Supersymmetric b-τ unification, gauge unification, and fixed points

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The equality assumption of the b and τ Yukawa couplings at the grand-unification scale can strongly constrain the allowed parameter space of supersymmetric models. We examine the constraints in the case that there is a discrepancy $\gtrsim 10\%$ in the gauge coupling unification assumption (which necessarily implies large perturbations at the grand scale). The constraints are shown to diminish in that case [most significantly so if $\alpha_s(M_Z)\approx 0.11$]. In particular, the requirement that the t Yukawa coupling h_t is near its quasifixed point may not be necessary. We discuss the colored-triplet threshold as a simple example of a source for the discrepancies, and comment on its possible implications. In addition, we point out that supersymmetric (as well as unification-scale) threshold corrections to h_t shift the fixed-point curve in the m_t-tanβ plane. The implications for the prediction of the Higgs boson mass are briefly discussed. [S0556-2821(96)01219-2]

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I. INTRODUCTION

Unification of the b and τ Yukawa couplings [1] is known to be consistent with the assumption of low-energy supersymmetry [2]. However, the allowed parameter space depends sensitively on the exact value of the strong coupling $\alpha_s(M_Z)=0.12\pm 0.01$ used in the calculation [3]. In particular, using the results from gauge coupling unification to calculate the b and τ Yukawa couplings, h_b and h_τ, respectively, strongly constrains the allowed range of the Higgs sector parameter $\tan\beta\equiv\langle H_2\rangle/\langle H_1\rangle$ to $\tan\beta\sim 1$ or $\tan\beta\gg 1$ [4,5].

Gauge coupling unification (including low-energy threshold corrections but neglecting corrections at the grand-unification scale) generically implies $\alpha_s(M_Z)\gtrsim 0.13$ and $\alpha_s(M_G)\sim 0.04$ [6,7] (where M_G denotes the unification point). The one-loop[1] expression for the weak-scale b to τ mass ratio is

$$\frac{m_b(M_Z)}{m_\tau(M_Z)}\sim 0.9\left[\frac{\alpha_s(M_Z)}{\alpha_s(M_G)}\right]^{8/9}Y, \qquad (1)$$

where the 0.9 factor is from hypercharge renormalization, $Y<1$ is a complicated function of the Yukawa couplings, which is important for large couplings, and $m_\tau(M_Z)=1.75$ GeV. Equation (1) and gauge unification imply (when neglecting Y) the prediction $m_b(M_Z)\sim 4.5$ GeV. In comparison, the allowed (one standard deviation) range is $m_b(M_Z)\lesssim 3.2$ GeV [8] (but because of low-energy renormalization the upper bound is a function of α_s). The QCD corrections are thus too large and need to be compensated by either large Yukawa coupling[2] which diminish Y (and also the prediction for α_s) [3–5,10] or finite one-loop supersymmetric threshold corrections to m_b (that are proportional to $\tan\beta$) [11,12]. Both mechanisms can be realized in the large $\tan\beta$ regime. On the other hand, in the small $\tan\beta$ regime only the former is relevant, and the allowed region is strongly constrained in $\tan\beta$ by requiring for the top Yukawa coupling $h_t(m_t)\gtrsim 1.1$, i.e., that h_t is near its quasifixed point [13,14]. It is interesting to note that for $\tan\beta\sim 1$ the Higgs sector imitates that of the standard model (SM) and contains a light SM-like Higgs boson,[3] $m_{h^0}^{\text{one loop}}\lesssim 100$ GeV, which is within reach of the CERN e^+e^- collider LEP II [17,18,15]. Hence, in this minimal framework, Higgs boson searches contain information about Yukawa unification.

However, the large predicted values of $\alpha_s(M_Z)$ (note that the prediction increases quadratically with m_t) are somewhat uncomfortable phenomenologically [19]. Particularly so if the $Z\to b\bar{b}$ width is significantly larger than what is predicted in the SM, as is currently implied by experiment [20]. (In that case, the predicted α_s is typically subject to large and positive low-energy threshold corrections [6], which further aggravate the potential problem.) Low-energy corrections could have a large and negative contribution to the α_s prediction only if (a) the low-energy spectrum is extremely heavy and degenerate, i.e., the correction parameters[4] M_1, M_2, and M_3 defined in Ref. [21] are large and equal, or (b) $M_2\gg M_1, M_3$ (see Fig. 5(a) of Ref. [21]). The former mechanism is not very likely, as it implies a degeneracy between colored (M_3) and noncolored (e.g., M_2) particles,

[1]In our numerical calculations of gauge and Yukawa couplings we will follow the procedure of Ref. [4] using two-loop renormalization group equations [three-loop equations for $\alpha_s(Q<M_Z)$]. The procedure is extended in a straightforward manner to include low-energy corrections to m_b (see below).

[2]A similar situation was discussed in the nonsupersymmetric case, e.g., in Ref. [9].

[3]This is when considering finite QCD corrections (but see a discussion below) to m_t and resummation of leading logarithms, which are the two most important higher-order corrections. The formal one-loop bound does not account for these effects by definition, and is higher by $10-15$ GeV (for example, see [15]). I thank Howard Haber for the discussion of this point. See also [16].

[4]The leading logarithm correction to $\alpha_i^{-1}(M_Z)$ is given by $(-\delta b_i/2\pi)\ln(M_i/M_Z)$ where $\delta b_i=25/10, 25/6, 4$ for $i=1, 2, 3$, respectively. $\alpha_{1,2,3}$ denotes the hypercharge (normalized by 5/3), weak and strong couplings, respectively.

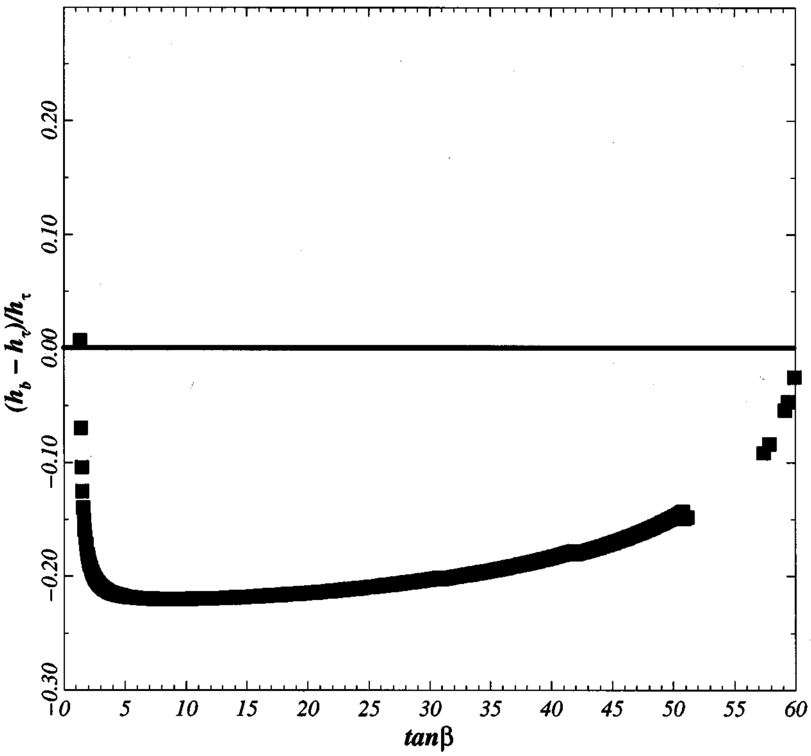

FIG. 1. The unification-scale difference $h_b - h_\tau$ is shown in h_τ units for $m_b(M_Z)=3$ GeV, $\alpha_s(M_Z)=0.12$, $m_t^{\rm pole}=170$ GeV and as a function of $\tan\beta$. Note the rapid change near the (naive) small and large $\tan\beta$ solutions, which is a measure of the required tuning.

