Meson masses within the model of induced nonlocal quark currents

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The model of induced quark currents being a kind of nonlocal extension of the bosonization procedure is developed. The model is based on the hypothesis that the QCD vacuum is realized by the (anti-)self-dual homogeneous gluon field. We study manifestations of confinement and chiral symmetry breaking due to the vacuum field in meson spectra and weak decay constants. It is shown that the confining properties of the vacuum field, chiral symmetry breaking, and localization of a composite field at the center of masses of quarks can explain the distinctive features of the meson spectrum: mass splitting between pseudoscalar and vector mesons, Regge trajectories, the asymptotic mass formulas in the heavy quark limit, $M_{Q\overline{Q}}\to 2m_Q$ for quarkonia and $M_{Q\overline{q}}\to m_Q$ for heavy-light mesons. Within the model, the chiral symmetry breaking due to the vacuum field is a dominating factor in forming the masses and decay constants of light mesons, while confinement is responsible for Regge trajectories, heavy quarkonia, and heavy-light mesons. With a minimal set of parameters (quark masses, vacuum field strength, and the quark-gluon coupling constant) the model describes to within ten percent inaccuracy the masses and weak decay constants of mesons from all qualitatively different regions of the spectrum. [S0556-2821(96)03917-3]

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I. INTRODUCTION

The underlining idea of this paper consists in the hypothesis that the QCD vacuum is realized by the (anti-)self-dual homogeneous gluon field. This vacuum gluon field leads to two important physical consequences for nonperturbative quark dynamics: Quarks are confined in the sense that the quark propagator has no poles corresponding to the free physical particles, and the chiral symmetry is broken via an interaction of quark spin with the vacuum gluon field. This spin-field interaction produces a zero mode (the lowest Landau level) in the spectrum of the operator $i(\partial_\mu - igt^aB^a_{\mu\nu}x_\nu)\gamma_\mu$, so that the quark condensate is nonzero in the massless limit

$$\lim_{m_q\to 0} m_q\langle\overline{q(x)}q(x)\rangle \neq 0.$$

Both consequences were noticed by Leutwyler in [1]. He has also demonstrated that the (anti-)self-dual homogeneous gluon field is a stable configuration in contrast to the purely chromomagnetic and chromoelectric fields [1,2]. Strong indications (not a rigorous proof) that this field could minimize the effective potential of Euclidean QCD were obtained by many authors in the beginning of the 1980s (see [3,4] and references therein). Although these studies do not prove the existence of an (anti-)self-dual homogeneous gluon vacuum field, they underline the key role of the non-Abelian nature of gluons in forming the nonzero vacuum field. Gluon self-interaction is the point in which QCD differs from QED cardinally. In this manner, it has been realized that the (anti-)self-dual homogeneous field could apply to the basic properties of QCD: quark confinement and chiral symmetry breaking.

At that time the manifestations of this vacuum field in hadron phenomenology were not studied in terms of bosonization and composite meson fields, although Flory approached a statement of the problem closely [4]. The field under consideration breaks the parity, rotational, and color symmetries of the QCD Lagrangian. This is the first difficulty to be overcome. An averaging of all physical amplitudes over the directions of the vacuum field, self-dual, and anti-self-dual configurations restores these symmetries in the matrix elements. Mathematical realization of this idea comes from instanton physics [5], and is reduced to the division of the QCD functional integral into integrations over given classical configurations and quantum fluctuations in the background field [6]. According to this concept there are no observable directions of the vacuum field. The second difficulty is that a self-consistent investigation of the problem of composite fields implies that the vacuum field has to be taken into account both in the quark and gluon propagators. Therefore, one has to deal with the bosonization of the nonlocal four-fermion interaction. The bosonization of the one-gluon exchange interaction was discussed in a series of papers [7].

In [8] we have developed a method of bosonization suitable for nonlocal interactions and have formulated the model of meson-meson interactions which incorporates the main nonperturbative effects produced by the gluon vacuum field under consideration: confinement and chiral symmetry breaking. The effective quark-quark coupling is described in the model by the nonlocal four-quark interaction induced by the one-gluon exchange in the presence of the (anti-)self-dual homogeneous gluon vacuum field.

The main features of the model are as follows [8].

There are the quark confinement and chiral symmetry breaking due to the vacuum field. The quark propagator, being an entire analytical function in the complex momentum plane [1], has the standard local ultraviolet behavior in the Euclidean region, and is modified essentially in the physical, i.e., Minkowski region.

One-gluon exchange is decomposed into an infinite sum of current-current interaction terms, in which the quark currents are nonlocal, colorless, and carry a complete set of quantum numbers including the orbital and radial ones. This effective quark-quark interaction generates a superrenormalizable perturbation expansion.

The bosonization of the nonlocal four-quark interaction leads to ultraviolet finite effective meson theory. Mesons are treated as extended nonlocal objects.

The model contains the minimal number of parameters: the quark masses, quark-gluon coupling constant, and the tension of the background gluon field.

It was shown also that the spectrum of the radial and orbital excitations is equidistant for sufficiently large angular momentum ℓ or radial quantum number n. In the heavy quark limit the mass of quarkonium tends to be equal to the sum of the masses of constituent quarks.

In [8] the main attention was paid to the mathematical details of obtaining the nonlocal quark currents induced by one-gluon exchange in the presence of the vacuum field and to formulation of the bosonization procedure based on these nonlocal currents. The motivation and connection of the model with QCD has been discussed as far as possible. The present paper concentrates on further development of the model and on application to systematic calculations of the weak decay constants and masses of mesons from different regions of the spectrum: the light mesons and their excited states, heavy quarkonia, and heavy-light mesons. We examine manifestations of the quark confinement and chiral symmetry breaking due to the vacuum field in the spectrum of mesonlike composite fields. This mechanism of symmetry breaking differs from the mechanism exploited in the standard local Nambu–Jona-Lasinio (NJL) model, in which the symmetry is broken due to the four-quark interaction [9–11]. It should be stressed that these two mechanisms do not contradict each other, but should be considered as mutually additional ones. The point is that NJL-type models eliminate the gluon degrees of freedom and cannot provide a basis for studying the physics produced by the gluon fields (both a vacuum field and quantum fluctuations). The pure effect of symmetry breaking by the vacuum field is interesting in itself and should be investigated separately. The next step consists in investigating the interplay of both mechanisms. A possibility of this kind is discussed in the last section of the paper.

The general result is that chiral symmetry breaking due to the interaction of the quark spin with the vacuum field affects strongly the masses and weak decay constants of light mesons, while confinement is responsible for Regge trajectories and heavy-heavy and heavy-light mesons.

With the minimal set of parameters (quark masses, vacuum field strength, and one quark-gluon coupling constant) the model describes all qualitatively different regions of the meson spectrum to within 10% inaccuracy. The reasons driving this successful description can be easily recovered.

The spin-field interaction leads to a splitting between the masses of the pseudoscalar and vector mesons (π-ρ, K-K^*) and provides the smallness of the pion mass. Simultaneously it excludes scalar mesons as the simple $q\bar{q}$ states. Within this model the scalar states appear in the superfine structure of the orbital excitations of vector mesons. The same interaction has a crucial influence on the weak decays of pion and kaon. These factors are qualitative and irrespective of the particular values of the model parameters.

Furthermore, the vacuum field produces three rigid asymptotic regimes for the spectrum of collective modes. The spectra of radial and orbital excitations of light mesons are equidistant for $\ell \gg 1$ or $n \gg 1$; i.e., they have Regge character. This is due to the specific form of nonlocality of the quark and gluon propagators determined by confining properties of the vacuum gluon field. Localization of the meson field at the center of masses of a quark system provides two other asymptotic regimes. In the limit of an infinitely heavy quark, the mass of quarkonium tends to be equal to the sum of the masses of constituent quarks, while the mass of a heavy-light meson approaches the mass of a heavy quark: $M_{Q\bar{Q}} \to 2m_Q - \Delta_{Q\bar{Q}}$, $M_{Q\bar{q}} \to m_Q + \Delta_{Q\bar{q}}$. The next-to-leading terms $\Delta_{Q\bar{Q}}$ and $\Delta_{Q\bar{q}}$ do not depend on the heavy quark mass. The same reasons provide the correct asymptotic behavior of the weak decay constant for the heavy-light pseudoscalar mesons: $f_P \sim 1/\sqrt{m_Q}$.

One can conclude that the (anti-)self-dual homogeneous background gluon field determines rather definitely the behavior of the masses and weak decay constants in all different regions of the meson spectrum, and this behavior is quantitatively consistent with experimental data.

Technically, these results are based on a decomposition of the bilocal colorless quark currents into a series of nonlocal currents with complete set of quantum numbers: spin, isospin, radial, and orbital numbers. The tensor structure of these nonlocal currents is represented by the irreducible tensors of the four-dimensional Euclidean rotational group, while their radial part is determined by a specific form of gluon propagator in the external vacuum field. The method of decomposition of the one-gluon exchange interaction into a series of nonlocal current-current interactions demonstrates how the form of the gluon propagator is reflected in the quark-meson vertices and, after all, in the meson spectrum.

This decomposition provides a new point of view on the renormalization problem: The Feynman diagrams appearing in each order of perturbation theory are ultraviolet finite due to the nonlocality of the meson-quark interaction.

The paper is organized as follows. In Sec. II we review the main points of the model with some modifications that relate to choosing the point of the localization of the meson field and a description of the superfine structure of the spectrum. These modifications reflect possibilities missed in [8]. In Sec. III we consider the masses of light mesons and their excitations, heavy quarkonia, and heavy-light mesons, as well as the weak decay constants. The basic approximations of the model and problems for further investigations are discussed in the last section.

II. MODEL OF INDUCED NONLOCAL QUARK CURRENTS

A. Basic assumptions, approximations, and notation

The representation of the Euclidean generating functional for QCD, in which the gluon and ghost fields are integrated out, serves as a starting point for many models of hadronization. Dyakonov and Petrov obtained this representation for the case of a nontrivial vacuum gluon field [5] (see also [6]):

$$Z=\int d\sigma_{\rm vac}\int DqD\overline{q}\exp\left\{\int d^4x\sum_f^{N_F}\overline{q_f}(x)\times(i\gamma_\mu\hat{\nabla}_\mu-m_f)q_f(x)+\sum_{n=2}^{\infty}L_n\right\}, \quad (1)$$

where N_F is the number of flavors corresponding to the $SU(N_F)$ flavor group and

$$L_n=\frac{g^n}{n!}\int d^4y_1\cdots\int d^4y_n j^{a_1}_{\mu_1}(y_1)\cdots j^{a_n}_{\mu_n}(y_n)\times G^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}(y_1,\ldots,y_n|B),$$

$$j^a_\mu(y)=\sum_f^{N_F}\overline{q_f}(x)\gamma_\mu t^a q_f(x),$$

$$\hat{\nabla}_\mu=\partial_\mu-it^aB^a_\mu.$$

The function $G^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}$ is the exact (up to the quark loops) n-point gluon Green function in the external field B^a_μ. We will investigate the mesonic $(q\overline{q})$-collective modes and consider Eq. (1) with the quark-quark interaction truncated up to the term L_2:

$$Z=\int d\sigma_{\rm vac}\int DqD\overline{q}\exp\left\{\int d^4x\sum_f^{N_F}\overline{q_f}(x)(i\gamma_\mu\hat{\nabla}_\mu-m_f)q_f(x)+\frac{g^2}{2}\int\int d^4xd^4y j^a_\mu(x)G^{ab}_{\mu\nu}(x,y|B)j^b_\nu(y)\right\}. \quad (2)$$

Representations (1) and (2) imply that there exists some vacuum (classical) gluon field $B^a_\mu(x)$ which minimizes the effective action (or effective potential) of the Euclidean QCD. In the general case, the vacuum field depends on a set of parameters $\{\sigma_{\rm vac}\}$, and the measure $d\sigma_{\rm vac}$ averages all physical amplitudes over a subset of $\{\sigma_{\rm vac}\}$, in respect to which the vacuum state is degenerate (for more details see [8] and references therein).

