Heavy baryon masses in the $1/m_Q$ and $1/N_c$ expansions

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The masses of baryons containing a single heavy quark are studied in a combined expansion in $1/m_Q$, $1/N_c$, and SU(3) flavor symmetry breaking. Heavy quark baryon mass splittings are related to mass splittings of the octet and decuplet baryons. The Σ_c^*, Ξ_c', and Ω_c^* are predicted to the level of a few MeV. A number of bottom baryon mass splittings are predicted very accurately. [S0556-2821(96)04117-3]

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I. INTRODUCTION

The mass spectrum of baryons containing a single heavy quark is tightly constrained by the presence of approximate symmetries. In the heavy quark limit, hadrons containing a single heavy quark respect a heavy quark spin-flavor symmetry [1]. For finite m_Q, this symmetry is broken by effects of order $1/m_Q$. In the large-N_c limit, baryons with an approximate flavor symmetry possess a larger spin-flavor symmetry [2]. For finite N_c, this symmetry is broken by effects of order $1/N_c$ [2–8]. The combined limit $m_b \to \infty$, $m_c \to \infty$, $N_c \to \infty$ for fixed (m_c/m_b) and $(N_c \Lambda_{\rm QCD}/m_b)$ results in a light quark and heavy quark spin-flavor symmetry $SU(6)_{\ell} \times SU(4)_h$ for baryons containing a single heavy quark. For finite m_Q and N_c, this symmetry is violated by effects of order $1/N_c$ and $(1/N_c m_Q)$, and by SU(3) flavor breaking. It is the purpose of this work to explore the implications of SU(3) flavor breaking and the combined $1/m_Q$ and $1/N_c$ expansions for the masses of heavy quark baryons.

Section II begins with a brief review of the $1/m_Q$ expansion of heavy hadron masses in heavy quark effective theory. Section III presents the operator analysis of the $1/N_c$ expansion. Heavy baryon masses are analyzed in the combined $1/m_Q$ and $1/N_c$ expansion for isospin flavor symmetry in Sec. IV. The generalization to SU(3) flavor symmetry is performed in Sec. V. The most precise predictions follow from the SU(3) mass relations of Sec. V. Readers interested only in these results may go directly to Sec. V A where they are presented. Isospin-violating mass splittings are considered in Sec. VI. Conclusions are presented in Sec. VII.

II. HEAVY QUARK BARYON MASSES IN THE $1/m_Q$ EXPANSION

The masses of hadrons containing a single heavy quark have been studied in the $1/m_Q$ expansion of heavy quark effective theory. A brief review of this expansion is presented in this section.

The heavy quark effective theory (HQET) Lagrangian

$$\mathcal{L}_{\rm HQET} = \overline{Q}_v (iv\cdot D) Q_v + \overline{Q}_v \frac{(iD)^2}{2m_Q} Q_v - Z_Q \overline{Q}_v \frac{g G_{\mu\nu}\sigma^{\mu\nu}}{4m_Q} Q_v + O\left(\frac{1}{m_Q^2}\right) \qquad (2.1)$$

describes the interactions of a heavy quark Q with fixed velocity v inside a hadron containing a single heavy quark. The heavy quark mass m_Q is removed from the QCD Lagrangian by the heavy quark field redefinition [9]. The residual mass term has been chosen so that there is no mass term in the HQET Lagrangian [10].

The mass of a hadron containing a single heavy quark has an expansion in $1/m_Q$,

$$M(H_Q) = m_Q + \overline{\Lambda} - \frac{\lambda_1}{2m_Q} - d_H \frac{\lambda_2}{2m_Q} + O\left(\frac{1}{m_Q^2}\right), \qquad (2.2)$$

where the order unity contribution $\overline{\Lambda}$ is the mass of the light degrees of freedom in the hadron, and the two $1/m_Q$ contributions are determined by the matrix elements

$$\lambda_1 = \langle H_Q(v) | \overline{Q_v} (iD)^2 Q_v | H_Q(v) \rangle \qquad (2.3)$$

and

$$d_H \lambda_2 = \frac{1}{2} Z_Q \langle H_Q(v) | \overline{Q_v} g G_{\mu\nu} \sigma^{\mu\nu} Q_v | H_Q(v) \rangle. \qquad (2.4)$$

In the above equation, d_H is the Clebsch factor, $d_H = -4(J_{\ell} \cdot J_Q)$, and Z_Q is a renormalization factor with $Z_Q(\mu = m_Q) = 1$. Renormalization group scaling between the scales m_b and m_c yields $Z_b/Z_c = [\alpha_s(m_b)/\alpha_s(m_c)]^{9/25}$.

Neglecting SU(3) flavor breaking, the masses of the lowest-lying pseudoscalar and vector mesons D and D^* for $Q = c$ and B and B^* for $Q = b$ are parametrized by

$$P_Q = m_Q + \overline{\Lambda}^{\rm meson} - \frac{\lambda_1^{\rm meson}}{2m_Q} - \frac{3\lambda_2^{\rm meson}}{2m_Q} + \cdots,$$

$$P_Q^* = m_Q + \overline{\Lambda}^{\rm meson} - \frac{\lambda_1^{\rm meson}}{2m_Q} + \frac{\lambda_2^{\rm meson}}{2m_Q} + \cdots, \qquad (2.5)$$

whereas the masses of the lowest-lying spin-$\frac{1}{2}$ antitriplet $(\overline{3})$ and spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextets (6) are parametrized by

$$T_Q = m_Q + \overline{\Lambda}_T^{\text{baryon}} - \frac{\lambda_{1T}^{\text{baryon}}}{2m_Q} + \cdots,$$

$$S_Q = m_Q + \overline{\Lambda}_S^{\text{baryon}} - \frac{\lambda_{1S}^{\text{baryon}}}{2m_Q} - \frac{4\lambda_{2S}^{\text{baryon}}}{2m_Q} + \cdots, \tag{2.6}$$

$$S_Q^* = m_Q + \overline{\Lambda}_S^{\text{baryon}} - \frac{\lambda_{1S}^{\text{baryon}}}{2m_Q} + \frac{2\lambda_{2S}^{\text{baryon}}}{2m_Q} + \cdots,$$

to order $1/m_Q^2$ in the $1/m_Q$ expansion. Note that different parameters Λ, λ_1, and λ_2 appear for mesons and baryons. When SU(3) flavor breaking is not neglected, different parameters also appear for different members of the meson and baryon SU(3) multiplets.

III. OPERATOR ANALYSIS

The $1/N_c$ operator analysis for baryons containing light quarks was derived in detail in Refs. [2–7]. The operator analysis is generalized to baryons containing heavy quarks in this section.

The approximate flavor symmetry of QCD with three light quarks u, d, and s and heavy quarks c and b is $SU(3) \times U(1)_c \times U(1)_b$. The corresponding spin-flavor symmetry for large-N_c baryons is $SU(6) \times SU(2)_c \times SU(2)_b$. There is a separate spin symmetry for each flavor of heavy quark in the baryon in the large-N_c limit. This heavy quark spin symmetry is operative for baryons containing multiple heavy quarks as well as baryons containing a single heavy quark. It is important to emphasize that large-N_c heavy quark spin symmetry is present for baryons even if the quark flavor in question is not heavy enough to have a valid $1/m_Q$ expansion. For this reason, it is possible to study the consequences of large-N_c heavy quark spin symmetry for baryons in the $1/N_c$ expansion without reference to the $1/m_Q$ expansion. We begin with this $1/N_c$ operator analysis and then generalize to the combined $1/m_Q$ and $1/N_c$ expansion.

The operator basis of the $1/N_c$ expansion for baryons containing heavy quarks is constructed out of the zero-body identity operator $\mathbb{1}$ and the generators of the spin-flavor algebra $SU(6) \times SU(2)_c \times SU(2)_b$. The SU(6) generators are defined by

$$J_\ell^i = q^\dagger \frac{\sigma^i}{2} q,$$

$$T^a = q^\dagger \frac{\lambda^a}{2} q,$$

$$G^{ia} = q^\dagger \frac{\sigma^i \lambda^a}{4} q,$$

where $J_\ell^i = J_u^i + J_d^i + J_s^i$ while the charm and bottom spin generators are defined by

$$J_c^i = c^\dagger \frac{\sigma^i}{2} c,$$

$$J_b^i = b^\dagger \frac{\sigma^i}{2} b. \tag{3.1}$$

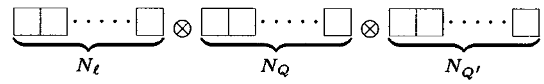

FIG. 1. Baryon representation of $SU(2N_F) \times SU(2)_Q \times SU(2)_{Q'}$ spin-flavor symmetry for large finite N_c. The total number of boxes equals $N_c = N_\ell + N_h$ where $N_h = N_Q + N_{Q'}$.

The light quark spin-flavor algebra is given in Ref. [7]. The heavy quark spin symmetry commutation relations are simply

$$[J_Q^i, J_Q^j] = i\epsilon^{ijk} J_Q^k, \tag{3.2}$$

where $Q = c$ or b.

Operators in the $1/N_c$ expansion are polynomials in the one-body operators. The set of polynomial operators is overcomplete. An operator basis which is complete and linearly independent is constructed by eliminating redundant operators using operator identities. The operator identities are derived in Ref. [7]. The structure of the operator identities is discussed in detail in Ref. [7], and will not be repeated here. For the case of baryons containing N_ℓ light quarks and N_h heavy quarks such that $N_c = N_\ell + N_h$, the $SU(2N_F)$ identities of Ref. [7] apply for J^i replaced by J_ℓ^i and N_c replaced by N_ℓ in Eq. (4.2) and Tables VII and VIII. Similarly, the $SU(2)_Q$ spin symmetry identities are the identities for one quark flavor with N_c replaced by N_Q, namely,

$$Q^\dagger Q = N_Q \mathbb{1},$$

$$\{J_Q^i, J_Q^i\} = \frac{1}{2} N_Q (N_Q + 2) \mathbb{1}. \tag{3.3}$$

The presence of N_ℓ and N_Q in the operator identities rather than N_c follows from the group theory of the baryon representation for a fixed number of light quarks and a fixed number of heavy quarks of each flavor. Figure 1 gives the $SU(6) \times SU(2)_Q \times SU(2)_{Q'}$ spin-flavor representation for baryons containing N_ℓ light quarks and $N_h = N_Q + N_{Q'}$ heavy quarks, with $N_\ell + N_h = N_c$, where Q and Q' represent two different flavors of heavy quark c and b. It is clear that the operator identities apply for the quark numbers N_ℓ and N_Q. For example, the total number of quarks in the baryon is described by the one-body $\rightarrow$ zero-body identities

$$\begin{aligned} q^\dagger q + Q^\dagger Q + Q'^\dagger Q' &= (N_\ell + N_Q + N_{Q'}) \mathbb{1} \\ &= (N_\ell + N_h) \mathbb{1} = N_c \mathbb{1}. \end{aligned} \tag{3.4}$$

The $1/N_c$ expansion of an l-body QCD operator acting on baryons containing N_ℓ light quarks and N_h heavy quarks has the form

$$\mathcal{O}_{\text{QCD}}^{(l)} = N_c^l \sum_{m=0}^{N_\ell} \sum_{n=0}^{N_h} c^{(m+n)} \frac{1}{N_c^{m+n}} \mathcal{O}_\ell^{(m)} \mathcal{O}_h^{(n)}, \tag{3.5}$$

where $\mathcal{O}_\ell^{(m)}$ denotes an m-body operator in the light quark spin-flavor generators J_ℓ^i, T^a, and G^{ia}, and $\mathcal{O}_h^{(n)}$ denotes an

n-body operator in the heavy quark generators.[1] The $1/N_c$ operator expansion only goes up to N_c-body operators since $N_c = N_\ell + N_h$. Each of the arbitrary coefficients of the $1/N_c$ expansion,

$$c^{(m+n)}\left(\frac{1}{N_c}\right), \tag{3.7}$$

is order one at leading order in the $1/N_c$ expansion. The coefficients are only a function of $1/N_c$ since N_ℓ, N_Q, and $N_{Q'}$ are to be regarded as fixed numbers, not operators, in the present expansion. The coefficients depend implicitly on the fixed ratios N_ℓ/N_c, N_Q/N_c, and $N_{Q'}/N_c$.

It is an important physical point that the operator expansion for baryons containing light and heavy quarks is suppressed by factors of $1/N_c$, rather than $1/N_\ell$ and $1/N_h$. This $1/N_c$ suppression is required for consistency of the operator expansion. One consistency requirement is that the $1/N_c$ power counting of the operator expansion (3.5) be preserved under commutation of the $(m+n)$-body operators $O^{(m+n)} \equiv O_\ell^{(m)} O_h^{(n)}$. This feature is present for a suppression factor of $1/N_c^{m+n}$, but not for $1/(N_\ell^m N_h^n)$. A second consistency requirement is that the operator expansion be invariant under a change of operator basis for the expansion. For example, it is possible to rewrite the operator basis using $J = J_\ell + J_h$, the total baryon spin, rather than J_ℓ. This recasting of the operator expansion is obviously possible if J, J_ℓ, and J_h are all suppressed by $1/N_c$, but it is not consistent if J_ℓ is suppressed by $1/N_\ell$ and J_h is suppressed by $1/N_h$. The conclusion that all the suppression factors are $1/N_c$ also follows from an analysis of the large-N_c Feynman diagrams.

