meter as a quantum nondemolition measuring device for force

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Braginsky has proposed a *speed meter* (a speed or momentum measuring device), consisting of a small Fabry-Perot cavity rigidly attached to a freely moving test mass. This work devises an optical readout strategy which enables the meter, when monitoring a classical force via speed changes, to beat the standard quantum limit—at least in principle. [S0556-2821(96)02818-4]

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I. INTRODUCTION

A laser interferometer gravitational wave detector is, in essence, a device for monitoring a classical force (the gravitational wave) that acts on freely moving test masses (the interferometer's suspended mirrors). ‘Advanced’ detectors, expected to operate in the LIGO-VIRGO interferometric network [1] in the middle or later part of the next decade, will be constrained by the *standard quantum limit* (SQL) for force measurements [2,3]:

$$\Delta F_{\text{SQL}} = \sqrt{\hbar m / \tau^3}. \quad (1)$$

Here \hbar is Planck's constant, m is the mass of the test body on which the force acts, and τ is the duration of the force.

It is well known that the SQL is not an absolute barrier to further sensitivity improvements [2]. With cleverness, one can devise so-called *quantum nondemolition* (QND) measurement schemes, which beat the SQL. Although fairly practical QND techniques have actually been devised for resonant-mass gravitational-wave detectors [4], no practical QND technique yet exists for the interferometric gravitational-wave detectors on which LIGO-VIRGO is based. The effort to devise such a technique is of great importance for the long-term future of the LIGO-VIRGO network.

Although a practical QND technique for such detectors is not yet known, several idealized techniques have been formulated [5] and are playing helpful roles in the search for a practical technique. Most of these idealized techniques are based on optical measurements of a test mass's position. One, however, is based on measurements of the mass's speed or momentum. This ‘speed meter’ has been devised in ini-

tial, conceptual form by Braginsky [6], and he has argued that it should be capable of beating the SQL.

The purpose of this paper is to demonstrate that, when coupled to a specific optical readout scheme that we have devised, Braginsky's speed meter does, indeed, beat the SQL, at least in principle.

II. THE BASIC IDEA OF THE SPEED METER

The fundamental idea underlying Braginsky's speed meter is to attach a small, rigid Fabry-Pérot cavity to the test mass, whose speed is to be measured. The cavity's two mirrors are to have identical transmissivities and negligible losses, in the idealized variant we shall analyze. This means that, when the cavity is at rest and is excited by laser light that is precisely in resonance with one of the cavity's modes, the light passes straight through the cavity without reflection and emerges from the other side unchanged. When the cavity starts moving, by contrast, it sees the incoming light Doppler shifted; and, as a result, the light emerging from the other side gets phase shifted by an amount

$$\Delta\phi = \frac{\omega_0 v \tau}{c}, \quad (2)$$

where v is the cavity's speed, ω_0 is its eigenfrequency, and τ is its ringdown time. Here and throughout we assume that $v \ll c/(\omega_0 \tau)$.

By measuring the phase of the emerging light, one can infer the speed of the cavity without learning anything about its position. This absence of information about position implies (Braginsky has argued) that such a device should be

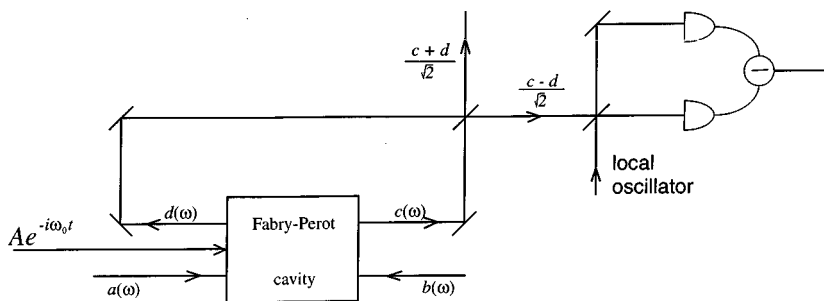


FIG. 1. The speed meter and its optical readout.

able to evade any backaction force of the measurement on the cavity's velocity (or momentum), and therefore should be capable of beating the SQL.

III. OUR READOUT SCHEME FOR THE SPEED METER, AND AN ANALYSIS OF ITS PERFORMANCE

In this section we shall exhibit an optical readout scheme for such a speed meter which does, indeed, enable it to beat the SQL. Our readout scheme is sketched in Fig. 1, whose details will become more clear in what follows.

