Isgur-Wise function for $\Lambda_b \to \Lambda_c$ in the BS approach

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(Received 24 May 1996)

In the heavy quark limit, the heavy baryon Λ_Q ($Q=b$ or c) can be regarded as composed of a heavy quark and a scalar light diquark which has good spin and flavor quantum numbers. Based on this picture we establish the Bethe-Salpeter (BS) equation for Λ_Q in the leading order of $1/m_Q$ expansion. With the kernel containing both the scalar confinement and one-gluon-exchange terms we solve the BS equation numerically. The Isgur-Wise function for $\Lambda_b \to \Lambda_c$ is obtained numerically from our model. A comparison with other model calculations is also presented. It seems that the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ drops faster than that for $B \to D$. The differential and total decay widths for $\Lambda_b \to \Lambda_c l \overline{\nu}$ are given in the limit $m_{b,c} \to \infty$. [S0556-2821(96)00219-6]

PACS number(s): 12.39.Hg, 11.10.St, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

The past few years have seen much progress in heavy flavor physics due to the discovery of the new flavor and spin symmetries $SU(2)_f \times SU(2)_s$ in the heavy quark limit and the establishment of the heavy quark effective theory (HQET) [1]. In the framework of the HQET the processes involving heavy quarks can be simplified since the HQET may reduce the number of form factors. For instance, in the leading order of $1/m_Q$ expansion only one form factor (the Isgur-Wise function) remains for $\Lambda_b \to \Lambda_c$ transition. However, to get the complete physics one still has to face the basic problem, the nonperturbative effects of QCD. As the formally exact equation to describe the relativistic bound state, the Bethe-Salpeter (BS) equation [2,3] can be simplified to a great extent in the heavy quark limit and has been applied to give many theoretical results concerning heavy mesons [4]. At present, there are not many experimental results for heavy baryons. But in the future we may expect more and more data coming out from the CERN e^+e^- collider LEP and the forthcoming B factory. Actually, recently the OPAL Collaboration has measured some physical quantities for Λ_b such as the lifetime of Λ_b and the product branching ratio for $\Lambda_b \to \Lambda l^- \overline{\nu} X$ [5]. Hence, the study of heavy baryons is of particular interest in the near future. It is the motivation of the present paper to establish the BS equation for the heavy baryon in the heavy quark limit and then solve it numerically by assuming some reasonable form of its kernel, and consequently give some phenomenological predictions.

When the quark mass is very heavy comparing with the QCD scale $\Lambda_{\rm QCD}$, the light degrees of freedom in a heavy baryon Λ_Q ($Q=b$ or c) becomes blind to the flavor and spin quantum numbers of the heavy quark because of the $SU(2)_f \times SU(2)_s$ symmetries. Therefore, the angular momentum and flavor quantum numbers of the light degrees of freedom (the light diquark) become good quantum numbers which can be used to classify heavy baryons. For example, Λ_Q and $\Sigma_Q^{(*)}$ correspond to the zero and one angular momentum states of the light diquark, respectively. Hence, it is natural to regard the heavy baryon to be composed of a heavy quark and a light diquark. In the present paper we concentrate on Λ_Q which has a scalar light diquark $S_{[ud]}$ with [ud] flavor quantum number and zero spin and isospin. Other heavy baryons such as $\Sigma_Q^{(*)}$ can be dealt with in the similar way.

Based on the above picture of the composition of the heavy baryon, the three-body system is simplified to two body system. We will establish the BS equation for the heavy baryon in this picture. Then, we will solve this equation numerically by assuming that the kernel contains the scalar confinement and one-gluon-exchange terms. Then, we will apply the BS equation to calculate the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ and the decay width for the semileptonic decay of Λ_b to Λ_c in the order $m_{b,c} \to \infty$.

The remainder of this paper is organized as follows. In Sec. II we establish the BS equation for the heavy quark and light scalar diquark system and discuss the form of its kernel. Consequently, this equation can be solved numerically. Then, in Sec. III we apply the solution of the BS equation to calculate the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ and also the differential and total decay widths for $\Lambda_b \to \Lambda_c l \overline{\nu}$. Comparison with other model calculations for the Isgur-Wise function will also be presented. Finally, Sec. IV is reserved for summary and discussions.