The quark-gluon interactions both in Eqs. (1) and (2) are local, and a decomposition over degrees of g^2 generates a renormalizable perturbation theory. It means that an appropriate regularization is implied in Eqs. (1) and (2). This point has to be stressed here, since our final technical aim is a transformation of the interaction term in Eq. (2), which generates a completely new superrenormalizable perturbation expansion of the functional integral (2).

Let us identify all ingredients of these general formulas for the particular case of a homogeneous (anti-)self-dual vacuum field:

$$\hat{B}_\mu(x)=\hat{n}B_\mu(x),\quad B_\mu(x)=B_{\mu\nu}x_\nu,$$

$$\hat{n}=n^at^a,\quad n^2=n^an^a=1.$$

The constant tensor $B_{\mu\nu}$ satisfies the conditions

$$B_{\mu\nu}=-B_{\nu\mu},\quad B_{\mu\rho}B_{\rho\nu}=-B^2\delta_{\mu\nu},$$

$$\widetilde{B}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B_{\alpha\beta}=\pm B_{\mu\nu},$$

where B is the gauge-invariant tension of the vacuum field. Since the chromomagnetic **H** and chromoelectric **E** fields relate to each other like $\mathbf{H}=\pm\mathbf{E}$, two spherical angles (φ,θ) define the direction of the field in Euclidean space. In the diagonal representation of $\hat{n}=n^at^a$, an additional angle ξ is needed to fix the direction of the field in color space:

$$\hat{n}=t^3\cos\xi+t^8\sin\xi,\quad 0\leqslant\xi<2\pi.$$

The one-loop calculations and some nonperturbative estimations of the effective potential for the homogeneous gluon field argue (do not prove) that the potential could have a minimum at a nonzero value of the field tension $B\neq0$ (e.g., see [1–3]). We will assume that the field under consideration realizes a nonperturbative QCD vacuum and study the manifestations of this field in the spectrum of collective modes. Since the effective potential is invariant under Euclidean rotations and parity and gauge transformations, this vacuum should be degenerate with respect to the directions of the field in color and Euclidean space and should be the same for anti-self-dual and self-dual configurations. According to this argumentation, the field B^a_μ in Eq. (2) corresponds to the tension B minimizing the effective potential, and the measure $d\sigma_{\rm vac}$ has the form

$$\int d\sigma_{\rm vac}=\frac{1}{(4\pi)^2}\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi\int_0^{2\pi}d\xi\sum_\pm, \quad (3)$$

where the sign $\Sigma_\pm$ denotes averaging over the self-dual and anti-self-dual configurations. To simplify calculations and to clarify the technical side of the bosonization procedure in the presence of the background field, we will omit the integral over ξ in Eq. (3) and fix a particular vector $n^a=\delta^{a8}$. In the fundamental (matrix t^8) and adjoint (matrix C^8) representations of $SU_c(3)$ one gets

$$\hat{n}=t^8={\rm diag}\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{2}{\sqrt{3}}\right),\quad \hat{B}_{\mu\rho}\hat{B}_{\rho\nu}=-(t^8)^2B^2\delta_{\mu\nu},$$

$$\check{n}=C^8=\frac{\sqrt{3}}{2}K,\quad \check{B}_{\mu\rho}\check{B}_{\rho\nu}=-\frac{3}{4}K^2B^2\delta_{\mu\nu},$$

$$K_{54}=-K_{45}=K_{76}=-K_{67}=i,\quad K^2={\rm diag}(0,0,0,1,1,1,1,0).$$

The rest of the elements of the matrix K are equal to zero. It is convenient to define the mass scale $\Lambda^2=\sqrt{3}B$:

$$\breve{B}_{\mu\rho}\breve{B}_{\rho\nu}=-\frac{1}{4}K^2\Lambda^4\delta_{\mu\nu},$$

$$\hat{B}_{\mu\rho}\hat{B}_{\rho\nu}=-v^2\Lambda^4\delta_{\mu\nu}, \quad v=\mathrm{diag}\left(\frac{1}{3},\frac{1}{3},\frac{2}{3}\right),$$

$$\frac{1}{v}=\mathrm{diag}\left(3,3,\frac{3}{2}\right).$$

We would like to stress that averaging over directions of the background field in color space should be incorporated into the formalism, and its role should be analyzed. But first of all, we would like to go as far as possible with the formalism, which is as simple as possible, and test how the technique proposed in [8] works in the meson phenomenology.

B. Quark and gluon propagators

The quark propagator $S_f(x,y|B)$ in Eq. (2) satisfies the equation

$$(i\gamma_\mu\hat{\nabla}_\mu-m_f)S_f(x,y|B)=-\delta(x-y),$$

and can be written in the form

$$S_f(x,y|B)=e^{i(x\hat{B}y)/2}H_f(x-y|B)e^{i(x\hat{B}y)/2},$$

$$H_f(z)=\frac{i\hat{\nabla}_\mu(z)\gamma_\mu+m_f}{\hat{\nabla}^2(z)+m_f^2+(\sigma\hat{B})}\delta(z), \tag{4}$$

$$\widetilde{H}_f(p|B)=\frac{1}{2v\Lambda}\int_0^1 dt e^{(-p^2/2v\Lambda^2)t}\left(\frac{1-t}{1+t}\right)^{\alpha_f^2/4v}\left[\alpha_f+\frac{1}{\Lambda}p_\mu\gamma_\mu + it\frac{1}{\Lambda}(\gamma f p)\right]\left[P_\pm+P_\mp\frac{1+t^2}{1-t^2}-\frac{i}{2}(\gamma f\gamma)\frac{t}{1-t^2}\right], \tag{5}$$

where

$$P_\pm=\frac{1}{2}(1\pm\gamma_5), \quad \alpha_f=\frac{m_f}{\Lambda}, \quad (xBy)=x_\mu B_{\mu\nu}y_\nu,$$

$$(pf\gamma)=p_\mu f_{\mu\nu}\gamma_\nu, \quad f_{\mu\nu}=\frac{t^8}{v\Lambda^2}B_{\mu\nu}, \quad f_{\mu\rho}f_{\rho\nu}=-\delta_{\mu\nu}.$$

The function $\widetilde{H}_f$ is the Fourier-transformed H_f. The upper (lower) sign in the matrix $P_\pm$ corresponds to the self-dual (anti-self-dual) field.

The term $(\sigma\hat{B})=\sigma_{\alpha\beta}\hat{B}_{\alpha\beta}$ in Eq. (4) [the second line in Eq. (5)] describes the interaction of a quark spin with the background field. One can see that this spin-field interaction leads to the singularity $1/m_f$ for $m_f\to 0$, which is a manifestation of the zero mode (the lowest Landau level) of the massless Dirac equation in the external (anti-)self-dual homogeneous field. The mathematical point is that the spectrum of the operator $\gamma_\mu\partial_\mu$ is continuous, whereas the spectrum of the operator $\gamma_\mu\hat{\nabla}_\mu(x)$ is discrete and the lowest eigennumber is equal to zero. Simple calculations give, for $m_f\to 0$,

$$\widetilde{H}_f(p|B)=2e^{-(p^2/2v\Lambda^2)}\left[\frac{1}{m_f}+\frac{1}{m_f^2}[p_\mu\gamma_\mu+i(\gamma f p)]\right]\times\left[P_\mp-\frac{i}{4}(\gamma f\gamma)\right]+\mathrm{const}, \tag{6}$$

and

$$\lim_{\varepsilon\to 0}\lim_{m_f\to 0} m_f\langle\overline{q_f}(x)q_f(x+\varepsilon)\rangle_B = -\lim_{\varepsilon\to 0}\lim_{m_f\to 0} m_f\mathrm{Tr}H_f(\varepsilon|B) = -\int\frac{d^4p}{(2\pi)^4}\lim_{m_f\to 0} m_f\mathrm{Tr}\widetilde{H}_f(p|B) = -\frac{4}{3\pi^2}\Lambda^4. \tag{7}$$

Because of the spin-field interaction, the quark condensate is nonzero in the limit of vanishing quark mass. This indicates that chiral symmetry is broken by the vacuum field in the limit $m_f\to 0$ (see also [1]). It will be clear below that just the spin-field interaction gives rise to the splitting between the masses of the pseudoscalar and vector mesons and provides a smallness of pion mass.

In terms of the variable $\zeta=p_\mu\gamma_\mu$ the propagator $\widetilde{H}_f(\zeta|B)$ is an entire analytical function in the complex ζ plane. There are no poles corresponding to the free quarks, which is treated as the confinement of quarks. The following asymptotic behavior takes place:

$$\widetilde{H}_f(\zeta|B)\to\begin{cases}\dfrac{m_f+\zeta}{-\zeta^2}=\dfrac{m_f+\gamma_\nu p_\nu}{p^2} & \text{if } \zeta\to\pm\infty \ (p^2\to\infty),\\ O\left(\exp\left(\dfrac{\zeta^2}{2v\Lambda^2}\right)\right)=O\left(\exp\left(\dfrac{-p^2}{2v\Lambda^2}\right)\right) & \text{if } \zeta\to\pm i\infty \ (p^2\to-\infty).\end{cases} \tag{8}$$

Equation (8) shows the standard local behavior of the fermion propagator in the Euclidean region ($p^2\to\infty$), while in the physical region ($p^2\to-\infty$) we see an exponential increase typical for nonlocal theories (for more details about the general theory of nonlocal interactions of quantized fields see [6,12]). Below, the absence of the poles and the exponential increase will be referred to as the confinement properties of a propagator.