Let

$$\psi_{\text{in}} = A e^{-i\omega_0(t-x/c)} + \int_0^\infty d\omega \sqrt{\frac{\hbar\omega}{sc}} a(\omega) e^{-i\omega(t-x/c)} \quad (3)$$

be the incoming field on the left side of the cavity. The first term on the right-hand side of Eq. (3) represents classical pumping, and the second term shows quantum fluctuations of the incoming field; x is the position of the cavity and acquires time dependence when the cavity is moving; s is the area of the beam. Also $a(\omega)$, $b(\omega)$, $c(\omega)$, and $d(\omega)$ represent annihilation operators for four modes as in Fig. 1, and $t(\omega)$ and $r(\omega)$ represent frequency-dependent transmission and reflection coefficients of the cavity, respectively. In our setup $t(\omega_0) = 1$, $r(\omega_0) = 0$.

Any motion of the cavity induces a time dependence of x in Eq. (3), and hence with respect to the cavity the classical pump acquires frequencies different from ω_0 . The resulting

effect of the cavity motion is to scatter the classical part of the incoming wave into modes which would otherwise carry only vacuum fluctuations. A simple calculation produces the relations

$$c(\omega) = t(\omega)a(\omega) + r(\omega)b(\omega) + iA \frac{\omega_0}{c} \sqrt{\frac{sc}{\hbar\omega}} [t(\omega) - 1] \tilde{X}(\omega - \omega_0), \quad (4)$$

$$d(\omega) = r(\omega)a(\omega) + t(\omega)b(\omega) + iA \frac{\omega_0}{c} \sqrt{\frac{sc}{\hbar\omega}} r(\omega) \tilde{X}(\omega - \omega_0), \quad (5)$$

where $\tilde{X}(\Omega)$ is defined by

$$x(t) = \int_{-\infty}^{\infty} \tilde{X}(\Omega) e^{-i\Omega t} d\Omega \quad (6)$$

We assume that the cavity is pushed by an external signal force $F_s(t)$ (due, e.g., to a gravitational wave). Then its position obeys the free-mass equation of motion

$$F = F_s + F_{\text{fl}} = m\ddot{x}, \quad (7)$$

where F_{fl} stands for the random force produced by quantum fluctuations of the light. This force, as evaluated using momentum conservation, has the Fourier transform

$$\tilde{F}_{\text{fl}}(\Omega) = \sqrt{\frac{W\hbar\omega_0}{2\pi c^2}} \{ [1 - t(\omega_0 + \Omega)] [a(\omega_0 + \Omega) + a^\dagger(\omega_0 - \Omega)] - r(\omega_0 + \Omega) [b(\omega_0 + \Omega) + b^\dagger(\omega_0 - \Omega)] \} \quad (8)$$

for $\Omega \ll \omega_0$, where $W = scA^2/2\pi$ is the power of the incoming wave. Formula (8) can be simplified by noting that for the Fabry-Perot cavity $t + r = 1$, so

$$\tilde{F}_{\text{fl}}(\Omega) = \sqrt{\frac{W\hbar\omega_0}{2\pi c^2}} r(\omega_0 + \Omega) [a(\omega_0 + \Omega) - b(\omega_0 + \Omega) + a^\dagger(\omega_0 - \Omega) - b^\dagger(\omega_0 - \Omega)]. \quad (9)$$

It is clear from Eq. (9) that $f(\omega) = [a(\omega) - b(\omega)]/\sqrt{2}$ is the only combination of the incoming modes which appears, multiplied by x , in the interaction part of the Hamiltonian. Therefore, all information about the motion of the cavity should be recorded in f and f^\dagger . The obvious suitable choice of readout is

$$e(\omega) = \frac{c(\omega) - d(\omega)}{\sqrt{2}}, \quad (10)$$

since putting Eqs. (6), (7), (9), into (10), we can express it solely as a function of f and the signal force:

$$e(\omega_0 + \Omega) = [t(\omega_0 + \Omega) - r(\omega_0 + \Omega)] f(\omega_0 + \Omega) + 2i \frac{W\omega_0}{m\Omega^2 c^2} r(\omega_0 + \Omega)^2 [f(\omega_0 + \Omega) + f^\dagger(\omega_0 - \Omega)] + \sqrt{2}i \frac{\omega_0}{c} \sqrt{\frac{2\pi W}{\hbar\omega_0}} r(\omega_0 + \Omega) \frac{F_s(\Omega)}{m\Omega^2}. \quad (11)$$

