Width difference in the B_s-$\overline{B_s}$ system

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We use the heavy quark expansion to investigate the width difference $\Delta\Gamma_{B_s}$ between the B_s mass eigenstates. The corrections of $O(\Lambda_{\rm QCD}/m_b)$ and $O(m_s/m_b)$ to the leading-order expression in the operator product expansion are derived and estimated to yield a sizable reduction of the leading result for $\Delta\Gamma_{B_s}$ by typically 30%. For completeness we also quantify small effects due to penguin operators and CKM-suppressed contributions. Based on our results we discuss the prediction for $(\Delta\Gamma/\Gamma)_{B_s}$ with particular emphasis on theoretical uncertainties. We find $(\Delta\Gamma/\Gamma)_{B_s}=0.16^{+0.11}_{-0.09}$, where the large error is dominated by the uncertainty in hadronic matrix elements. An accuracy of about 10% in $(\Delta\Gamma/\Gamma)_{B_s}$ should be within reach, assuming continuing progress in lattice calculations. In addition, we address phenomenological issues and implications of a $\Delta\Gamma_{B_s}$ measurement for constraints on ΔM_{B_s} and CKM parameters. We further consider in some detail the lifetime ratio $\tau(B_s)/\tau(B_d)$ and estimate that, most likely, $|\tau(B_s)/\tau(B_d)-1|<1\%$. [S0556-2821(96)01519-6]

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I. INTRODUCTION

Mixing phenomena in neutral B meson systems provide an important testing ground for standard model flavordynamics. The mass difference between the B_d eigenstates, ΔM_{B_d}, gave the first evidence for a large top-quark mass and provides a valuable constraint on $|V_{td}|$ and the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle. A direct measurement of ΔM_{B_s}, the corresponding quantity for B_s mesons, through B_s-$\overline{B_s}$ oscillations, would yield further information and help to reduce hadronic uncertainties in the extraction of CKM parameters. Complementary insight can be gained from the width difference $\Delta\Gamma_{B_s}$ between the B_s mass eigenstates [1,2]. This width difference is expected to be the largest among bottom hadrons [3], and it may be large enough to be accessible by experiment in the near future. The width difference for B_d mesons, on the other hand, is CKM suppressed and experimentally much harder to determine.

If $\Delta\Gamma_{B_s}$ is indeed found to be sizable, the observation of CP violation and the extraction of CKM phases from untagged B_s data samples can be contemplated [1,4,5]. This possibility could be important in two respects. First, tagging any B_s data sample costs in statistics and in purity. Second, the rapid oscillations dependent on $\Delta M_{B_s}t$ all cancel in time evolutions of untagged B_s data samples, which are governed by the two exponentials $\exp(-\Gamma_L t)$ and $\exp(-\Gamma_H t)$ alone.

The present article continues previous work of one of us [1] on the phenomenological potential of $\Delta\Gamma_{B_s}$, and focuses on theoretical uncertainties and improvements of the prediction. We compute the width difference in the heavy quark expansion and include explicit $1/m_b$ corrections, which improves over previous estimates of $\Delta\Gamma_{B_s}$ based on a partonic [6–11] or exclusive [12] approach and allows us to assess the remaining uncertainties more reliably. Combined with future measurements of $\Delta\Gamma_{B_s}$, these predictions can be used to derive indirect constraints on $|V_{ts}/V_{td}|$ [2] and ΔM_{B_s}. Non-standard-model sources of CP violation in the B_s system would reduce $\Delta\Gamma_{B_s}$ compared to its standard model value, as explained in [13], so that a lower bound on the standard model prediction is especially interesting.

Starting from the flavor eigenstates $\{|B_s\rangle,|\overline{B_s}\rangle\}$, B_s-$\overline{B_s}$ mixing is determined by the 2×2 matrix

$$\mathcal{M}=\mathbf{M}-\frac{i}{2}\mathbf{\Gamma}, \tag{1}$$

with Hermitian $\mathbf{M}$ and $\mathbf{\Gamma}$. Because of CPT conservation, $M_{11}=M_{22}\equiv M_{B_s}$, $\Gamma_{11}=\Gamma_{22}\equiv\Gamma_{B_s}$. We recall that for the B_s system the off-diagonal elements obey the pattern

$$\left|\frac{\Gamma_{12}}{M_{12}}\right|=O\left(\frac{m_b^2}{m_t^2}\right). \tag{2}$$

The mass and lifetime difference between eigenstates are given by (H for ''heavy,'' L for ''light'')

$$\Delta M_{B_s}\equiv M_H-M_L=2|M_{12}|, \tag{3}$$

$$\Delta\Gamma_{B_s}\equiv\Gamma_L-\Gamma_H=-\frac{2\,\mathrm{Re}(M_{12}^*\Gamma_{12})}{|M_{12}|}. \tag{4}$$

The corrections to Eqs. (3) and (4) are extremely suppressed. They enter only at order $|\Gamma_{12}/M_{12}|^2$ and vanish altogether in the limit of exact CP symmetry. Anticipating the actual hierarchy of eigenvalues, we have defined both ΔM_{B_s} and $\Delta\Gamma_{B_s}$ to be positive quantities.

Neglecting CP-violating corrections, which are very small in the standard model (SM), the mass eigenstates are

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CP eigenstates (up to corrections of at most 10^{-3}), and with the phase convention $CP|B_s\rangle = -|\overline{B_s}\rangle$ one has $|B_{H/L}\rangle = (|B_s\rangle \pm |\overline{B_s}\rangle)/\sqrt{2}$. Then,[1] using standard CKM phase conventions [14],

$$\Delta\Gamma_{B_s} = -2\,\Gamma_{12} = -2\,\Gamma_{21}. \tag{5}$$

Note that the lighter state is CP even [1] and decays more rapidly than the heavier state. This also follows from the fact that most of the decay products in the $b \to c\overline{c}\overline{s}$ transition which are common to B_s and $\overline{B}_s$ are CP even [12].

Both the mass and lifetime difference are determined by the familiar box diagrams that give rise to an effective $\Delta B = 2$ Hamiltonian (B denotes b-quark number). On distance scales larger than $1/M_W$, but still smaller than $1/m_b$, this effective Hamiltonian contains a local $\Delta B = 2$ interaction as well as a bilocal part constructed from two (local) $\Delta B = 1$ transitions. The mass difference is given by the real part of the box diagram and is dominated by the top-quark contribution. For this reason, M_{12} is generated by an interaction that is local already on scales $x > 1/M_W$ and theoretically well under control. The short-distance contribution has been calculated to next-to-leading order in QCD [15]. The long-distance contribution is parametrized by the matrix element of a single four-quark operator between B_s and $\overline{B}_s$ states. Corrections to this result are suppressed by powers of m_b^2/M_W^2 and completely irrelevant for all practical purposes.

The lifetime difference is given by the imaginary part of the box diagram and determined by real intermediate states, which correspond to common decay products of B_s and $\overline{B}_s$, so that only the bilocal part of the $\Delta B = 2$ Hamiltonian can contribute. The presence of long-lived (on hadronic scales) intermediate states would normally preclude a short-distance treatment of the lifetime difference as indeed it does for neutral kaons. But for bottom mesons, the b-quark mass m_b provides an additional short-distance scale that leads to a large energy release (compared to $\Lambda_{\rm QCD}$) into the intermediate states. Thus, at typical hadronic distances $x > 1/m_b$, the decay is again a local process. The bilocal $\Delta B = 2$ Hamiltonian can be expanded in inverse powers of the heavy quark mass, schematically:

$$\mathrm{Im}\, i \int d^4x\, T[O^{\Delta B=1}(x) O^{\Delta B=1}(0)] = \sum_n \frac{C_n}{m_b^n} O_n^{\Delta B=2}(0). \tag{6}$$