contradictory to the different nature of the radiative corrections in both sectors.[5] It was suggested, however, that the latter mechanism could be realized if the QCD gauge fermions (the gluinos) are much lighter than the weak gauge fermions (the winos) [23]. While possible, this would imply that supersymmetry breaking is transmitted to the observable sector at a much lower scale than the breaking of the grand-unified group: If the supersymmetry breaking is transmitted to the visible sector gravitationally at Planckian scales, then the ratio of the different gaugino masses is dictated by the grand-unified symmetry to be approximately equal to that of the respective gauge couplings.[6] Such models [25] must contain new exotic matter beyond the minimal supersymmetric extension (MSSM), and are not discussed in this work (but see Ref. [26]).

Thus, if indeed $\alpha_s(M_Z) \lesssim 0.12$, then one expects (aside from the above-mentioned caveat) significant perturbations to the naive grand-unification relations at the unification scale. This is a crucial point when discussing Yukawa unification. It is straightforward to show that low-energy corrections to the α_s prediction constitute only a second-order perturbation in the $m_b(M_Z)$ prediction [4] (but they could affect the M_Z-m_b renormalization). However, corrections at the unification scale are multiplied by a large logarithm and can, depending on the way in which they propagate into the m_b/m_τ relation, correct the m_b prediction significantly.

In this work we investigate the possible implications of such a scenario to Yukawa unification. Our purpose is not to define the allowed parameter space with any high precision, but rather examine whether such a precision is possible beyond the minimal framework (which is not favored by the data). In Sec. II, we discuss two examples of corrections: nonrenormalizable operators (NRO's) and colored-triplet thresholds. (We also include in our numerical analysis low-energy corrections to m_b.) We examine the allowed parameter space as a function of α_s and of h_t. The latter is a useful measure of the parameter space which is independent of the size of the low-energy corrections to m_t, discussed in Sec. III. We find that the gap between the allowed small and large $\tan\beta$ regions is a sensitive function of α_s, the low-energy corrections to m_b (and thus, the soft parameters), m_t, and of the unification-scale perturbation to h_b/h_τ. Outside the minimal framework (which constrains α_s and the perturbations), none of these parameters is significantly constrained and the range of the allowed $\tan\beta \gg 1$ region is ambiguous. In particular, the gap nearly vanishes if $\alpha_s(M_Z) \sim 0.11$, or if the unification scale perturbation is $\sim 20\%$. Even though one can, in general, distinguish two different branches, the distinction is less significant as the gap diminishes, undermining the motivation to consider one branch rather than the other. Thus, the strong constraints on b-τ unification are intimately linked to the large values of α_s predicted in the minimal framework. In Sec. III we discuss the sensitivity of the h_t

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[5]When including the radiative corrections, the leading-logarithm correction to the prediction is typically proportional to the logarithm of the supersymmetric Higgs boson mass $|\mu|$ [5] and is more likely to be positive. It is negative if $|\mu|$ is very large. On the other hand, a large $|\mu|$ typically implies large mixing between left- and right-handed scalars and possibly a light scalar. The inclusion of finite corrections results now in a positive shift of the one-loop correction [6,7]. Because of this anticorrelation between the finite and logarithmic corrections, it is very difficult to obtain a negative one-loop correction [22]. The Roszkowski-Shifman proposal described below does not affect the proportionality to $|\mu|$, but only its coefficient [5].

[6]If the gauge kinetic function is grossly nonminimal, then this relation, and also gauge coupling unification, can be altered [24].

fixed-point curve to different threshold and other corrections, and stress that one-loop supersymmetric corrections to h_t are as important as the standard QCD correction. We conclude in Sec. IV where we summarize our results and, in addition, point out the implications to the prediction of the Higgs boson mass in Yukawa unified models.

Throughout this work it is assumed that the reader is familiar with the MSSM framework and with the framework of coupling unification (see references for background and other recent discussions). However, the main issues and results are reviewed and summarized in Sec. IV in a nontechnical manner.

II. GAUGE vs YUKAWA UNIFICATION

Before discussing examples of possible unification-scale corrections to the α_s prediction, it is important to realize the smallness of typical couplings at that scale and its implications.

$\alpha_s(M_G)\sim 0.04$. Because of the QCD enhancement[7] of small unification-scale perturbations in the value of $\alpha_s(M_G)$, the allowed $\sim\pm 8\%$ range of $\alpha_s(M_Z)=0.12\pm 0.01$ corresponds to only a $\sim\pm 3\%$ (or $\sim\pm 0.0015$) range at the unification scale.

$h_\tau(M_G)\sim 1/100\cos\beta$ ($y_\tau = h_\tau^2/4\pi \sim 10^{-5}$), and similarly $h_b(M_G)\sim 0.01$ (for $\tan\beta\sim 1$).

In extrapolating h_τ we used the near flatness of its renormalization curve (for not too large $\tan\beta$). Note also that when using the data as boundary conditions, $h_b(M_G) < h_\tau(M_G)$ by $\sim 10^{-3}$. In Fig. 1 it is shown that typically [for $\alpha_s(M_Z)=0.12$] $(h_b - h_\tau)/h_\tau \sim -0.2$ at M_G. The ratio is ~ -0.3 for $\alpha_s(M_Z)=0.13$ and ~ -0.1 for $\alpha_s(M_Z)=0.11$. Hence, a small numerical perturbation constitutes a large percentile perturbation.

The smallness and near flatness of h_τ is of particular importance in our case [27]. It implies that small shifts in $h_\tau(M_G)$ correspond to an apparent large violation of h_b- h_τ unification. One can visualize this as shifting the initial point of a nearly flat line (the h_τ renormalization curve). A small shift can drastically change its intersection with the moderately sloped h_b renormalization curve (the slope of the QCD renormalized h_b curve decreases at high energies where the couplings are small), leading to an apparent (or effective) unification point which could be many orders of magnitude below M_G. (Recall that the renormalization curve is a function of $\ln Q$ and not Q.) One can control such shifts by requiring that the apparent Yukawa-unification scale is not more than 2 or 3 orders of magnitude below M_G [4]. Such a constraint, however, is not motivated if one allows large shifts elsewhere [e.g., in $\alpha_s(M_G)$]. If one eliminates such (''no-conspiracy'') constraints, then there could be corrections of $\sim 100\%$ in the case that h_b and h_τ are still numerically small (i.e., for $\tan\beta\sim 1$). On the other hand, from Fig. 1, one observes that already $\sim 20\%$ corrections remove many of the constraints. We return to this point below.