II. THE BS EQUATION FOR Λ_Q

As discussed in Introduction, Λ_Q is regarded as the bound state of a heavy quark and a light scalar diquark. Based on this picture we can define the BS wave function of Λ_Q by

$$\chi(x_1,x_2,P)=\langle 0|T\psi(x_1)\varphi(x_2)|\Lambda_Q(P)\rangle, \tag{1}$$

where $\psi(x_1)$ and $\varphi(x_2)$ are the wave functions of the heavy quark and the light scalar diquark, respectively, $P=m_{\Lambda_Q}v$ is the momentum of Λ_Q, and v is its velocity. Let m_Q and m_D be the masses of the heavy quark and the light diquark in

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the baryon, $\lambda_1 = m_Q/(m_Q+m_D), \lambda_2 = m_D/(m_Q+m_D)$, and p be the relative momentum of the two constituents. The BS wave function in the momentum space is defined as

$$\chi(x_1,x_2,P) = e^{iPX}\int \frac{d^4p}{(2\pi)^4} e^{ipx}\chi_P(p), \tag{2}$$

where $X = \lambda_1 x_1 + \lambda_2 x_2$ is the coordinate of the center of mass and $x = x_1 - x_2$. The momentum of the heavy quark is $p_1 = \lambda_1 P + p$ and that of the diquark is $p_2 = -\lambda_2 P + p$.

The derivation of the BS equation for the two fermion systems can be found in textbooks [3]. In the same way we can prove that for the fermion and scalar object system the form of the BS equation is still valid. $\chi_P(p)$ satisfies the BS equation

$$\chi_P(p) = S_F(\lambda_1 P + p)\int \frac{d^4q}{(2\pi)^4} G(P,p,q)\chi_P(q) \times S_D(-\lambda_2 P + p), \tag{3}$$

where $G(P,p,q)$ is the kernel which is defined as the sum of the two particle irreducible diagrams. In the following we will use the variables $p_l = v\cdot p - \lambda_2 m_{\Lambda_Q}, p_t = p - (v\cdot p)v$. Then, in the leading order of $1/m_Q$ expansion, we have

$$S_F(\lambda_1 P + p) = \frac{i(1+\not v)}{2(p_l + E_0 + m_D + i\epsilon)},$$
$$S_D(-\lambda_2 P + p) = \frac{i}{p_l^2 - W_p^2 + i\epsilon}, \tag{4}$$

where $W_p = \sqrt{p_t^2 + m_D^2}$ and E_0 is the binding energy. The corrections to Eq. (4) are from $O(1/m_Q)$.

In general, $\chi_P(p)$ can be expanded as

$$\chi_P(p) = (A + B\not v + C\not p + D\not v\not p)u_{\Lambda_Q}(v,s), \tag{5}$$

where $u_{\Lambda_Q}(v,s)$ is the spinor of Λ_Q with helicity s and A, B, C, D are Lorentz scalar functions.

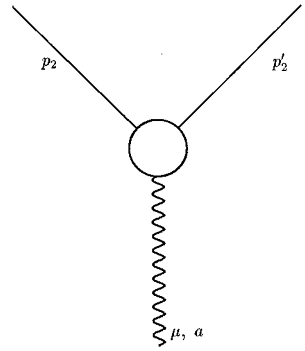

FIG. 1. The diquark-gluon-diquark vertex.

From Eqs. (3) and (4) it can be seen that

$$\not v\chi_P(p) = \chi_P(p). \tag{6}$$

Combining Eqs. (5) and (6), we immediately see that

$$\chi_P(p) = \phi_P(p)u_{\Lambda_Q}(v,s), \tag{7}$$

where $\phi_P(p)$ is a scalar field. This form is consistent with our picture about the heavy baryon.