The matrix elements of local $\Delta B = 2$ operators that appear here and in the mass difference are not independent of m_b. Their mass dependence could be made explicit with the help of heavy quark effective theory (HQET). The difference between the mass and lifetime difference is that for the lifetime difference explicit $1/m_b$ corrections arise from the expansion (6) even before expanding the matrix elements of local operators. The heavy quark expansion applies as well to the diagonal elements $\Gamma_{ii} \equiv \Gamma_{B_s} \equiv (\Gamma_H + \Gamma_L)/2$ and has been used to predict the total width of bottom hadrons [3]. A contribution to Γ_{12} requires that the spectator strange quark and the bottom quark come together within a distance $1/m_b$ in a meson of size $1/\Lambda_{\rm QCD}$. This volume suppression together with the phase space enhancement, leads to the estimate

$$\left|\frac{\Gamma_{12}}{\Gamma_{11}}\right| \sim 16\pi^2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3. \tag{7}$$

The application of heavy quark expansions to nonleptonic decays assumes local duality. The accuracy of this assumption cannot be quantified within the framework itself, at least not to finite order in the heavy quark expansion. The assumption that the sum over exclusive modes is accurately described by the heavy quark expansion might be especially troubling for $\Delta\Gamma_{B_s}$, since it is saturated by only a few $D_s^{(*,**)}\overline{D}_s^{(*,**)}$ intermediate states and the energy release is only slightly larger than one GeV. On the other hand, in the small-velocity limit $\Lambda_{\rm QCD} \ll m_b - 2m_c \ll m_c$, and the $N_c \to \infty$-limit,[2] local duality with only a few intermediate states can indeed be verified explicitly [12].

This article starts from the hypothesis that duality violations should be less than 10% for $\Delta\Gamma_{B_s}$. Aiming at an accuracy of 10%, the following corrections to the leading-order result have to be considered: (i) $1/m_b$ corrections from dimension-seven operators in Eq. (6); (ii) deviations from the "vacuum insertion" ("factorization") assumption for matrix elements of four-fermion operators; (iii) radiative corrections of order α_s/π; (iv) penguin and Cabibbo-suppressed contributions.

The major part of this paper is devoted to $1/m_b$ corrections. We hope to return to radiative corrections in a subsequent publication. These would bring the short-distance part of the calculation for $\Delta\Gamma_{B_s}$ on the same level that has already been achieved for ΔM_{B_s}. The result for $\Delta\Gamma_{B_s}$ to next-to-leading order in the $1/m_b$ expansion is obtained in Sec. II. We use the vacuum insertion approximation for the dimension-seven operators, and express the result in terms of two nonperturbative parameters that have to be computed with lattice methods. Section III is devoted to the phenomenology of $\Delta\Gamma_{B_s}$. Numerical results are discussed in Sec. III A, together with the theoretical uncertainties in $\Delta\Gamma_{B_s}/\Gamma_{B_s}$. In Sec. III B a generally valid upper bound on $\Delta\Gamma_{B_s}$ is derived. Section III C describes potential strategies to measure the width difference in experiment. Some phenomenological applications of such a measurement are considered in Sec. III D.

An issue related to $\Delta\Gamma_{B_s}$ concerns the total decay rate Γ_{B_s} of B_s mesons, averaged over the long-lived and short-lived component. For experimental investigations of $\Delta\Gamma_{B_s}$ [1] it would be helpful to know to what extent the average

[1] Subsequently, we present the result of our calculation of Γ_{21} as a result for $\Delta\Gamma_{B_s}$ using Eq. (5). If one does not want to assume standard model CP violation, Eq. (5) must be generalized to Eq. (4), but our result for Γ_{21} is still valid, provided non-standard-model CP violation modifies only M_{12}, but not Γ_{12}. Since Γ_{12} results predominantly from tree decays, this is reasonable to assume.

[2] This limit is necessary to justify the factorization assumption for four-fermion operators.

B_s decay rate Γ_{B_s} differs from Γ_{B_d}. These decay widths are estimated to coincide to a high accuracy [3]. We quantify this expectation and detail the contributions that could give rise to a difference between Γ_{B_s} and Γ_{B_d} in Sec. IV. A summary is presented in Sec. V. Penguin and Cabibbo-suppressed contributions turn out to shift $\Delta\Gamma_{B_s}$ by less then 10% and are discussed in the appendices, along with a comment on the lifetime ratio of B^+ to B_d mesons.

II. $\Delta\Gamma_{B_s}$: BASIC FORMALISM

The optical theorem relates the total decay width of a particle to its forward scattering amplitude. The off-diagonal element Γ_{21} of the decay width matrix is given by

$$\Gamma_{21}=\frac{1}{2M_{B_s}}\langle\overline{B_s}|\mathcal{T}|B_s\rangle. \tag{8}$$

The normalization of states is $\langle B_s|B_s\rangle=2EV$ (conventional relativistic normalization) and the transition operator $\mathcal{T}$ is defined by

$$\mathcal{T}=\mathrm{Im}\; i\int d^4x\,T\mathcal{H}_{\mathrm{eff}}(x)\mathcal{H}_{\mathrm{eff}}(0). \tag{9}$$

Here, $\mathcal{H}_{\mathrm{eff}}$ is the low energy effective weak Hamiltonian mediating bottom quark decay. The component that is relevant for Γ_{21} reads explicitly

$$\mathcal{H}_{\mathrm{eff}}=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{cs}[C_1(\mu)(\overline{b_i}c_j)_{V-A}(\overline{c_j}s_i)_{V-A}$$
$$+C_2(\mu)(\overline{b_i}c_i)_{V-A}(\overline{c_j}s_j)_{V-A}], \tag{10}$$

where we are neglecting Cabibbo-suppressed channels and the contributions from penguin operators, whose coefficients are small numerically. These contributions will be considered in the appendices. We use the notation $(\overline{q_1}q_2)_{V-A}=\overline{q_1}\gamma_\mu(1-\gamma_5)q_2$ and similar notation for other combinations of Dirac matrices. The indices i,j refer to color. The Wilson coefficient functions $C_{1,2}$ read in the leading logarithmic approximation

$$C_{2,1}=\frac{C_+\pm C_-}{2},\quad C_+(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{6/23},$$
$$C_-(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{-12/23}, \tag{11}$$

with scale μ of order m_b.

The leading contribution to the $\Delta B=2$ transition operator is shown in Fig. 1, where the vertices correspond to the interaction terms in Eq. (10). The operator product expansion is constructed using standard methods [3]. Because of the large momentum flowing through the fermion loop, it can be contracted to a point. To leading order in $1/m_b$, the strange momentum can be neglected and the b-quark momentum identified with the meson momentum. The result can be expressed in terms of two dimension-six operators

$$Q=(\overline{b_i}s_i)_{V-A}(\overline{b_j}s_j)_{V-A}, \tag{12}$$

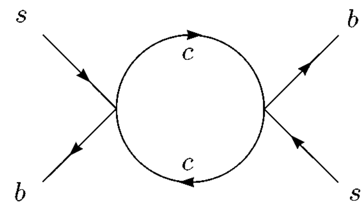

FIG. 1. Diagram that gives the leading- and next-to-leading order in $1/m_b$ terms in the heavy quark expansion of the forward scattering amplitude.

$$Q_S=(\overline{b_i}s_i)_{S-P}(\overline{b_j}s_j)_{S-P}. \tag{13}$$

The first operator coincides with the single operator that contributes to the mass difference. The appearance of a second operator can be traced to the fact that in the calculation of Γ_{21} the external b momentum cannot be neglected, because its zero component (in the meson rest frame) provides the large momentum scale.