[7]This is similar to the scaling between the QCD and weak scales that drastically reduces large uncertainties in the α_s measurements at ~ 1 GeV when propagated to M_Z. (The smaller coupling is compensated in our case by a larger logarithm.)

Next, we elaborate on possible corrections to α_s. One mechanism that could possibly shift $\alpha_s(M_G)$ is gravitational smearing (i.e., gravitationally induced NRO's), originally proposed as a nonperturbative mechanism [28,24] and later realized as an efficient perturbation (or smearing) to unification relations [29–31]. Requiring that the effect is perturbative typically constrains the coefficient of the (leading) operator such that the absolute value of the correction to the $\alpha_s(M_Z)$ prediction (which depends on the correlated shifts of all three gauge couplings) is $\lesssim 0.010-0.015$. (The exact number depends on the group theory structure.) One could argue for a larger correction, depending on the perturbativity criterion imposed. On the other hand, one typically expects a smaller correction, e.g., in Ref. [21] it was estimated that the absolute value of the correction is $\lesssim 0.006$. The correction can be propagated to the m_b/m_τ ratio as a constant shift in $\alpha_s(M_Z)$ [4] (see also Ref. [30]). In addition, other operators could now shift the boundary conditions of other couplings, e.g., $h_\tau(M_G)$, generating the perturbations discussed above.

A different mechanism for lowering the α_s prediction is by introducing an SU(5) breaking between (colored and non-colored) heavy chiral supermultiplet thresholds. In extended models many candidates could exist (for examples, see Refs. [21,32–36]). However, the most obvious candidate is the colored Higgs triplet T that has to be split from the light Higgs doublets (see also Ref. [37]). Indeed, the doublet-triplet splitting problem, even though solvable by fine-tuning of the superpotential and of the scalar potential, calls for nongeneric solutions that may affect the properties of the triplet threshold [38]. We consider this generic threshold as an example only.

Typically, one assumes $M_T \gtrsim M_G$ so that the loop-level (dimension-five) colored-Higgsino mediated proton decay [39] is sufficiently suppressed [40]. Nevertheless, the effectiveness of $M_T\sim 10^{-2}M_G$ in lowering the prediction for α_s may suggest a different mechanism for suppression of the dimension-five proton decay operator. One possibility[8] is that all Yukawa couplings of T are suppressed [46], in which case the only correction to Yukawa unification is via the modification of α_s:[9,10]

$$\Delta_{\alpha_s}\sim \frac{9\alpha_s^2(M_Z)}{14\pi}\ln\frac{M_T}{M_G}. \tag{2}$$

A different possibility is that some of the Yukawa couplings of T to the third generation are not suppressed. This assumption is particularly motivated here, since naive Yukawa unification is successful only in the case of the third family, and thus, provides no information on the Yukawa couplings (and mixing angles) of the two light families. If these are the only couplings which are not suppressed, then proton decay constraints on M_T are diminished. In addition to diminishing

[8]Other possibilities involve suppression due to symmetries [41], group theory [32,42–44], and the structure of the soft terms [45].

[9]Ignoring proton decay constraints, one could entertain the idea that an intermediate-scale triplet drives $\alpha_s(M_Z)<0.11$, which is then corrected to $\alpha_s(M_Z)>0.11$ by low-energy thresholds.

[10]In the light triplet models of Ref. [46] the correction is proportional to the logarithm of the triplet to (new) doublet mass ratio.

$\alpha_s(M_T)$, the triplet threshold in this case (i) introduces a Yukawa coupling correction to h_b/h_τ, (ii) shifts the h_t fixed point (see Sec. III), and (iii) renormalizes the soft parameters (i.e., the scalar potential) corresponding to the third family, an effect which is particularly important for the mass of the scalar τ, which could become too light or tachionic [47]. From (i) one has a $\sim\{1-[(h_t^2(M_G)/(16\pi^2)]\ln(M_T/M_G)\}$ correction factor to Eq. (1) [12], which can be absorbed as a shift in the boundary conditions. (We will include it explicitly in the numerical integration, i.e., in the numerical calculation of Y.) From (iii), there could be an enhancement[11] of low-energy lepton-number violation processes,[12] e.g., $\mu\to e\gamma$ [47,48].

In fact, both mechanisms, the operators and the triplet threshold, may be linked. Perturbations of some form or another are required in order to explain the failure of Yukawa unification for the two light families. One common mechanism to generate these perturbations is NRO's which are either gravitational or higher-symmetry remnants. Such operators most probably shift also the third family Yukawa couplings, and could allow only extra suppressed couplings for the colored triplet.

We examine the parameter space in Figs. 2 and 3, where we fixed $m_t^{\rm pole}=170$ GeV (consistent with direct [50] and indirect [20] determinations). In order to examine the smearing of the allowed[13] $\tan\beta$ range for $\alpha_s(M_Z)=0.12$, we require in Fig. 2(a) that b-τ unification at the α_1 - α_2 unification point ($M_G\approx3\times10^{16}$ GeV) holds to a precision of either 5%, 15%, or 25%. In practice, this would typically mean $h_b(M_G)\to0.8h_\tau(M_G)$, leading to a better agreement with the data. For example, a perturbation of 15% [or $h_b(M_G)\sim0.85h_\tau(M_G)$] corresponds in some cases to an apparent Yukawa-unification point as low as 10^{10} GeV. Low-energy corrections to m_b [11] are also included and calculated explicitly assuming, for simplicity, ''universal'' boundary conditions to the soft parameters at the grand scale,[14] and radiative symmetry breaking, agreement with experimental lower bounds on the masses (and an imposed upper bound of ~2 TeV), and using a Monte Carlo scan of the parameter space (for further details, see [15]). We account for NRO's (or other corrections whose main effect is to shift α_s at high energies) by fixing $\alpha_s(M_Z)=0.120$ [and $\alpha_s(M_Z)=0.110,0.130$ in Fig. 3]. For comparison, we show the respective allowed points when not including the low-energy corrections to $m_b(M_Z)$ in Fig. 2(b).

As implied by Fig. 1, for a 25% perturbation, no constraints exist on small $\tan\beta$. It is interesting to note that it is extremely difficult to find very large $\tan\beta$ solutions. The exclusion of $\tan\beta\gtrsim45\sim m_t/m_b$ results from the simultaneous requirement of radiative symmetry breaking and acceptable threshold corrections to m_b (and may be overcome by excessive tuning of parameters [52,53], in particular, in nonuniversal schemes [54,53]). When not including the low-energy corrections [Fig. 2(b)], these points are again allowed, but the intermediate $\tan\beta$ range is excluded [unless there is a $\gtrsim20\%$ perturbation]. The extreme tuning (for small perturbations) of very small and very large $\tan\beta$ solutions [e.g., see Figs. 1 and 2(b)] may suggest that the allowed region of intermediate $\tan\beta$ solutions is preferred. However, one has to be cautious, as such solutions depend sensitively on the soft parameters.[15] In Fig. 4 we show the possible low-energy corrections to m_b, where points which constitute the 5% perturbation curve in Fig. 2 are indicated by bullets. Only a small fraction of points has the required $\sim-20\%$ correction. Therefore, for small perturbations, all solutions for Yukawa unification require some tuning. (In principle, one could distinguish three allowed regions, but because of their complimentary nature, we will keep identifying both the intermediate and the very large $\tan\beta$ branches as the large $\tan\beta$ solution.)