We assume the kernel has the form

$$-iG = I\otimes IV_1 + v_\mu \otimes (p_2 + p_2')^\mu V_2, \tag{8}$$

where the first term arises from scalar confinement and the second one is from one-gluon-exchange diagram. The vertex of gluon with two scalar diquarks is proportional to $(p_2+p_2')_\mu$ (p_2 and p_2' are the momenta of the two scalar diquarks on the vertex) and a form factor $F[(p_2+p_2')^2]$ (see Fig. 1). Substituting Eqs. (4), (7), and (8) into Eq. (3) we have,

$$\phi_P(p) = -\frac{1}{(p_l + E_0 + m_D + i\epsilon)(p_l^2 - W_p^2 + i\epsilon)}\int \frac{d^4q}{(2\pi)^4} G(P,p,q)\phi_P(q). \tag{9}$$

Now, consider the vertex of two heavy quarks with gluon. The momenta of the two heavy quarks are $p_1 = \lambda_1 m_{\Lambda_Q} v + p$ and $p_1' = \lambda_1 m_{\Lambda_Q} v + q$, respectively. p and q are of the order $\Lambda_{\rm QCD}$. In the heavy quark limit the heavy quark is almost on-shell, therefore, $p_l = q_l$ on this vertex. Hence, we can make the covariant instantaneous approximation [4] in the kernel. Then, Eq. (9) becomes

$$\phi_P(p) = -\frac{i}{(p_l + E_0 + m_D + i\epsilon)(p_l^2 - W_p^2 + i\epsilon)}\int \frac{d^4q}{(2\pi)^4} \times(\tilde V_1 + 2p_l \tilde V_2)\phi_P(q), \tag{10}$$

where $\tilde V$ is defined as $\tilde V \equiv V|_{p_l = q_l}$.

In general, $\phi_P(p)$ can be the function of p_l and p_t. Defining $\tilde\phi_P(p_t) = \int (dp_l/2\pi)\phi_P(p)$, one gets immediately the BS equation for $\tilde\phi_P(p_t)$:

$$\tilde\phi_P(p_t) = -\frac{1}{2(E_0 - W_p + m_D)W_p}\int \frac{d^3q_t}{(2\pi)^3} \times(\tilde V_1 - 2W_p \tilde V_2)\tilde\phi_P(q_t). \tag{11}$$

If one knows the form for the kernel $\tilde V_1$ and $\tilde V_2$, then

$\widetilde{\phi}_P(p_t)$ can be obtained. Consequently, $\phi_P(p)$ can be solved since after integrating q_l in Eq. (10), we get the relation between $\phi_P(p)$ and $\widetilde{\phi}_P(q_t)$:

$$\phi_P(p)=\frac{i}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\int\frac{d^3q_t}{(2\pi)^3}\times(\widetilde{V}_1+2p_l\widetilde{V}_2)\widetilde{\phi}_P(q_t). \tag{12}$$

The kernel $\widetilde{V}_1$ and $\widetilde{V}_2$ for the BS equation in the meson case was given in [4] as

$$\widetilde{V}_1|_{\text{meson}}=\frac{8\pi\kappa'}{[(p_t-q_t)^2+\mu^2]^2}-(2\pi)^3\delta^3(p_t-q_t)\int\frac{d^3k}{(2\pi)^3}\frac{8\pi\kappa'}{(k^2+\mu^2)^2}, \tag{13}$$

$$\widetilde{V}_2|_{\text{meson}}=-\frac{16\pi}{3}\frac{\alpha_{seff}}{(p_t-q_t)^2+\mu^2}, \tag{14}$$

where κ' and α_{seff} are coupling parameters related to scalar confinement and one-gluon-exchange diagram, respectively. The second term on the right-hand side of Eq. (13) is the counterterm which removes the infrared divergence in the integral equation. From the potential model [6] and the BS equation analysis in the meson case [4], κ' is around 0.2. The parameter μ is introduced to avoid the infrared divergence in numerical calculations. The limit $\mu\to0$ is taken in the end.

In the baryon case, since confinement is still due to scalar interaction, the form of $\widetilde{V}_1$ need not be changed. Only the parameter κ' has to be replaced by κ which describes the confinement interaction between the quark and diquark. However, since the diquark is not a pointlike object, there should be a form factor $F(Q^2)(Q=p_2-p_2')$ to describe the vertex of the gluon and two scalar diquarks. In general, this vertex is (Fig. 1)

$$ig_s\frac{\lambda^a}{2}(p_2+p_2')^\mu F(Q^2)$$

with λ's being the Gell-Mann color matrices and g_s the strong coupling constant. The form of $F(Q^2)$ was assumed as [7]

$$F(Q^2)=\frac{\alpha_{seff}Q_0^2}{Q^2+Q_0^2}, \tag{15}$$

where Q_0^2 is a parameter which freezes $F(Q^2)$ when Q^2 is very small. In the high energy region the form factor is proportional to $1/Q^2$ which is consistent with perturbative QCD calculations [8]. By analyzing the electromagnetic form factor for the proton, it was found that $Q_0^2=3.2\ \text{GeV}^2$ can lead to consistent results with the experimental data [7].