Although factors of $1/N_\ell$ and $1/N_h$ do not appear as explicit suppression factors in the $1/N_c$ operator expansion, N_ℓ and N_h do appear implicitly through the matrix elements of O_ℓ and O_h. Matrix elements of the operator $O_\ell^{(m)}$ are at most order $(N_\ell)^m$, whereas matrix elements of $O_h^{(n)}$ are at most $(N_h)^n$, so that

$$\frac{1}{N_c^{m+n}} \langle O_\ell^{(m)} O_h^{(n)} \rangle \lesssim \left(\frac{N_\ell}{N_c}\right)^m \left(\frac{N_h}{N_c}\right)^n. \tag{3.8}$$

Both N_ℓ/N_c and N_h/N_c are suppression factors as long as neither N_ℓ nor N_h is equal to N_c. Thus, for baryons containing both light and heavy quarks, higher-body operators in the $1/N_c$ expansion *are* suppressed relative to lower-body operators if the operator basis is written in terms of light-quark and heavy-quark one-body operators. Alternative operator bases are less ideal since they will not have this feature. For example, the highest spin baryon states have $J \sim N_c$, so that the use of the one-body operator J instead of J_ℓ will not lead to an operator basis with this feature. For the physical situation of principle interest in this work, namely baryons containing a single heavy quark, the N_ℓ/N_c and N_h/N_c suppression factors will be 2/3 and 1/3, respectively.

The large-N_c heavy quark spin symmetry is independent of whether the quark flavor in question is heavy enough to have a valid $1/m_Q$ expansion. If the quark flavor is sufficiently heavy that it has a valid $1/m_Q$ expansion, then heavy quark spin symmetry is also a consequence of the $m_Q \to \infty$ limit, independent of whether the large-N_c limit is taken. It thus follows that heavy quark spin symmetry violation is suppressed by $(1/N_c m_Q)$ for very heavy quarks.

For the special case of baryons containing a single heavy quark, there is a heavy quark flavor symmetry in the limit that each of the heavy quark flavors is regarded as infinitely heavy. More specifically, for baryons with $N_h = 1$ in the limit $m_b \to \infty$ and $m_c \to \infty$ with m_c/m_b held fixed, heavy quark spin symmetry for each of the heavy flavors is promoted to a heavy-quark spin-flavor symmetry $SU(4)_h$ [1]. In the presence of heavy quark flavor symmetry (regardless of the origin of the heavy quark flavor symmetry[2]), heavy-quark spin-flavor symmetry for baryons is a consequence of the large-N_c limit. Thus, when the large-N_c limit $N_c \to \infty$, with $N_c \Lambda_{\rm QCD}/m_Q$ held fixed, is added to the heavy quark limit, there is a $SU(6)_\ell \times SU(4)_h$ spin-flavor symmetry for baryons with one heavy-quark [3]. The heavy-quark spin-flavor algebra is generated by the one-body operators

$$\begin{aligned} J_h^i &= Q^\dagger \frac{\sigma^i}{2} Q, \\ I_h^a &= Q^\dagger \frac{\tau^a}{2} Q, \\ G_h^{ia} &= Q^\dagger \frac{\sigma^i \tau^a}{4} Q, \end{aligned} \tag{3.9}$$

where $a = 1,2,3$ is the heavy flavor quark index. The charm and bottom quark spin and number operators are related to the heavy quark operators and N_h through

$$\begin{aligned} J_h^i &= J_c^i + J_b^i, \\ I_h^3 &= \frac{1}{2}(N_{\rm charm} - N_b), \\ G_h^{i3} &= \frac{1}{2}(J_c^i - J_b^i), \end{aligned} \tag{3.10}$$

and $N_h = N_{\rm charm} + N_b$. The heavy quark flavor symmetry only applies to baryons with $N_h = 1$, and so operators in the $1/N_c$ expansion for these baryons will contain no more than one heavy-quark one-body operator J_h^i, I_h^a, and G_h^{ia}.

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[1]For baryons containing N_Q heavy quarks of type Q and $N_{Q'}$ heavy quarks of type Q', $O_h^{(n)}$ is an n-body operator which is at most N_Q-body in J_Q^i and at most $N_{Q'}$-body in $J_{Q'}^i$:

$$\sum_{p+q=0}^{N_h} \frac{1}{N_c^{p+q}} O_h^{(p+q)} \to \sum_{p=0}^{N_Q} \sum_{q=0}^{N_{Q'}} \frac{1}{N_c^{p+q}} O_Q^{(p)} O_{Q'}^{(q)}. \tag{3.6}$$

[2]For example, if there were two moderately heavy quarks of similar mass in QCD, there would be a flavor symmetry among these quarks and a spin-flavor symmetry for baryons containing these quarks in the large-N_c limit. There would not be a spin-flavor symmetry for mesons in this situation.

Finally, the expansion (3.5) can easily be generalized to encompass baryons with differing number of light quarks and heavy quarks Q and Q' by including all operators up to N_c-body in the light-quark one-body operators and the heavy-quark one-body operators, which now consist of N_Q, J^i_Q, $N_{Q'}$, and $J^i_{Q'}$. Notice that the heavy quark number operators N_Q and $N_{Q'}$ are now to be regarded as heavy-quark one-body operators since one is not restricted to baryons with fixed numbers of Q and Q' quarks. (Because of the constraint $N_c = N_{\ell\!\!/} + N_Q + N_{Q'}$, it is not necessary to introduce $N_{\ell\!\!/}$ as an additional one-body operator as well.) The coefficients of the expansion (3.5) contain implicit N_Q/N_c and $N_{Q'}/N_c$ dependences, which appear as operators in the generalized expansion.

IV. MASSES OF HEAVY BARYONS WITH FIXED STRANGENESS

In this section, the masses of baryons containing heavy quarks are analyzed in the combined $1/m_Q$ and $1/N_c$ expansion for the special case of fixed strangeness. It is useful to study this special case because it illustrates the structure of the combined expansion without the complication of flavor SU(3) breaking. The analysis here depends only on the presence of isospin flavor symmetry. The general SU(3) mass analysis is given in Secs. V and VI.

The baryons containing a single heavy quark Q with zero strangeness are the Λ_Q, Σ_Q, and Σ_Q^*. The HQET $1/m_Q$ expansion of these masses is given by

$$\begin{aligned}
\Lambda_Q &= m_Q + \overline{\Lambda}_T - \frac{\lambda_{1T}}{2m_Q} + \cdots,\\
\Sigma_Q &= m_Q + \overline{\Lambda}_S - \frac{\lambda_{1S}}{2m_Q} - \frac{4\lambda_2}{2m_Q} + \cdots,\\
\Sigma_Q^* &= m_Q + \overline{\Lambda}_S - \frac{\lambda_{1S}}{2m_Q} + \frac{2\lambda_2}{2m_Q} + \cdots,
\end{aligned} \tag{4.1}$$

to order $1/m_Q^2$ in the $1/m_Q$ expansion. The leading term in this expansion is exactly m_Q to all orders in the $1/N_c$ expansion. Each of the subleading parameters $\overline{\Lambda}$, λ_1, and λ_2 has an expansion in $1/N_c$,

$$\begin{aligned}
\overline{\Lambda} &= c_0 N_c \mathbb{1} + c_2 \frac{1}{N_c} J_{\ell\!\!/}^2,\\
\frac{\lambda_1}{2m_Q} &= c_0' \frac{1}{2m_Q} N_Q + c_2' \frac{1}{N_c^2} \frac{1}{2m_Q} N_Q J_{\ell\!\!/}^2,\\
-d_H \frac{\lambda_2}{2m_Q} &= c_2'' \frac{2}{N_c m_Q} (J_{\ell\!\!/} \cdot J_Q),
\end{aligned} \tag{4.2}$$

where the factor of 2 in the λ_2 expansion follows from the factor of 4 in the definition of $d_H = -4(J_{\ell\!\!/} \cdot J_Q)$. The unknown coefficients in Eq. (4.2) are a function of $1/N_c$ and the QCD scale $\Lambda_{\rm QCD}$. Each coefficient has an expansion in $1/N_c$ beginning at order unity in the $1/N_c$ expansion. (The coefficients do not depend on $1/m_Q$ since the HQET parameters are defined to occur at a definite order in the $1/m_Q$ expansion.) Without loss of generality, it is possible to set $c_0 \equiv \Lambda$, and use this relation as the definition of a QCD scale. The remaining coefficients are now given by a dimensional power of Λ times a dimensionless function of $1/N_c$ beginning at order unity in the $1/N_c$ expansion. For example, the coefficient c_2 is proportional to Λ, whereas the $1/m_Q$-suppressed operators have coefficients proportional to Λ^2.

The numbers of operators of the $1/N_c$ expansions for $\overline{\Lambda}$, λ_1, and λ_2 are in one-to-one correspondence with the numbers of parameters $\overline{\Lambda}_T$, $\overline{\Lambda}_S$, λ_{1T}, λ_{1S}, and λ_2 appearing in Eq. (4.1). The $1/N_c$ expansion predicts that the $\overline{\Lambda}$ parameters of the $\overline{3}$ and 6 are equal to $N_c\Lambda$ with a splitting of relative order $1/N_c^2$ compared to the leading contribution. Similarly, the λ_1 parameters of the $\overline{3}$ and 6 are order unity in the $1/N_c$ expansion with a splitting of relative order $1/N_c^2$. Finally, the λ_2 parameter for baryons is order $1/N_c$. The $1/N_c$ scaling of λ_2 can be tested by comparing the meson and baryon hyperfine splittings resulting from the heavy-quark spin-symmetry-violating chromomagnetic operator. The baryon mass splitting is only measured in the charm system at present.[3] The charm meson hyperfine mass splitting is given by

$$(D^* - D) = \frac{2\lambda_2^{\rm meson}}{m_c} + \cdots, \tag{4.3}$$

whereas the analogous charm baryon mass splitting is

$$(\Sigma_c^* - \Sigma_c) = \frac{3\lambda_2^{\rm baryon}}{m_c} + \cdots. \tag{4.4}$$

The naive $1/N_c$ scaling predicted by the baryon $1/N_c$ expansion implies

$$\lambda_2^{\rm baryon} \sim \frac{1}{N_c} \lambda_2^{\rm meson}. \tag{4.5}$$

The measured charm meson and baryon mass differences $(D^* - D) = 141$ MeV and $(\Sigma_c^* - \Sigma_c) = 77.1 \pm 5 \pm 5$ MeV are in remarkable agreement with this scaling. The $(D^* - D)$ mass difference determines the canonical heavy quark symmetry suppression factor $\Lambda_{\rm QCD}^2/m_c$:

$$\frac{\lambda_2^{\rm meson}}{m_c} \sim \frac{\Lambda_{\rm QCD}^2}{m_c} = \frac{1}{2}(D^* - D). \tag{4.6}$$

The λ_1 parameters for baryons and mesons also can be related. The $1/N_c$ expansion implies

$$\lambda_1^{\rm baryon} \sim \lambda_1^{\rm meson}. \tag{4.7}$$

Equation (4.7) could be used to estimate the magnitude of $\lambda_1^{\rm baryon}$ if $\lambda_1^{\rm meson}$ is known accurately. The most recent extraction of $\lambda_1^{\rm meson}$ [11] is too uncertain, however, so the naive estimate

[3] Recent measurements of the $\Sigma_b^{(*)}$ masses by DELPHI are probably unreliable and will be ignored.

TABLE I. Mass splittings of baryons containing a single heavy quark Q for strangeness $S=0$ and $S=-1$ baryons. Each operator is an isospin singlet; the $J_\ell \cdot J_Q$ operator violates heavy quark spin symmetry. The operator matrix element and $1/m_Q$, $1/N_c$, and isospin-flavor-breaking suppressions of each mass combination are tabulated. The singlet mass combination has a contribution m_Q and a contribution of order N_c at leading order, i.e., $m_Q+N_c\Lambda$.

Operator	$(SU(2),J_Q)$	Mass combination	$\langle O\rangle$	$1/m_Q$	$1/N_c$	Flavor
$\mathbb{1}$	(1,0)	Λ_Q	1	*	*	1
J_ℓ^2	(1,0)	$\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)-\Lambda_Q$	2	1	$1/N_c$	1
$J_\ell \cdot J_Q$	(1,1)	$\Sigma_Q^*-\Sigma_Q$	$\frac{3}{2}$	$2/m_Q$	$1/N_c$	1
$\mathbb{1}$	(1,0)	Ξ_Q	1	*	*	1
J_ℓ^2	(1,0)	$\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)-\Xi_Q$	2	1	$1/N_c$	1
$J_\ell \cdot J_Q$	(1,1)	$\Xi_Q^*-\Xi_Q'$	$\frac{3}{2}$	$2/m_Q$	$1/N_c$	1

$$\frac{\lambda_1^{\text{baryon}}}{2m_Q} \sim \frac{\Lambda_{\text{QCD}}^2}{2m_Q}, \tag{4.8}$$

is used in Eq. (4.2).

We now proceed to analyze the masses of baryons containing heavy quarks using the combined $1/m_Q$ and $1/N_c$ operator expansion developed in the previous section. We consider several different mass expansions.