To include $1/m_b$ corrections, the forward scattering amplitude, evaluated between on-shell quark states, is expanded in the small strange quark momentum and matched onto operators with derivatives or with a factor of m_s, the strange quark mass, which we count as Λ_{QCD}. Operators with additional gluon fields contribute only to corrections of order $(\Lambda_{\mathrm{QCD}}/m_b)^2$ and need not be considered. It is more direct (and rather trivial at this order) to use the background field method [16]. Since we do not scale out the ''kinematic'' part of order m_b in derivatives acting on b fields, we do not have immediate power counting. Some operators of higher dimension in Eq. (6) have to be kept, if they contain derivatives on b fields, such as R_2 below. Using the equations of motion, we are left with operators with at most one derivative on b fields and obtain

$$\Gamma_{21}=-\frac{G_F^2m_b^2}{12\pi(2M_{B_s})}(V_{cb}^*V_{cs})^2\sqrt{1-4z}$$
$$\times\left[\left((1-z)K_1+\frac{1}{2}(1-4z)K_2\right)\langle Q\rangle\right.$$
$$\left.+(1+2z)(K_1-K_2)\langle Q_S\rangle+\hat{\delta}_{1/m}\right], \tag{14}$$

where $z=m_c^2/m_b^2$ and

$$K_1=N_cC_1^2+2C_1C_2,\quad K_2=C_2^2. \tag{15}$$

The brackets denote the matrix element of an operator O between a $\overline{B_s}$ and B_s state, $\langle O\rangle\equiv\langle\overline{B_s}|O|B_s\rangle$. The $1/m_b$ corrections are summarized in

$$\hat{\delta}_{1/m}=(1+2z)[K_1(-2\langle R_1\rangle-2\langle R_2\rangle)+K_2(\langle R_0\rangle$$
$$-2\langle\widetilde{R}_1\rangle-2\langle\widetilde{R}_2\rangle)]-\frac{12z^2}{1-4z}[K_1(\langle R_2\rangle+2\langle R_3\rangle)$$
$$+K_2(\langle\widetilde{R}_2\rangle+2\langle\widetilde{R}_3\rangle)]. \tag{16}$$

The subdominant operators are denoted by R_i and $\widetilde{R}_i$ and read (R_4 will be needed below)

$$R_0=Q_S+\tilde{Q}_S+\frac{1}{2}Q,\quad \tilde{Q}_S=(\overline{b_i}s_j)_{S-P}(\overline{b_j}s_i)_{S-P}, \tag{17}$$

$$R_1=\frac{m_s}{m_b}(\overline{b_i}s_i)_{S-P}(\overline{b_j}s_j)_{S+P}, \tag{18}$$

$$R_2=\frac{1}{m_b^2}[\overline{b_i}\overleftarrow{D}_\rho\gamma^\mu(1-\gamma_5)D^\rho s_i][\overline{b_j}\gamma_\mu(1-\gamma_5)s_j], \tag{19}$$

$$R_3=\frac{1}{m_b^2}[\overline{b_i}\overleftarrow{D}_\rho(1-\gamma_5)D^\rho s_i][\overline{b_j}(1-\gamma_5)s_j], \tag{20}$$

$$R_4=\frac{1}{m_b}[\overline{b_i}(1-\gamma_5)iD_\mu s_i][\overline{b_j}\gamma^\mu(1-\gamma_5)s_j]. \tag{21}$$

The $\tilde{R}_i$ denote the color-rearranged operators that follow from the expressions for R_i by interchanging s_i and s_j. In deriving Eq. (14) we omitted total derivative terms, because four-momentum is conserved in the forward scattering amplitude.

The operators R_i and $\tilde{R}_i$ are not all independent at order $1/m_b$. Relations can be derived by using the equations of motion and omitting total derivatives. To reduce R_0, one can start from the Fierz identity

$$\begin{aligned}[\overline{b_i}\gamma_\mu(1-\gamma_5)s_i][\overline{b_j}\gamma_\nu(1-\gamma_5)s_j] &= -[\overline{b_i}\gamma_\mu(1-\gamma_5)s_j][\overline{b_j}\gamma_\nu(1-\gamma_5)s_i]\\ &\quad+\frac{1}{2}g_{\mu\nu}[\overline{b_i}\gamma^\lambda(1-\gamma_5)s_j][\overline{b_j}\gamma_\lambda(1-\gamma_5)s_i]\end{aligned} \tag{22}$$

and apply derivatives in an appropriate way. Up to corrections of $1/m_b$ (or less), we find

$$\begin{aligned}R_0&=2\tilde{R}_1-R_2+2R_4,\\ \tilde{R}_0&=R_0,\\ \tilde{R}_2&=-R_2,\\ \tilde{R}_3&=R_3+R_2/2,\\ \tilde{R}_4&=R_4+\tilde{R}_1-R_1-R_2.\end{aligned} \tag{23}$$

The first of these relations shows explicitly that the matrix element of R_0 is $1/m_b$ suppressed compared to Q, which is not directly evident from its definition above.

At this point, we have expressed the $1/m_b$ corrections to $\Delta\Gamma_{B_s}$ in terms of five new unknown parameters, in addition to the two nonperturbative parameters that appear already at leading order, and which also contain implicit $1/m_b$ corrections. In principle, they can all be obtained within the framework of lattice gauge theory.[3] Unfortunately, results accurate to 10% are not yet available, especially not for $\langle Q_S\rangle$ (and all the subleading operators). We therefore adopt the following strategy: we parametrize the two operators that appear at leading order. They can be estimated in vacuum insertion or the large N_c limit, but should ultimately be computed on the lattice. The operators R_i, $\tilde{R}_i$, on the other hand, are only of subleading importance and we shall content ourselves here with the factorization approximation.

Following standard conventions we express the matrix elements of Q and Q_S in terms of the corresponding ‘‘bag’’ parameters B and B_S:

$$\langle Q\rangle=f_{B_s}^2M_{B_s}^2 2\left(1+\frac{1}{N_c}\right)B, \tag{24}$$

$$\langle Q_S\rangle=-f_{B_s}^2M_{B_s}^2\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S, \tag{25}$$

where M_{B_s} and f_{B_s} are the mass and decay constant of the B_s meson and N_c is the number of colors. The parameters B and B_S are defined such that $B=B_S=1$ corresponds to the factorization (or ‘‘vacuum insertion’’) approach, which can provide a first estimate. Factorization of four-fermion operators is a controlled approximation only for large N_c or for a nonrelativistic system. In the large N_c limit, $B=3/4$ and $B_S=6/5$. In the sense of these limiting cases, factorization for realistic B_s mesons can be expected to yield the correct order of magnitude and, in particular, the right sign of these matrix elements. Existing nonperturbative calculations such as lattice simulations for $\langle Q\rangle$, and for its counterpart in the $K-\overline{K}$ system, are in agreement with this expectation. Beyond these limits factorization does not reproduce the correct renormalization scale and scheme dependence, necessary to cancel the corresponding, unphysical dependences in the Wilson coefficients. This raises the additional question, to which we return below, at what scale factorization should be employed to estimate the matrix elements. Without further information, a certain variation of the parameters B, B_S should be allowed in performing a numerical analysis.

Next, we consider the subleading operators R_i, $\tilde{R}_i$, where we apply factorization. Using relations such as (α,β refer to spinor indices, i,j to color as before)

$$\langle \overline{B_s}|\overline{b}_{\alpha i}\overleftarrow{D}_\rho D^\rho s_{\beta j}|0\rangle=\tfrac{1}{2}(m_b^2-M_{B_s}^2)\langle\overline{B_s}|\overline{b}_{\alpha i}s_{\beta j}|0\rangle, \tag{26}$$

valid to first order in $1/m_b$, all matrix elements can be expressed in terms of f_{B_s}, M_{B_s}, and quark masses. We find

$$\langle R_0\rangle=f_{B_s}^2M_{B_s}^2\left(1+\frac{1}{N_c}\right)\left(1-\frac{M_{B_s}^2}{(m_b+m_s)^2}\right),$$

$$\langle R_1\rangle=f_{B_s}^2M_{B_s}^2\frac{m_s}{m_b}\left(2+\frac{1}{N_c}\right),$$

$$\langle \tilde{R}_1\rangle=f_{B_s}^2M_{B_s}^2\frac{m_s}{m_b}\left(1+\frac{2}{N_c}\right),$$

[3] The matrix elements of the subleading operators could be evaluated in the static limit. However, to consistently include all $1/m_b$ corrections, $\langle Q\rangle$ and $\langle Q_S\rangle$ must be computed either in full QCD or in heavy quark effective theory including $1/m_b$ corrections to the Lagrangian as well as to the effective theory operators. The parametrization of $1/m_b$ corrections to $\langle Q\rangle$ has been analyzed in [17].