The function $D^{ab}_{\mu\nu}(x,y|B)$ in representation (2) is the exact gluon propagator for pure gluodynamics in the presence of the vacuum field B^a_μ. This function is unknown, and some approximation has to be introduced. For instance, the local NJL model corresponds to the choice $D^{ab}_{\mu\nu}=\delta^{ab}\delta_{\mu\nu}\delta(x-y)$. We go beyond this approximation and replace the function $D^{ab}_{\mu\nu}(x,y|B)$ by the confined part

$$D_{\mu\nu}(x,y|B)=\delta_{\mu\nu}K^2e^{i(x\breve{B}y)/2}D(x-y|\Lambda^2)e^{i(x\breve{B}y)/2}, \quad (9)$$

$$D(z|\Lambda^2)=\frac{1}{(2\pi)^2z^2}\exp\left\{-\frac{\Lambda^2z^2}{4}\right\}$$

of the gluon propagator

$$G^{ab}_{\mu\nu}(x,y|B)=D^{ab}_{\mu\nu}(x,y|B)+R^{ab}_{\mu\nu}(x,y|B),$$

which is a solution of the equation (for details see [8])

$$(\breve{\nabla}^2\delta_{\mu\nu}+4i\breve{B}_{\mu\nu})G_{\nu\rho}(x,y|B)=-\delta_{\mu\rho}\delta(x-y).$$

The Fourier transform of the function $D(z|\Lambda^2)$ is an entire analytical function in momentum space. It has local behavior in the Euclidean region, but increases exponentially in the physical region. This function describes the propagation of the confined modes of the gluon field. Other terms $R^{ab}_{\mu\nu}$, which contain a contribution of the zero modes and an antisymmetric part, will be omitted.

C. Color singlet bilocal quark currents

Substituting gluon propagator (10) to the interaction term in representation (2), using the Fierz transformation of the color, flavor, and Dirac matrices, and keeping only the scalar J^{aS}, pseudoscalar J^{aP}, vector J^{aV}, and axial-vector J^{aA} colorless currents, we arrive at the expression [8]

$$L_2=\frac{g^2}{2}\sum_{bJ} C_J\int\int d^4xd^4yJ^{bJ}(x,y)D(x-y|\Lambda^2)J^{bJ}(y,x), \quad (10)$$

$$J^{bJ}(x,y)=\overline{q_f}(x)M^b_{ff'}\Gamma^Je^{i(x\hat{B}y)}q_{f'}(y), \quad (11)$$

$$\Gamma^S=\mathbf{1},\quad \Gamma^P=i\gamma_5,\quad \Gamma^V=\gamma_\mu,\quad \Gamma^A=\gamma_5\gamma_\mu,$$

$$C_S=C_P=\tfrac{1}{9},\quad C_V=C_A=\tfrac{1}{18}.$$

Here $M^b_{ff'}$ are the flavor-mixing matrices $(b=0,\dots,N_F^2-1)$ corresponding to the SU(N_F) flavor group. In the case of SU(2) and SU(3) they are given by the matrices τ^b and λ^b, respectively.

Because of the phase factor $\exp\{i(x\hat{B}y)\}$, bilocal quark currents (11) are the scalars under the local gauge transformations

$$q(x)\to e^{-i\hat{\omega}(x)}q(x),\quad \overline{q}(x)\to\overline{q}(x)e^{i\hat{\omega}(x)},$$

$$\hat{B}_\mu\to e^{-i\hat{\omega}(x)}\hat{B}_\mu e^{i\hat{\omega}(x)}+\frac{i}{g}e^{-i\hat{\omega}(x)}\partial_\mu e^{i\hat{\omega}(x)}. \quad (12)$$

Let us transform integration variables x and y in Eq. (10) to the coordinate system corresponding to the center of masses of quarks $q_f(x)$ and $q_{f'}(y)$:

$$x\to x+\xi_f y, y\to x-\xi_{f'}y,\quad \xi_f=\frac{m_f}{m_f+m_{f'}},\quad \xi_{f'}=\frac{m_{f'}}{m_f+m_{f'}}. \quad (13)$$

The corresponding transformation of the quark currents looks as

$$J^{bJ}(x,y)=\overline{q_f}(x)M^b_{ff'}\Gamma^Je^{i(x\hat{B}y)}q_{f'}(y)$$
$$\to\overline{q_f}(x+\xi_f y)M^b_{ff'}\Gamma^Je^{i(x\hat{B}y)}q_{f'}(x-\xi_{f'}y)$$
$$=\overline{q_f}(x)M^b_{ff'}\Gamma^Je^{y\overleftrightarrow{\nabla}_{ff'}(x)/2}q_{f'}(x)\overset{\text{ds}}{=}J^{bJ}(x,y), \quad (14)$$

where $\overleftrightarrow{\nabla}_{ff'}$ is a linear combination of the left and right covariant derivatives:

$$\overleftrightarrow{\nabla}_{ff'}(x)=\xi_f(\overleftarrow{\partial}+i\hat{B}(x))-\xi_{f'}(\overrightarrow{\partial}-i\hat{B}(x)).$$

These covariant derivatives indicate that the currents (14) are nonlocal and colorless. The interaction term (10) takes the form

$$L_2=\frac{g^2}{8\pi^2}\sum_{bJ} C_J\int\int d^4xd^4y\frac{1}{y^2}\times\exp\left\{-\frac{\Lambda^2y^2}{4}\right\}J^{bJ}(x,y)J^{bJ}(y,x), \quad (15)$$

where we have made use of the representation (9). The currents are defined by Eq. (14). Transformation (13) turns out to be crucial for a simultaneous description of the light-light, heavy-light, and heavy-heavy mesons.

D. Decomposition of bilocal currents

The idea of our next step consists in a decomposition of the bilocal currents (14) over some complete set of orthonormalized polynomials in such a way that the relative coordinate of two quarks y in Eq. (15) would be integrated out. One can see that a particular form of this set is determined by the form of the gluon propagator [$\exp\{-\Lambda^2y^2/4\}$ in Eq. (15)]. The propagator plays the role of a weight function in the orthogonality condition. The physical meaning of the decomposition consists in classifying the relative motion of two quarks in the bilocal currents over a set of radial n and angular ℓ quantum numbers. In other words, according to the general principles of quantum mechanics the bilocal currents have to be represented as a set of quark currents with definite radial n and angular ℓ quantum numbers. Thus, we are looking for a decomposition of the form

$$J^{bJ}(x,y)=\sum_{n\ell}(y^2)^{\ell/2}f^{n\ell}_{\mu_1\cdots\mu_\ell}(y)\mathcal{J}^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x), \quad (16)$$

$$f^{\ell n}_{\mu_1\cdots\mu_\ell}(y)=L_{n\ell}(y^2)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y),\quad n_y=y/\sqrt{y^2}.$$

The angular part of $f^{\ell n}$ is given by the irreducible tensors of the four-dimensional rotational group $T^{(\ell)}_{\mu_1\cdots\mu_\ell}$, which are orthogonal,

$$\int_\Omega\frac{d\omega}{2\pi^2}T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y)T^{(k)}_{\nu_1\cdots\nu_k}(n_y)=\frac{1}{2^\ell(\ell+1)}\delta^{\ell k}\delta_{\mu_1\nu_1}\cdots\delta_{\mu_\ell\nu_\ell}, \quad (17)$$

and satisfy the conditions

$$T^{(\ell)}_{\mu_1\cdots\mu\cdots\nu\cdots\mu_\ell}(n_y)=T^{(\ell)}_{\mu_1\cdots\nu\cdots\mu\cdots\mu_\ell}(n_y),$$

$$T^{(\ell)}_{\mu\cdots\mu\cdots\mu_\ell}(n_y)=0,$$

$$T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_y)T^{(\ell)}_{\mu_1\cdots\mu_\ell}(n_{y'})=\frac{1}{2^\ell}C^{(1)}_\ell(n_y n_{y'}). \qquad (18)$$

The measure $d\omega$ in Eq. (17) relates to integration over the angles of unit vector n_y, and $C^{(1)}_\ell$ in Eq. (18) are the Gegenbauer's (ultraspherical) polynomials. The polynomials $L_{n\ell}(u)$ obey the condition

$$\int_0^\infty du\,\rho_\ell(u)L_{n\ell}(u)L_{n'\ell}(u)=\delta_{nn'}.$$

The weight function $\rho_\ell(u)$ arising from the exponential term in Eq. (15) looks like

$$\rho_\ell(u)=u^\ell e^{-u};$$

hence, $L_{n\ell}(u)$ are the generalized Laguerre's polynomials.

The details of the calculation of the currents $\mathcal{J}^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x)$ in Eq. (16) can be found in [8]. As a result, the interaction term L_2 takes the form

$$L_2=\frac{1}{2}\sum_{bJ\ell n}\left(\frac{G_{J\ell n}}{\Lambda}\right)^2\int d^4x[\mathcal{J}^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x)]^2,$$

$$G^2_{J\ell n}=C_J g^2\frac{(\ell+1)}{2^\ell n!(\ell+n)!}, \qquad (19)$$

$$\mathcal{J}^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x)=\overline{q(x)}V^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x)q(x), \qquad (20)$$

$$V^{bJ\ell n}_{\mu_1\cdots\mu_\ell}(x)\equiv V^{bJ\ell n}_{\mu_1\cdots\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)=M^b\Gamma^J\left\{\left\{F_{n\ell}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\right)T^{(\ell)}_{\mu_1\cdots\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)\right\}\right\}, \qquad (21)$$

$$F_{n\ell}(4s)=s^n\int_0^1 dt\,t^{\ell+n}e^{st}. \qquad (22)$$

The double curly brackets in Eq. (21) mean that the covariant derivatives commute inside these brackets. The form factors $F_{n\ell}(s)$ are entire analytical functions in the complex s plane, which is a manifestation of the gluon confinement.