Let us first consider the operator expansion for strangeness zero baryons containing a single flavor of heavy quark Q. The three baryons masses Λ_Q, Σ_Q, and Σ_Q^* are parametrized by the three operators [3]

$$M=(m_Q+N_c\Lambda+\cdots)\mathbb{1}+\frac{1}{N_c}J_\ell^2+\frac{2}{N_c m_Q}(J_\ell\cdot J_Q), \tag{4.9}$$

where the leading N_c and m_Q dependence of each operator appears explicitly. The leading operator $\mathbb{1}$ has a coefficient given exactly by $m_Q+N_c\Lambda$ up to a correction of order $1/m_Q$ in the combined $1/m_Q$ and $1/N_c$ expansion. It is to be understood that every other operator is accompanied by a coefficient

$$c\left(\Lambda,\frac{1}{m_Q},\frac{1}{N_c}\right), \tag{4.10}$$

with an expansion in $1/m_Q$ and $1/N_c$ beginning at order unity. For example, expanding to order $1/m_Q^2$ in the $1/m_Q$ expansion, there is an order $(1/2m_Q)$ contribution to the operator $\mathbb{1}$ from the λ_1 contribution proportional to c_0' in Eq. (4.2) and an order $(1/N_c^2)(1/2m_Q)$ contribution to the operator J_ℓ^2 from the λ_1 contribution proportional to c_2'. Each of the operators in Eq. (4.9) contributes to one specific linear combination of the three baryon masses. The operators and their corresponding mass combinations are tabulated in Table I, together with the operator matrix element and the leading $1/m_Q$ and $1/N_c$ dependence for each mass splitting. The operator matrix elements depend implicitly on $N_\ell=2$ and $N_Q=1$. Up to unknown coefficients of order unity, the combined $1/m_Q$ and $1/N_c$ expansion predicts that the mass combinations Λ_Q, $\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)-\Lambda_Q$, and $(\Sigma_Q^*-\Sigma_Q)$ satisfy a hierarchy given by $(m_Q+N_c\Lambda)$, $2\Lambda/N_c$, and $\frac{3}{2}(2\Lambda^2/N_c m_Q)$. For $Q=c$, the experimental values of the mass splittings are 2285.0 ± 0.6 MeV, $219\pm3\pm3$ MeV, and $77\pm5\pm5$ MeV, which compares very favorably[4] with the theoretical hierarchy for canonical values of m_c and Λ. (Note that the experimental values and errors of the hyperfine splittings are dominated by the uncertainty in the mass measurement $\Sigma_c^*=2530\pm5\pm5$ MeV.)

For two flavors of heavy quark $Q=c$ and $Q'=b$, the previous analysis of the baryon mass spectrum holds for each flavor of heavy quark separately. In the presence of heavy quark flavor symmetry, a combined analysis of the six heavy baryon masses using heavy-quark spin-flavor operators is possible. This mass expansion for baryons containing a single heavy quark of flavor Q or Q' is given by

$$\begin{aligned}M=&\left[\frac{1}{2}(m_Q+m_{Q'})+N_c\Lambda+\dots\right]\mathbb{1}+I_h^3(m_Q-m_{Q'})\\&+\frac{1}{N_c}J_\ell^2+\frac{1}{N_c^2}I_h^3(J_\ell)^2\left(\frac{1}{2m_Q}-\frac{1}{2m_{Q'}}\right)+\frac{1}{N_c}(J_\ell\cdot J_h)\\&\times\left(\frac{1}{m_Q}+\frac{1}{m_{Q'}}\right)+\frac{1}{N_c}J_\ell^i G_h^{i3}\left(\frac{2}{m_Q}-\frac{2}{m_{Q'}}\right),\end{aligned} \tag{4.11}$$

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients of the form Eq. (4.10) are understood to accompany each operator [with the exception of the order m_Q and $N_c\Lambda$ terms given explicitly on the first line of Eq. (4.11)]. The important new operators of the heavy-quark spin-flavor analysis are the heavy-quark flavor-violating operators which do not violate heavy quark spin symmetry: I_h^3 and $I_h^3(J_\ell)^2$. The most interesting operator is $I_h^3(J_\ell)^2$. The leading order J_ℓ^2 hyperfine splitting for heavy baryons is a heavy-quark flavor-independent splitting of order $1/N_c$. The $I_h^3(J_\ell)^2$ operator represents heavy quark flavor symmetry breaking in the J_ℓ^2 hyperfine splittings. This heavy-quark flavor-symmetry-breaking hyperfine mass splitting is order $1/N_c^2$ times $(1/2m_Q-1/2m_{Q'})$, and is sensitive to the subleading $(1/N_c^2)(1/2m_Q)$ contribution to the J_ℓ^2 hy-

[4]The parameter values $m_c=1450$ MeV, $\Lambda_{\text{QCD}}=\Lambda=310$ MeV, $\Lambda_{\text{QCD}}^2/m_c=66$ MeV, $\Lambda_{\text{QCD}}/m_c=0.21$, $m_b=4757$ MeV, and $m_c/m_b=0.3$ are used throughout this paper. These parameter values follow from the charm baryon mass analysis of this work, and are consistent with canonical values used in the heavy quark literature.

TABLE II. $I_h=1$ heavy baryon mass splittings which violate heavy quark flavor symmetry, but preserve heavy quark spin symmetry for strangeness $S=0$ and $S=-1$ baryons. The mass splittings are suppressed by $(1/2m_c-1/2m_b)$ and one additional factor of $1/N_c$. The I_h^3 mass combination has a leading contribution of (m_c-m_b).

Operator	(SU(2),J_Q)	Mass combination	$\langle O\rangle$	$1/m_Q$	$1/N_c$	Flavor
I_h^3	(1,0)	$(\Lambda_c-\Lambda_b)$	1	(m_c-m_b)	1	1
$I_h^3(J_\ell)^2$	(1,0)	$[\frac{1}{3}(\Sigma_c+2\Sigma_c^*)-\Lambda_c]-[\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b]$	2	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c^2$	1
I_h^3	(1,0)	$(\Xi_c-\Xi_b)$	1	(m_c-m_b)	1	1
$I_h^3(J_\ell)^2$	(1,0)	$[\frac{1}{3}(\Xi_c'+2\Xi_c^*)-\Xi_c]-[\frac{1}{3}(\Xi_b'+2\Xi_b^*)-\Xi_b]$	2	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c^2$	1

perfine splitting. Similar remarks hold for the operator I_h^3 which represents heavy quark flavor symmetry breaking in the singlet $\mathbb{1}$ mass, except that this case is less interesting since there is a leading contribution equal to $(m_Q-m_{Q'})$ which dominates the subleading contribution proportional to $(1/2m_Q-1/2m_{Q'})$.

The mass combinations corresponding to the operators I_h^3 and $I_h^3(J_\ell)^2$ are tabulated in Table II. Both operators probe heavy flavor violation in λ_1. The latter operator is sensitive to the the λ_1 contribution proportional to c_2', while the former operator depends on the λ_1 operator proportional to c_0' only at subleading order. The $\Sigma_b^{(*)}$ masses are not reliably measured at present, and so the $I_h^3(J_\ell)^2$ splitting cannot be evaluated. Instead, the mass relation

$$\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b=\frac{1}{3}(\Sigma_c+2\Sigma_c^*)-\Lambda_c \qquad (4.12)$$

can be used to predict the bottom baryon J_ℓ^2 hyperfine splitting to a theoretical accuracy of order $2/N_c^2$ times $\Lambda^2(1/2m_c-1/2m_b)$, which is about 5 MeV. Thus, the mass splitting $[\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b]$ is predicted to be equal to 219 ± 7 MeV using Eq. (4.12). The measured $(\Lambda_c^+-\Lambda_b^0)$ splitting of $-3338\pm5\pm4$ MeV gives a measure of (m_c-m_b), up to corrections of order $\Lambda^2(1/2m_c-1/2m_b)$ which is about 23 MeV.

The remaining operators in the expansion (4.12) correspond to sum and difference combinations of Q and Q' splittings. There is no advantage to studying these combinations rather than the Q and Q' splittings separately, since there is no cancellation of leading order contributions in these terms. For example, there is no reason to study $J_\ell\cdot J_h$ and $J_\ell^i G_h^{i3}$ rather than $J_\ell\cdot J_Q$ and $J_\ell\cdot J_{Q'}$, which are order $2/N_c m_Q$ and $2/N_c m_{Q'}$, respectively.

Additional information about the heavy quark baryon mass spectrum can be obtained by generalizing the expansion to encompass baryons with differing numbers of heavy quarks. The strangeness zero baryons for one flavor of heavy quark Q consist of the $N_Q=0$ baryons N and Δ; the $N_Q=1$ baryons Λ_Q, Σ_Q, and Σ_Q^*; the $N_Q=2$ baryons Ξ_{QQ} and Ξ_{QQ}^*; and the $N_Q=3$ baryon Ω_{QQQ}^*. The $1/N_c$ operator basis for strangeness zero baryons with $N_Q=0,1,2,3$ heavy quarks consists of the eight operators $\mathbb{1}$, N_Q, J_ℓ^2, $(J_\ell\cdot J_Q)$, $N_Q J_\ell^2$, N_Q^2, $N_Q(J_\ell\cdot J_Q)$, and N_Q^3. Notice that N_Q is to be regarded as a heavy-quark one-body operator along with J_Q when considering baryons with differing numbers of heavy quarks. Although it is interesting to study this expansion for $N_Q=0,1,2,3$ baryons, the principal interest of this work will be to relate mass splittings of baryons containing a single heavy quark to the well-measured mass splittings of baryons containing no heavy quarks, namely, baryons in the spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decuplet. Thus, we will restrict the operator expansion to baryons with $N_Q=0$ and 1. The operator expansion for strangeness zero baryons containing no heavy quark and baryons containing a single heavy quark of flavor Q is given by

$$M=N_Q(m_Q+\cdots)+N_c\Lambda\mathbb{1}+\frac{1}{N_c}J_\ell^2+\frac{2}{N_c m_Q}(J_\ell\cdot J_Q)+\frac{1}{N_c^2}N_Q J_\ell^2, \qquad (4.13)$$

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients of the form Eq. (4.10) are understood to accompany each operator with the exception of the order m_Q and $N_c\Lambda$ terms. Since one is only considering baryons containing upto one heavy quark, each operator in the expansion has at most one heavy-quark one-body operator N_Q or J_Q. Comparing with Eq. (4.9), there are two new operators in the present expansion: N_Q and $N_Q J_\ell^2$. These operators reduce to $\mathbb{1}$ and J_ℓ^2 for baryons with fixed $N_Q=1$, reproducing expansion (4.9). The most interesting operator is $N_Q J_\ell^2$ which represents the heavy-quark number-dependent contribution to the J_ℓ^2 hyperfine splittings. This heavy-quark number-dependent mass splitting is of order $(1/N_c^2)$. The N_Q operator is less interesting since it corresponds to the leading N_Q-dependent mass splitting which is equal to m_Q. There is a subleading order unity contribution to this splitting.

The mass combinations corresponding to N_Q and $N_Q J_\ell^2$ are tabulated in Table III. The $[\Lambda_Q-\frac{1}{4}(5N-\Delta)]$ splittings of 1419.2 ± 0.7 and 4757.2 ± 6.4 for $Q=c$ and b, respectively, give a measure of m_c and m_b, up to corrections of order Λ. The $N_Q J_\ell^2$ mass relation

$$\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)-\Lambda_Q=\frac{2}{3}(\Delta-N) \qquad (4.14)$$

TABLE III. Mass splittings between baryons containing a single heavy quark Q and baryons containing no heavy quarks for strangeness $S=0$ and $S=-1$ baryons. The mass splittings are suppressed by an additional factor of $1/N_c$. The N_Q mass combination has a leading order contribution of m_Q.

Operator	$(\mathrm{SU}(2),J_Q)$	Mass combination	$\langle O\rangle$	$1/m_Q$	$1/N_c$	Flavor
N_Q	(1,0)	$\Lambda_Q-\frac{1}{4}(5N-\Delta)$	1	m_Q	1	1
$N_Q J_\ell^2$	(1,0)	$[\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)-\Lambda_Q]-\frac{2}{3}(\Delta-N)$	2	1	$1/N_c^2$	1
N_Q	(1,0)	$\Xi_Q-\frac{1}{4}[\frac{5}{4}(3\Sigma+\Lambda)-\Sigma^*]$	1	m_Q	1	1
$N_Q J_\ell^2$	(1,0)	$[\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)-\Xi_Q]-\frac{2}{3}[\Sigma^*-\frac{1}{4}(3\Sigma+\Lambda)]$	2	1	$1/N_c^2$	1

is predicted to be satisfied to of order $(1/N_c^2)$. Relation (4.15) has been obtained previously in the context of the Skyrme model where heavy baryons appear as bound states of Skyrmions and heavy mesons [12,13]. The present derivation is model independent, and provides a prediction for the accuracy to which the relation is satisfied. The theoretically predicted accuracy can be tested in the charm baryon sector where the relevant masses are measured. The $(\Delta-N)$ splitting of 292 ± 1 MeV and the $[\frac{1}{3}(\Sigma_c+2\Sigma_c^*)-\Lambda_c]$ splitting of 219 ± 4.6 MeV are both J_ℓ^2/N_c splittings with J_ℓ^2 matrix elements of 3 and 2, respectively. The mass relation (4.14) is satisfied to 24 ± 5 MeV, to be compared with the predicted violation of $(1/N_c^2)\Lambda$ times the matrix element $\langle N_Q J_\ell^2\rangle=2$, which is about 69 MeV. [Alternatively, the violation should be suppressed by a factor of $(1/N_c)$ compared to the charm baryon J_ℓ^2 splitting of 219 MeV.] Relation (4.14) can be used to predict the bottom baryon J_ℓ^2 hyperfine splitting $[\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b]$ to an accuracy of $(1/N_c^2)$ as well. The theoretical prediction is

$$\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b=195\pm69\ \ \mathrm{MeV}, \qquad (4.15)$$

which is consistent with, but much less accurate than, the prediction made from the previous expansion using the $I_h^3(J_\ell)^2$ mass combination.