$$\langle R_2\rangle = f_{B_s}^2 M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)\left(-1+\frac{1}{N_c}\right),$$

$$\langle \widetilde{R}_2\rangle = f_{B_s}^2 M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)\left(1-\frac{1}{N_c}\right),$$

$$\langle R_3\rangle = f_{B_s}^2 M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)\left(1+\frac{1}{2N_c}\right),$$

$$\langle \widetilde{R}_3\rangle = f_{B_s}^2 M_{B_s}^2\left(\frac{M_{B_s}^2}{m_b^2}-1\right)\left(\frac{1}{2}+\frac{1}{N_c}\right). \tag{27}$$

Combining the above results, one can obtain $\Delta\Gamma_{B_s}$ from Eq. (14). The sensitivity to V_{cb} may be eliminated by normalizing to the total decay rate Γ_{B_s} expressed in terms of the semileptonic width and branching ratio

$$\Gamma_{B_s} = \frac{\Gamma(B_s\to Xe\nu)}{B(B_s\to Xe\nu)} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\frac{g(z)\,\widetilde{\eta}_{\rm QCD}}{B(B_s\to Xe\nu)}, \tag{28}$$

$$g(z)=1-8z+8z^3-z^4-12z^2\ln z, \tag{29}$$

where $B(B_s\to Xe\nu)$ is to be taken from experiment[4] and $z=m_c^2/m_b^2$ as before. $\widetilde{\eta}_{\rm QCD}$ denotes the one-loop QCD corrections (m_b refers to the b-quark pole mass). Their analytic expression can be found in [18]. At $m_b=4.8$ GeV, $m_c=1.4$ GeV, $\mu=m_b$, and with $\alpha_s(m_b)=0.216$ one has $\widetilde{\eta}_{\rm QCD}=0.88$. Since radiative corrections to $\Delta\Gamma_{B_s}$ are not yet known, the inclusion of radiative corrections to the semileptonic width seems somewhat arbitrary. On the other hand, with $V_{cb}=0.04$ and $\Gamma_{B_s}^{-1}=1.54$ ps, one obtains $m_b\approx 4.8$ GeV from Eq. (28), compared to $m_b\approx 4.5$ GeV without QCD corrections. We prefer the first value as our central choice for m_b in the numerical analysis, but repeat that, in the absence of radiative corrections to $\Delta\Gamma_{B_s}$, $\widetilde{\eta}_{\rm QCD}$ can as well be considered as a normalization uncertainty that replaces the normalization uncertainty due to the errors in V_{cb} and Γ_{B_s}. Finally, one arrives at the expression

$$\begin{aligned}\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} &= 16\pi^2 B(B_s\to Xe\nu)\frac{\sqrt{1-4z}}{g(z)\,\widetilde{\eta}_{\rm QCD}}\frac{f_{B_s}^2 M_{B_s}}{m_b^3}V_{cs}^2\\ &\times\Bigg[[2(1-z)K_1+(1-4z)K_2]\left(1+\frac{1}{N_c}\right)B\\ &+(1+2z)(K_2-K_1)\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S\\ &+\delta_{1/m}+\delta_{\rm rem}\Bigg].\end{aligned} \tag{30}$$

$\delta_{1/m}$ is related to $\hat{\delta}_{1/m}$, defined in Eq. (16), through

$$\hat{\delta}_{1/m} = f_{B_s}^2 M_{B_s}^2 \delta_{1/m}, \tag{31}$$

and from now on we imply that Eq. (27) is used. We have indicated by $\delta_{\rm rem}$ the contributions from CKM-suppressed intermediate states $(u\bar{c}, \bar{u}c, u\bar{u})$ and from penguin operators in the $\Delta B=1$ effective Hamiltonian, which are estimated in Appendices A and B to be below $\pm 3\%$ and about -5%, respectively, relative to the leading-order contribution. We shall neglect $\delta_{\rm rem}$ in the analysis to follow.

Since $f_{B_s}\sim\Lambda_{\rm QCD}^{3/2}/m_b^{1/2}$, $\Delta\Gamma_{B_s}/\Gamma_{B_s}\sim 16\pi^2(\Lambda_{\rm QCD}/m_b)^3$ as in the estimate (7). Equation (30) is valid to leading-$[O(1/m_b^3)]$ and next-to-leading order $[O(1/m_b^4)]$ in the heavy quark expansion. The most important neglected terms are radiative corrections of order $O(\alpha_s/m_b^3)$. Implicit here is the assumption that the quantity $(\Delta\Gamma/\Gamma)_{B_s}$ can indeed be represented to reasonable accuracy by the series in powers of $\Lambda_{\rm QCD}/m_b$ that is generated by the heavy quark expansion. As mentioned earlier, this assumption is equivalent to the assumption of local quark hadron duality.

The leading term in Eq. (30), represented by the contributions proportional to B and B_S, agrees with the results that have been given previously in the literature[5] [6–10]. Note that we have consistently kept the distinction between quark masses, arising from the short-distance loops or the equations of motion, and the meson mass M_{B_s} from hadronic matrix elements, since we are aiming at effects beyond leading order in the heavy quark expansion.

In Eq. (30), K_1, K_2 and B, B_S should be evaluated at a scale of order m_b. If we wanted to use vacuum insertion to estimate the bag factors, it is physically clear, especially in the heavy quark limit $m_b\to\infty$, that vacuum insertion should be applied not at the scale m_b, but at a typical hadronic scale $\mu_h\sim 1$ GeV. This still leaves us with an ambiguity as to the choice of μ_h and, in addition, with the question how $B(\mu_h)=B_S(\mu_h)=1$ are related to $B(m_b)$ and $B_S(m_b)$. This latter question can be answered in the limit $\mu_h\ll m_b$ and corresponds to the inclusion of ''hybrid logarithms'' [19,20], as done in [10]. The evolution from m_b to μ_h is performed in the leading logarithmic approximation in the static theory and leads to[6]

$$B(m_b)=1,$$

$$B_S(m_b)=1-\frac{3}{5}\left(1-\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_h)}\right]^{8/25}\right). \tag{32}$$

The first equation in Eqs. (32) reflects the well-known result that the matrix element of the operator Q has the same leading logarithmic corrections in the static theory (HQET) as the square of the decay constant $f_{B_s}^2$. Taking $\mu_h=0.5, 1, 2$ GeV results in $B_S(m_b)=0.80, 0.88, 0.94$. (The scale $\mu_h=0.5$ GeV might already be too low for a perturbative evolution.)

[4]Since we show in Sec. IV that the lifetime difference between B_s and B_d is tiny, no attention has to be paid to the flavor content of the B meson.

[5]Often factorization is assumed for the leading-order term, so that B and B_S have to be set to unity to recover the result.

[6]We have checked the calculation of hybrid logarithms and agree with the findings of [10].

The b-quark mass $m_b \approx 4.8$ GeV is probably not large enough to make this estimate realistic, even if factorization held at the scale μ_h. The logarithm $\ln m_b/\mu_h$ is not very large, so that other contributions such as nonlogarithmic $O(\alpha_s)$ terms which are omitted in Eq. (32), can be expected to be numerically of the same order as the hybrid logarithms that are retained, especially since summing hybrid logarithms amounts to a moderate 10% effect (with $\mu_h = 1$ GeV). The one-loop matching of Q on its counterpart(s) in heavy quark effective theory indeed exhibits sizable cancellations between logarithms and constants, at least in the particular matching scheme considered in [21]. Furthermore, the QCD renormalization between m_b and μ_h in Eq. (32) is only valid at leading order in HQET and neglects $1/m_b$ corrections in the matrix elements, which is not consistent with our keeping of explicit $1/m_b$ corrections. On the other hand, the B factors are, in principle, calculable in full QCD. In this case they will automatically include $1/m_b$ corrections as well as the hybrid logarithms, among further important contributions. For these reasons we prefer to keep the expression for $(\Delta\Gamma/\Gamma)_{B_s}$ in the form given in Eq. (30) and do not include hybrid renormalization explicitly, with the understanding that the bag factors will eventually be available from lattice QCD. In our numerical analysis, we take the conservative, but perhaps too agnostic attitude that $B_S(m_b)$ could take any value between 0.7 and 1.3, keeping in mind Eq. (32) as a particular model estimate of B and B_S. The upper end of this range is motivated by the $N_c \to \infty$ limit, in which $B_S = 6/5$.