The classification of the currents will be complete if we will decompose $\mathcal{J}^{aJ\ell n}_{\alpha,\mu_1\cdots\mu_\ell}$ with $J=V,A$ and $\ell>0$ into the sum of orthogonal currents $\mathcal{I}^{bJ\ell nj}_{\alpha\mu_1\cdots\mu_\ell}$ with different total angular momenta $j=\ell-1,\ell,\ell+1$. The index α relates to the matrices $\Gamma^V_\alpha=\gamma_\alpha$ and $\Gamma^A_\alpha=\gamma_5\gamma_\alpha$ in Eq. (21). This decomposition can be arranged by the division

$$\mathcal{J}^{bJ\ell n}_{\alpha,\mu_1\cdots\mu_\ell}=\sum_{j=\ell-1}^{\ell+1}\mathcal{I}^{bJ\ell nj}_{\alpha\mu_1\cdots\mu_\ell}, \qquad (23)$$

where

$$\mathcal{I}^{bJ\ell nj}_{\alpha\mu_1\cdots\mu_\ell}=\begin{cases}\dfrac{1}{(\ell+1)^2}\mathcal{P}_{\alpha\mu_1\cdots\mu_\ell}[\delta_{\alpha\mu_1}\mathcal{J}^{bJ\ell n}_{\rho,\rho\mu_2\cdots\mu_\ell}], & j=\ell-1,\\ \dfrac{1}{\ell+1}\sum_{i=1}^{\ell}[\mathcal{J}^{bJ\ell n}_{\alpha,\mu_i\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_\ell}-\mathcal{J}^{bJ\ell n}_{\mu_i,\alpha\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_\ell}], & j=\ell,\\ \dfrac{1}{\ell+1}\mathcal{P}_{\alpha\mu_1\cdots\mu_\ell}\left[\mathcal{J}^{bJ\ell n}_{\alpha,\mu_1\cdots\mu_\ell}-\dfrac{1}{\ell+1}\delta_{\alpha\mu_1}\mathcal{J}^{bJ\ell n}_{\rho,\rho\mu_2\cdots\mu_\ell}\right], & j=\ell+1.\end{cases} \qquad (24)$$

The symbol $\mathcal{P}_{\alpha\mu_1\cdots\mu_\ell}$ in Eq. (24) denotes a cyclic permutation of the indices $(\alpha\mu_1\cdots\mu_\ell)$. Let s_J be defined as

$$s_P=s_S=0, \quad s_V=s_A=1;$$

then, using the orthogonality of the currents with different j, we can rewrite interaction term L_2 as

$$L_2=\sum_{aJ\ell n}\sum_{j=|\ell-s_J|}^{\ell+s_J}\frac{1}{2\Lambda^2}G^2_{J\ell n}\int d^4x[\mathcal{I}^{aJ\ell n}_j(x)]^2,$$

where we have introduced the notation

$$\mathcal{I}^{bJ0n1}_\alpha=\mathcal{J}^{bJ0n}_\alpha \quad \text{for } J=V,A, \quad \ell=0,$$

$$\mathcal{I}^{bJ\ell n\ell}_{\mu_1\cdots\mu_\ell}=\mathcal{J}^{bJ\ell n}_{\mu_1\cdots\mu_\ell} \quad \text{for } J=S,P,\ell\geqslant 0.$$

This form is equivalent to Eq. (10), but now the interaction between quarks is expressed in terms of the nonlocal quark currents, which are elementary currents of the system in the sense of the classification over quantum numbers.

For large Euclidean momentum the vertices $\widetilde{V}^{aJ\ell n}$ behave as $1/(p^2)^{1+\ell/2}$. Therefore, only the "bubble" diagrams, shown in Fig. 1, are divergent. These divergences can be removed by counterterms of the form $-2\mathcal{I}(x)\mathrm{Tr}VS$.

To avoid an unnecessary complication of notation, it is convenient to introduce the condensed index $\mathcal{N}$ enumerating

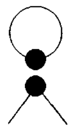

FIG. 1. Divergent bubble diagram.

the currents with all different combinations of the quantum numbers a, J, ℓ, n, and j. The renormalized vacuum amplitude Z takes the form

$$Z=\int d\sigma_{\rm vac}\int DqD\overline{q}\exp\left\{-\int\int d^4xd^4y\overline{q(x)}S^{-1}(x,y|B)\times q(y)+\sum_{\mathcal{N}}\frac{1}{2\Lambda^2}G_{\mathcal{N}}^2\int d^4x[\mathcal{I}_{\mathcal{N}}(x)-{\rm Tr}V_{\mathcal{N}}S]^2\right\}. \quad (25)$$

E. Bosonization

By means of the standard bosonization procedure [9,10] applied to Eq. (25) the amplitude Z can be represented in terms of the composite meson fields $\Phi_{\mathcal{N}}$ [8]:

$$Z=N\int\prod_{\mathcal{N}}D\Phi_{\mathcal{N}}\exp\left\{\frac{1}{2}\int\int d^4xd^4y\Phi_{\mathcal{N}}(x)[(\Box-M_{\mathcal{N}}^2)\times\delta(x-y)-h_{\mathcal{N}}^2\Pi_{\mathcal{N}}^R(x-y)]\Phi_{\mathcal{N}}(y)+I_{\rm int}[\Phi]\right\}, \quad (26)$$

$$I_{\rm int}=-\frac{1}{2}\int d^4x_1\int d^4x_2h_{\mathcal{N}}h_{\mathcal{N}'}\Phi_{\mathcal{N}}(x_1)[\Gamma_{\mathcal{N}\mathcal{N}'}(x_1,x_2)-\delta_{\mathcal{N}\mathcal{N}'}\Pi_{\mathcal{N}}(x_1-x_2)]\Phi_{\mathcal{N}'}(x_2)-\sum_{m=3}\frac{1}{m}\int d^4x_1\cdots\int d^4x_m\times\prod_{k=1}^{m}h_{\mathcal{N}_k}\Phi_{\mathcal{N}_k}(x_k)\Gamma_{\mathcal{N}_1\cdots\mathcal{N}_m}(x_1,\ldots,x_m),$$

$$\Gamma_{\mathcal{N}_1\cdots\mathcal{N}_m}=\int d\sigma_{\rm vac}{\rm Tr}\{V_{\mathcal{N}_1}(x_1)\times S(x_1,x_2|B)\cdots V_{\mathcal{N}_m}(x_m)S(x_m,x_1|B)\}.$$

The meson masses $M_{\mathcal{N}}$ are defined by the equations

$$1+\left(\frac{G_{\mathcal{N}}}{\Lambda}\right)^2\widetilde{\Pi}_{\mathcal{N}}(-M_{\mathcal{N}}^2)=0, \quad (27)$$

where $\widetilde{\Pi}_{\mathcal{N}}(-M_{\mathcal{N}}^2)$ is the diagonal part of the two-point function $\widetilde{\Gamma}_{\mathcal{N}\mathcal{N}'}$, which corresponds to the diagram shown in Fig. 2(a). The fields $\Phi_{\mathcal{N}}$ ($\mathcal{N}=\{a,J,\ell,n,j\}$) with $j>0$ satisfy the on-shell condition

$$p_\mu\widetilde{\Phi}_{\mathcal{N}}^{\cdots\mu\cdots}(p)=0 \quad \text{if } p^2=-M_{\mathcal{N}}^2,$$

which excludes all extra degrees of freedom of the field, so that the numbers ℓ and j can be treated as the O(3) orbital momentum and total momentum, respectively [8]. The total momentum j plays the role of an observable spin of the state with a given $\mathcal{N}=\{b,J,\ell,n,j\}$.

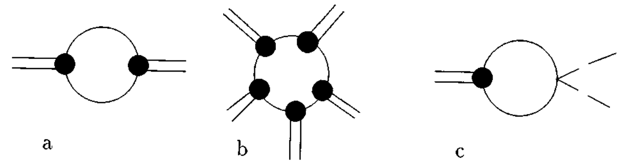

FIG. 2. Diagrams describing different processes in effective nonlocal meson theory in the lowest (one-loop) order.

The constants

$$h_{\mathcal{N}}=1/\sqrt{\widetilde{\Pi}'_{\mathcal{N}}(-M_{\mathcal{N}}^2)} \quad (28)$$

play the role of the effective coupling constants of the meson-quark interaction.

The quark masses m_f, the scale Λ (strength of the background field), and the quark-gluon coupling constant g are the free parameters of the effective meson theory (26)–(28).

In the one-loop approximation, the interactions between mesons with given quantum numbers $\mathcal{N}=\{b,J,\ell,n,j\}$ are described by quark loops like the diagram in Fig. 2(b). Because of the nonlocality of meson-quark vertices, the quark loops are ultraviolet finite. The whole diagram is averaged by integration over the measure $d\sigma_{\rm vac}$.

Figure 3 illustrates the central idea of the method of induced nonlocal currents, which has been realized in this section. An effective four-quark interaction is represented as an infinite series of interactions between the nonlocal quark currents characterized by the complete set of quantum numbers $\{b,J,\ell,n,j\}$. The form of the currents is induced by a particular form of the gluon propagator. This new representation of the four-quark interaction generates an expansion of any amplitude into a series of partial amplitudes with a particular value of the quantum numbers. Each partial amplitude is ultraviolet finite at any order of expansion over degrees of the coupling constant g. The composite meson fields in Eq. (26) are nothing else but "elementary" collective excitations, which are classified according to the complete set of quantum numbers of the relativistic two-quark system.

It should be stressed that the model (26) satisfies all demands of the general theory of nonlocal interactions of quantum fields [12], which means that Eq. (26) defines a nonlocal, relativistic, unitary, and ultraviolet finite quark model of meson-meson interactions.

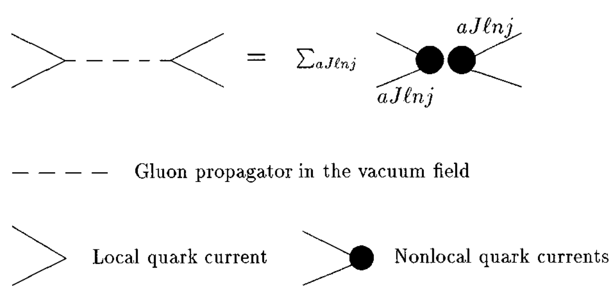

FIG. 3. The decomposition of the four-quark interaction.

Now we would like to test how this formalism works in the meson phenomenology.

III. MESON SPECTRUM AND WEAK DECAY CONSTANTS

Let us rewrite Eq. (27) in a more detailed form

$$\Lambda^2+G^2_{J\ell n}\tilde{\Pi}_{bJ\ell nj}(-M^2_{bJ\ell nj};m_f,m_{f'};\Lambda)=0. \qquad (29)$$

The function $\tilde{\Pi}_{bJ\ell nj}$ in Eq. (29) is given by the diagonal part of the tensor (in the momentum representation):

$$\Pi^{bJ\ell nj}_{\mu_1\cdots,\nu_1\cdots}(x-y;m_f,m_{f'};\Lambda)$$
$$=\int d\sigma_{\rm vac}{\rm Tr}[V^{bJ\ell nj}_{\mu_1\cdots}(x)S(x,y|B)V^{bJ\ell nj}_{\nu_1\cdots}(y)S(y,x|B)]. \qquad (30)$$

Relation (29) is the master equation for meson masses. The function $\tilde{\Pi}$ can be calculated using the representations (4) and (5) for the quark propagator and Eq. (21) for the vertices. The only point that requires a comment is an averaging over the space directions of the vacuum field. Actually we have to average the tensors $f_{\mu\nu}$, $f_{\mu\nu}f_{\alpha\beta}$, and so on. The generating formula looks as

$$\langle\exp(if_{\mu\nu}J_{\mu\nu})\rangle=\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\sin\theta\exp(if_{\mu\nu}J_{\mu\nu})$$
$$=\frac{\sin\sqrt{2(J_{\mu\nu}J_{\mu\nu}\pm\tilde{J}_{\mu\nu}J_{\mu\nu})}}{\sqrt{2(J_{\mu\nu}J_{\mu\nu}\pm\tilde{J}_{\mu\nu}J_{\mu\nu})}},$$

where φ and θ are spherical angles, $J_{\mu\nu}$ is an antisymmetric tensor, $\tilde{J}_{\mu\nu}$ is a dual tensor, and $\pm$ corresponds to the self-dual and anti-self-dual vacuum field. In particular this general representation gives

$$\langle f_{\mu\nu}\rangle=0,$$
$$\langle f_{\mu\nu}f_{\alpha\beta}\rangle=\frac{1}{3}(\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu}\pm\varepsilon_{\alpha\beta\mu\nu}). \qquad (31)$$

A. Light pseudoscalar and vector mesons

First of all, let us fit the free parameters of the model, taking the masses of π, ρ, K, and K^* mesons as the basic quantities. Below, we will sometimes use the symbol of a given meson instead of the corresponding set of quantum numbers [for example, π instead of $(3,P,0,0;0)$].