It is instructive to generalize the expansion (4.13) to two heavy quark flavors Q and Q'. The mass expansion for strangeness zero baryons containing zero or one heavy quark of flavor Q or Q' is given by

$$\begin{aligned}M=&N_Q(m_Q+\cdots)+N_{Q'}(m_{Q'}+\cdots)+N_c\Lambda\mathbb{1}+\frac{1}{N_c}J_\ell^2\\&+\frac{1}{N_c^2}N_QJ_\ell^2+\frac{1}{N_c^2}N_{Q'}J_\ell^2+\frac{2}{N_cm_Q}(J_\ell\cdot J_Q)\\&+\frac{2}{N_cm_{Q'}}(J_\ell\cdot J_{Q'}),\end{aligned} \qquad (4.16)$$

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients are understood to accompany all operators except the m_Q and $N_c\Lambda$ terms. This expansion can be rewritten in terms of heavy-quark spin-flavor operators as

$$\begin{aligned}M=&\frac{1}{2}N_h[(m_Q+m_{Q'})+\cdots]+I_h^3(m_Q-m_{Q'})+N_c\Lambda\mathbb{1}\\&+\frac{1}{N_c}J_\ell^2+\frac{1}{N_c^2}N_hJ_\ell^2+\frac{1}{N_c^2}I_h^3(J_\ell)^2\left(\frac{1}{2m_Q}-\frac{1}{2m_{Q'}}\right)\\&+\frac{1}{N_c}(J_\ell\cdot J_h)\left(\frac{1}{m_Q}+\frac{1}{m_{Q'}}\right)\\&+\frac{1}{N_c}J_\ell^iG_h^{i3}\left(\frac{2}{m_Q}-\frac{2}{m_{Q'}}\right),\end{aligned} \qquad (4.17)$$

where $N_h=N_Q+N_{Q'}$. The interesting suppressions of the heavy-quark flavor-symmetry-violating and heavy-quark number-dependent J_ℓ^2 splittings discussed in the preceeding paragraphs are manifest.

The above analysis which applies to $S=0$ baryons in the isospin limit also can be performed in the $S=-1$ sector. These mass combinations are provided in Tables I, II, and III as well. The charm baryon masses Ξ_c and Ξ_c^* are well measured, but the lone Ξ_c' mass measurement is unpublished and possibly unreliable. A precise measurement of the Ξ_c' is likely at CLEO in the not too distant future. A prediction of the Ξ_c' mass using the $N_QJ_\ell^2$ mass relation

$$\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)-\Xi_Q=\frac{2}{3}\left[\Sigma^*-\frac{1}{4}(3\Sigma+\Lambda)\right] \qquad (4.18)$$

is possible, but not terribly accurate. Relation (4.18) is predicted to be satisfied to an accuracy of order $(2\Lambda/N_c^2)$, which is about 69 MeV. The $[\Sigma^*-\frac{1}{4}(3\Sigma+\Lambda)]$ mass splitting is 210.8 ± 0.4 MeV, and so Eq. (4.18) predicts $[\frac{1}{3}(\Xi_c'+2\Xi_c^*)-\Xi_c]=140.5\pm69$ MeV. The $\Xi_c\equiv\frac{1}{2}(\Xi_c^++\Xi_c^0)$ mass is 2467.7 ± 1.2 MeV and the analogous Ξ_c^* mass is 2644.0 ± 1.6 MeV, so the Ξ_c' prediction is not very useful. A much more precise prediction of the Ξ_c' mass is made in Sec. V. The bottom baryon $S=-1$ J_ℓ^2 splitting also is predicted by relation (4.18) to the same theoretical accuracy. Once the Ξ_c' mass is measured, the bottom baryon J_l^2 splitting can be accurately predicted from the charm baryon mass splitting.

The results of this section follow from the combined $1/m_Q$ and $1/N_c$ expansion using only isospin symmetry of the u and d quarks. The SU(3) analysis including SU(3) flavor breaking is provided in the next section. The SU(3) analysis leads to more precise predictions than the isospin analysis. The isospin symmetry predictions of this section are consistent with the mass predictions of Sec. V. The isospin-breaking analysis is given in Sec. VI.

TABLE IV. Mass splittings of baryons containing a single heavy quark Q. SU(3) flavor and heavy quark spin J_Q symmetry quantum numbers of each operator and mass combination are given explicitly. The operator matrix element for each mass combination and suppression factors of $1/m_Q$, $1/N_c$, and SU(3) flavor breaking ϵ are tabulated. The singlet operator is $m_Q+N_c\Lambda$ at leading order.

Operator	$(SU(3),J_Q)$	Mass combination	$\langle O\rangle$	$1/m_Q$	$1/N_c$	Flavor
$\mathbb{1}$	(1,0)	$\frac{1}{3}(\Lambda_Q+2\Xi_Q)$	1	*	*	1
J_ℓ^2	(1,0)	$-\frac{1}{3}(\Lambda_Q+2\Xi_Q)$ $+\frac{1}{18}[3(\Sigma_Q+2\Sigma_Q^*)+2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]$	2	1	$1/N_c$	1
$J_\ell\cdot J_Q$	(1,1)	$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)+2(\Xi_Q^*-\Xi_Q')+(\Omega_Q^*-\Omega_Q)]$	$\frac{3}{2}$	$2/m_Q$	$1/N_c$	1
T^8	(8,0)	$(\Lambda_Q-\Xi_Q)$	$\frac{3}{2\sqrt{3}}$	1	1	ϵ
$J_\ell^i G^{i8}$	(8,0)	$-\frac{5}{8}(\Lambda_Q-\Xi_Q)$ $+\frac{1}{24}[3(\Sigma_Q+2\Sigma_Q^*)-(\Xi_Q'+2\Xi_Q^*)-2(\Omega_Q+2\Omega_Q^*)]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}$	1	$1/N_c$	ϵ
$J_Q^i G^{i8}$	(8,1)	$\Xi_Q\Xi_Q'$	$\frac{3}{8}$	$2/m_Q$	$1/N_c$	ϵ
$(J_\ell\cdot J_Q)T^8$	(8,1)	$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)-(\Xi_Q^*-\Xi_Q')-2(\Omega_Q^*-\Omega_Q)]-\frac{5}{2\sqrt{3}}\Xi_Q\Xi_Q'$	$\frac{1}{2\sqrt{3}}\frac{15}{4}$	$2/m_Q$	$1/N_c^2$	ϵ
$\{T^8,T^8\}$	(27,0)	$\frac{1}{6}[(\Sigma_Q+2\Sigma_Q^*)-2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]$	$\frac{3}{2}$	1	$1/N_c$	ϵ^2
$J_Q^i\{T^8,G^{i8}\}$	(27,1)	$\frac{1}{4}[(\Sigma_Q^*-\Sigma_Q)-2(\Xi_Q^*-\Xi_Q')+(\Omega_Q^*-\Omega_Q)]$	$\frac{9}{16}$	$2/m_Q$	$1/N_c^2$	ϵ^2

V. SU(3) ANALYSIS OF HEAVY BARYON MASSES

The SU(3) analysis of the masses of baryons containing a single heavy quark in the combined $1/m_Q$ and $1/N_c$ expansion is a straightforward generalization of the isospin analysis of the previous section. The lowest-lying baryons containing a single heavy quark are the spin-$\frac{1}{2}$ $\bar{3}$ which consists of the isosinglet Λ_Q and the isodoublet Ξ_Q; the spin-$\frac{1}{2}$ 6 which consists of the isotriplet Σ_Q, the isodoublet Ξ_Q', and the isosinglet Ω_Q; and the spin-$\frac{3}{2}$ 6 which consists of the isotriplet Σ_Q^*, the isodoublet Ξ_Q^*, and the isosinglet Ω_Q^*. Heavy baryon masses will be denoted by their particle labels. Each of the $I=0$ masses refers to the average mass of the isomultiplet. For example, $\Xi_c=\frac{1}{2}(\Xi_c^+ + \Xi_c^0)$, etc. In addition to the eight $I=0$ heavy baryon masses, there is an $I=0$ off-diagonal mass $\Xi_Q\Xi_Q'=\Xi_Q'\Xi_Q$ between the spin-$\frac{1}{2}$ isodoublets, which will be referred to as $\Xi_Q\Xi_Q'$ throughout the rest of the paper. The $I=0$ off-diagonal mass is defined to be the average off-diagonal mass of the spin-$\frac{1}{2}$ $S=-1$ isodoublets:

$$\Xi_c\Xi_c'=\frac{1}{2}(\Xi_c^+\Xi_c'^+ + \Xi_c^0\Xi_c^{0\prime}),$$

$$\Xi_b\Xi_b'=\frac{1}{2}(\Xi_b^0\Xi_b'^0 + \Xi_b^-\Xi_b^{-\prime}). \tag{5.1}$$

Many of the charm baryon masses are now measured. The measured $I=0$ masses are

$$\Lambda_c=2285.0\pm0.6 \text{ MeV } [14],$$

$$\Xi_c=2467.7\pm1.2 \text{ MeV } [14],$$

$$\Sigma_c=2452.9\pm0.6 \text{ MeV } [14\text{–}16], \tag{5.2}$$

$$\Omega_c^0=2704\pm4 \text{ MeV } [14],$$

$$\Xi_c^*=2644.0\pm1.6 \text{ MeV } [17].$$

The masses $\Xi_c'\sim2560$ MeV [18] and $\Sigma_c^*=2530\pm5\pm5$ MeV [14] are measured, but are less reliable or precise. The Ω_c^* mass and the off-diagonal mass $\Xi_c\Xi_c'$ are unmeasured. At present, only the bottom baryon

$$\Lambda_b=5623\pm5\pm4 \text{ MeV } [19] \tag{5.3}$$

is accurately measured. As stated earlier, the reported $\Sigma_b^{(*)}$ measurements by DELPHI are not used.

The operator analysis including SU(3) flavor breaking decomposes[5] into SU(3) flavor singlet, octet, and 27 expansions:

$$M=M^{(1)}+\epsilon M^{(8)}+\epsilon^2 M^{(27)}, \tag{5.4}$$

where ϵ is an SU(3)-violating parameter whose magnitude is governed by the quark mass difference $(m_s-\hat{m})$ divided by the chiral-symmetry-breaking scale Λ_χ. The symmetry-breaking parameter $\epsilon\sim0.25$. The flavor singlet, octet, and 27 mass expansions are given by

$$M^{(1)}=(m_Q+N_c\Lambda+\cdots)\mathbb{1}+\frac{1}{N_c}J_\ell^2+\frac{1}{N_c}\frac{2}{m_Q}(J_\ell\cdot J_Q),$$

$$M^{(8)}=T^8+\frac{1}{N_c}J_\ell^i G^{i8}+\frac{1}{N_c}\frac{2}{m_Q}J_Q^i G^{i8}+\frac{1}{N_c^2}\frac{2}{m_Q}(J_\ell\cdot J_Q)T^8, \tag{5.5}$$

[5]The analytic flavor dependence of the flavor-27 expansion is given here. The nonanalytic contribution of order $\epsilon^{3/2}$ is proportional to $(\frac{1}{3}m_\pi^2-\frac{4}{3}m_K^2+m_\eta^2)$ which is numerically very small, and does not dominate the ϵ^2 counterterm [20,21]. The order $\epsilon^{3/2}$ contribution is computed in chiral perturbation theory in Ref. [22].

TABLE V. Heavy baryon mass hierarchy for $Q=c$ and $Q=b$. The experimental charm baryon mass splittings are evaluated using theoretically extracted values for the three masses $\Sigma_c^*=2532.5\pm5.3$ MeV, $\Xi_c'=2582.9\pm2.5$ MeV, and $\Omega_c^*=2746.6\pm6.1$ MeV. Agreement between the theory values appearing with an asterisk and experiment is imposed by the determination of the parameter values m_c, Λ, $(\epsilon\Lambda_\chi)$, and $(\epsilon^2\Lambda_\chi)$. Comparison of the remaining theory and experimental values provides support for the theoretical hierarchy.