Based on the above analysis we find that the kernel for the BS equation in the baryon case is of the form

TABLE I. Values of κ and α_{seff} for $m_D=700$ MeV.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08	0.1
α_{seff}	0.67	0.70	0.72	0.74	0.75

$$\widetilde{V}_1=\frac{8\pi\kappa}{[(p_t-q_t)^2+\mu^2]^2}-(2\pi)^3\delta^3(p_t-q_t)\int\frac{d^3k}{(2\pi)^3}\frac{8\pi\kappa}{(k^2+\mu^2)^2},$$

$$\widetilde{V}_2=-\frac{16\pi}{3}\frac{\alpha_{seff}^2Q_0^2}{[(p_t-q_t)^2+\mu^2][(p_t-q_t)^2+Q_0^2]}. \tag{16}$$

There are two parameters κ and α_{seff} in the kernel. However, there should be some relation between them since when we solve the BS equation (11) numerically, the binding energy should satisfy the relation

$$m_{\Lambda_Q}=m_Q+m_D+E_0, \tag{17}$$

where we have omitted corrections from $O(1/m_Q)$ since we are working in the heavy quark limit. From the BS equation solutions in meson case it has been found that the values $m_b=5.02$ GeV and $m_c=1.58$ GeV give predictions which are in good agreement with experiments [4]. Hence, in the baryon case we expect

$$m_D+E_0=0.62\ \text{GeV}. \tag{18}$$

On the other hand, the dimension of κ is three and that of κ' is two. This extra dimension in κ should be caused by nonperturbative diagrams which include the frozen form factor $F(Q^2)$ at low momentum region. Since Λ_{QCD} is the only parameter which is related to confinement, we expect that

$$\kappa\sim\Lambda_{\text{QCD}}\kappa'. \tag{19}$$

Therefore, in our numerical calculations we let κ vary in the region between 0.02 GeV^3 and 0.1 GeV^3. The diquark mass m_D is chosen to vary from 650 MeV to 800 MeV. In order to satisfy the relation (18) we obtain the parameters for different values of m_D. The results for $m_D=700$ MeV are shown in Table I.

When $m_D=650$ MeV, we find that $\alpha_{seff}=0.60$ for $\kappa=0.02\ \text{GeV}^3$ and $\alpha_{seff}=0.71$ for $\kappa=0.1\ \text{GeV}^3$. When $m_D=800$ MeV, $\alpha_{seff}=0.79$ for $\kappa=0.02\ \text{GeV}^3$ and $\alpha_{seff}=0.84$ for $\kappa=0.1\ \text{GeV}^3$. Having these two parameters we obtain the numerical solution for the BS wave function directly.

III. THE ISGUR-WISE FUNCTION FOR $\Lambda_b\to\Lambda_c$

In this section we will apply the BS equation for heavy baryons to obtain the numerical result for the Isgur-Wise function for $\Lambda_b\to\Lambda_c$. The transition diagram is plotted in Fig. 2. In the limit $m_{b,c}\to\infty$, there is only one form factor, the Isgur-Wise function, to describe the weak transition from Λ_b to Λ_c:

$$\langle\Lambda_c(v')|\overline{c}\gamma_\mu b|\Lambda_b(v)\rangle=\xi(\omega)\overline{u_{\Lambda_c}}(v')\gamma_\mu u_{\Lambda_b}(v), \tag{20}$$

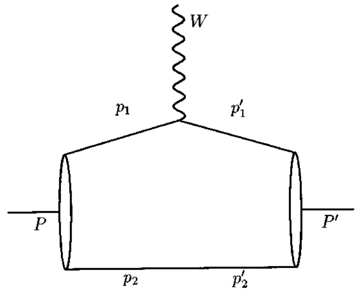

FIG. 2. The weak transition diagram for $\Lambda_b \to \Lambda_c$.

where $\omega = v \cdot v'$ is the velocity transfer and u_{Λ_b} and u_{Λ_c} are the Dirac spinors of Λ_b and Λ_c, respectively.