We further examine solutions with small (5%) perturbations in Fig. 3, where we fix $\alpha_s(M_Z)=0.110,0.130$. The latter is roughly the value one would get when requiring gauge coupling unification and $M_T\gtrsim M_G$, i.e., the minimal framework. We also present curves requiring gauge coupling unification but fixing $M_T=10^{15},10^{14}$ GeV [$\alpha_s(M_Z)\approx0.118,0.112$, respectively]. (The triplet threshold is included numerically and the correlation between the shifts in $\alpha_s(M_Z)$, $\alpha_s(M_G)$, and M_G [4] is automatically accounted for.)

In the minimal framework, even when including the low-energy corrections, the two branches, $\tan\beta\sim1.3$ and $\tan\beta\gtrsim15$ are clearly distinguished. However, the small $\tan\beta$ solution is extremely tuned in this case because of the large QCD correction (see Sec. III) and because of the M_Z - m_b QCD renormalization. (An $\sim1-2\%$ low-energy correction can now exclude an otherwise consistent solution.) While a significant gap remains in this case, it is smeared almost completely for $\alpha_s\sim0.110$. It is worth stressing, however, that some gap remains (for small perturbations) in all cases. Thus, one can still distinguish two allowed branches, as in the minimal framework. This is because of the fixed point relative insensitivity for corrections to α_s and the proportionality of the m_b corrections to $\tan\beta$, which lead to only negligible smearing of the $\tan\beta\sim1$ branch. Nevertheless, smearing of the large $\tan\beta$ branch down to $\tan\beta\sim8(4)$ for $\alpha_s(M_Z)\sim0.120(0.110)$ significantly diminishes the excluded region, as well as undermines arguments

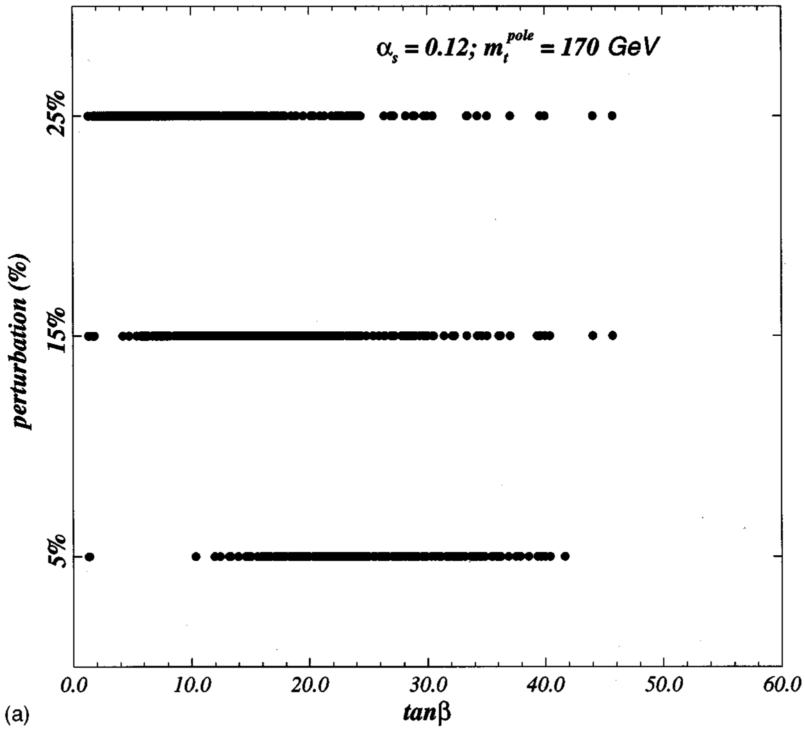
[11] In principle, one could obtain a (model-dependent) lower bound on M_T, independent of proton decay and of the α_s prediction.

[12] We find [48], for example, an enhancement as large as 2 orders of magnitude (for $M_T/M_G\gtrsim10^{-3}$) to the $\mu\to e\gamma$ branching ratios of the models considered in Ref. [49].

[13] We require $4.00\leqslant m_b(m_b)\leqslant4.45$ GeV (e.g., see Ref. [8]). More points would be allowed had we imposed this constraint, but for $m_b(m_b^{\rm pole}\sim5$ GeV) rather than for $m_b(m_b)$. For a discussion, see also Refs. [4,11].

[14] For simplicity, we do not include renormalization effects above M_G [51].

[15] There is also a correlation (which we do not treat in this work) between the m_b correction and the size of the chargino loop contribution to $b\to s\gamma$, and a negative correction typically implies an enhancement of the $b\to s\gamma$ rate [52]. This effect is generally important for $\tan\beta\gtrsim25-30$ and a too high $b\to s\gamma$ rate may exclude some of the allowed points in that region, depending on the charged Higgs boson mass.



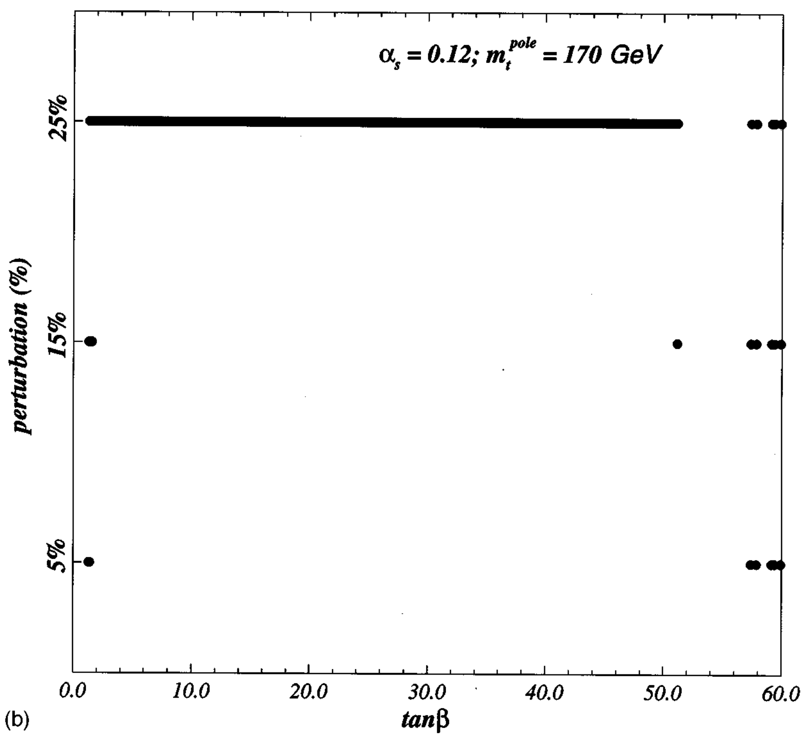


FIG. 2. The MSSM points which are consistent with b-τ unification for $\alpha_s(M_Z)=0.12$ and $m_t^{\rm pole}=170$ GeV are shown as a function of $\tan\beta$ when including (a) and when omitting (b) low-energy corrections to m_b. The different curves correspond to $h_b/h_\tau=1\pm0.05, 1\pm 0.15, 1\pm0.25$, at the unification scale, respectively.