Mass combination	Theory	$Q=c$	Expt. $Q=c$	$Q=b$
$\frac{1}{3}(\Lambda_Q+2\Xi_Q)$	$m_Q+N_c\Lambda$	2380*	2406.8 ± 0.8	5687
$-\frac{1}{3}(\Lambda_Q+2\Xi_Q)+\frac{1}{18}[3(\Sigma_Q+2\Sigma_Q^*)+2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]$	$2\frac{1}{N_c}\Lambda$	207	176.1 ± 1.9	207
$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)+2(\Xi_Q^*-\Xi_Q')+(\Omega_Q^*-\Omega_Q)]$	$3\frac{1}{N_c}\frac{\Lambda^2}{m_Q}$	66*	67.3 ± 3.1	20
$(\Lambda_Q-\Xi_Q)$	$\frac{3}{2\sqrt{3}}(\epsilon\Lambda_\chi)$	-195*	-182.7 ± 1.3	-195
$-\frac{5}{8}(\Lambda_Q-\Xi_Q)+\frac{1}{24}[3(\Sigma_Q+2\Sigma_Q^*)-(\Xi_Q'+2\Xi_Q^*)-2(\Omega_Q+2\Omega_Q^*)]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}\frac{1}{N_c}(\epsilon\Lambda_\chi)$	40.6	42.9 ± 1.9	40.6
$\Xi_Q\Xi_Q'$	$\frac{3}{4}\frac{1}{N_c}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.8	$--$	3.5
$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)-(\Xi_Q^*-\Xi_Q')-2(\Omega_Q^*-\Omega_Q)]-\frac{5}{2\sqrt{3}}\Xi_Q\Xi_Q'$	$\frac{1}{2\sqrt{3}}\frac{15}{2}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon\Lambda_\chi)$	11.3	$(15.4\pm3.6)-\frac{5}{2\sqrt{3}}\Xi_c\Xi_c'$	3.4
$\frac{1}{6}[(\Sigma_Q+2\Sigma_Q^*)-2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]$	$\frac{3}{2}\frac{1}{N_c}(\epsilon^2\Lambda_\chi)$	-4.4*	-4.4 ± 3.1	-4.4
$\frac{1}{4}[(\Sigma_Q^*-\Sigma_Q)-2(\Xi_Q^*-\Xi_Q')+(\Omega_Q^*-\Omega_Q)]$	$\frac{9}{8}\frac{1}{N_c^2}\frac{\Lambda}{m_Q}(\epsilon^2\Lambda_\chi)$	0.23	0 ± 2.7	0.07

$$M^{(27)}=\frac{1}{N_c}\{T^8,T^8\}+\frac{1}{N_c^2}\frac{2}{m_Q}J_Q^i\{T^8,G^{i8}\},$$

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients are understood to accompany all operators (except for the leading terms of the $\mathbb{1}$ operator). The operators appearing in the flavor-27 expansion require flavor singlet and octet subtractions: these subtractions are implied, but not given explicitly throughout this paper. The relevant flavor-27 projection operator for an operator O^{ab} with two symmetric flavor indices (ab) is

$$\delta^{a8}\delta^{b8}-\frac{1}{8}\delta^{ab}-\frac{3}{5}d^{ab8}d^{888}. \tag{5.6}$$

The operators and their corresponding mass combinations are tabulated in Table IV, together with the SU(3) and J_Q representations of the operator, the operator matrix element, and the leading $1/m_Q$, $1/N_c$, and SU(3)-flavor-breaking factors. Table V combines these factors to predict a hierarchy of mass combinations. The numerical values of the theoretical hierarchy are evaluated for canonical values of parameters[6] m_Q, Λ, $(\epsilon\Lambda_\chi)$, and $(\epsilon^2\Lambda_\chi)$ in the columns labeled $Q=c$ and $Q=b$. It is not possible to evaluate seven of the nine mass combinations using the measured charm baryon masses without knowledge of Ω_c^* and $\Xi_c\Xi_c'$. The measured mass combinations $\frac{1}{3}(\Lambda_c+2\Xi_c)$ and $(\Lambda_c-\Xi_c)$ which appear in column 4 were used to determine the parameters $(m_c+N_c\Lambda)$ and $(\epsilon\Lambda_\chi)$, and so do not test the theoretical hierarchy.

Table V predicts a hierarchy of mass relations obtained by successively neglecting operators in the mass expansion (5.4). The most accurate mass relation is the flavor-27 heavy-quark spin-violating relation

$$\frac{1}{4}[(\Sigma_Q^*-\Sigma_Q)-2(\Xi_Q^*-\Xi_Q')+(\Omega_Q^*-\Omega_Q)]=0, \tag{5.7}$$

which is satisfied up to a correction of order $\frac{9}{8}(1/N_c^2)(\Lambda^2/m_Q)(\epsilon^2\Lambda_\chi)$, which is about 0.23 MeV for $Q=c$ and 0.07 for $Q=b$. Thus, Eq. (5.7) is essentially an exact relation for both the charm and the bottom baryons. The physical content of this relation is that the three chromomagnetic mass splittings of the sextet, $(\Sigma_Q^*-\Sigma_Q)$, $(\Xi_Q^*-\Xi_Q')$, and $(\Omega_Q^*-\Omega_Q)$, contain only a singlet contribution and a contribution which is linear in strangeness. Differences of the splittings are equal:

$$(\Sigma_Q^*-\Sigma_Q)-(\Xi_Q^*-\Xi_Q')=(\Xi_Q^*-\Xi_Q')-(\Omega_Q^*-\Omega_Q). \tag{5.8}$$

Equation (5.8) is the equal spacing rule of Savage [22] for the chromomagnetic mass splittings of the sextet.

The next most accurate relation is the flavor-27 mass relation

[6]Canonical values for the flavor octet and flavor-27 mass splittings are $\epsilon\Lambda_\chi=225$ MeV and $(\epsilon^2\Lambda_\chi)=8.8$ MeV. These values are obtained from the octet and decuplet baryon masses. Note that the flavor-27 mass parameter is considerably smaller than ϵ times the octet mass parameter and may be underestimated.

$$\frac{1}{6}[(\Sigma_Q+2\Sigma_Q^*)-2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]=0, \tag{5.9}$$

which is expected to be satisfied to order $\frac{3}{2}(1/N_c)(\epsilon^2\Lambda_\chi)$, or about 4.4 MeV for $Q=c,b$. This mass relation respects heavy quark spin symmetry. It implies that the three spin-averaged masses of the sextet, $\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)$, $\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)$, and $\frac{1}{3}(\Omega_Q+2\Omega_Q^*)$, receive a singlet contribution and a contribution linear in strangeness. Differences of the splittings are predicted to be equal

$$\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)-\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)=\frac{1}{3}(\Omega_Q+2\Omega_Q^*)-\frac{1}{3}(\Xi_Q'+2\Xi_Q^*), \tag{5.10}$$

at the 8.8 MeV level. Equation (5.10) is an equal spacing rule for the spin-averaged masses of the sextet.

The two flavor-27 mass relations Eqs. (5.7) and (5.9) together imply the vanishing of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet flavor-27 combinations

$$\Sigma_Q-2\Xi_Q'+\Omega_Q=0,$$

$$\Sigma_Q^*-2\Xi_Q^*+\Omega_Q^*=0, \tag{5.11}$$

to a precision of about 8.8 MeV. The first of these equations was obtained previously by Savage [22].

There are two additional mass relations

$$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)-(\Xi_Q^*-\Xi_Q')-2(\Omega_Q^*-\Omega_Q)]-\frac{5}{2\sqrt{3}}\Xi_Q\Xi_Q'=0, \tag{5.12}$$

$$\Xi_Q\Xi_Q'=0,$$

satisfied to orders $\frac{15}{4}(2/N_c^2)(\Lambda/m_Q)(\epsilon\Lambda_\chi)/2\sqrt{3}$ and $\frac{3}{4}(1/N_c)(\Lambda/m_Q)(\epsilon\Lambda_\chi)$, respectively, which are numerically comparable: about 11.5 MeV for $Q=c$ and 3.5 for $Q=b$. The second relation gives an order of magnitude estimate of the $I=0$ off-diagonal mass $\Xi_Q\Xi_Q'$ while the first relation implies a determination of this off-diagonal mass in terms of the sextet masses. The two relations together yield the mass relation

$$\frac{1}{6}[3(\Sigma_Q^*-\Sigma_Q)-(\Xi_Q^*-\Xi_Q')-2(\Omega_Q^*-\Omega_Q)]=0, \tag{5.13}$$

which is predicted to hold to 16.6±11.5MeV for $Q=c$ and to 5.1±3.5 MeV for $Q=b$. This relation combined with the flavor-27 relation (5.7) implies that the chromomagnetic mass splittings of the sextet are equal,

$$(\Sigma_Q^*-\Sigma_Q)=(\Xi_Q^*-\Xi_Q')=(\Omega_Q^*-\Omega_Q). \tag{5.14}$$

Equation (5.14) was obtained previously by Savage [22]. The combined $1/m_Q$, $1/N_c$, and SU(3)-flavor-breaking expansion predicts that this equation need not be very accurate. The two equalities in Eq. (5.14) are predicted to be satisfied only to 19.9±13.8 MeV for $Q=c$ and to 6.1±4.2 MeV for $Q=b$. The equality $(\Sigma_Q^*-\Sigma_Q)=(\Omega_Q^*-\Omega_Q)$ is predicted to be satisfied only to 39.8±27.6 MeV for $Q=c$ and 12.2 ±8.4 MeV for $Q=b$.

The remaining mass combinations in Table V are not suppressed enough to yield useful mass relations.

The two flavor-27 mass relations can be used to predict the poorly measured Ξ_c' mass and the unmeasured Ω_c^* mass. [The mass $\Sigma_c^*=2530\pm7$ MeV is known to greater precision than the mass relation (5.13) which would be needed to extract a third charm baryon mass.] The masses $\Xi_c'=2578.5\pm4.8$ MeV and $\Omega_c^*=2758.0\pm11.7$ MeV are obtained using the flavor-27 sextet mass relations (5.11). A different, more precise, extraction of these masses will be obtained shortly.

The analysis of Sec. IV showed that additional constraints on the heavy baryon hierarchy follow from studying heavy quark flavor symmetry violation and heavy quark number dependence in the heavy baryon mass splittings which do not violate heavy quark spin symmetry. For the SU(3) analysis, these mass splittings correspond to the operators $\mathbb{1}$, J_{ℓ}^2, T^8, $J_{\ell}^iG^{i8}$, and $\{T^8,T^8\}$ times I_h^3 and N_Q, respectively. It is useful to study the mass combinations corresponding to these operators since the leading order contribution of each $J_Q=0$ SU(3) splitting cancels out of the corresponding heavy-quark flavor-violating and heavy-quark number-dependent splitting.

The $I_h=1$ heavy-quark flavor-symmetry-violating mass splittings are tabulated in Table VI. These mass combinations correspond to splittings between heavy charm and bottom baryons, and are suppressed by a relative factor of $1/N_c$ times $(1/2m_c-1/2m_b)$ in comparison to the leading heavy-quark flavor-symmetric contribution to the heavy baryon mass splitting [except for the $I_h^3\mathbb{1}$ splitting which is proportional to (m_c-m_b)]. The most accurate $I_h=1$ mass relations in Table VI can be used to predict the bottom baryon mass splittings from the corresponding charm baryon mass splittings.

Heavy-quark number-violating mass splittings between baryons containing a single heavy quark Q and baryons containing no heavy quarks are given in Table VII. These mass combinations relate heavy baryon mass splittings to splittings among the spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decuplet baryons. The mass combinations are suppressed by a relative factor of $1/N_c$ in comparison to the leading contribution to the heavy baryon mass splitting (except for the $N_Q\mathbb{1}$ mass splitting which is proportional to m_Q). The N_Q and $N_QJ_{\ell}^2$ mass combinations are the SU(3) generalizations of the SU(2) mass combinations studied in Sec. IV. The linear combination of octet and decuplet masses appearing for these operators are the $\mathbb{1}$ and J_{ℓ}^2 mass combinations for $N_Q=0$ baryons derived in Ref. [8]. The derivation of the flavor octet and flavor-27 mass combinations is more subtle. There are five flavor octet $(J_Q=0)$ operators among the $N_Q=0$ and $N_Q=1$ baryons: T^8, $J_{\ell}^iG^{i8}$, $(J_{\ell})^2T^8$, N_QT^8, and $N_QJ_{\ell}^iG^{i8}$. It is important to note that the four-body operator $N_Q(J_{\ell})^2T^8$ is a redundant operator. The mass combination corresponding to $N_QJ_{\ell}^iG^{i8}$ is the mass combination with vanishing matrix elements of the remaining four operators. Since the matrix element of

TABLE VI. $I_h=1$ heavy baryon mass splittings which violate heavy quark flavor symmetry, but preserve heavy quark spin symmetry. The mass splittings are suppressed by $(1/2m_c-1/2m_b)$ and one additional factor of $1/N_c$. The I_h^3 mass combination has a leading contribution of (m_c-m_b).