Four equations for the masses of the basic mesons can be written in the form

$$2\tilde{\Pi}_{\pi}(-M^2_{\pi};m_u,m_u;\Lambda)=\tilde{\Pi}_{\rho}(-M^2_{\rho};m_u,m_u;\Lambda), \qquad (32)$$

$$2\tilde{\Pi}_{K}(-M^2_{K};m_s,m_u;\Lambda)=\tilde{\Pi}_{K*}(-M^2_{K*};m_s,m_u;\Lambda), \qquad (33)$$

$$2\tilde{\Pi}_{\pi}(-M^2_{\pi};m_u,m_u;\Lambda)=\tilde{\Pi}_{K}(-M^2_{K};m_s,m_u;\Lambda), \qquad (34)$$

$$g^2=-9\Lambda^2/\tilde{\Pi}_{\pi}(-M^2_{\pi};m_u,m_u;\Lambda). \qquad (35)$$

If M_π, M_ρ, M_K, and M_{K*} are taken to be equal to the experimental values, then the masses m_u and m_s of the u and s quarks as functions of Λ are defined by Eqs. (32) and (33). Using $m_u(\Lambda)$ and $m_s(\Lambda)$ in Eq. (34), we find the value of Λ, which provides a simultaneous description of the strange and nonstrange mesons. An optimal value of the coupling constant g is calculated by means of Eq. (35). By this way we arrive at the values

$$m_u=198.28\ \text{MeV},\quad m_s=412.96\ \text{MeV},$$
$$\Lambda=319.46\ \text{MeV},\quad g=9.96. \qquad (36)$$

Solution (36) is unique.

It is well known that there should be a special reason which provides a small pion mass and splits the masses of pseudoscalar and vector mesons. Breaking of chiral symmetry due to the four-quark interaction and two independent coupling constants for pseudoscalar and vector mesons ($g_P\neq g_V$ instead of our parameter g) plays the role of a such reason in the local NJL model. As has already been pointed out, the interaction of quark spin with the vacuum field leads to the singular behavior of the quark propagator in the massless limit and generates a nonzero quark condensate, which indicates breaking of chiral symmetry by the vacuum gluon field. Now let us illustrate that in our case the same spin-field interaction is responsible for a small pion mass and for the mass splitting between P and V mesons.

The polarization function $\tilde{\Pi}_J$ ($\ell=0$, $n=0$, $J=P,V$) can be represented in the form

$$\tilde{\Pi}_J(-M^2;m_f,m_{f'};\Lambda)=-\frac{\Lambda^2}{4\pi^2}{\rm Tr}_v\int_0^1dt_1\int_0^1dt_2\int_0^1ds_1\int_0^1ds_2\left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4v\Lambda^2}\left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4v\Lambda^2}$$
$$\times\left[\frac{M^2}{\Lambda^2}\frac{F_1^{(J)}(t_1,t_2,s_1,s_2)}{\Phi_2^4(t_1,t_2,s_1,s_2)}+\frac{m_fm_{f'}}{\Lambda^2}\frac{F_2^{(J)}(s_1,s_2)}{(1-s_1^2)(1-s_2^2)\Phi_2^2(t_1,t_2,s_1,s_2)}+\frac{2v(1-4v^2t_1t_2)F_3^{(J)}(s_1,s_2)}{\Phi_2^3(t_1,t_2,s_1,s_2)}\right]$$
$$\times\exp\left\{\frac{M^2}{2v\Lambda^2}\Phi(t_1,t_2,s_1,s_2)\right\}, \qquad (37)$$

where

$$\Phi=\frac{\Phi_1(t_1,t_2,s_1,s_2)}{\Phi_2(t_1,t_2,s_1,s_2)}, \tag{38}$$

$$\Phi_1=2v(t_1+t_2)[s_1\xi_1^2+s_2\xi_2^2]+s_1s_2[1+4v^2t_1t_2(\xi_1-\xi_2)^2],$$

$$\Phi_2=2v(t_1+t_2)(1+s_1s_2)+(1+4v^2t_1t_2)(s_1+s_2),$$

$$F_1^{(P)}=(1+s_1s_2)[A_1A_2+4v^2(t_1-t_2)^2\xi_1\xi_2s_1s_2],$$

$$F_1^{(V)}=\frac{1}{3}[(3-s_1s_2)A_1A_2+4v^2(t_1-t_2)^2\xi_1\xi_2s_1s_2(1-3s_1s_2)],$$

$$A_1=[1-4v^2t_1t_2(\xi_1-\xi_2)]s_1+2v(t_1+t_2)\xi_2,$$

$$A_2=[1+4v^2t_1t_2(\xi_1-\xi_2)]s_2+2v(t_1+t_2)\xi_1,$$

$$F_2^{(P)}=(1+s_1s_2)^2,\quad F_2^{(V)}=1-s_1^2s_2^2, \tag{39}$$

$$F_3^{(P)}=2(1+s_1s_2),\quad F_3^{(V)}=1-s_1s_2.$$

Equations (37)–(39) show that the singularity $(1-s_1)^{-1}(1-s_2)^{-1}$, arising from the spin-field interaction [see the second line of Eq. (5)], is accumulated in the term of Eq. (37) proportional to the quark masses. Other terms are free from this singularity, although the spin-field interaction contributes to them. This is due to the structure of the trace of the Dirac matrices.

Let us compare behavior of pion and ρ-meson polarization functions in the limit

$$m_f=m_{f'}=m_u\ll\Lambda.$$

Using the singularity of the integrand in Eq. (37) at $s_1\to1$ and $s_2\to1$, one can check that the pion polarization function is singular in this limit and behaves as $1/m_u^2$

$$\tilde{\Pi}_\pi(-M^2;m_u,m_u;\Lambda)=-\mathrm{Tr}_v\frac{4v^2\Lambda^4}{\pi^2m_u^2}\int_0^1\int_0^1\frac{dt_1dt_2}{\Phi_2^2(t_1,t_2,1,1)}\times\exp\left\{\frac{M^2}{2v\Lambda^2}\Phi(t_1,t_2,1,1)\right\}+\mathrm{const.} \tag{40}$$

On the contrary, the ρ-meson polarization is regular at $m_u=0$ and looks like

$$\tilde{\Pi}_\rho(-M^2;m_u,m_u;\Lambda)=-\frac{\Lambda^2}{4\pi^2}\mathrm{Tr}_v\int_0^1dt_1\int_0^1dt_2\int_0^1ds_1\int_0^1ds_2\left(\exp\left\{\frac{M^2}{2v\Lambda^2}\Phi(t_1,t_2,s_1,s_2)\right\}\left[\frac{M^2}{\Lambda^2}\frac{F_1^{(V)}(t_1,t_2,s_1,s_2)}{\Phi_2^4(t_1,t_2,s_1,s_2)}+\frac{2v(1-4v^2t_1t_2)F_3^{(V)}(s_1,s_2)}{\Phi_2^3(t_1,t_2,s_1,s_2)}\right]+\exp\left\{\frac{M^2}{2v\Lambda^2}\Phi(t_1,t_2,s_1,1)\right\}\frac{2v}{\Phi_2^2(t_1,t_2,s_1,1)}\right)+O(m_u). \tag{41}$$

This difference appears owing to the factors $F_2^{(P)}$ and $F_2^{(V)}$ in Eq. (39) and leads to the inequality

$$|\tilde{\Pi}_\pi(-M^2;m_u,m_u;\Lambda)|\gg|\tilde{\Pi}_\rho(-M^2;m_u,m_u;\Lambda)|.$$

This relation shows that the masses of pion and ρ meson, satisfying Eq. (29), are strongly split and $M_\rho^2\gg M_\pi^2$ if the quark mass goes to zero. A similar picture takes place for K and K^* mesons, but since the strange quark mass is not so small, the effect is more smooth.

The above consideration is illustrated in Table II. The pion mass is much larger, and the difference in the masses of pseudoscalar and vector mesons is smaller, if the spin-field interaction in the quark propagator is eliminated (compare the first and the last lines in the table).

Thus, we can conclude that the large splitting between the masses of P and V mesons is explained in our case by the spin-field interaction. This splitting is the reason why Eqs. (32) have an appropriate solution (36).

It should be noted that the scalar polarization function $\tilde{\Pi}_S$ differs from the pseudoscalar $\tilde{\Pi}_P$ only by the sign before $m_fm_{f'}$ in Eq. (37). Because of the above-mentioned singularity, the term proportional to $m_fm_{f'}$ is leading, and $\tilde{\Pi}_S$ is positive for a wide range of parameter values. As a result, Eq. (29) has no real solutions for the case of scalar mesons. It looks interesting that the scalar $(q\bar{q})$ bound states do not appear due to the same spin-field interaction that diminishes the pion mass and provides a nonzero quark condensate.

Consideration of the $SU_F(3)$ singlet and the eighth octet states shows an ideal mixing both for vector and pseudoscalar mesons. The masses of ω and ϕ, calculated with the parameter values (36), are

$$M_\omega=M_\rho=770\ \mathrm{MeV},\quad M_\phi=1034\ \mathrm{MeV},$$

which is in good agreement with the experimental values. The ideal mixing of the pseudoscalar states is not the case that can provide an appropriate description of η and η' mesons. It is well known that the problem of η and η' masses can be solved by taking into account another Euclidean gluon configuration, the instanton vacuum field [13]. The instantons can be incorporated into our formalism without any principal problems, and we hope to realize this idea in forthcoming publications.

Now let us consider the weak decays of π and K mesons. In the lowest approximation, amplitude of the decay $P\to l\bar{\nu}$

TABLE I. Parameters of the model.

m_u (MeV)	m_d (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	Λ (MeV)	g
198.3	198.3	413	1650	4840	319.5	9.96

is given by the diagram in Fig 2(c). The weak decay constant f_P is defined by the standard formulas

$$A_{P\to l\bar{\nu}}(k,k')=i\frac{G_F}{\sqrt{2}}\mathcal{K}h_P F(k^2)\Phi_P(k)k_\mu\overline{l(k')}\times(1-\gamma_5)\gamma_\mu\nu(k+k'),$$

$$f_P=h_P F(-M^2), \tag{42}$$

where the meson-quark coupling constant h_P is calculated via Eq. (28), and $\mathcal{K}$ is the KKM matrix element corresponding to a given meson. For an arbitrary pseudoscalar meson the diagram in Fig. 2(c) gives the following expression for f_P:

$$f_P=h_P\frac{1}{4\pi^2}\mathrm{Tr}_v\int\int\int_0^1\frac{dtds_1ds_2(1+s_1s_2)}{[2vt(1+s_1s_2)+s_1+s_2]^3}\times\left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4v\Lambda^2}\left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4v\Lambda^2}\times\left[m_f\frac{s_1+2vt[1-\xi_1(1+s_1^2)]}{1-s_1^2}+m_{f'}\frac{s_2+2vt[1-\xi_2(1+s_2^2)]}{1-s_2^2}\right]\times\exp\left\{\frac{M^2}{2v\Lambda^2}\Psi(t,s_1,s_2)\right\},$$

$$\Psi=\frac{s_1s_2+2vt(s_1\xi_1^2+s_2\xi_2^2)}{2vt(1+s_1s_2)+s_1+s_2}. \tag{43}$$

The singularity of the integrand of Eq. (43) at $s_1\to1$ and $s_2\to1$ appears from the above-mentioned spin-field interaction in the quark propagator and plays the main role in regulating the value of f_P for the light mesons. Calculation of pion and kaon decay constants by means of Eq. (43) with the values of the parameters (36) gives

$$f_\pi=126\ \text{MeV},\quad f_K=145\ \text{MeV}.$$

Note that the coupling constants h_π and h_K depend on the meson mass, quark masses, and parameter Λ [see Eq. (28) and Table II].