It is attractive to read out $e(\omega)$ using homodyne detection as sketched in Fig. 1. The measurement output then is the homodyne quadrature

$$y(\Omega) = e(\omega_0 + \Omega)e^{i\psi(\Omega)} + e^\dagger(\omega_0 - \Omega)e^{-i\psi(\Omega)}, \quad (12)$$

where $\psi(\Omega)$ is a phase factor that we shall fix below so as to minimize the noise (see also [7]). Then the quantum noise spectral density in this measured quantity, as computed from the formula $\langle y(\Omega)y(\Omega') \rangle = S_y(\Omega)\delta(\Omega + \Omega')$, is

$$S_y(\Omega) = 2\alpha^2(\Omega)\{1 - \cos[2\psi(\Omega)] - 2\alpha(\Omega)\sin[2\psi(\Omega)]\} + 1, \quad (13)$$

where $\alpha(\Omega) = (2W\omega_0/m\Omega^2c^2)|r(\omega_0 + \Omega)|^2$. This noise is minimized for

$$\tan(2\psi_{\min}(\Omega)) = \frac{1}{\alpha(\Omega)}. \quad (14)$$

Putting in explicitly $r(\Omega) = i\Omega\tau_{\text{ringdown}}/(1 + i\Omega\tau_{\text{ringdown}})$ we see that for large power $W \gg mc^2/\omega_0\tau_{\text{ringdown}}^2$ the minimum noise is

$$S_y(\Omega) = \frac{1}{4\alpha(\Omega)^2} = \frac{m^2\Omega^4c^4}{16W^2\omega_0^2|r(\omega_0 + \Omega)|^2}. \quad (15)$$

Here τ_{ringdown} is the e -folding time for resonant light to escape from the cavity. Now suppose that the form of the signal, $F_s(\Omega)$, is known and we use an optimal filter to search in the output $y(\Omega)$ to see whether the signal is actually present. The signal-to-noise ratio for this search is given by

$$\frac{S}{N} = \frac{1}{2\pi} \int_{\Omega_1}^{\Omega_2} \frac{16\pi W\omega_0|r(\omega_0 + \Omega)|^2}{mc^2\Omega^2} \frac{|F_s(\Omega)|^2}{\hbar m\Omega^2} d\Omega. \quad (16)$$

For a narrow-band measurement ($\tau_{\text{meas}} \gg \tau_{\text{ringdown}}$) $\psi(\Omega)$ is a constant ($\psi \rightarrow mc^2/4W\omega_0\tau_{\text{ringdown}}^2$) which makes homodyne detection technically easier (see Fig. 1), and we have

$$\frac{S}{N} = \frac{W}{W_{\text{SQL}}} \left(\frac{S}{N} \right)_{\text{SQL}}, \quad (17)$$

where

$$\left(\frac{S}{N} \right)_{\text{SQL}} = \frac{1}{\hbar m} \int \frac{|F_s|^2}{\Omega^2} d\Omega \quad (18)$$

and

$$W_{\text{SQL}} = \frac{mc^2}{16\pi\omega_0\tau_{\text{ringdown}}^2}. \quad (19)$$

Thus the minimum detectable force may be lower than the standard quantum limit by a factor of $\sqrt{W_{\text{SQL}}/W}$:

$$F_{\min} = \sqrt{\frac{W_{\text{SQL}}}{W}} \Delta F_{\text{SQL}}. \quad (20)$$

The minimum detectable force is proportional to $1/\sqrt{W}$. Equation (20) obviously holds for a broadband signal as well, but the expression for W_{SQL} is different and the homodyne phase acquires frequency dependence, which makes the detection very difficult.

IV. CONCLUSION

We have shown that, for the readout scheme of Fig. 1, the minimum measureable force F_{\min} is given by

$$F_{\min} = \sqrt{\frac{W_{\text{SQL}}}{W}} \Delta F_{\text{SQL}}, \quad (21)$$

where ΔF_{SQL} is the standard quantum limit [Eq. (1)], W is the laser power, and W_{SQL} is the minimum laser power required for beating the SQL:

$$\begin{aligned} W_{\text{SQL}} &= \frac{mc^2}{16\pi\omega_0\tau^2} \\ &= 5 \times 10^4 \text{ W} \frac{m}{10 \text{ kg}} \frac{4 \times 10^{15} \text{ s}^{-1}}{\omega_0} \left(\frac{0.01 \text{ s}}{\tau} \right)^2. \end{aligned} \quad (22)$$

Although it is not outrageous to imagine achieving the laser powers $W > W_{\text{SQL}}$ at which Eq. (21) reports a beating of the SQL, there are many serious practical obstacles to implementing such a speed meter in a real interferometric gravitational-wave detector. Nevertheless, this speed meter might contain the conceptual seeds from which will grow a practical QND scheme for the LIGO-VIRGO network.

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