Operator	$(SU(3),J_Q)$	Mass combination	$\langle O\rangle$	$1/m$	$1/N_c$	Flavor
I_h^3	(1,0)	$\frac{1}{3}(\Lambda_c+2\Xi_c)-\frac{1}{3}(\Lambda_b+2\Xi_b)$	1	(m_c-m_b)	1	1
$I_h^3(J_{\ell})^2$	(1,0)	$[-\frac{1}{3}(\Lambda_c+2\Xi_c)+\frac{1}{18}(3\Sigma_c+2\Xi_c'+\Omega_c)+\frac{1}{9}(3\Sigma_c^*+2\Xi_c^*+\Omega_c^*)]-[-\frac{1}{3}(\Lambda_b+2\Xi_b)+\frac{1}{18}(3\Sigma_b+2\Xi_b'+\Omega_b)+\frac{1}{9}(3\Sigma_b^*+2\Xi_b^*+\Omega_b^*)]$	2	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c^2$	1
$I_h^3T^8$	(8,0)	$(\Lambda_c-\Xi_c)-(\Lambda_b-\Xi_b)$	$\frac{3}{2\sqrt{3}}$	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c$	ϵ
$I_h^3J_{\ell}^iG^{i8}$	(8,0)	$[-\frac{5}{8}(\Lambda_c-\Xi_c)+\frac{1}{24}(3\Sigma_c-\Xi_c'-2\Omega_c)+\frac{1}{12}(3\Sigma_c^*-\Xi_c^*-2\Omega_c^*)]-[-\frac{5}{8}(\Lambda_b-\Xi_b)+\frac{1}{24}(3\Sigma_b-\Xi_b'-2\Omega_b)+\frac{1}{12}(3\Sigma_b^*-\Xi_b^*-2\Omega_b^*)]$	$\frac{1}{2\sqrt{3}}\frac{15}{8}$	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c^2$	ϵ
$I_h^3\{T^8,T^8\}$	(27,0)	$[\frac{1}{6}(\Sigma_c-2\Xi_c'+\Omega_c)+\frac{1}{3}(\Sigma_c^*-2\Xi_c^*+\Omega_c^*)]-[\frac{1}{6}(\Sigma_b-2\Xi_b'+\Omega_b)+\frac{1}{3}(\Sigma_b^*-2\Xi_b^*+\Omega_b^*)]$	$\frac{3}{2}$	$\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)$	$1/N_c^2$	ϵ^2

$(J_{\ell})^2T^8$ must vanish in this combination, the linear combination of octet and decuplet masses appearing in the $N_QJ_{\ell}^iG^{i8}$ mass combination is not simply the $J_{\ell}^iG^{i8}$ mass combination for $N_Q=0$ baryons [8]. Similar remarks apply for the flavor-27 mass combinations. There are three flavor-27 $(J_Q=0)$ operators among the $N_Q=0$ and $N_Q=1$ baryons: $\{T^8,T^8\}$, $\{T^8,J_{\ell}^iG^{i8}\}$, and $N_Q\{T^8,T^8\}$. The four-body operator $N_Q\{T^8,J_{\ell}^iG^{i8}\}$ is a redundant operator. The mass combination corresponding to the $N_Q\{T^8,T^8\}$ operator is the mass combination with vanishing matrix elements of the two other operators.

There are two accurate mass relations obtained by neglecting heavy-quark number-violating operators. Neglect of the flavor-27 operator $N_Q\{T^8,T^8\}$ yields the mass relation

$$\frac{1}{6}[(\Sigma_Q+2\Sigma_Q^*)-2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]=\frac{1}{3}\left[\frac{1}{4}(2N-\Sigma-3\Lambda+2\Xi)+\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)\right], \tag{5.15}$$

which is exact up to corrections of order $(3/2N_c^2)(\epsilon^2\Lambda_\chi)$. The estimate of this correction is 1.5 MeV for $Q=c,b$. Equation (5.15) is a factor of $1/N_c$ more accurate than relation (5.9), which sets the heavy baryon mass combination equal to zero. The linear combination of octet and decuplet masses on the right-hand side of the equation equals -4.43 MeV with negligible error, which agrees with the estimated accuracy of Eq. (5.9). Neglect of the $N_QJ_{\ell}^iG^{i8}$ operator yields the mass relation

$$\left\{-\frac{5}{8}(\Lambda_Q-\Xi_Q)+\frac{1}{24}[3(\Sigma_Q+2\Sigma_Q^*)-(\Xi_Q'+2\Xi_Q^*)-2(\Omega_Q+2\Omega_Q^*)]\right\}=-\frac{1}{24}(8N-9\Sigma+3\Lambda-2\Xi)+\frac{1}{12}(2\Delta-\Xi^*-\Omega), \tag{5.16}$$

which is satisfied up to a correction of order $15/8(1/N_c^2)(\epsilon\Lambda_\chi)/2\sqrt{3}$, which is about 13.5 MeV for $Q=c,b$. The linear combination of octet and decuplet masses on the right-hand side of this equation equals 42.89 ± 0.19 MeV, and so there was not a useful mass relation involving this heavy baryon mass splitting in expansion (5.5). Finally, a third mass relation following from the neglect of the N_QT^8 operator provides a test of the theoretical hierarchy. This mass relation

$$(\Lambda_Q-\Xi_Q)=\frac{1}{8}(6N-3\Sigma+\Lambda-4\Xi)-\frac{1}{20}(2\Delta-\Xi^*-\Omega) \tag{5.17}$$

is predicted to hold to order $3(1/N_c)(\epsilon\Lambda_\chi)/2\sqrt{3}$, which is about 65 MeV for $Q=c,b$. Evaluation of the mass difference of $(\Lambda_c-\Xi_c)$ and the right-hand side of Eq. (5.17) yields 43 MeV, which is a factor of 0.6 times the coefficient unity estimation.

TABLE VII. Mass splittings between baryons containing a single heavy quark Q and baryons containing no heavy quarks. The mass splittings are suppressed by an additional factor of $1/N_c$. The N_Q mass combination has a leading order contribution of m_Q.

Operator	$(\mathrm{SU}(3),J_Q)$	Mass combination	$\langle O\rangle$	$1/m_Q$	$1/N_c$	Flavor
N_Q	(1,0)	$\frac{1}{3}(\Lambda_Q+2\Xi_Q)-\frac{1}{4}[\frac{5}{8}(2N+3\Sigma+\Lambda+2\Xi)-\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)]$	1	m_Q	1	1
$N_QJ_\ell^2$	(1,0)	$\{-\frac{1}{3}(\Lambda_Q+2\Xi_Q)+\frac{1}{18}[3(\Sigma_Q+2\Sigma_Q^*)+2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]\}-\frac{2}{3}[\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)-\frac{1}{8}(2N+3\Sigma+\Lambda+2\Xi)]$	2	1	$1/N_c^2$	1
N_QT^8	(8,0)	$(\Lambda_Q-\Xi_Q)-\frac{1}{8}(6N-3\Sigma+\Lambda-4\Xi)+\frac{1}{20}(2\Delta-\Xi^*-\Omega)$	$\frac{3}{2\sqrt{3}}$	1	$1/N_c$	ϵ
$N_QJ_\ell^iG^{i8}$	(8,0)	$\{-\frac{5}{8}(\Lambda_Q-\Xi_Q)+\frac{1}{24}[3(\Sigma_Q+2\Sigma_Q^*)+(\Xi_Q'+2\Xi_Q^*)-2(\Omega_Q+2\Omega_Q^*)]\}+\frac{1}{24}(8N-9\Sigma+3\Lambda-2\Xi)-\frac{1}{12}(2\Delta-\Xi^*-\Omega)$	$\frac{1}{2\sqrt{3}}\frac{15}{8}$	1	$1/N_c^2$	ϵ
$N_Q\{T^8,T^8\}$	(27,0)	$\frac{1}{6}[(\Sigma_Q+2\Sigma_Q^*)-2(\Xi_Q'+2\Xi_Q^*)+(\Omega_Q+2\Omega_Q^*)]+\frac{1}{3}[-\frac{1}{4}(2N-\Sigma-3\Lambda+2\Xi)-\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)]$	$\frac{3}{2}$	1	$1/N_c^2$	ϵ^2

Elimination of the mass $\frac{1}{3}(\Omega_Q+2\Omega_Q^*)$ between Eqs. (5.15) and (5.16) yields a prediction for the heavy baryon mass splitting,

$$\left[\frac{1}{3}(\Sigma_Q+2\Sigma_Q^*)-\Lambda_Q\right]-\left[\frac{1}{3}(\Xi_Q'+2\Xi_Q^*)-\Xi_Q\right], \tag{5.18}$$

in terms of octet and decuplet baryon mass splittings.

Predictions

The two new mass relations (5.15) and (5.16) together with the most precise relation (5.7) can be used to extract values for the Σ_c^*, Ξ_c', and Ω_c^*:

$$\begin{aligned}\Sigma_c^*&=2532.5\pm5.3\ \text{MeV},\\ \Xi_c'&=2582.9\pm2.5\ \text{MeV},\\ \Omega_c^*&=2746.6\pm6.1\ \text{MeV}.\end{aligned}\tag{5.19}$$

The predicted Σ_c^* mass is consistent with the current Particle Data Group (PDG) value of $2530\pm5\pm5$ MeV, while the predicted Ξ_c' mass is significantly larger than the unpublished WA-89 measurement of $\sim$2560 MeV. All of the other charm baryon masses are accurately measured, and so the predicted masses for Σ_c^*, Ξ_c', and Ω_c^* together with the measured charm baryon masses can be used to evaluate the mass combinations of the combined $1/m_Q$ and $1/N_c$ expansion. The numerical values of the mass combinations then can be compared with the theoretical hierarchy predicted by $1/m_Q$, $1/N_c$, and SU(3)-flavor-breaking suppressions.

Using the derived masses and the measured masses of all the other charmed baryons, it is possible to evaluate the mass hierarchy of Table V. These numbers are listed in the column labeled ''Expt.'' in Table V. Good agreement with the theoretical hierarchy is found. Only the predictions for the J_ℓ^2 and $J_\ell^iG^{i8}$ mass combinations and the two mass combinations involving $\Xi_c\Xi_c'$ can be regarded as pure predictions since the choice of parameters m_c, Λ, $(\epsilon\Lambda_\chi)$, and $(\epsilon^2\Lambda_\chi)$ affects four of the mass combinations, and relation (5.7) was imposed to extract the unknown and inaccurately measured charm baryon masses.

It also is worthwhile to study the implications of the predictions (5.19) for the charm baryon mass spectrum. The chromomagnetic mass splittings are evaluated to be

$$\begin{aligned}(\Sigma_c^*-\Sigma_c)&=79.6\pm5.3\ \text{MeV},\\ (\Xi_c^*-\Xi_c')&=61.1\pm3.0\ \text{MeV},\\ (\Omega_c^*-\Omega_c)&=42.6\pm7.3\ \text{MeV},\end{aligned}\tag{5.20}$$

which implies that the linear in strangeness number contribution to the chromomagnetic splittings is *negative*, so that $(\Sigma_c^*-\Sigma_c)>(\Xi_c^*-\Xi_c')>(\Omega_c^*-\Omega_c)$. This ordering seems reasonable based on intuition from the quark model: In the quark model, the hyperfine operator $J_q\cdot J_Q$ is suppressed by $1/m_qm_Q$, which is a greater suppression for $q=s$ than for $q=u,d$. The differences of these splittings are equal as dictated by Eq. (5.7), and given by 18.5 ± 4.6 MeV, and so Eq. (5.14) which holds at this level is not very accurate. The spin-averaged sextet masses are evaluated to be

$$\begin{aligned}\frac{1}{3}(\Sigma_c+2\Sigma_c^*)&=2506.0\pm3.5\ \text{MeV},\\ \frac{1}{3}(\Xi_c'+2\Xi_c^*)&=2623.6\pm1.4\ \text{MeV},\\ \frac{1}{3}(\Omega_c+2\Omega_c^*)&=2732.4\pm4.3\ \text{MeV}.\end{aligned}\tag{5.21}$$

The sextet mass differences

$$\frac{1}{3}(\Xi_c'+2\Xi_c^*)-\frac{1}{3}(\Sigma_c+2\Sigma_c^*)=117.6\pm3.8\ \text{MeV},$$

$$\frac{1}{3}(\Omega_c+2\Omega_c^*)-\frac{1}{3}(\Xi_c'+2\Xi_c^*)=108.8\pm 4.5\ \text{MeV} \tag{5.22}$$

are equal at the 8.8 MeV level. The $J_{\not\ell}^2$ hyperfine splittings in each strangeness sector are

$$\frac{1}{3}(\Sigma_c+2\Sigma_c^*)-\Lambda_c=221.0\pm 3.6\ \text{MeV},$$
$$\frac{1}{3}(\Xi_c'+2\Xi_c^*)-\Xi_c=155.9\pm 1.8\ \text{MeV}. \tag{5.23}$$

The difference of these splittings is predicted to be large:

$$\left[\frac{1}{3}(\Sigma_c+2\Sigma_c^*)-\Lambda_c\right]-\left[\frac{1}{3}(\Xi_c'+2\Xi_c^*)-\Xi_c\right]$$
$$=65.1\pm 4.0\ \text{MeV}. \tag{5.24}$$