(based on Yukawa unification) in favor of the $\tan\beta\sim1$ branch. Furthermore, as $m_t^{\rm pole}$ increases, the h_t fixed-point curve is flatter in $\tan\beta$, further diminishing the gap (see Fig. 5). Also, given the smallness of h_b and h_τ for $\tan\beta\sim1$, $\sim20\%$ perturbations are reasonable, as discussed above, and the $\tan\beta\sim1$ branch could also be smeared (see Fig. 2).

In Fig. 5 we allow $m_t^{\rm pole}=180\pm12$ GeV [50] (with a Gaussian distribution) and show the allowed values of the top Yukawa coupling h_t as a function of $\tan\beta$ for $\alpha_s(M_Z)=0.120$ and a 5% perturbation. (Note that for large values of $m_t^{\rm pole}\gtrsim190-200$ GeV, h_t could be near its fixed point for intermediate values of $\tan\beta$.) The requirement $h_t\gtrsim1.1$ holds for $\tan\beta\lesssim8$. This is a reflection of the respective excluded region (the gap) in Fig. 2 where $m_t^{\rm pole}=170$ GeV (and $h_t\lesssim1.1$ for $\tan\beta\gtrsim1.4$). The fact that now there is no gap is due to the higher values of $m_t^{\rm pole}$.

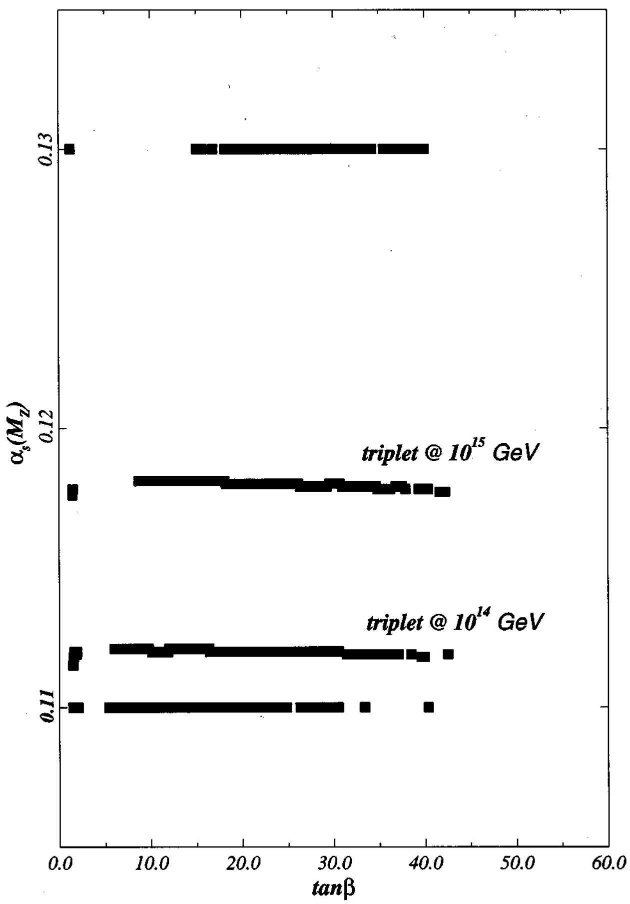


FIG. 3. The MSSM points which are consistent with b-τ unification for $m_t^{\text{pole}}=170$ GeV and when requiring $h_b/h_\tau=1\pm0.05$ are shown as a function of $\tan\beta$ (including low-energy corrections to m_b). The upper and lower curves correspond to $\alpha_s(M_Z)=0.13$, 0.11, respectively. In the two middle curves $\alpha_s(M_Z)$ is predicted when a colored-triplet threshold at $M_T=10^{15}$, 10^{14} GeV (with Yukawa couplings to the third family) is assumed (and accounted for in the numerical integration).

III. THE FIXED-POINT CURVE

Points near the h_t fixed point were shown above to provide a solution to b-τ Yukawa unification. That solution is the least sensitive to either enhancement or suppression of the low-energy corrections to m_b (the sensitivity grows with α_s, as discussed above). However, the solution is a result of the large numerical value of h_t only, and because of the h_t convergence to its fixed-point value this result is relatively insensitive to $\alpha_s(M_Z)=0.12\pm0.01$. The translation of this value to a curve in the m_t^{pole} - $\tan\beta$ plane contains a few ambiguities, which are worth recalling.

More precisely, this is only a quasifixed point [14] (i.e., flatness due to cancellations between gauge and Yukawa terms, which leads to convergence from above). If the low-energy h_t exceeds its fixed-point value, then it becomes nonperturbative at some higher scale. In a consistent calculation the quasifixed point has to be defined numerically, e.g., that renormalization from two loops is smaller than a certain fraction of that from one loop. This leads, e.g., to the condition $h_t\lesssim3$ at all scales below the cutoff scale [3]. Therefore, the cutoff scale for the calculation enters the definition. For example, using 10^{18} GeV rather than M_G as a cutoff, leads [in SU(5)] to the requirement $h_t(M_G)\lesssim2$, shifting the fixed point curve to slightly higher values of $\tan\beta$. (In fact, there may be another quasifixed point $h_t\sim2$ at M_G [49].) In addition, the fixed-point value of h_t depends on the other large couplings in the renormalization group equations, i.e., α_s. The lower α_s is, the lower is that value, and again, the curve slides to slightly larger values of $\tan\beta$ (e.g., this can be seen in Fig. 3).

If there are other large couplings, i.e., new large Yukawa couplings (or a large number of new couplings), then the fixed-point value of h_t also changes. The quasifixed point is reached by a cancellation of gauge and Yukawa terms. Since the size of the former is roughly fixed, any new Yukawa coupling modifies the upper bound on all other Yukawa couplings (that enter the same set of renormalization group equations). New Yukawa couplings could renormalize (i) h_τ, (ii) h_b, and (iii) h_t. In most examples all three are relevant and a fixed-point value of $h_t<1$ is possible (i.e., $\tan\beta$ slides to larger values) while still maintaining Yukawa unification. Some examples include (a) low-energy singlets [15,55], (b) fourth family [56,57], and (c) baryon and lepton-number violating couplings [58].

A most interesting case is that of (d) an intermediate-scale right-handed neutrino where only (i) and (iii) occur. Before its decoupling at the intermediate scale, the new Yukawa coupling, h_ν, renormalizes h_τ in the same way that h_t renormalizes h_b. The two Yukawa corrections roughly cancel in the ratio [assuming $h_\nu(M_G)\approx h_t(M_G)$], and the Yukawa correction function Y in Eq. (1) is closer to unity (depending on the right-handed neutrino scale), unless h_b, itself, is significantly large [59,34]. The small $\tan\beta$ solution is excluded in this case, regardless of the exact location of the h_t fixed point.