III. $\Delta\Gamma_{B_s}$: PHENOMENOLOGY

A. Numerical analysis of $(\Delta\Gamma/\Gamma)_{B_s}$

We first turn to a numerical analysis and discussion of $(\Delta\Gamma/\Gamma)_{B_s}$ based on Eq. (30). It is useful to separate the dependence on the long-distance parameters f_{B_s}, B, and B_S and write $(\Delta\Gamma/\Gamma)_{B_s}$ as

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = [aB + bB_S + c]\left(\frac{f_{B_s}}{210\ \mathrm{MeV}}\right)^2, \tag{33}$$

where c incorporates the explicit $1/m_b$ corrections. To estimate the sensitivity of $(\Delta\Gamma/\Gamma)_{B_s}$ on the short-distance input parameters, we keep the following parameters fixed: $m_b - m_c = 3.4$ GeV, $m_s = 200$ MeV, $\Lambda_{LO}^{(5)} = 200$ MeV. In addition, $M_{B_s} = 5.37$ GeV and the semileptonic branching ratio is $B(B_s \to X e \nu) = 10.4\%$. Then a, b, and c depend only on m_b and the renormalization scale μ. For some values of m_b and μ, the coefficients a, b, c are listed in Table I. For a central choice of parameters, which we take as $m_b = 4.8$ GeV, $\mu = m_b$, $B = B_S = 1$, and $f_{B_s} = 210$ MeV, we obtain

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.220 - 0.065 = 0.155, \tag{34}$$

where the leading term and the $1/m_b$ correction are separately displayed. As seen from Table I, the dependence on m_b is weak, but $(\Delta\Gamma/\Gamma)_{B_s}$ increases by almost 20% when the renormalization scale is lowered to $m_b/2$, at fixed B and B_S. These dependences are not specific to the values $B = B_S = 1$. The weak m_b dependence is a somewhat accidental consequence of using the semileptonic branching ratio to eliminate V_{cb}. If instead, we normalize to $\Gamma_{B_s}^{-1} = 1.54$ ps and take $V_{cb} = 0.04$, $(\Delta\Gamma/\Gamma)_{B_s}$ would vary from 0.143 to 0.166 under the same variation of m_b as in Table I. Let us also add the following more general observations.

TABLE I. Dependence of a, b, and c on the b-quark mass and renormalization scale for fixed values of all other short-distance parameters. The last column gives $(\Delta\Gamma/\Gamma)_{B_s}$ for $B = B_S = 1$ (at the given scale μ), $f_{B_s} = 210$ MeV.

m_b/GeV	μ	a	b	c	$(\Delta\Gamma/\Gamma)_{B_s}$
4.8	m_b	0.009	0.211	-0.065	0.155
4.6	m_b	0.015	0.239	-0.096	0.158
5.0	m_b	0.004	0.187	-0.039	0.151
4.8	$2\,m_b$	0.017	0.181	-0.058	0.140
4.8	$m_b/2$	0.006	0.251	-0.076	0.181

(i) The theoretical expression for $\Delta\Gamma_{B_s}$ in Eq. (30) predicts the sign of this quantity, which *a priori* could have either value. $\Delta\Gamma_{B_s}$ is positive and implies a larger decay rate for the CP even (lighter) state [10,12] (see the conventions in Introduction). The typical magnitude of $(\Delta\Gamma/\Gamma)_{B_s}$ to leading order in the heavy quark expansion is about 0.2, larger than other width differences among bottom hadrons with the possible exception of the case of Λ_b (depending on whether theory or present experiments turn out to be right on Λ_b).

(ii) The explicit $1/m_b$ corrections are numerically important and vary strongly with m_b. For our central parameter choice they reduce the leading order prediction by about 30%. Essentially, all the various $1/m_b$ correction terms add with the same sign and make the result somewhat larger than the natural size of the corrections, $\Lambda_{\mathrm{QCD}}/m_b \approx (M_{B_s} - m_b - m_s)/m_b \approx 8\%$ and $m_s/m_b \approx 4\%$.

(iii) The contribution from the scalar operator Q_S by far dominates over the contribution from Q, because there is a strong cancellation between terms of different sign in the Wilson coefficient of the latter operator. This has important implications for $(\Delta M/\Delta\Gamma)_{B_s}$, which we discuss below, because hadronic uncertainties cancel only partially in the ratio B/B_S.

(iv) If $B_S = 1.3$, a $(\Delta\Gamma/\Gamma)_{B_s}$ of as much as 0.25 is not excluded, although this appears unlikely. On the other hand, if $B_S < 1$, as suggested by the estimate from hybrid logarithms, and if f_{B_s} turns out to be merely 180 MeV, $(\Delta\Gamma/\Gamma)_{B_s}$ could be as small as 0.07, making its experimental detection more difficult.

This discussion shows that to resolve the theoretical uncertainties, a reliable calculation of B_S is mandatory. Further improvement then requires a full next-to-leading order calculation of short-distance corrections.

B. Upper limit on $\Delta\Gamma_{B_s}$

Since the $b \to c\bar{c}s$ transition is the dominant contributor to $(\Delta\Gamma)_{B_s}$, one obtains the upper bound [22,1,23]

$$\left(\frac{|\Delta\Gamma|}{\Gamma}\right)_{B_s} \leqslant 2B(b\to c\overline{cs})_{B_s}. \tag{35}$$

It can be readily understood by considering the limit in which only $b\to c\overline{cs}$ transitions were generated by the effective Hamiltonian. Equation (35) then follows from the requirement that the decay rates be non-negative, $\Gamma_\pm = \Gamma(b\to c\overline{cs}) \pm \Delta\Gamma/2 \geqslant 0$. $B(b\to c\overline{cs})_{B_s}$ denotes the fraction of B_s-meson decays governed by the $b\to c\overline{cs}$ transitions in the absence of mixing. Within the heavy quark expansion, $(|\Delta\Gamma|)/\Gamma)_{B_s}$ is suppressed by m_b^{-3} relative to spectator branching ratios, such as $B(b\to c\overline{cs})$. From this point of view a bound such as Eq. (35) might appear trivial. However, the virtue of relation (35) is its very general validity. It would hold even if a heavy quark expansion were not applicable to the underlying process.

We can obtain a numerical estimate of the right-hand side of Eq. (35), assuming $B(b\to c\overline{cs})_{B_s} \approx B(b\to c\overline{cs})_{B_d}$. This approximation should be accurate to a few percent, the expected size of the weak annihilation contribution to unmixed B_s decay (see Sec. IV). CLEO [24] recently confirmed the prediction [25] of a significant ''wrong'' charm yield in B decays, thereby completing the first direct measurement of

$$B(b\to c\overline{cs}')_{B_d} \approx B(b\to \overline{c}) = 0.227 \pm 0.035, \tag{36}$$

where $B(b\to\overline{c})$ is the average number of $\overline{c}$ produced per b decay. The Cabibbo-allowed transition is

$$B(b\to c\overline{cs}) = |V_{cs}|^2 B(b\to c\overline{cs}') = 0.22 \pm 0.03. \tag{37}$$

This yields the upper limit

$$\left(\frac{|\Delta\Gamma|}{\Gamma}\right)_{B_s} \leqslant 0.44 \pm 0.06. \tag{38}$$

C. Measuring $\Delta\Gamma_{B_s}$

We hope to have convinced the reader about the importance of an accurate measurement of $\Delta\Gamma$. One method is to substitute Γ_{B_d} for the average B_s width Γ_{B_s} and to extract $\Delta\Gamma_{B_s}$ from the time dependences of untagged flavor specific B_s data samples [1]. Time-dependent studies of angular distributions of untagged $\overset{(-)}{B_s} \to J/\psi\phi$ decays allow the extraction of Γ_L, and also of Γ_H if the CP-odd component is non-negligible [5,26]. These and other methods using decay length distributions of fully reconstructed B_s mesons are at present statistically limited [1,5,26].