Finally, now that all the charm baryon masses have been determined, it is possible to return to the problem of predicting the bottom baryon masses from the charm baryon masses. The heavy-quark spin-violating splittings of the bottom baryons can be obtained by rescaling the charm baryon mass splittings by a factor $(m_c/m_b)\sim 0.3$. Thus, the chromomagnetic mass splittings of the bottom baryons are predicted to be

$$(\Sigma_b^*-\Sigma_b)\sim 23.8\pm 1.6\ \text{MeV},$$
$$(\Xi_b^*-\Xi_b')\sim 18.3\pm 0.9\ \text{MeV}, \tag{5.25}$$
$$(\Omega_b^*-\Omega_b)\sim 12.8\pm 2.2\ \text{MeV},$$

where the precise numerical values in Eq. (5.25) will change outside errors if a different scale factor is used since the error on the scale factor has not been taken into account. For example, using the more rigorous scale factor $(Z_b/Z_c)(m_c/m_b)\sim 0.24$ reduces the central values by about 5 MeV. The mass relation Eq. (5.15) and the second flavor octet mass relation of Table VI yield precise predictions for two additional bottom baryon mass combinations

$$\frac{1}{6}[(\Sigma_b+2\Sigma_b^*)-2(\Xi_b'+2\Xi_b^*)+(\Omega_b+2\Omega_b^*)]=-4.43\pm 1.5\ \text{MeV},$$
$$\left\{-\frac{5}{8}(\Lambda_b-\Xi_b)+\frac{1}{24}[3(\Sigma_b+2\Sigma_b^*)-(\Xi_b'+2\Xi_b^*)-2(\Omega_b+2\Omega_b^*)]\right\}=42.89\pm 2.1\ \text{MeV}, \tag{5.26}$$

where the errors represent the combined theoretical and experimental accuracy of each relation. There are two additional mass relations which are less accurate, namely the second and third mass relations in Table VI, with theoretical accuracies of about 4.8 and 5.1 MeV, respectively:

$$(\Lambda_b-\Xi_b)=-182.7\pm 4.9\ \text{MeV},$$
$$-\frac{1}{3}(\Lambda_b+2\Xi_b)+\frac{1}{18}[3(\Sigma_b+2\Sigma_b^*)+2(\Xi_b'+2\Xi_b^*)+(\Omega_b+2\Omega_b^*)]=176.1\pm 5.4\ \text{MeV}. \tag{5.27}$$

Combined with the measured Λ_b mass, these seven constraints determine the seven masses Ξ_b, Σ_b, Ξ_b' Ω_b, Σ_b^* Ξ_b^*, and Ω_b^*. The three chromomagnetic splittings were given already in Eq. (5.25). The four mass relations in Eq. (5.26) and (5.27) together with the Λ_b measurement determine the four spin-averaged mass combinations

$$\frac{1}{3}(\Lambda_b+2\Xi_b)=5744.8\pm 5.8\ \text{MeV},$$
$$\frac{1}{3}(\Sigma_b+2\Sigma_b^*)=5844.0\pm 8.9\ \text{MeV},$$
$$\frac{1}{3}(\Xi_b'+2\Xi_b)=5961.6\pm 10.8\ \text{MeV}, \tag{5.28}$$
$$\frac{1}{3}(\Omega_b+2\Omega_b^*)=6070.3\pm 23.6\ \text{MeV},$$

where the precision of the extraction is limited by the theoretical accuracy of the least accurate mass relations. Once two additional bottom baryon masses are measured, a more precise extraction of the remaining unmeasured masses will be possible using only the most accurate mass relations in Eq. (5.26). Some linear combinations of the spin-averaged mass combinations are determined accurately, however. For example, the two mass relations in Eq. (5.26) imply

$$\left[\frac{1}{3}(\Sigma_b+2\Sigma_b^*)-\Lambda_b\right]-\left[\frac{1}{3}(\Xi_b'+2\Xi_b^*)-\Xi_b\right]$$
$$=65.1\pm 3.6\ \text{MeV}. \tag{5.29}$$

VI. ISOSPIN-VIOLATING MASS SPLITTINGS

Isospin-violating mass splittings are analyzed in this section for completeness. Almost all isospin splittings will be at the sub-MeV level, and so isospin symmetry is a very good symmetry for the heavy baryon masses. Isospin breaking arises due to differences in the u and d quark masses and electromagnetic charges. These two sources of isospin breaking will be denoted by the parameters ϵ' and ϵ''. Isospin breaking due to quark mass differences is purely $I=1$, whereas the electromagnetic mass splittings are second order in the quark charge matrix and can be $I=1,2$. Both sources of isospin breaking produce comparable mass splittings in QCD, and so $\epsilon' \sim \epsilon''$. All $I=2$ mass splittings are electromagnetic mass splittings suppressed by $\alpha_{\rm EM}/4\pi$, and will be proportional to the parameter $(\epsilon'' \Lambda_\chi)$. The $I=1$ mass splittings can arise from either source of isospin breaking. Since these effects are comparable, the splittings will be written in terms of the parameter $(\epsilon' \Lambda_\chi)$.

The $I=1$ flavor octet, $(10+\overline{10})$, and flavor-27 mass expansions are given by

$$M^{(8)}=T^3+\frac{1}{N_c}J^i_{\ell}G^{i3}+\frac{1}{N_c}\frac{2}{m_Q}J^i_Q G^{i3}+\frac{1}{N_c^2}\frac{2}{m_Q}(J_{\ell}\cdot J_Q)T^3,$$

$$M^{(10+\overline{10})}=\frac{1}{N_c^2}\frac{1}{m_Q}(\{T^3,G^{i8}\}-\{T^8,G^{i3}\}), \tag{6.1}$$

$$M^{(27)}=\frac{1}{N_c}\{T^3,T^8\}+\frac{1}{N_c^2}\frac{1}{m_Q}J^i_Q(\{T^3,G^{i8}\}+\{T^8,G^{i3}\}),$$

whereas the $I=2$ flavor-27 mass expansion is given by

$$M^{(27)}=\frac{1}{N_c}\{T^3,T^3\}+\frac{1}{N_c^2}\frac{2}{m_Q}J^i_Q\{T^3,G^{i3}\}. \tag{6.2}$$

The mass combinations corresponding to these operators are tabulated in Table VIII for $Q=c$ and Table IX for $Q=b$. Table X combines the operator matrix element, $1/m_Q$, $1/N_c$, and flavor-breaking suppression factors to predict a hierarchy of mass relations. The numerical values of the theoretical hierarchy are evaluated for canonical values of the parameters in the columns labeled $Q=c$ and $Q=b$. The largest isospin mass splitting is $(\Xi_c^+-\Xi_c^0)$ or $(\Xi_b^0-\Xi_b^-)$, which are equal and of order a few MeV in magnitude. These splittings can be used to determine the isospin-breaking parameter $(\epsilon' \Lambda_\chi)$ which sets the scale of the mass hierarchy. Experimentally, $(\Xi_c^0-\Xi_c^+)=5.2\pm 2.2$ MeV [14]. The next largest mass splitting is the $I=2$ mass combination

$$\frac{1}{10}(\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0})+\frac{2}{5}(\Sigma_c^{*++}-2\Sigma_c^{*+}+\Sigma_c^{*0}) \tag{6.3}$$

or the analogous splitting for $Q=b$, which will be about 3.5 ± 1.5 MeV. The $I=1$ mass splitting

$$-\frac{5}{8}(\Xi_c^+-\Xi_c^0)+\frac{1}{24}[2(\Sigma_c^{++}-\Sigma_c^0)+(\Xi_c^{\prime +}-\Xi_c^{\prime 0})]$$
$$+\frac{1}{12}[2(\Sigma_c^{*++}-\Sigma_c^{*0})+(\Xi_c^{*+}-\Xi_c^{*0})] \tag{6.4}$$

or the $Q=b$ analogue will both be about 1.0 ± 0.5 MeV. All remaining isospin mass combinations are predicted to be sub-MeV. The mass relations obtained by neglect of these operators are given in Tables VIII and IX.

VII. CONCLUSIONS

It has been shown that there are light-quark and heavy-quark spin-flavor symmetries for baryons containing a single

TABLE VIII. Isospin-violating mass splittings of baryons containing a single charm quark.

Operator	(SU(3), J_c)	Mass combination	$\langle O \rangle$	$1/m_c$	$1/N_c$	Flavor
		$I=1$				
T^3	(8,0)	$(\Xi_c^+-\Xi_c^0)$	1	1	1	ϵ'
$J^i_{\ell}G^{i3}$	(8,0)	$-\frac{5}{8}(\Xi_c^+-\Xi_c^0)+\frac{1}{24}[2(\Sigma_c^{++}-\Sigma_c^0)+(\Xi_c^{\prime +}-\Xi_c^{\prime 0})]$ $+\frac{1}{12}[2(\Sigma_c^{*++}-\Sigma_c^{*0})+(\Xi_c^{*+}-\Xi_c^{*0})]$	$\frac{5}{8}$	1	$1/N_c$	ϵ'
$J^i_c G^{i3}$	(8,1)	$\frac{1}{2}[\Lambda_c^+\Sigma_c^+ +\frac{1}{2}(\Xi_c^+\Xi_c^{\prime +}-\Xi_c^0\Xi_c^{\prime 0})]$	$\frac{3\sqrt{3}}{8}$	$2/m_c$	$1/N_c$	ϵ'
$(J_{\ell}\cdot J_c)T^3$	(8,1)	$-\frac{1}{6}[2(\Sigma_c^{++}-\Sigma_c^0)+(\Xi_c^{\prime +}-\Xi_c^{\prime 0})]$ $-\frac{5}{3\sqrt{3}}[\Lambda_c^+\Sigma_c^+ +\frac{1}{2}(\Xi_c^+\Xi_c^{\prime +}-\Xi_c^0\Xi_c^{\prime 0})]$ $+\frac{1}{6}[2(\Sigma_c^{*++}-\Sigma_c^{*0})+(\Xi_c^{*+}-\Xi_c^{*0})]$	$\frac{5}{4}$	$2/m_c$	$1/N_c^2$	ϵ'
$\{T^3,T^8\}$	(27,0)	$\frac{1}{18}[(\Sigma_c^{++}-\Sigma_c^0)-2(\Xi_c^{\prime +}-\Xi_c^{\prime 0})]$ $+\frac{1}{9}[(\Sigma_c^{*++}-\Sigma_c^{*0})-2(\Xi_c^{*+}-\Xi_c^{*0})]$	$\frac{1}{\sqrt{3}}$	1	$1/N_c$	$\epsilon\epsilon'$
$J^i_c(\{T^3,G^{i8}\}+\{T^8,G^{i3}\})$	(27,1)	$-\frac{1}{6}[(\Sigma_c^{++}-\Sigma_c^0)-2(\Xi_c^{\prime +}-\Xi_c^{\prime 0})]$ $+\frac{1}{6}[(\Sigma_c^{*++}-\Sigma_c^{*0})-2(\Xi_c^{*+}-\Xi_c^{*0})]$	$\frac{\sqrt{3}}{2}$	$2/m_c$	$1/N_c^2$	$\epsilon\epsilon'$
$J^i_c(\{T^3,G^{i8}\}-\{T^8,G^{i3}\})$	$(10+\overline{10},1)$	$-\frac{1}{2}[\Lambda_c^+\Sigma_c^+ -(\Xi_c^+\Xi_c^{\prime +}-\Xi_c^0\Xi_c^{\prime 0})]$	$\frac{3}{4}$	$2/m_c$	$1/N_c^2$	$\epsilon\epsilon'$
		$I=2$				
$\{T^3,T^3\}$	(27,0)	$\frac{1}{10}(\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0})+\frac{2}{5}(\Sigma_c^{*++}-2\Sigma_c^{*+}+\Sigma_c^{*0})$	2	1	$1/N_c$	ϵ''
$J^i_c\{T^3,G^{i3}\}$	(27,1)	$-\frac{1}{4}(\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0})+\frac{1}{4}(\Sigma_c^{*++}-2\Sigma_c^{*+}+\Sigma_c^{*0})$	$\frac{5}{8}$	$2/m_c$	$1/N_c^2$	ϵ''

TABLE IX. Isospin-violating mass splittings of baryons containing a single bottom quark.