On the other hand, the transition matrix element of $\Lambda_b \to \Lambda_c$ is related to the BS wave functions of Λ_b and Λ_c by the equation

$$\langle \Lambda_c(v')|\overline{c}\gamma_\mu b|\Lambda_b(v)\rangle = \int \frac{d^4p}{(2\pi)^4} \overline{\chi_{P'}}(p')\chi_P(p)S_D^{-1}(p_2), \tag{21}$$

where P (P') is the momentum of Λ_b (Λ_c). $\overline{\chi_{P'}}(p')$ is the wave function of the final state $\Lambda_c(v')$ which satisfies the constraint

$$\overline{\chi_{P'}}(p)\not{v}' = \overline{\chi_{P'}}(p). \tag{22}$$

The scalar part of the final state BS wave function obeys the same BS equation as Eq. (9). Then from Eqs. (20) and (21), one gets immediately

$$\xi(\omega) = \int \frac{d^4p}{(2\pi)^4} \phi_{P'}(p')\phi_P(p)S_D^{-1}(p_2). \tag{23}$$

Since in the weak transition the diquark acts as a spectator, its momentum in the initial and final baryons should be the same, $p_2 = p_2'$. Then, we can show that

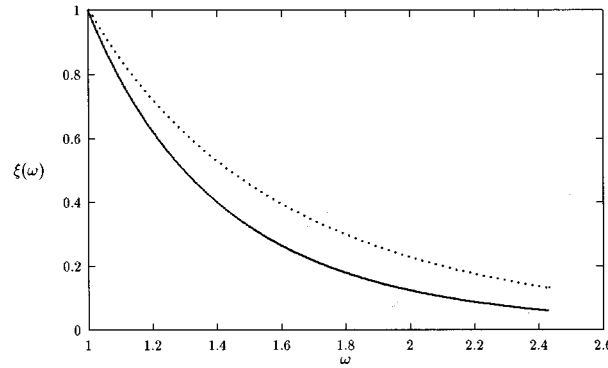

FIG. 3. The Isgur-Wise function for $\Lambda_b \to \Lambda_c (m_D = 700$ MeV). For the solid line $\kappa = 0.02$ GeV^3 and for the dotted line $\kappa = 0.1$ GeV^3.

$$p' = p + m_D(v' - v), \tag{24}$$

where again we omitted the $O(1/m_Q)$ corrections. From Eq. (24) we can get the relations between p_l' p_t' and p_l, p_t straightforwardly

$$\begin{aligned} p_l' &= p_l\omega - p_t\sqrt{\omega^2-1}\cos\theta, \\ p_t'^2 &= p_t^2 + p_t^2(\omega^2-1)\cos^2\theta + p_l^2(\omega^2-1) \\ &\quad - 2p_l p_t \omega\sqrt{\omega^2-1}\cos\theta, \end{aligned} \tag{25}$$

where θ is the angle between p_t and v_t'.

Substituting the relation between $\phi_P(p)$ and $\widetilde{\phi}_P(p_t)$ [Eq. (12)] into Eq. (23) and after integrating the p_l component by selecting the proper contour, we have

$$\begin{aligned} \xi(\omega) = \int \frac{d^3p_t}{(2\pi)^3} & \frac{1}{2W_p(E_0+m_D-W_p)(E_0+m_D-\omega W_p - p_t\sqrt{\omega^2-1}\cos\theta)} \int \frac{d^3r_t}{(2\pi)^3} [\widetilde{V}_1(p_t'-r_t) - 2(\omega W_p \\ & + p_t\sqrt{\omega^2-1}\cos\theta)\widetilde{V}_2(p_t'-r_t)]|_{k_l=-W_p} \widetilde{\phi}_{P'}(r_t) \int \frac{d^3l_t}{(2\pi)^3} [\widetilde{V}_1(p_t-l_t) - 2\omega W_p \widetilde{V}_2(p_t-l_t)]\widetilde{\phi}_P(l_t), \end{aligned} \tag{26}$$

where the relation (25) has been substituted into the above equation.