One could get a definite impression, that a simultaneous description of the masses of π, K, ρ, K^*, ω, and ϕ mesons and quite accurate values of f_π and f_K are obtained mostly due to the breakdown of chiral symmetry by the spin-field interaction [see also Eq. (7)].

In order to clarify the status of this impression, one needs to investigate the chiral limit of the model. Whether or not and in what form the Goldberger-Treiman and Oakes-Reiner-Treiman relations are fulfilled in this limit? Naively, it seems that the chiral limit corresponds to the case $m_u\ll\Lambda$. However, the situation is much more complicated.

Just for illustration of this statement, consider the pion mass in the limit $m_u\ll\Lambda$. The integrals in Eq. (40) can be evaluated, and we get the following asymptotic form of Eq. (29) for the pion mass:

$$1-g^2\frac{16\Lambda^6}{9\pi^2m_u^2M_\pi^4}\mathrm{Tr}_v v^2\left[\exp\left\{\frac{M_\pi^2}{8v\Lambda^2}\right\}-\exp\left\{\frac{M_\pi^2}{8v(1+2v)\Lambda^2}\right\}\right]^2=0. \tag{44}$$

For any values of g there is a real positive or negative solution M_π^2 to Eq. (44). The pion mass is equal to zero if the values of m_u, g, and Λ satisfy the relation

$$\frac{m_u^2}{\Lambda^2}=\frac{g^2}{9\pi^2}\mathrm{Tr}_v\frac{v^2}{(1+2v)^2}\approx\left(\frac{2g}{15\pi}\right)^2.$$

For a fixed value of g and $m_u/\Lambda\to0$, the solution M_π to Eq. (44) is purely imaginary and behaves as

$$\frac{M_\pi}{\Lambda}=i\frac{4}{3}\sqrt{7\ln\frac{\Lambda}{m_u}}+O\left[\frac{\ln\ln(\Lambda/m_u)}{\sqrt{\ln(\Lambda/m_u)}}\right],$$

which has no physically reasonable interpretation, but just indicates that the limit $g=$const and $m_u/\Lambda\to0$ is ill defined.

From our point of view, a correct transition to the chiral limit has to be based on a simultaneous changing of m_u, g, and Λ as functions of an actual physical parameter like the temperature or particle density. The quark masses, coupling constant, and the vacuum field strength as the functions of this parameter have to be extracted from consideration of QCD dynamics at nonzero temperature and density. Unfortunately, this is a quite complicated problem, and we leave it for further investigations.

The main result of this subsection is very simple: We have demonstrated by explicit model calculations that the spin-field interaction contained in the quark propagator [in presence of the homogeneous (anti-)self-dual vacuum gluon field] can be responsible for the observable masses of the light pseudoscalar and vector mesons (with the exception of η, η'), and for the values of the weak decay constants of pions and kaons. All numerical results are given in Tables I and II.

B. Regge trajectories

It has been shown in our previous paper [8] that the spectrum of radial and orbital excitations of the light mesons is asymptotically equidistant:

TABLE II. The masses (MeV), weak decay constants (MeV), and meson-quark coupling constants h of the light mesons. M^*, calculation without taking into account the spin-field interaction.

Meson	π	ρ	K	K^*	ω	ϕ
M	140	770	496	890	770	1034
M^{expt}	140	770	496	890	786	1020
f_P	126	-	145	-	-	-
f_P^{expt}	132	-	157	-	-	-
h	6.51	4.16	7.25	4.48	4.16	4.94
M^*	630	864	743	970	864	1087

$$M^2_{aJ\ell n}=\frac{8}{3}\ln\left(\frac{5}{2}\right)\Lambda^2 n+O(\ln n) \quad \text{for } n\gg\ell, \tag{45}$$

$$M^2_{aJ\ell n}=\frac{4}{3}\ln 5\Lambda^2\ell+O(\ln\ell) \quad \text{for } \ell\gg n. \tag{46}$$

Technically this result is based on the exponential behavior of the quark propagator, Eq. (8), and vertex function $F_{n\ell}$, Eq. (22), in the Minkowski region ($p^2/\Lambda^2\to-\infty$) like

$$\begin{aligned}\tilde{H}_f(p|B)&\to O\left(\exp\left\{\frac{|p^2|}{2v\Lambda^2}\right\}\right),\\ F_{n\ell}(p^2)&\to O\left(\exp\left\{\frac{|p^2|}{4\Lambda^2}\right\}\right),\end{aligned} \tag{47}$$

and on the specific dependence of the coupling constant $G_{J\ell n}$ [see Eq. (19)] on the orbital and radial quantum numbers ℓ and n arising from the decomposition of the bilocal quark currents over the generalized Lagguerre polynomials, which is determined in its order by the form of gluon propagator (9) (for details see Sec. II D). In general the Regge character of the spectrum is determined in our model by the confining properties of the vacuum field.

Numerical calculation of masses of the first orbital excitations of π, K, ρ, and K^* mesons by means of Eq. (29) with the parameters (36) gives the masses shown in Table III. The superfine structure of the excited states of ρ and K^* mesons coming from classification of currents over total momentum [Eq. (23)] is qualitatively correct. Superfine splitting of the levels with $\ell=1$ is not very large.

C. Heavy quarkonia

Exponential behavior of the quark propagator and vertices (47) is responsible for the following relation between the masses of heavy quarkonia $M_{Q\bar{Q}}$ and heavy quark m_Q in the leading approximation [8]:

TABLE III. The masses (MeV) of orbital excitations of π, K, ρ, and K^* mesons. The superfine structure of the $\ell=1$ excitation of ρ and K^* is shown [ℓ is the orbital momentum and j is the total momentum (an observable spin) of a state].

Meson	ℓ	j	M	M^{exp}
π	0	0	140	140
b_1	1	1	1252	1235
K	0	0	496	496
$K_1(1270)$	1	1	1263	1270
ρ	0	1	770	770
	1	0	1238	
a_1	1	1	1311	1260
a_2	1	2	1364	1320
K^*	0	1	890	890
	1	0	1274	
$K_1(1400)$	1	1	1342	1400
K_2^*	1	2	1388	1430

$$M_{Q\bar{Q}}=2m_Q \quad \text{for } m_Q\gg\Lambda.$$

Now let us calculate the next-to-leading term in the mass formula. In other words, we have to solve Eq. (29) with the polarization function $\tilde{\Pi}_J$ defined by Eq. (37) with

$$m_f=m_{f'}=m_Q\gg\Lambda, \quad M_{Q\bar{Q}}=2m_Q-\Delta_{Q\bar{Q}} \tag{48}$$

in the next-to-leading approximation over $1/m_Q$. Since the masses of quarks are equal to each other, we have [see Eq. (13)]

$$\xi_1=\xi_2=1/2,$$

which means that the composite quarkonium field $\Phi_{Q\bar{Q}}(x)$ is localized at the center of masses of two heavy quarks (in Euclidean four-dimensional space). It is convenient to transform the variables s_1 and s_2 in Eq. (37):

$$r_1=(s_1+s_2)/\sqrt{2}, \quad r_2=(s_1-s_2)/\sqrt{2}. \tag{49}$$

The term with $F_3^{(J)}$ in Eq. (37) does not contribute to the leading and next-to-leading behavior of the integral and can be omitted. After the transformation we arrive at the expression

$$\begin{aligned}\tilde{\Pi}_J(-M^2;m_Q,m_Q;\Lambda)=&-\frac{m_Q^2}{4\pi^2}\mathrm{Tr}_v\int\int_0^1 dt_1dt_2\left(\int_0^{1/\sqrt{2}}dr_1\int_{-r_1}^{r_1}dr_2+\int_{1/\sqrt{2}}^{\sqrt{2}}dr_1\int_{-\sqrt{2}+r_1}^{\sqrt{2}-r_1}dr_2\right)\left[\frac{R_1^{(J)}(t_1,t_2,r_1,r_2)}{\varphi_2^4(t_1,t_2,r_1,r_2)}\right.\\&\left.+\frac{R_2^{(J)}(r_1,r_2)}{[2-(r_1-r_2)^2][1-(r_1+r_2)^2]\varphi_2^2(t_1,t_2,r_1,r_2)}\right]\exp\left\{\frac{M^2}{4v\Lambda^2}\varphi(t_1,t_2,r_1,r_2)\right\}+O\left(\frac{\Lambda}{m_Q}\right),\end{aligned} \tag{50}$$

where

$$\varphi=\frac{\varphi_1(t_1,t_2,r_1,r_2)}{\varphi_2(t_1,t_2,r_1,r_2)}-\frac{m_Q^2}{M^2}\ln\frac{(\sqrt{2}+r_1)^2-r_2^2}{(\sqrt{2}-r_1^2)^2-r_2^2}, \quad (51)$$

$$\varphi_1=\sqrt{2}v(t_1+t_2)r_1+r_1^2-r_2^2,$$

$$\varphi_2=v(t_1+t_2)(2+r_1^2-r_2^2)+\sqrt{2}(1+4v^2t_1t_2)r_1,$$

$$R_1^{(P)}=\frac{1}{4}(2+r_1^2-r_2^2)[A_1A_2+v^2(t_1-t_2)^2(r_1^2-r_2^2)],$$

$$R_1^{(V)}=\frac{1}{12}[(6-r_1^2+r_2^2)A_1A_2+v^2(t_1-t_2)^2(r_1^2-r_2^2)\times(1-3r_1^2+3r_2^2)],$$

$$A_1=r_1-r_2+\sqrt{2}v(t_1+t_2),\quad A_2=r_1+r_2+\sqrt{2}v(t_1+t_2),$$

$$R_2^{(P)}=(2+r_1^2-r_2^2)^2,\quad R_2^{(V)}=4-(r_1^2+r_2^2)^2.$$

The asymptotic value of the integral over r_2 in the limit $M\gg\Lambda$ can be evaluated by the Laplace method. One can check that the function φ has a maximum at the point $r_2=0$ for any values of r_1, t_1, t_2:

$$\frac{\partial}{\partial r_2}\varphi(t_1,t_2,r_1,r_2)|_{r_2=0}=0,$$

$$\frac{\partial^2}{\partial r_2^2}\varphi(t_1,t_2,r_1,r_2)|_{r_2=0}=-\frac{m_Q^2}{M^2}\frac{8\sqrt{2}r_1}{(2-r_1^2)^2}-\frac{4v(t_1+t_2)+\sqrt{2}r_1(1-v^2(t_1-t_2)^2)}{\varphi_2(t_1,t_2,r_1,0)}<0,$$

$$\varphi(t_1,t_2,r_1,0)=\frac{r_1[r_1+2\sqrt{2}v(t_1+t_2)]}{\varphi_2(t_1,t_2,r_1,0)}-\frac{2m_Q^2}{M^2}\ln\frac{\sqrt{2}+r_1}{\sqrt{2}-r_1}, \quad (52)$$

which means that the leading terms can be obtained by evaluating the Gaussian integral over r_2. Furthermore, one can see that the largest value of the function $\varphi(t_1,t_2,r_1,0)$ in the interval $r_1\in[0,\sqrt{2}]$ corresponds to $r_1=0$ for any t_1, t_2; moreover,

TABLE IV. The spectrum of charmonium.