Operator	(SU(3), J_b)	Mass combination	$\langle O\rangle$	$1/m_b$	$1/N_c$	Flavor
		$I=1$				
T^3	(8,0)	$(\Xi_b^0-\Xi_b^-)$	1	1	1	ϵ'
$J^i_\ell G^{i3}$	(8,0)	$-\frac{5}{8}(\Xi_b^0-\Xi_b^-)+\frac{1}{24}[2(\Sigma_b^+-\Sigma_b^-)+(\Xi_b'^0-\Xi_b'^-)]$ $+\frac{1}{12}[2(\Sigma_b^{*+}-\Sigma_b^{*-})+(\Xi_b^{*0}-\Xi_b^{*-})]$	$\frac{5}{8}$	1	$1/N_c$	ϵ'
$J^i_b G^{i3}$	(8,1)	$\frac{1}{2}[\Lambda_b^0\Sigma_b^0+\frac{1}{2}(\Xi_b^0\Xi_b'^0-\Xi_b^-\Xi_b'^-)]$	$\frac{3\sqrt{3}}{8}$	$2/m_b$	$1/N_c$	ϵ'
$(J_\ell\cdot J_b)T^3$	(8,1)	$-\frac{1}{6}[2(\Sigma_b^+-\Sigma_b^-)+(\Xi_b'^0-\Xi_b'^-)]$ $-\frac{5}{3\sqrt{3}}[\Lambda_b^0\Sigma_b^0+\frac{1}{2}(\Xi_b^0\Xi_b'^0-\Xi_b^-\Xi_b'^-)]$ $+\frac{1}{6}[2(\Sigma_b^{*+}-\Sigma_b^{*-})+(\Xi_b^{*0}-\Xi_b^{*-})]$	$\frac{5}{4}$	$2/m_b$	$1/N_c^2$	ϵ'
$\{T^3,T^8\}$	(27,0)	$\frac{1}{18}[(\Sigma_b^+-\Sigma_b^-)-2(\Xi_b'^0-\Xi_b'^-)]$ $+\frac{1}{9}[(\Sigma_b^{*+}-\Sigma_b^{*-})-2(\Xi_b^{*0}-\Xi_b^{*-})]$	$\frac{1}{\sqrt{3}}$	1	$1/N_c$	$\epsilon\epsilon'$
$J^i_b(\{T^3,G^{i8}\}+\{T^8,G^{i3}\})$	(27,1)	$-\frac{1}{6}[(\Sigma_b^+-\Sigma_b^-)-2(\Xi_b'^0-\Xi_b'^-)]$ $+\frac{1}{6}[(\Sigma_b^{*+}-\Sigma_b^{*-})-2(\Xi_b^{*0}-\Xi_b^{*-})]$	$\frac{\sqrt{3}}{2}$	$2/m_b$	$1/N_c^2$	$\epsilon\epsilon'$
$J^i_b(\{T^3,G^{i8}\}-\{T^8,G^{i3}\})$	$(10+\overline{10},1)$	$-\frac{1}{2}[\Lambda_b^0\Sigma_b^0-(\Xi_b^0\Xi_b'^0-\Xi_b^-\Xi_b'^-)]$	$\frac{3}{4}$	$2/m_b$	$1/N_c^2$	$\epsilon\epsilon'$
		$I=2$				
$\{T^3,T^3\}$	(27,0)	$\frac{1}{10}(\Sigma_b^+-2\Sigma_b^0+\Sigma_b^-)+\frac{2}{5}(\Sigma_b^{*+}-2\Sigma_b^{*0}+\Sigma_b^{*-})$	2	1	$1/N_c$	ϵ''
$J^i_b\{T^3,G^{i3}\}$	(27,1)	$-\frac{1}{4}(\Sigma_b^+-2\Sigma_b^0+\Sigma_b^-)+\frac{1}{4}(\Sigma_b^{*+}-2\Sigma_b^{*0}+\Sigma_b^{*-})$	$\frac{5}{8}$	$2/m_b$	$1/N_c^2$	ϵ''

TABLE X. Hierarchy of heavy baryon mass splittings in decreasing order of magnitude for $Q=c$ and $Q=b$. Superscripts refer to the isospin I_z of the baryon rather than the electromagnetic charge. The parameters $(\epsilon'\Lambda_\chi)$ and $(\epsilon''\Lambda_\chi)$ are expected to be comparable and of the order of a few MeV.

Mass combination	Theory	$Q=c$	$Q=b$
$(\Xi_Q^{+\frac{1}{2}}-\Xi_Q^{-\frac{1}{2}})$	$(\epsilon'\Lambda_\chi)$	$1.0(\epsilon'\Lambda_\chi)$	$1.0(\epsilon'\Lambda_\chi)$
$\frac{1}{10}(\Sigma_Q^{+1}-2\Sigma_Q^0+\Sigma_Q^{-1})+\frac{2}{5}(\Sigma_Q^{*+1}-2\Sigma_Q^{*0}+\Sigma_Q^{*-1})$	$2\frac{1}{N_c}(\epsilon''\Lambda_\chi)$	$0.67(\epsilon''\Lambda_\chi)$	$0.67(\epsilon''\Lambda_\chi)$
$-\frac{5}{8}(\Xi_Q^{+\frac{1}{2}}-\Xi_Q^{-\frac{1}{2}})+\frac{1}{24}[2(\Sigma_Q^{+1}-\Sigma_Q^{-1})+(\Xi_Q'^{+\frac{1}{2}}-\Xi_Q'^{-\frac{1}{2}})]$ $+\frac{1}{12}[2(\Sigma_Q^{*+1}-\Sigma_Q^{*-1})+(\Xi_Q^{*+\frac{1}{2}}-\Xi_Q^{*-\frac{1}{2}})]$	$\frac{5}{8}\frac{1}{N_c}(\epsilon'\Lambda_\chi)$	$0.21(\epsilon'\Lambda_\chi)$	$0.21(\epsilon'\Lambda_\chi)$
$\frac{1}{18}[(\Sigma_Q^{+1}-\Sigma_Q^{-1})-2(\Xi_Q'^{+\frac{1}{2}}-\Xi_Q'^{-\frac{1}{2}})]$ $+\frac{1}{9}[(\Sigma_Q^{*+1}-\Sigma_Q^{*-1})-2(\Xi_Q^{*+\frac{1}{2}}-\Xi_Q^{*-\frac{1}{2}})]$	$\frac{1}{\sqrt{3}}\frac{1}{N_c}\epsilon(\epsilon'\Lambda_\chi)$	$0.05(\epsilon'\Lambda_\chi)$	$0.05(\epsilon'\Lambda_\chi)$
$\frac{1}{2}[\Lambda_Q^0\Sigma_Q^0+\frac{1}{2}(\Xi_Q^{+\frac{1}{2}}\Xi_Q'^{+\frac{1}{2}}-\Xi_Q^{-\frac{1}{2}}\Xi_Q'^{-\frac{1}{2}})]$	$\frac{3\sqrt{3}}{8}\frac{1}{N_c}\frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)$	$0.09(\epsilon'\Lambda_\chi)$	$0.03(\epsilon'\Lambda_\chi)$
$-\frac{1}{6}[2(\Sigma_Q^{+1}-\Sigma_Q^{-1})+(\Xi_Q'^{+\frac{1}{2}}-\Xi_Q'^{-\frac{1}{2}})]$ $-\frac{5}{3\sqrt{3}}[\Lambda_Q^0\Sigma_Q^0+\frac{1}{2}(\Xi_Q^{+\frac{1}{2}}\Xi_Q'^{+\frac{1}{2}}-\Xi_Q^{-\frac{1}{2}}\Xi_Q'^{-\frac{1}{2}})]$ $+\frac{1}{6}[2(\Sigma_Q^{*+1}-\Sigma_Q^{*-1})+(\Xi_Q^{*+\frac{1}{2}}-\Xi_Q^{*-\frac{1}{2}})]$	$\frac{5}{4}\frac{1}{N_c^2}\frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)$	$0.06(\epsilon'\Lambda_\chi)$	$0.02(\epsilon'\Lambda_\chi)$
$-\frac{1}{4}(\Sigma_Q^{+1}-2\Sigma_Q^0+\Sigma_Q^{-1})+\frac{1}{4}(\Sigma_Q^{*+1}-2\Sigma_Q^{*0}+\Sigma_Q^{*-1})$	$\frac{5}{8}\frac{1}{N_c^2}\frac{2\Lambda}{m_Q}(\epsilon''\Lambda_\chi)$	$0.03(\epsilon''\Lambda_\chi)$	$0.009(\epsilon''\Lambda_\chi)$
$-\frac{1}{6}[(\Sigma_Q^{+1}-\Sigma_Q^{-1})-2(\Xi_Q'^{+\frac{1}{2}}-\Xi_Q'^{-\frac{1}{2}})]$ $+\frac{1}{6}[(\Sigma_Q^{*+1}-\Sigma_Q^{*-1})-2(\Xi_Q^{*+\frac{1}{2}}-\Xi_Q^{*-\frac{1}{2}})]$	$\frac{\sqrt{3}}{2}\frac{1}{N_c^2}\frac{2\Lambda}{m_Q}\epsilon(\epsilon'\Lambda_\chi)$	$0.01(\epsilon'\Lambda_\chi)$	$0.003(\epsilon'\Lambda_\chi)$
$-\frac{1}{2}[\Lambda_Q^0\Sigma_Q^0-(\Xi_Q^{+\frac{1}{2}}\Xi_Q'^{+\frac{1}{2}}-\Xi_Q^{-\frac{1}{2}}\Xi_Q'^{-\frac{1}{2}})]$	$\frac{3}{4}\frac{1}{N_c^2}\frac{2\Lambda}{m_Q}\epsilon(\epsilon'\Lambda_\chi)$	$0.009(\epsilon'\Lambda_\chi)$	$0.0026(\epsilon'\Lambda_\chi)$

heavy quark in the combined heavy quark and large-N_c limits. A (spin$\otimes$flavor) operator expansion in $1/N_c$ and $1/m_Q$ has been constructed for heavy quark baryons at finite m_Q and N_c. Heavy quark spin symmetry is present for $N_c \to \infty$ or for $m_Q \to \infty$, and so any violation of heavy quark spin symmetry is suppressed by $(1/N_c m_Q)$. In the presence of heavy-quark flavor symmetry, heavy-quark spin-flavor symmetry is present for $N_c \to \infty$. For the physical situation of two heavy flavors $Q=c$ and $Q=b$, heavy quark flavor symmetry is justified by the heavy quark limit [1]. Heavy-quark spin-flavor symmetry $SU(4)_h$ is a better symmetry for baryons than for mesons because violation of the symmetry is suppressed by $(1/N_c m_Q)$, rather than $1/m_Q$. In addition to heavy-quark spin-flavor symmetry, there is a light-quark SU(6) spin-flavor symmetry for heavy quark baryons in the large-N_c limit [3].

The masses of baryons containing a single heavy quark have been analyzed in a combined expansion in $1/m_Q$, $1/N_c$, and SU(3) flavor symmetry breaking. The naive $1/N_c$ scalings

$$\bar{\Lambda}^{\text{baryon}} \sim N_c \bar{\Lambda}^{\text{meson}},$$

$$\lambda_1^{\text{baryon}} \sim \lambda_1^{\text{meson}},$$

$$\lambda_2^{\text{baryon}} \sim \frac{1}{N_c} \lambda_2^{\text{meson}} \tag{7.1}$$

work beautifully for the baryon masses. A mass hierarchy is predicted by the $1/m_Q$, $1/N_c$, and flavor-breaking expansion. The most suppressed operators yield mass relations which are well satisfied experimentally. The precision of the mass relations and the magnitude of mass combinations are predicted by the expansion. Heavy baryon mass splittings have been related to octet and decuplet mass splittings. The most accurate mass relations, Eqs. (5.7), (5.15), and (5.16), have been used to predict the unmeasured or poorly measured charm baryon masses Σ_c^*, Ξ_c', and Ω_c^* to a precision of several MeV. A number of interesting features of the charm baryon mass spectrum are found. These include the following. (i) The mass splitting

$$\left[\frac{1}{3}(\Sigma_c + 2\Sigma_c^*) - \Lambda_c\right] - \left[\frac{1}{3}(\Xi_c' + 2\Xi_c^*) - \Xi_c\right] \tag{7.2}$$

is predicted to be large, (ii) the chromomagnetic hyperfine mass splittings satisfy a reverse hierarchy,

$$(\Sigma_c^* - \Sigma_c) > (\Xi_c^* - \Xi_c') > (\Omega_c^* - \Omega_c) \tag{7.3}$$

and (iii) the equal spacing rule [22]

$$(\Sigma_c^* - \Sigma_c) - (\Xi_c^* - \Xi_c') = (\Xi_c^* - \Xi_c') - (\Omega_c^* - \Omega_c) \tag{7.4}$$

is almost exact. Bottom baryon mass splittings are predicted in terms of charm baryon mass splittings and mass splittings of the octet and decuplet baryons. Some bottom baryon mass splittings are predicted very accurately.

As this work neared completion, Ref. [23] appeared. The present analysis shows that Eq. (2d) of [23] is not well satisfied.

Note added in proof. There are no renormalon ambiguities in the operator coefficients which correspond to physical mass splittings. The renormalon ambiguity of the heavy quark mass m_Q and $\bar{\Lambda}^{\text{baryon}} \sim N_c \lambda_{\text{QCD}}$ is of order Λ_{QCD}, and formally appears as a term $N_Q \Lambda_{\text{QCD}}$ in expansion (4.13).

The two very accurate mass relations Eqs. (5.7) and (5.15) yield very precise predictions for Ξ_c' and $(\Sigma_c^* + \Omega_c^*)$. A new CLEO measurement of $\Sigma_c^* = 2518.6 \pm 2.2$ MeV is significantly lower than previous measurements. In addition, there is a somewhat more accurate value for $\Omega_c = 2699.9 \pm 2.9$ MeV from the E687 Collaboration. The more accurate Ω_c value changes the Ξ_c' prediction slightly to

$$\Xi_c' = 2580.8 \pm 2.1 \ \text{MeV}.$$

The lower Σ_c^* mass implies a larger value for the predicted Ω_c^* mass:

$$\Omega_c^* = 2760.6 \pm 6.4 \ \text{MeV}.$$

The mass combination quoted in Eqs. (5.20)–(5.29) will be modified accordingly. The only significant modification is that the differences of the chromomagnetic mass splittings $(\Sigma_Q^* - \Sigma_Q)$, $(\Xi_Q^* - \Xi_Q')$, and $(\Omega_Q^* - \Omega_Q)$ are much smaller than before, which implies that relation (5.13) is as small as possible. This occurs if the magnitude of the first mass combination in Eq. (5.12) is dominated by the $\Xi_Q \Xi_Q'$ mixing term.

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