As an illustration one may consider the measurement of

$$\tau(B_s\to J/\psi\phi) = 1.34^{+0.23}_{-0.19} \pm 0.05 \ \text{ps}, \tag{39}$$

recently obtained by the Collider Detector at Fermilab (CDF) Collaboration from a single lifetime fit of their $\overset{(-)}{B_s} \to J/\psi\phi$ data sample [27]. Next, we can write

$$1/\Gamma_L \leqslant \tau(B_s\to J/\psi\phi), \tag{40}$$

which holds only as an inequality, because $B_s\to J/\psi\phi$ is not necessarily a pure CP-even final state. The world average B_d lifetime [28]

$$\tau_{B_d} = 1.54 \pm 0.04 \ \text{ps} \tag{41}$$

together with the result of Sec. IV, informs us about the inverse of the average B_s width $1/\Gamma_{B_s} = \tau_{B_d}$. We then use

$$\frac{\Delta\Gamma}{\Gamma} = 2\left(\frac{\Gamma_L}{\Gamma} - 1\right) \tag{42}$$

and obtain

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} \geqslant 0.3 \pm 0.4, \tag{43}$$

which is still inconclusive, but can serve to indicate the present status.

Just establishing a nonvanishing difference in decay length distributions for partially reconstructed B_s mesons in comparison to other B mesons would constitute progress. The ideal inclusive b-hadron data sample should have large statistics and be highly enriched in B_s decay products originating predominantly from a single mass eigenstate B_L (or B_H). The last requirement maximizes differentiation between B_s and other B mesons. The $\phi\phi X$ final state serves as an example [29]. The probable decay chain is $B_s\to D_s^+ D_s^- X$, which is dominantly CP even [12]. Both D_s's then decay into ϕ's. While D_s is seen significantly in ϕ's, the D^+ is seen in ϕ's by about a factor of 10 less and the D^0 even less than that [30]. The background due to B-meson decays is thus controllable and further suppressed because B's prefer to be seen as D^0 over D^+ by a ratio of 2.7 [31]. If sufficient statistics is available, the $D_s^\pm \phi X$ sample would be even better.

The inclusive $B_s\to \phi \ell^+ X$ sample with a high $P_{T,\mathrm{re}\ell}$ lepton, is flavor specific. Its time dependence is governed by the sum of two exponentials, $\exp(-\Gamma_L t) + \exp(-\Gamma_H t)$. Theory predicts $(\Gamma_L + \Gamma_H)/2 = 1/\tau_{B_d}$, but the observation of the two exponents requires precise decay length and boost information, whose accuracy increases the more fully the B_s is reconstructed.

The less reconstructed the B_s data sample, the more important it is to have a monoenergetic source of B_s mesons. Thus, the more inclusive techniques tend to be more useful for $e^+e^-\to Z^0$ experiments than at hadron accelerators. Of course, fully reconstructed B_s data samples allow clean measurements of $\Delta\Gamma_{B_s}$.

D. B_s-$\overline{B_s}$ mixing and CKM elements

The traditional methods for observation of CP violation and the extraction of CKM phases require to resolve the rapid $\Delta M_{B_s} t$ oscillations of tagged B_s data samples [32]. Current vertexing technology allows one to resolve such oscillations for $\Delta M_{B_s} \lesssim 10\ \text{ps}^{-1}$. Thus the recent lower limit from the ALEPH Collaboration [33]

$$\Delta M_{B_s} > 6.6 \ \text{ps}^{-1} (95\% \text{ C.L.}) \tag{44}$$

is significant. It may indicate the need to develop new methods capable of a higher resolving power. Reliable predictions of ΔM_{B_s} are therefore important in order to plan future B_s experiments, in particular if only lower limits will be available with current vertex techniques. The most straightforward method makes use of [34]

$$\Delta M_{B_s}=\frac{G_F^2 M_W^2}{6\pi^2}\,\eta_B S_0(x_t) M_{B_s} B_{B_s} f_{B_s}^2 |V_{ts}|^2, \tag{45}$$

where $x_t=m_t^2/M_W^2$. The current relative uncertainty is about 50% and is dominated by the uncertainty in B_{B_s} ($\pm 30\%$), $f_{B_s}^2$ ($\pm 40\%$), $|V_{ts}|^2$ ($\pm 15\%$), and $S_0(x_t)$ ($\pm 8\%$). The fractional uncertainty on ΔM_{B_s} can be expected to decrease to $\sim 15\%$ by the year 2002, anticipating improvements in the accuracy of the relevant parameters B_{B_s} ($\pm 10\%$), $f_{B_s}^2$ ($\pm 5\%$), $|V_{ts}|^2$ ($\pm 5\%$), and $S_0(x_t)$ ($\pm 3\%$).

A variant of this method uses the experimental value for ΔM_{B_d} and the ratio

$$\frac{(\Delta M)_{B_s}}{(\Delta M)_{B_d}}=\frac{M_{B_s}}{M_{B_d}}\,\frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2}\left|\frac{V_{ts}}{V_{td}}\right|^2 \tag{46}$$

to predict ΔM_{B_s}. This approach will be useful only if the CKM ratio $|V_{ts}/V_{td}|^2$ is accurately known.

If the first observation of B_s-$\overline{B}_s$ mixing is a nonvanishing $\Delta\Gamma_{B_s}$ rather than ΔM_{B_s}, then a complementary method to predict ΔM_{B_s} opens up, based on the quantity [see Eq. (30)]

$$\begin{aligned}\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} &= \frac{\pi}{2}\,\frac{m_b^2}{M_W^2}\left|\frac{V_{cb}V_{cs}}{V_{ts}V_{tb}}\right|^2 \frac{\sqrt{1-4z}}{\eta_B S_0(x_t)}\Bigg[[2(1-z)K_1 \\ &\quad +(1-4z)K_2]\left(1+\frac{1}{N_c}\right)+(1+2z) \\ &\quad \times (K_2-K_1)\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)\frac{B_S}{B}+\delta_{1/m}\Bigg].\end{aligned} \tag{47}$$

This result is valid to next-to-leading order in the $1/m_b$ expansion and to leading logarithmic accuracy in QCD. We have again used factorization for the subleading $1/m_b$ corrections. Note that with the bag parameter B as defined in Eq. (24), the appropriate QCD correction factor η_B is identical to $C_+(\mu)$ from Eq. (11) in the leading logarithmic approximation.

In the ratio $\Delta\Gamma/\Delta M$ the decay constant cancels and the CKM uncertainty is almost completely removed since

$$\left|\frac{V_{cb}V_{cs}}{V_{ts}V_{tb}}\right|^2=1\pm 0.03. \tag{48}$$

At present, the accuracy of $\Delta\Gamma/\Delta M$ is still rather poor, $\Delta\Gamma/\Delta M=(5.6\pm 2.6)\times 10^{-3}$. The breakdown of errors is as follows: ± 2.3 from varying B_S/B between 0.7 and 1.3, ${}^{+1.1}_{-0.7}$ from varying μ between $m_b/2$ and $2m_b$, ± 0.4 from $m_b=4.8\pm 0.2$ GeV, and ± 0.4 from $m_t=176\pm 9$ GeV. The dominant uncertainty is due to B_S/B, which has never been studied before. It is conceivable that a lattice study could actually calculate B_S/B more accurately than the bag parameters themselves, because some systematic uncertainties may be expected to cancel in the ratio. The quantity $\Delta\Gamma/\Delta M$ might thus be calculable rather precisely in the future and ΔM_{B_s} could then be estimated from the observed $\Delta\Gamma_{B_s}$. In conjunction with ΔM_{B_d}, this would provide an alternative way of determining the CKM ratio $|V_{ts}/V_{td}|$, especially if the latter is around its largest currently allowed value [2]. The width difference, and hence its observability, increases the larger $|V_{ts}|\approx|V_{cb}|$ becomes. In contrast, the ratio $\Gamma(B\to K^*\gamma)/\Gamma(B\to\{\varrho,\omega\}\gamma)$ is best suited for extracting small $|V_{ts}/V_{td}|$ ratios, provided the long-distance effects can be sufficiently well understood [35].