Meson	η_c	J/ψ	χ_{c_0}	χ_{c_1}	χ_{c_2}	ψ'	ψ''
n	0	0	0	0	0	1	2
ℓ	0	0	1	1	1	0	0
j	0	1	0	1	2	1	1
M (MeV)	3000	3161	3452	3529	3531	3817	4120
$M^{\rm expt}$ (MeV)	2980	3096	3415	3510	3556	3770	4040

$$\frac{\partial}{\partial r_1}\varphi(t_1,t_2,r_1,0)|_{r_1=0}=-\frac{1}{\sqrt{2}}\left(\frac{4m_Q^2}{M^2}-1\right)<0, \quad (53)$$

and in the leading approximation the integrand is reduced to an exponential function in r_1. Using Eqs. (52) and (53), and taking into account Eq. (48), one can integrate over r_2 and r_1 with the result

$$\tilde{\Pi}_J(-M^2;m_Q,m_Q;\Lambda)|=-\frac{3\Lambda^3}{4\pi\sqrt{\pi}\Delta_{Q\bar{Q}}}\int\int_0^1\frac{dt_1dt_2}{\sqrt{t_1+t_2}}+O\left(\frac{\Lambda}{m_Q}\right). \quad (54)$$

Integrating over t_1 and t_2 in Eq. (54) and substituting the result to Eq. (29), one can find

$$\frac{\Delta_{Q\bar{Q}}^{(J)}}{\Lambda}=\frac{2(\sqrt{2}-1)}{\pi\sqrt{\pi}}C_Jg^2+O\left(\frac{\Lambda}{m_Q}\right), \quad (55)$$

where $C_P=1/9$, $C_V=1/18$ [see Eq. (19)]. It should be stressed that the difference in the constants,

$$\Delta_{Q\bar{Q}}^{(P)}=2\Delta_{Q\bar{Q}}^{(V)}, \quad (56)$$

originates from the Fierz transformation of the Dirac matrices in the interaction term L_2 in representation (2). Relation (56) means that the vector quarkonium state is always heavier than the pseudoscalar one.

The results of numerical calculation of the masses of different heavy quarkonia states are summarized in Tables IV and V. The parameters Λ and g are equal to the values (36) fitting the light meson masses, and $m_c=1650$ MeV, $m_b=4840$ MeV. The agreement with the experimental values is rather satisfactory. The superfine splitting (χ_{c0}, χ_{c1}, χ_{c2}, and so on) is very small, since it is regulated by the terms $O(1/m_Q)$ in Eq. (29). Its description is qualitatively correct. The splitting is generated in our model by dividing the quark currents with γ_μ and $\ell>0$ into antisymmetric,

TABLE V. The spectrum of bottomonium.

Meson	Υ	χ_{b_0}	χ_{b_1}	χ_{b_2}	Υ'	χ'_{b_0}	χ'_{b_1}	χ'_{b_2}	Υ''
n	0	0	0	0	1	1	1	1	2
ℓ	0	1	1	1	0	1	1	1	0
j	1	0	1	2	1	0	1	2	1
M (MeV)	9490	9767	9780	9780	10052	10212	10215	10215	10292
$M^{\rm expt}$ (MeV)	9460	9860	9892	9913	10230	10235	10255	10269	10355

symmetric traceless, and diagonal parts [see Eq. (23)], which extracts the states with different total angular momenta, mixed in the currents $\overline{q}M\gamma_\alpha T^{(\ell)}_{\mu_1\cdots\mu_\ell}F_{n\ell}q$.

We conclude that the correct description of heavy quarkonia in our model is provided by the specific form of the nonlocality of the quark and gluon propagators induced by the vacuum field, localization of the meson field at the center of masses of constituent quarks, and by a separation of the nonlocal currents with different total momentum. In general, the spectrum is driven by the rigid asymptotic formulas (48) and (55).

D. Heavy-light mesons

Another interesting sector of the meson spectrum is heavy-light mesons, characterized by a rich physics [14,15]. In this subsection we will consider the masses and weak decay constants of heavy-light mesons. First of all, let us obtain the asymptotic formulas in the limit of infinitely heavy quark. Namely, we have to investigate the behavior of the polarization function $\widetilde{\Pi}_J(-M;m_Q,m_q;\Lambda)$ [Eq. (37)] and the weak decay constant f_P [Eq. (43)] in the case

$$m_f=m_Q\gg\Lambda,\quad m_{f'}=m_q=O(\Lambda),$$
$$\xi_f=\frac{m_Q}{m_Q+m_q}=1+O(m_q/m_Q),$$
$$\xi_{f'}=\frac{m_q}{m_Q+m_q}=O(m_q/m_Q). \tag{57}$$

Equations (57) indicate that in the heavy quark limit the composite meson field $\Phi_{Q\overline{q}}(x)$ is localized at the point in which the heavy quark Q is situated.

Let us show that in the limit (57) the leading and next-to-leading terms of the solution to Eq. (29) read

$$M_{Q\overline{q}}=m_Q+\Delta^{(J)}_{Q\overline{q}}+O(1/m_Q), \tag{58}$$

where the next-to-leading term $\Delta^{(J)}_{Q\overline{q}}$ does not depend on the heavy quark mass m_Q. This term is a function of a light quark mass m_q and coupling constant G_{J00} [see Eq. (19)].

Omitting the term with $F^{(J)}_3$, which does not contribute to the leading and next-to-leading behavior of the integral, and taking into account conditions (57), one can rewrite Eq. (37) in the form

$$\widetilde{\Pi}_J(-M^2;m_Q,m_q;\Lambda)=-\frac{1}{4\pi^2}\mathrm{Tr}_v\int\int\int\int_0^1 dt_1dt_2ds_1ds_2\left(\frac{1-s_2}{1+s_2}\right)^{m_q^2/4v\Lambda^2}\left[\frac{(1-4v^2t_1t_2)Y(t_1,t_2,s_2)T^{(J)}_1(s_1,s_2)s_1M^2}{[s_1X(t_1,t_2,s_2)+Y(t_1,t_2,s_2)]^4}\right.$$
$$\left.+\frac{T^{(J)}_2(s_1,s_2)m_Qm_q}{(1-s_1^2)(1-s_2^2)[s_1X(t_1,t_2,s_2)+Y(t_1,t_2,s_2)]^2}\right]\exp\left\{\frac{M^2}{2v\Lambda^2}\phi(t_1,t_2,s_1,s_2)\right\}+O\left(\frac{\Lambda}{m_Q}\right), \tag{59}$$

where

$$\phi=\frac{s_1Y(t_1,t_2,s_2)}{s_1X(t_1,t_2,s_2)+Y(t_1,t_2,s_2)}-\frac{m_Q^2}{2M^2}\ln\frac{1+s_1}{1-s_1},$$
$$X=1+4v^2t_1t_2+2v(t_1+t_2)s_2,$$
$$Y=2v(t_1+t_2)+(1+4v^2t_1t_2)s_2,$$
$$T^{(P)}_1=1+s_1s_2,\quad T^{(V)}_1=\frac{1}{3}(3-s_1s_2),$$
$$T^{(P)}_2=(1+s_1s_2)^2,\quad T^{(V)}_2=1-s_1^2s_2^2.$$

One can check that for any t_1, t_2, and s_2 the function $\phi(t_1,t_2,s_1,s_2)$ has a maximum at $s_1=s_1^{\max}$:

$$s_1^{\max}=\frac{Y(t_1,t_2,s_2)}{2X(t_1,t_2,s_2)}\left(1-\frac{m_Q^2}{M^2}\right)+O\left(\frac{\Lambda^2}{m_Q^2}\right),$$
$$\phi(t_1,t_2,s_1^{\max},s_2)=\frac{Y(t_1,t_2,s_2)}{2X(t_1,t_2,s_2)}\left(1-\frac{m_Q^2}{M^2}\right)^2+O\left(\frac{\Lambda^3}{m_Q^3}\right),$$

$$\frac{\partial^2}{\partial s_1^2}\phi(t_1,t_2,s_1,s_2)|_{s_1=s_1^{\max}}=-\frac{2X(t_1,t_2,s_2)}{Y(t_1,t_2,s_2)}\left(1-\frac{m_Q^2}{M^2}\right)^2+O\left(\frac{\Lambda^3}{m_Q^3}\right). \tag{60}$$

Therefore, we can write

$$\widetilde{\Pi}_J(-M^2;m_Q,m_q;\Lambda)=-\frac{1}{4\pi^2}\mathrm{Tr}_v\int\int\int_0^1 dt_1dt_2ds_2\left(\frac{1-s_2}{1+s_2}\right)^{m_q^2/4v\Lambda^2}$$
$$\times\exp\left\{\frac{M^2}{4v\Lambda^2}\left(1-\frac{m_Q^2}{M^2}\right)^2\frac{Y(t_1t_2,s_2)}{X(t_1,t_2,s_2)}\right\}\left[\frac{(1-4v^2t_1t_2)s_1^{\max}M^2}{Y^3(t_1,t_2,s_2)}\right.$$
$$\left.+\frac{m_Qm_q}{(1-s_2^2)Y^2(t_1,t_2,s_2)}\right]\int_{-\infty}^{\infty}ds_1\exp\left\{-\frac{M^2X(t_1,t_2,s_2)}{2v\Lambda^2Y(t_1,t_2,s_2)}s_1^2\right\}+O\left(\frac{\Lambda}{m_Q}\right). \tag{61}$$