The generic heavy threshold corrections follow similar patterns. The adjoint field, like the singlet [case (a)], is coupled to the ''Higgs boson leg'' of the Yukawa operators, and the effect cancels in the h_b/h_τ ratio [4]. However, it also affects h_t, and hence, affects h_b/h_τ indirectly. However, unlike the low-energy singlet case, the indirect correction here is suppressed by a small logarithm. It could shift the fixed point if its coupling to the Higgs doublets, which renormalizes h_t, is large [i.e., in SU(5) it is the case that the color triplet is heavy], and its self-coupling (that determines its own mass) is small. The color triplet has leptoquark couplings that unify with h_t, and is a special example of (c). Because of its large mass (i.e., the small logarithm) the effect is again moderate. We find that for $M_T\gtrsim10^{14}$ GeV the fixed-point value of $\tan\beta$ increases (including the modification of the α_s prediction) by less than 0.18 (and less than 0.06 for a fixed value of α_s).

Lastly, supersymmetric threshold corrections to m_b play a crucial rule in expanding the allowed parameter space: They generate the allowed intermediate $\tan\beta$ region in the case of small perturbations. Similar corrections have been shown to affect other parameters [60], an observation which is related to renewed interest [61,44] in (weak-scale) radiative fermion masses [62]. In fact, it is doubtful that one can consider predictions for the SM fermionic sector parameters independently from the supersymmetric spectrum parameters. The corrections that are relevant for our discussion are those for

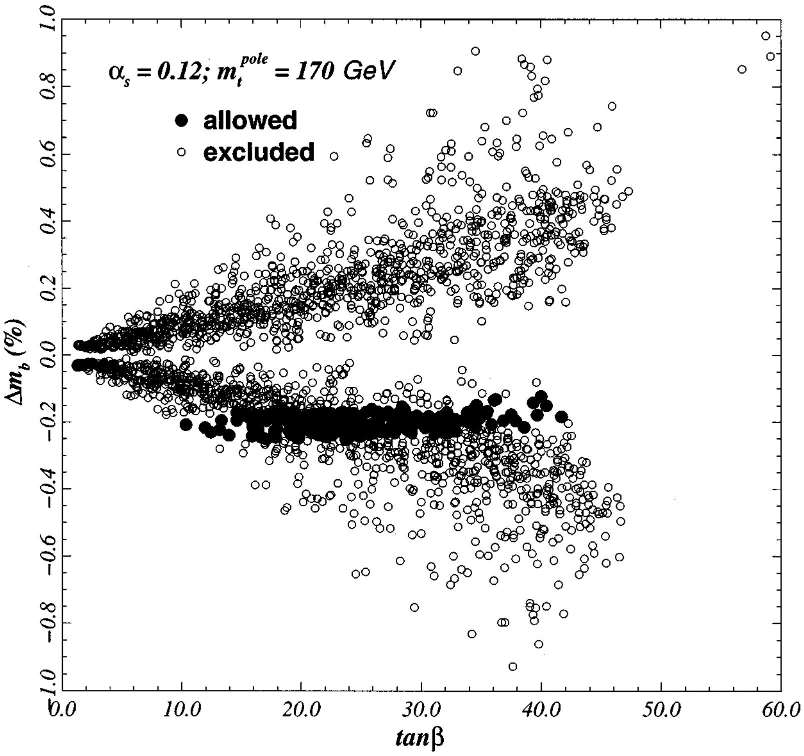


FIG. 4. The low-energy threshold corrections to m_b for the MSSM points considered in Fig. 2. Only the points indicated by bullets correspond to the 1 ± 0.05 curve in Fig. 2.

the $m_t^{\rm pole}/m_t^{\overline{\rm DR}}$ ratio ($\overline{\rm DR}$ stands for the dimensional-reduction scheme):

$$h_t = \frac{m_t^{\overline{\rm DR}}}{174\ {\rm GeV}} \frac{\sqrt{1+\tan^2\beta}}{\tan\beta}. \tag{3}$$

We defined the parameter $m_t^{\overline{\rm DR}}$ to absorb all threshold corrections, i.e., at one loop,

$$m_t^{\overline{\rm DR}} = m_t^{\rm pole}[1 - \Delta^t_{\rm QCD} - \Delta^t_{\rm SUSY\ QCD} - \Delta^t_{\rm EW}], \tag{4}$$

where[16] [63]

$$\Delta^t_{\rm QCD} = \frac{5}{3}\frac{\alpha_s(m_t)}{\pi}, \tag{5}$$

and $\Delta^t_{\rm EW}$ includes electroweak and Yukawa contributions [12,64] that we neglect hereafter. $\Delta^t_{\rm SUSY\ QCD}$ includes new QCD contributions in the MSSM (which are only implicitly dependent on $\tan\beta$), that have been calculated using three- [12] and two-point [64] functions and shown to be potentially of the order of magnitude of $\Delta^t_{\rm QCD}$. Recently, it has been further shown [45] that $\Delta^t_{\rm SUSY\ QCD}$ does not have a fixed sign[17] and introduces a significant ambiguity in the fixed-point curve. In particular, this correction can be more important than the $\sim 2\%$ two-loop QCD contribution to Eq. (5) that many authors include while neglecting supersymmetric loops.

In Fig. 6 we examine the corrections for the point $(m_t^{\rm pole}, \tan\beta)$ = (170 GeV, 1.4), i.e., in the vicinity of the ''naive'' fixed-point curve, and for $\alpha_s(M_Z) = 0.12$ (using the vertex formalism of Ref. [12] and imposing the same assumptions on the parameter space as above). By fixing h_t to its fixed-point value, the corrections are absorbed in the invariant combination $m_t^{\rm pole}/\sin\beta$. (Note that the corrections, though represented by a mass parameter, are in fact corrections to the Yukawa coupling.) It is straightforward to absorb the corrections in $m_t^{\rm pole}$ (vertical line), in which case the correction in our example is $-2\% \lesssim \Delta^t_{\rm SUSY\ QCD} \lesssim 5\%$ or between -3 and 8 GeV. (The asymmetry is due to the fixed sign of the leading logarithms.) However, if $m_t^{\rm pole}$ is known with high precision, then the corrections are to be absorbed[18] in $\sin\beta$ (horizontal line). [A similar procedure could be used to treat the uncertainty in α_s in (5).] The two lines define a region in the parameter space that corresponds to one point on the ''naive'' fixed-point curve. Figure 5 is insensitive to this ambiguity, but the interpretation of Figs. 2 and 3 is sensitive. The ambiguity in $m_t^{\rm pole}/\sin\beta$ diminishes the required tuning of the $\tan\beta \sim 1$ solutions (at the price of dependence on the soft parameters) in a similar way to the smearing of the large $\tan\beta$ solutions due to the corrections to m_b. The correction (absorbed in $m_t^{\rm pole}$) is shown in Fig. 7 for any $\tan\beta$ for $\alpha_s(M_Z) = 0.12$ (and requiring b - τ unification with a 5% perturbation). The dependence on $\tan\beta$ is from the supersymmetric Higgs mass $\mu = \mu(\tan\beta, \ldots)$, the left-right

[16]One also needs to include a $\Delta^b_{\rm QCD} = [1/3][\alpha_s(M_Z)/\pi]$ when converting $m_b(M_Z)$ from its $\overline{\rm DR}$ definition to its modified minimal-subtraction definition, which is the relevant one for $m_b(m_b)$. This correction is important, e.g., for $\alpha_s(M_Z) = 0.13$.