These approaches could complement other methods to determine $|V_{td}/V_{ts}|$. Such additional possibilities would be to relate $|V_{ts}|$ to the accurate $|V_{cb}|$ measurements and to obtain $|V_{td}|$ from ΔM_{B_d}, CKM unitarity constraints [36], and in particular $B(K^+\to\pi^+\nu\overline{\nu})$ [34,37], which has the unique advantage of being exceptionally clean from a theoretical point of view.

IV. THE B_s-B_d WIDTH DIFFERENCE

The ratio of the B_s- and B_d-meson decay widths $\Gamma_{B_s}/\Gamma_{B_d}$ is expected to be very close to unity [3,38]. Deviations arise predominantly from SU(3)-breaking effects in already small corrections to the leading spectator decay of the bottom quark. In the following we will discuss the mechanisms that differentiate between Γ_{B_s} and Γ_{B_d} and estimate their numerical importance. The decay rate of B_d, B_s mesons has the general form ($q=d,\,s$)

$$\Gamma_{B_q}=\Gamma_0+\Delta\Gamma_{\rm kin}^{(q)}+\Delta\Gamma_{\rm mag}^{(q)}+\Delta\Gamma_{\rm WA}^{(q)}. \tag{49}$$

Here, Γ_0 denotes the leading, universal, free b-quark decay rate, $\Delta\Gamma_{\rm kin}$ is the time dilatation correction, $\Delta\Gamma_{\rm mag}$ the contribution from the chromomagnetic interaction of the heavy quark spin, and $\Delta\Gamma_{\rm WA}$ describes the weak annihilation of b with q. $\Delta\Gamma_{\rm kin}$ and $\Delta\Gamma_{\rm mag}$ are of the order $O(1/m_b^2)$ relative to Γ_0 and $\Delta\Gamma_{\rm WA}$ enters at order $O(1/m_b^3)$. Higher orders have been neglected in Eq. (49). There is no linear correction in $1/m_b$ [3]. Through order $O(1/m_b^3)$ one may thus write

$$\begin{aligned}\frac{\Gamma_{B_s}}{\Gamma_{B_d}} &= 1+\frac{\Delta\Gamma_{\rm kin}^{(s)}-\Delta\Gamma_{\rm kin}^{(d)}}{\Gamma}+\frac{\Delta\Gamma_{\rm mag}^{(s)}-\Delta\Gamma_{\rm mag}^{(d)}}{\Gamma} \\ &\quad +\frac{\Delta\Gamma_{\rm WA}^{(s)}-\Delta\Gamma_{\rm WA}^{(d)}}{\Gamma}.\end{aligned} \tag{50}$$

We will now discuss the three different corrections which contribute to $\Gamma_{B_s}/\Gamma_{B_d}-1$ in turn.

The first two can be related to meson mass differences. For this purpose we define

$$\overline{M}_H=\tfrac{1}{4}\,(M_H+3M_{H^*}), \tag{51}$$

where M_H and M_{H^*} are the masses of a pseudoscalar heavy-light meson H ($^1S_0$) and of its vector-meson partner

H^* ($^3S_1$). In the weighted average $\overline{M_H}$ the spin-splitting contribution cancels in the HQET mass formula which then takes the form ($Q=b,\, c$)

$$\overline{M}_{H_q}=m_Q+\overline{\Lambda}_q+\frac{\langle\vec{p}^{\,2}\rangle_q}{2m_Q}+O\left(\frac{\Lambda^3_{\mathrm{QCD}}}{m_Q^2}\right). \tag{52}$$

Here, $\langle\vec{p}^{\,2}\rangle_q$ is the average momentum squared of the heavy quark inside the meson and $\overline{\Lambda}_q$ may be viewed as the constituent mass of the light degrees of freedom. Both quantities depend on the light quark flavor q but are independent of the heavy quark mass. Combining Eq. (52) for the cases of D_s, D^+, B_s, and B_d and recalling that $\Delta\Gamma^{(q)}_{\mathrm{kin}}/\Gamma=-\langle\vec{p}^{\,2}\rangle_q/(2m_b^2)$ one finds

$$\frac{\Delta\Gamma^{(s)}_{\mathrm{kin}}-\Delta\Gamma^{(d)}_{\mathrm{kin}}}{\Gamma}=-\frac{m_c/m_b}{m_b-m_c}[\overline{M}_{D_s}-\overline{M}_{D^+}-(\overline{M}_{B_s}-\overline{M}_{B_d})]$$
$$\approx-(3\pm6)\times10^{-4}. \tag{53}$$

All required meson masses can be obtained from [14], except for $M_{B_s^*}$. In this case we use the heavy quark symmetry relation

$$M_{B_s^*}-M_{B_s}=\frac{M_{D_s^*}-M_{D_s}}{M_{D^{*+}}-M_{D^+}}(M_{B_d^*}-M_{B_d})=(46\pm1)\ \ \mathrm{MeV} \tag{54}$$

to find $M_{B_s^*}=(5421\pm6)$ MeV. This expectation is in accordance with direct measurements of the $B_s^*\to B_s\gamma$ transition, which yield $M_{B_s^*}-M_{B_s}=(47.0\pm2.6)$ MeV [39]. We see that the correction in Eq. (53) is exceedingly small. This number, however, should probably not be taken at face value. Given the smallness of the effect it is conceivable that terms neglected in Eq. (52) could have an impact on the precise estimate of Eq. (53). The typical size of such a correction would be (here we use $\Lambda_{\mathrm{QCD}}=0.3$ GeV)

$$\left|\frac{\Delta\Gamma^{(s)}_{\mathrm{kin}}-\Delta\Gamma^{(d)}_{\mathrm{kin}}}{\Gamma}\right|\approx\frac{m_c/m_b}{m_b-m_c}\left[\frac{\Lambda^3_{\mathrm{QCD}}}{m_c^2}\right]\approx12\times10^{-4}. \tag{55}$$

At any rate, while Eq. (53) might not be a completely accurate estimate of this correction, it seems safe to conclude that the effect on $\Gamma_{B_s}/\Gamma_{B_d}$ due to $\Delta\Gamma^{(s)}_{\mathrm{kin}}-\Delta\Gamma^{(d)}_{\mathrm{kin}}$ is well below 1% and thus negligible for all practical purposes.

Next, the chromomagnetic correction $\Delta\Gamma^{(q)}_{\mathrm{mag}}$ can be related to the spin splitting in S-wave B mesons and is proportional to $M_{B_q^*}-M_{B_q}$. Hence, we may write

$$\frac{\Delta\Gamma^{(s)}_{\mathrm{mag}}-\Delta\Gamma^{(d)}_{\mathrm{mag}}}{\Gamma}=\frac{\Delta\Gamma^{(d)}_{\mathrm{mag}}}{\Gamma}\,\frac{M_{B_s^*}-M_{B_s}-(M_{B_d^*}-M_{B_d})}{M_{B_d^*}-M_{B_d}}$$
$$\approx-(3\pm8)\times10^{-4}. \tag{56}$$

The quantity $\Delta\Gamma^{(d)}_{\mathrm{mag}}/\Gamma$ is known [3] and can be calculated to be -0.012. Using $M_{B_d^*}-M_{B_d}=(46.0\pm0.6)$ MeV [14] and $M_{B_s^*}-M_{B_s}=(47.0\pm2.6)$ MeV [39], one finds the numerical estimate quoted in Eq. (56). Clearly, this effect on the B_s-B_d lifetime difference is negligible as well.