In Eq. (26) all the three-dimensional integral can be simplified to one-dimensional integral. Furthermore, the normalization constant of the BS wave function should be chosen such that the Isgur-Wise function is equal to one at zero momentum transfer. In Fig. 3 we plot numerical result for the Isgur-Wise function with the parameter $m_D = 700$ MeV. For other values of m_D in the range of 650 MeV to 800 MeV, the shape of the Isgur-Wise function does not change a lot.

The slope of the Isgur-Wise function at $\omega = 1$, which is of particular interest, is defined as

$$\left.\frac{d\xi(\omega)}{d\omega}\right|_{\omega=1} = -\rho^2. \tag{27}$$

In Table II we list the values of ρ^2 for different m_D and κ. The values of ρ^2 outside (inside) the brackets correspond to $\kappa = 0.02$ GeV^3 (0.1 GeV^3), respectively.

TABLE II. Values of ρ^2.

m_D(GeV)	0.65	0.7	0.8
ρ^2	1.4 (2.4)	1.4 (2.4)	1.6 (2.4)

It can be seen from Table II that the slope is insensitive to the values of m_D.

The Isgur-Wise function has also been calculated in other models. In Ref. [9] Guo and Kroll use the Drell-Yan-type overlap integrals for the model hadronic wave functions of Λ_b and Λ_c to obtain the following form of the Isgur-Wise function:

$$\xi(\omega)=\left(\frac{2}{\omega+1}\right)\exp\left(-2\varepsilon^2 b^2\frac{\omega-1}{\omega+1}\right)\frac{K_6(2\varepsilon b/\sqrt{\omega+1})}{K_6(\sqrt{2}\varepsilon b)}, \tag{28}$$

where ε is the light scalar diquark mass and b is related to the average transverse momentum of the constituents in the heavy baryon. K_l is defined as

$$K_l(x)=\int_{-x}^{\infty} dz e^{-z^2}(z+x)^l.$$

The Isgur-Wise function obtained by Jenkins, Manohar, and Wise [10] from the soliton model has the form

$$\xi(\omega)=0.99\exp[-1.3(\omega-1)]. \tag{29}$$

The MIT bag model calculation by Sadzikowski and Zalewski [11] gives the result

$$\xi(\omega)=\left(\frac{2}{\omega+1}\right)^{3.5+1.2/\omega}. \tag{30}$$

The values of ρ^2 from the above models are listed in the following:

$$\rho^2=\begin{cases} 2.9(3.7)[9] & \text{for } b=1.18(1.77)\ \text{GeV}^{-1}, \\ 1.3\ [10], \\ 2.4\ [11]. \end{cases} \tag{31}$$

We can see that different models may give results for the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ with some difference. $\rho^2 \geqslant 1.3$ in all these model calculations. The ρ parameter of the Isgur-Wise function for $B \to D$ has also been calculated.

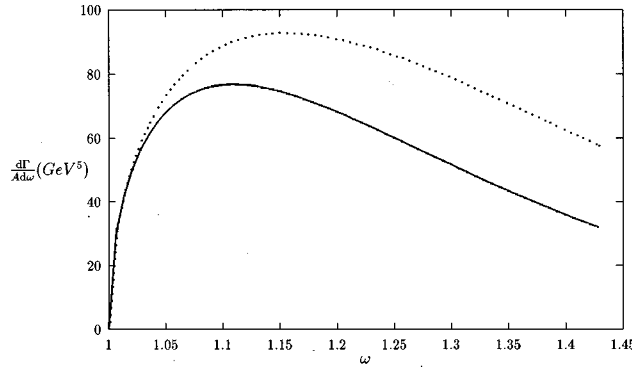

FIG. 4. The differential decay width for $\Lambda_b \to \Lambda_c l \overline{\nu}$ ($m_D = 700$ MeV). For the solid line $\kappa = 0.02$ GeV^3 and for the dotted line $\kappa = 0.1$ GeV^3.

In the BS equation approach [4] ρ^2 varies from 1.0 to 1.1. Other model calculations for ρ in $B \to D$ transition give the following values: $\rho^2 = 1.19 \pm 0.25$ [12]; $\rho = 1.13 \pm 0.11$ [13]; and $\rho = 1.20 \pm 0.17$ [14]. Because of the large uncertainties, we cannot draw a definite conclusion by comparing the slopes of the Isgur-Wise function at the zero recoil point in the meson and baryon cases. However, it seems that the Isgur-Wise function drops faster in the baryon case than in the meson case. At least in the BS approach the ρ parameter is bigger in the baryon case than that in the meson case.