Integrating out the variable s_1 in Eq. (61), substituting $M=m_Q+\Delta_{Q\bar{q}}^{(J)}$ to the resulting expression, and using Eq. (29), we arrive at the equation

$$1=g^2C_J\frac{1}{(2\pi)^{3/2}}\mathrm{Tr}_v\sqrt{v}\int\int\int_0^1\frac{dt_1dt_2ds_2[(1-s_2)/(1+s_2)]^{m_q^2/4v\Lambda^2}}{[X(t_1,t_2,s_2)Y(t_1,t_2,s_2)]^{3/2}}\left[\frac{\Delta_{Q\bar{q}}^{(J)}}{\Lambda}+\frac{X(t_1,t_2,s_2)}{1-s_2^2}\frac{m_q}{\Lambda}\right]$$
$$\times\exp\left\{\frac{[\Delta_{Q\bar{q}}^{(J)}]^2}{v\Lambda^2}\frac{Y(t_1t_2,s_2)}{X(t_1,t_2,s_2)}\right\}+O\left(\frac{\Lambda}{m_Q}\right). \tag{62}$$

Equation (62) describes dependence of $\Delta_{Q\bar{q}}^{(J)}$ in the mass formula (58) on the coupling constant g, the light quark mass m_q, and vacuum field strength B (Λ). There is a single real solution to Eq. (62) for any positive g, m_q, and Λ. In particular, for the values (36) we get

$$\Delta_{Q\bar{u}}^{(P)}=20\ \text{MeV},\quad \Delta_{Q\bar{u}}^{(V)}=155\ \text{MeV},$$
$$\Delta_{Q\bar{s}}^{(P)}=63\ \text{MeV},\quad \Delta_{Q\bar{s}}^{(V)}=191\ \text{MeV}.$$

As is seen from Eq. (62), the difference between the pseudoscalar $\Delta_{Q\bar{q}}^{(P)}$ and vector $\Delta_{Q\bar{q}}^{(V)}$ is due to the constant C_J, which appears from the Fierz transformation of the Dirac matrices. This is the same situation as in the case of heavy quarkonia [see Eq. (55)].

Table VI demonstrates the reasonably good agreement between the experimental data and the masses of the heavy-light mesons calculated by means of Eq. (29) with the parameters (36). The masses of heavy quarks are the same as in the description of the heavy quarkonia (see Table I).

Now let us turn to the calculation of the weak decay constant for the pseudoscalar heavy-light mesons. Under conditions (57), the integral over s_1 in Eq. (43) can be evaluated by the Laplace method. The result is

$$f_P=h_P\frac{\Lambda^2}{m_Q}\mathcal{A}_f\left(\frac{\Delta_{Q\bar{q}}^{(P)}}{\Lambda},\frac{m_q}{\Lambda}\right), \tag{63}$$

where

$$\mathcal{A}_f=\mathrm{Tr}_v\frac{\sqrt{v}}{(2\pi)^{3/2}}\int\int_0^1\frac{dtds_2[(1-s_2)/(1+s_2)]^{m_q^2/4v\Lambda^2}}{[s_2+2vt]^{3/2}[1+2vts_2]^{3/2}}$$
$$\times\left[\frac{\Delta_{Q\bar{q}}^{(P)}}{\Lambda}+\frac{m_q}{\Lambda}\frac{1+2vts_2}{1-s_2^2}\right]\exp\left\{\frac{[\Delta_{Q\bar{q}}^{(P)}]^2}{v\Lambda^2}\frac{s_2+2vt}{1+2vts_2}\right\}$$
$$+O\left(\frac{\Lambda}{m_Q}\right), \tag{64}$$

and the difference $\Delta_{Q\bar{q}}^{(P)}$ between the masses of heavy quark and heavy-light meson is given by Eq. (62). The procedure for obtaining Eqs. (63) and (64) is very similar to the calculations providing Eq. (62). To get the final formula for f_P, the asymptotic form of the meson-quark coupling constant h_P has to be defined in the case (57). Performing calculations analogous to those which lead to Eq. (62), we arrive at

$$h_P=\sqrt{\frac{m_Q}{\Lambda}}\mathcal{A}_h^{-1}\left(\frac{\Delta_{Q\bar{q}}^{(P)}}{\Lambda},\frac{m_q}{\Lambda}\right),$$

$$\mathcal{A}_h^2=\frac{\Delta_{Q\bar{q}}^{(P)}}{2(2\pi)^{3/2}\Lambda}\mathrm{Tr}_v\frac{1}{\sqrt{v}}\int\int\int_0^1\frac{dt_1dt_2ds_2[(1-s_2)/(1+s_2)]^{m_q^2/4v\Lambda^2}}{[X(t_1,t_2,s_2)Y(t_1,t_2,s_2)]^{3/2}}\left[\frac{\Delta_{Q\bar{q}}^{(J)}}{\Lambda}+\frac{X(t_1,t_2,s_2)}{1-s_2^2}\frac{m_q}{\Lambda}\right]$$
$$\times\exp\left\{\frac{[\Delta_{Q\bar{q}}^{(J)}]^2}{v\Lambda^2}\frac{Y(t_1,t_2,s_2)}{X(t_1,t_2,s_2)}\right\}+O\left(\frac{\Lambda}{m_Q}\right). \tag{65}$$

One can see that Eqs. (63)–(65) give the following asymptotic relation in the heavy quark limit (57):

$$f_P=\frac{\Lambda^{3/2}}{\sqrt{m_Q}}\frac{\mathcal{A}_f}{\mathcal{A}_h}, \tag{66}$$

where $\mathcal{A}_f$ and $\mathcal{A}_h$ do not depend on the heavy quark mass in the leading approximation over Λ/m_Q, as is indicated in Eqs. (64), and (65). Relation (66) agrees with the accepted notion about the behavior of the weak decay constants of the heavy-light mesons [15]. Results of the numerical calculation of the weak decay constants for different pseudoscalar mesons are given in Table VI.

IV. DISCUSSION

In conclusion, we would like to point out several problems that require more profound studying.

TABLE VI. The masses and weak decay constants (MeV) of heavy-light mesons.

Meson	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*
M	1766	1991	1910	2142	4965	5143	5092	5292
$M^{\rm expt}$	1869	2010	1969	2110	5278	5324	5375	5422
f_P	149	-	177	-	123	-	150	-

We have assumed from the very beginning that the nonperturbative QCD vacuum is characterized by a nonzero background (anti-)self-dual homogeneous field. In other words, the minimum of the QCD effective potential (the free energy density) for this gluon configuration is assumed to be at nonzero field strength. Different estimations of the effective potential indicate that this situation can be realized (see [3] and numerous references therein). Although these estimations cannot be used as a basis for a more or less rigorous proof, they underline the key role of the gluon self-interaction in forming the effective potential for a homogeneous gluon field. Just the self-interaction of gauge bosons is the distinctive feature of non-Abelian theories such as QCD. Gluon-gluon coupling is manifested also in the nontrivial form of the gluon propagator (9). Although the background field is quasi-Abelian, the non-Abelian nature of the gluon field plays the crucial role for the model under consideration.

In order to clarify the basic assumption of this paper, one needs to get a reliable nonperturbative estimation of the free energy density or effective potential of QCD for the background field under consideration. Lattice calculations seem to be the most promising approach to this problem.

In this paper we have demonstrated how the singularity $1/m_f$ of the quark propagator affects the masses and weak decay constants of light mesons. However, more detailed consideration of chiral symmetry breaking by the background field is needed. This can be achieved by investigating the Dirac equation in the presence of the homogeneous (anti-)self-dual field.

Another source of chiral symmetry violation is the effective four-quark interaction. This is the main idea of the NJL-type models. The vacuum gluon field does not remove the mechanism of the NJL type. The divergent diagram in Fig. 1 should play the key role in studying this mechanism of symmetry breaking. Taking this into account one can rewrite representation (25) in the form

$$Z=e^{-\Omega E(\varphi_0|\Lambda,g,m)}\int d\sigma_{\rm vac}\int Dq D\overline{q}$$
$$\times\exp\left\{-\int\int d^4x d^4y \overline{q}(x)\mathcal{S}^{-1}(x,y|B)q(y)\right.$$
$$\left.+\sum_{\mathcal{N}}\frac{1}{2\Lambda^2}G_{\mathcal{N}}^2\int d^4x[\mathcal{I}_{\mathcal{N}}(x)-{\rm Tr}V_{\mathcal{N}}\mathcal{S}]^2\right\}.$$

Here Ω is the Euclidean volume. The approximated free energy density $E(\varphi_0|\Lambda,g,m)$ depends on the vacuum expectation value φ_0 of the scalar nonlocal quark current ($N_F=2$ for simplicity). The energy density is ultraviolet finite and can be calculated. It is normalized as $E(0|\Lambda,g,m)=0$. The modified quark propagator $\mathcal{S}(x,y|B,\varphi_0)$ depends on φ_0 and satisfies the equation

$$\left[i\gamma_\mu\hat{\nabla}_\mu-m-G_{S00}\varphi_0F_{00}\left(4\frac{\vec{\nabla}^2}{\Lambda^2}\right)\right]\mathcal{S}(x,y|B,\varphi_0)$$
$$=-\delta(x-y),$$

where G_{S00} and F_{00} are given by Eqs. (19) and (22). The constant vacuum expectation value φ_0 is a solution to the equation

$$\varphi_0^2\left[1+\left(\frac{G_{S00}}{\Lambda}\right)^2\widetilde{\mathcal{R}}(0|\varphi_0;\Lambda,g,m)\right]=0,$$

where $\widetilde{\mathcal{R}}(p^2|\varphi_0;\Lambda,g,m)$ is the Fourier transform of the function

$$\mathcal{R}(x-y|\varphi_0;\Lambda,g,m)=\int d\sigma_{\rm vac}{\rm Tr}F_{00}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\right)S(x,y|B)$$
$$\times F_{00}\left(\frac{\overleftrightarrow{\nabla}^2(y)}{\Lambda^2}\right)\mathcal{S}(y,x|B,\varphi_0).$$

These formulas are free from ultraviolet divergences. There exist two possible phases with $\varphi_0\equiv 0$ and $\varphi_0\neq 0$. The phase with zero φ_0 corresponds to the case investigated in this paper. Another phase with $\varphi_0\neq 0$ concerns symmetry breaking due to the four-quark interaction.

By means of this representation the interplay of both mechanisms of chiral symmetry breaking can be studied. This problem is under our consideration now, but is not yet finished. In any case, the pure effect of symmetry breaking by the vacuum field should be investigated separately. The study of the problem including both mechanisms is technically difficult, and, at least, an attempt to solve all the problems in one paper is unreliable. It is better to solve them one by one.

One can expect that an additional breakdown of chiral symmetry by the four-quark interaction could diminish the quark masses. In view of this, the large values of u-, d-, and s-quark masses (see Table I) should be considered as the question for further investigations rather than the argument against the physical effects produced by the vacuum field under consideration.

One can see that the coupling constant g in Table I is rather large. A possible origin of this unpleasant feature could be covered in the elimination of some terms of the gluon propagator (9) (for more details see [8]). In other words, some truncations in the gluon propagator were compensated for by the rising of the coupling constant. This point also has to be investigated carefully.

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