[17]The leading logarithm terms agree in sign with $\Delta^t_{\rm QCD}$ but the overall sign is model dependent.

[18]This is a similar procedure to absorbing radiative corrections in the weak angle rather than in M_Z.

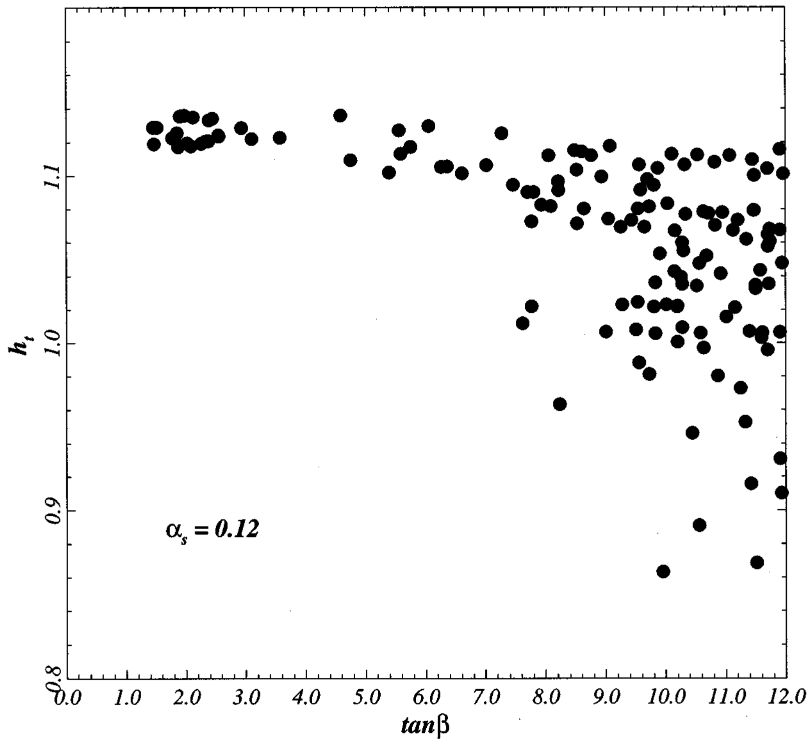


FIG. 5. The MSSM points which are consistent with b-τ unification for $\alpha_s(M_Z)=0.12$, $m_t^{\rm pole}=180\pm 12$ GeV and when requiring $h_b/h_\tau=1\pm 0.05$, are shown as a function of $\tan\beta$ and of the Yukawa coupling h_t (which is calculated including only its QCD correction). Because of the larger values of $m_t^{\rm pole}$, larger values of h_t (and thus, solutions for b-τ unification) are obtained for $\tan\beta > 2$. The correspondence between h_t and $m_t^{\rm pole}$ could change when including the SUSY-QCD corrections of Sec. III.

t-scalar mixing, and a correlation between the m_t and m_b corrections (which we do not explore in detail in this work).

IV. SUMMARY

Requiring supersymmetric b-τ Yukawa unification could impose strong constraints on the parameter space. The constraints typically lead to two allowed branches of $\tan\beta\sim 1$ and $\tan\beta\gg 1$ in the m_t-$\tan\beta$ plane, where the parameter $\tan\beta=\langle H_2\rangle/\langle H_1\rangle$ and m_t is the t-quark mass. In this work we addressed the possible implications of (1) unification-scale corrections to gauge-coupling unification, and of (2) weak-scale corrections to the t-quark Yukawa coupling h_t for the above constraints.

We stressed that typically one predicts from gauge-coupling unification $\alpha_s(M_Z)\gtrsim 0.13$, and that (aside from a minor caveat) values of $\alpha_s(M_Z)\lesssim 0.12$ would require the introduction of sizable perturbations at the unification scale. We then argued that this is a crucial issue for Yukawa unification, as the b-Yukawa coupling h_b is sensitive to unification scale (but not weak-scale) corrections to α_s. In addition, because of their typical smallness at the unification scale, the Yukawa couplings are also sensitive to corrections to their boundary conditions at that scale. Such corrections should be allowed if the discrepancy in gauge-coupling unification [i.e., in the $\alpha_s(M_Z)$ prediction] is a measure of the relative importance of unification-scale perturbations.

Two specific examples of unification-scale corrections, nonrenormalizable operators and a color-triplet threshold, were considered. (The latter was also shown to correlate issues such as proton decay, scalar lepton mass predictions, and $\mu\to e\gamma$.) We then showed that the constraints imposed by the Yukawa unification assumption on $\tan\beta$ are indeed a sensitive function of α_s (Fig. 3), unification-scale perturbations (Fig. 2), and low-energy corrections to m_b (Fig. 4) [and of m_t (Fig. 5)], and nearly vanish for $\alpha_s(M_Z)=0.11$ or an $\sim 20\%$ low- or high-scale correction to m_b. In particular, the constraints are sensitive to the specific corrections to α_s that we considered.

The above sensitivity is due to (a) the h_b renormalization which is extremely sensitive to the α_s and to h_t values (and also to h_b itself), and (b) the low-energy renormalization of $m_b(Q<M_Z)$ which is sensitive to α_s and to the supersymmetric corrections to its boundary condition $m_b(M_Z)$. In both cases, i.e., the high-energy unification-weak scale renormalization and the low-energy weak-QCD scale renormalization, small perturbations to the boundary conditions are usually enhanced by the large QCD renormalization.

The uncertainties in the constraints were shown to be absorbed, for the most part, in the definition of the $\tan\beta\gg 1$ branch. From our figures one can obtain a qualitative description of the excluded region (the gap) between the two allowed branches in terms of the lower bound on the large $\tan\beta$ branch (for $m_t^{\rm pole}=170$ GeV):

$$\tan\beta\gtrsim\frac{1}{2}\left\{\frac{[\alpha_s(M_Z)-0.100]}{0.001}+\frac{h_b(M_G)-h_\tau(M_G)}{0.010\times h_\tau(M_G)}\right\}. \qquad (6)$$

(We assume that the left-hand side of (6) is ≥ 1, otherwise $\tan\beta\geq 1$.) Thus, the success of ‘‘simple’’ gauge unification $[\alpha_s(M_Z)>0.12]$ and the constraints on Yukawa unification are intimately linked, and the difference between the predicted and measured α_s values should indeed be viewed as a sensitive measure of the typical size of perturbation at the unification scale, as was suggested above.