The decay width for the semileptonic transition $\Lambda_b \to \Lambda_c l \overline{\nu}$ can also be predicted by using the Isgur-Wise function obtained. The differential decay width has the form [9–15]

$$\frac{d\Gamma}{d\omega}=\frac{2}{3}m_{\Lambda_c}^4 m_{\Lambda_b} A \xi^2(\omega)\sqrt{\omega^2-1}[3\omega(\eta+\eta^{-1})-2-4\omega^2], \tag{32}$$

where $\eta = m_{\Lambda_b}/m_{\Lambda_c}$ and $A = G_F^2/(2\pi)^3 |V_{cb}|^2 B(\Lambda_c \to ab)$. $|V_{cb}|$ is the Kobayashi-Maskawa matrix element. $B(\Lambda_c \to ab)$ is the branching ratio for the decay $\Lambda_c \to a([1/2]^+) + b(0^-)$ through which Λ_c is detected since the structure for such decay is already well known. The plot for $A^{-1}(d\Gamma/d\omega)$ is shown in Fig. 4 for $m_D = 700$ MeV. Again, for other values of m_D the results change only a little.

After integrating ω in Eq. (32), we have the total decay width for $\Lambda_b \to \Lambda_c l \overline{\nu}$. For $m_D = 700$ MeV,

$$\Gamma(\Lambda_b \to \Lambda_c l \overline{\nu})=\begin{cases} 4.2B(\Lambda_c \to ab)\times 10^{10}\ \text{s}^{-1} & \text{when } \kappa=0.02\ \text{GeV}^3, \\ 5.7B(\Lambda_c \to ab)\times 10^{10}\ \text{s}^{-1} & \text{when } \kappa=0.1\ \text{GeV}^3. \end{cases} \tag{33}$$

When $m_D = 650$ MeV, $\Gamma(\Lambda_b \to \Lambda_c l \overline{\nu})$ is $4.3(5.9)B(\Lambda_c \to ab) \times 10^{10}$ s^{-1} for $\kappa = 0.02(0.1)$ and when $m_D = 800$ MeV, $\Gamma(\Lambda_b \to \Lambda_c l \overline{\nu})$ is $4.1(5.0)B(\Lambda_c \to ab) \times 10^{10}$ s^{-1} for $\kappa = 0.02(0.1)$.

IV. SUMMARY AND DISCUSSIONS

In this paper we established the BS equation for the heavy baryon which is considered as composed of a heavy quark and a light scalar diquark. By assuming the kernel containing scalar confinement and one-gluon-exchange terms we solve the BS equation numerically. Furthermore, we applied the obtained results to calculate the Isgur-Wise function for $\Lambda_b \to \Lambda_c$. It is found that in the BS approach, the Isgur-Wise function for $\Lambda_b \to \Lambda_c$ decreases faster than for $B \to D$. A comparison with other model calculations is also presented. The slope parameter ρ^2 at the zero recoil point for $\Lambda_b \to \Lambda_c$ is larger than 1.3 in all the present model calculations. In the limit $m_{b,c} \to \infty$, we also calculated the differential and total decay widths for $\Lambda_b \to \Lambda_c l \overline{\nu}$.

Although, the BS equation is formally the exact equation to describe the bound state, there is much difficulty in applying it to the real physical state. The most difficult point is that we cannot solve out the form of the kernel. Hence, we have to use some phenomenological kernel which is reasonable because of the success of the potential model. This leads to some uncertainties. In our approach, the parameters κ and α_{seff} in the kernel are not exactly determined. Therefore, we let them to vary in some reasonable range. The experimental data concerning Λ_Q in the future can help to fix the parameters in our model.

We have worked in the heavy quark limit. The physical predictions such as $\Gamma(\Lambda_b \to \Lambda_c l \overline{\nu})$ will be subjective to $1/m_Q$ corrections. The study on the $1/m_Q$ corrections will appear in our later work.

ACKNOWLEDGMENTS

One of us (X.-H.G.) would like to thank the Japan Society for the Promotion of Science (JSPS) for the support. He is also indebted to Dr. H.-Y. Jin for helpful discussions.

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