The $\tan\beta\sim 1$ branch was shown, however, to be sensitive to an ambiguity in the location (in parameter space) of the h_t-quasifixed point, and we demonstrated the need to consider threshold corrections to m_t when discussing the quasifixed point curve. (The quasifixed point corresponds to flat

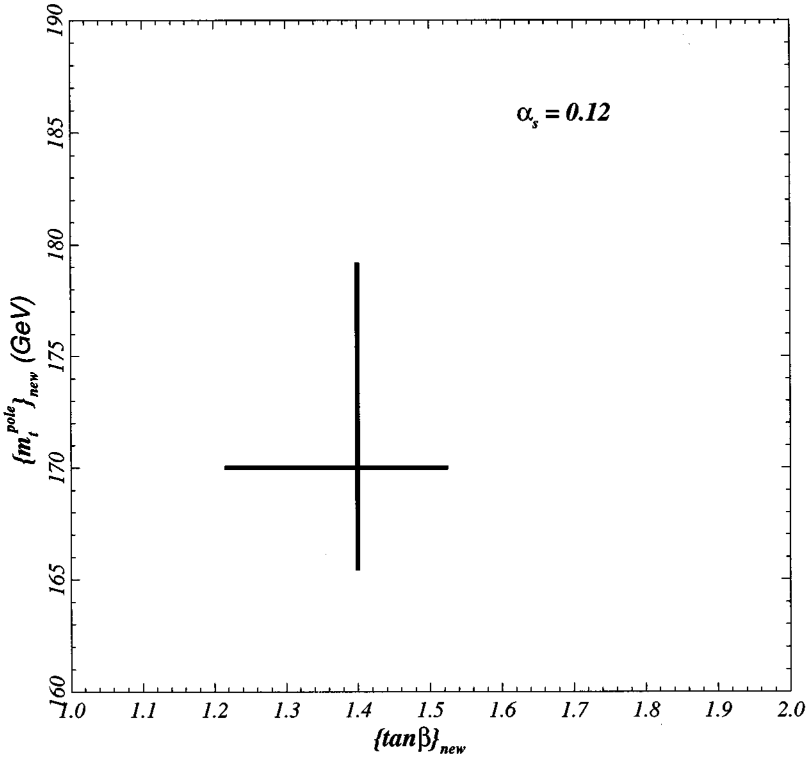


FIG. 6. The SUSY-QCD corrections to h_t are absorbed in $m_t^{\rm pole}$ (vertical line) and in $\tan\beta$ (horizontal line), smearing the naive point $m_t^{\rm pole} = 170$ GeV and $\tan\beta = 1.4$ (assuming a fixed h_t value and $\alpha_s(M_Z) = 0.12$). b-τ unification is not required.

renormalization of h_t. Its location determines the upper bound on h_t at low energies.) In particular, this effect has more general implications and also affects the h_t-perturbativity lower bound on $\tan\beta$ (for a given cutoff scale).

The required properties of the unification-scale perturbations, which we simply assumed when discussing examples, can, on the one hand, put severe constraints on model building and enhance the predictive power in the high-scale theory (see, for example, Ref. [36]). On the other hand, it implies loss of some predictive power in the low-energy theory, i.e., unlike the minimal framework, now there are no generic predictions but only model-dependent ones (which depend on additional parameters). The loss of low-energy predictive power may be compensated in some cases by the effects of threshold corrections (due to these perturbations)

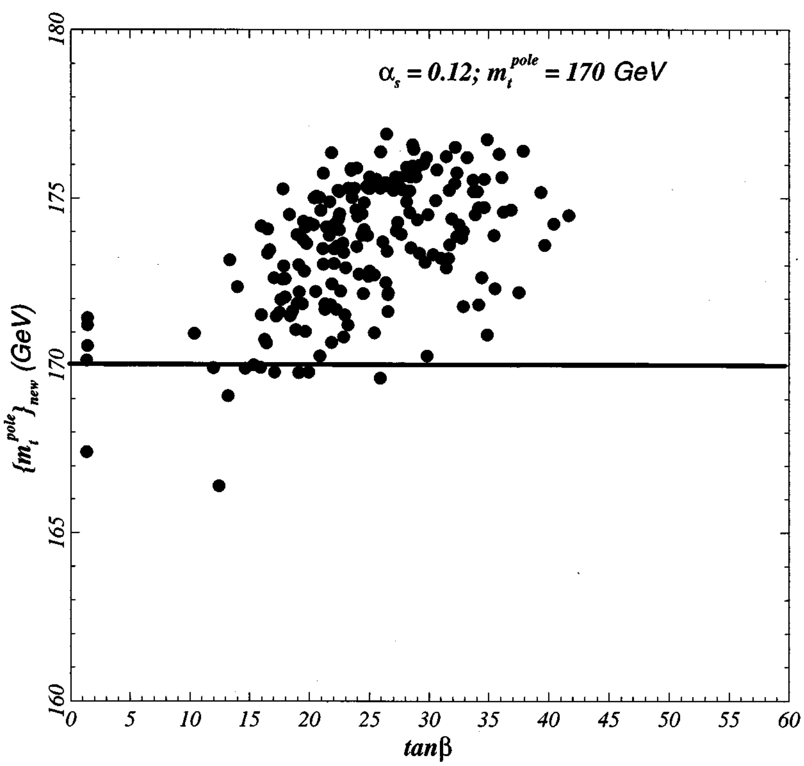


FIG. 7. The SUSY-QCD corrections to h_t are absorbed in $m_t^{\rm pole}$ for the points of the 1 ± 0.05 curve in Fig. 2.

in the soft parameters on flavor-changing neutral-current processes, but these are again strongly model dependent.

Regarding the light Higgs boson mass m_{h^0}, its lightness is due to the accidental proximity of the h_t quasi-fixed-point curve to the flat direction in the Higgs scalar potenial for $\tan\beta=1$. The latter implies $m_{h^0}^{\rm tree}<M_Z|\cos 2\beta|\to 0$ near the fixed-point curve. If the curve slides to larger values of $\tan\beta$, $m_{h^0}^{\rm tree}$ increases. However, unless new Yukawa couplings are introduced [e.g., examples (a) – (c) above], the increase is $\lesssim 10$ GeV, and since the mass $m_{h^0}^{\rm one\ loop}$ is a sum in quadrature of tree and loop terms, it has no significant ambiguity. The ambiguity due to $\Delta^t{}_{\rm SUSY\ QCD}$ is more of an interpretational ambiguity, since h_t (or $m_t^{\overline{\rm DR}}$) is the relevant parameter for the calculation of the loop correction in $m_{h^0}^{\rm one\ loop}$. (As commented above, this is actually one of the more important higher-order refinements of the calculation.) The prediction of $m_{h^0}^{\rm one\ loop}$ is thus insensitive to the corrections (if absorbed in $m_t^{\rm pole}$). However, the correspondence between $m_t^{\rm pole}$ and $m_{h^0}^{\rm one\ loop}$ is now ambiguous. We thus conclude that, indeed, Higgs searches can probe the MSSM fixed-point region. However, while this region may be motivated by various reasons (not the least, the existence of a fixed-point structure),[19] the diminished gap between the two allowed branches for Yukawa unification undermines the uniqueness of the $\tan\beta\sim 1$ branch and the motivation to consider this region based on b-τ unification, unless α_s is large and unification-scale perturbations are small.

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