Finally, we turn to the corrections due to weak annihilation. These contributions arise from the annihilation reactions $\overline{b}s\to\overline{c}c$ and $\overline{b}d\to\overline{c}u$ in the case of a B_s and a B_d meson, respectively. Neglecting Cabibbo-suppressed modes and penguin contributions, they are readily calculated to be[7]

$$\frac{\Delta\Gamma^{(s)}_{\mathrm{WA}}}{\Gamma}=16\pi^2B(B\to Xe\nu)\frac{f^2_{B_s}}{m_b^2}V^2_{cs}\frac{\sqrt{1-4z}}{g(z)\,\tilde{\eta}_{\mathrm{QCD}}}$$
$$\times\left[-(1-z)\left(K_1B^{(s)}_1+\frac{1}{N_c}K_2B^{(s)}_2\right)\right.$$
$$\left.+(1+2z)\left(K_1B^{(s)}_3+\frac{1}{N_c}K_2B^{(s)}_4\right)\right], \tag{57}$$

$$\frac{\Delta\Gamma^{(d)}_{\mathrm{WA}}}{\Gamma}=16\pi^2B(B\to Xe\nu)\frac{f^2_{B_d}}{m_b^2}V^2_{ud}\frac{(1-z)^2}{g(z)\,\tilde{\eta}_{\mathrm{QCD}}}$$
$$\times\left[-\left(1+\frac{z}{2}\right)\left(K_1B^{(d)}_1+\frac{1}{N_c}K_2B^{(d)}_2\right)\right.$$
$$\left.+(1+2z)\left(K_1B^{(d)}_3+\frac{1}{N_c}K_2B^{(d)}_4\right)\right]. \tag{58}$$

Here, we have again used Eq. (28) to eliminate the V_{cb} dependence. The leading log QCD coefficients $K_{1,2}$ are defined in Eq. (15). The bag factors $B^{(q)}_i$ parametrize the matrix elements

$$\langle B_q|(\overline{b}_iq_i)_{V-A}(\overline{q}_jb_j)_{V-A}|B_q\rangle=f^2_{B_q}m_b^2B^{(q)}_1,$$

$$\langle B_q|(\overline{b}_iq_j)_{V-A}(\overline{q}_jb_i)_{V-A}|B_q\rangle=\frac{1}{N_c}f^2_{B_q}m_b^2B^{(q)}_2,$$

$$\langle B_q|(\overline{b}_iq_i)_{S-P}(\overline{q}_jb_j)_{S+P}|B_q\rangle=f^2_{B_q}m_b^2B^{(q)}_3,$$

$$\langle B_q|(\overline{b}_iq_j)_{S-P}(\overline{q}_jb_i)_{S+P}|B_q\rangle=\frac{1}{N_c}f^2_{B_q}m_b^2B^{(q)}_4, \tag{59}$$

where we have assumed $M_{B_q}\approx m_b$.

Using the strict factorization estimate $B^{(q)}_i\equiv1$ would yield the result (taking $f_{B_d}\approx f_{B_s}$ and expanding in $z\approx0.1$)

$$\left[\frac{\Delta\Gamma^{(s)}_{\mathrm{WA}}-\Delta\Gamma^{(d)}_{\mathrm{WA}}}{\Gamma}\right]_{\mathrm{fact}}\simeq24\pi^2B(B\to Xe\nu)\frac{f^2_B}{m_b^2}\frac{1-2z}{g(z)\,\tilde{\eta}_{\mathrm{QCD}}}z$$
$$\times\left(K_1+\frac{1}{N_c}K_2\right). \tag{60}$$

Note that, in "vacuum insertion," this expression coincides with $\Delta\Gamma^{(d)}_{\mathrm{WA}}/\Gamma$ while $\Delta\Gamma^{(s)}_{\mathrm{WA}}/\Gamma$ is twice as large. For our central parameter set, Eq. (60) amounts to 2×10^{-4}. The ex-

[7]Our results are in agreement with the expressions recently obtained in [38].

treme smallness of this number is the result of two effects. The first is helicity suppression, manifesting itself in the factor of $z = m_c^2/m_b^2$ in Eq. (60). Second, a further suppression comes from a, somewhat accidental, cancellation between QCD coefficients in $K_1 + K_2/3 \approx -0.39 + 0.42 = 0.03$. It is important to realize that both features are a consequence of the factorization assumption. Even with small deviations from factorization, the factor $z(K_1 + K_2/N_c)$ would be substituted by a number almost 100 times larger. To get an idea of the typical order of magnitude, we approximate Eqs. (57) and (58) to

$$\frac{\Delta\Gamma_{\rm WA}^{(q)}}{\Gamma} = 16\pi^2 B(B\to Xe\nu)\frac{f_{B_q}^2}{m_b^2}V_{ud}^2\frac{1-2z}{g(z)\,\tilde{\eta}_{\rm QCD}} \times\left[K_1(B_3^{(q)}-B_1^{(q)}) + \frac{1}{N_c}K_2(B_4^{(q)}-B_2^{(q)}) + O(z)\right], \tag{61}$$

where we have used $\sqrt{1-4z} \approx (1-z)^2 \approx 1-2z$ and neglected small helicity-suppressed contributions proportional to z in the square brackets. Taking $-K_1 \approx K_2/3 \approx 0.4$ and $|B_3^{(q)} - B_1^{(q)}|, |B_4^{(q)} - B_2^{(q)}| < 0.6$, the modulus of the term in square brackets is 0.5 or less, which yields $\Delta\Gamma_{\rm WA}^{(q)}/\Gamma \leqslant 0.023$. Assuming 40% of SU(3) breaking then gives

$$\left|\frac{\Delta\Gamma_{\rm WA}^{(s)} - \Delta\Gamma_{\rm WA}^{(d)}}{\Gamma}\right| \leqslant 0.9\%. \tag{62}$$

Although with extreme variations, allowing also $|K_1|$ and $|K_2/3|$ to differ (for example, by choosing a renormalization scale μ different from m_b), this difference could be up to 2.5%, it is more likely that the correction (62) will actually be much smaller because of various possible cancellations in Eq. (61) and because 40% is probably an overestimate of the magnitude of SU(3) breaking. Furthermore, from previous experience with lattice calculations of bag parameters in the B-meson system it seems likely that the $B_i^{(q)}$ will not differ too dramatically from one, so that Eq. (62), although admittedly somewhat crude, is probably on the safe side.

Summarizing the discussion of the various contributions to Eq. (50) we conclude that, most likely, the ratio of rates of B_s and B_d mesons should differ from unity by no more than one percent:

$$\left|\frac{\Gamma_{B_s}}{\Gamma_{B_d}} - 1\right| < 1\%. \tag{63}$$

V. SUMMARY

In this paper we have analyzed the theoretical prediction for $\Delta\Gamma_{B_s}$ within the framework of the heavy quark expansion. We have calculated the explicit next-to-leading $O(1/m_b)$ corrections in the operator product expansion for the transition matrix element. In addition to the two leading dimension-six operators, five new operators of dimension seven appear at this level. The matrix elements of the latter operators were evaluated using factorization, which should give a fair estimate of these subleading corrections. Their effect on $\Delta\Gamma_{B_s}$, formally of order $O(\Lambda_{\rm QCD}/m_b)$ and $O(m_s/m_b)$, turned out to be sizable numerically, causing a 30% reduction of the leading order prediction.

We performed a numerical investigation of $\Delta\Gamma_{B_s}$ with emphasis on theoretical errors, which are presently dominated by the uncertainties in hadronic matrix elements. These errors are still rather large and lead to a prediction of $(\Delta\Gamma/\Gamma)_{B_s} = 0.16^{+0.11}_{-0.09}$ However, a systematic improvement of this result is possible, in particular by progress in lattice QCD. In future it would be desirable to measure on the lattice the S-P four-fermion operator along with the V-A operator that has received most attention in the past due to its connection with the mass difference. Eventually, an accuracy of 10% for $\Delta\Gamma_{B_s}$ should be feasible when the next-to-leading analysis of short-distance corrections is also completed.

The effects of penguin operators and contributions from CKM-suppressed modes have also been considered. They were shown to give only a few percent relative correction in $(\Delta\Gamma/\Gamma)_{B_s}$ and are thus negligible in view of other uncertainties.

We further studied the B_s-B_d lifetime difference and quantified the expectation $\tau_{B_s} \approx \tau_{B_d}$, estimating $|\tau(B_s)/\tau(B_d) - 1| < 1\%$. This result is useful input for experimental analyses of $\Delta\Gamma_{B_s}$.

To put our theoretical analysis into perspective, we have included a short discussion of the current experimental situation concerning $\Delta\Gamma_{B_s}$. Using information on $\tau(B_s \to J/\psi\phi)$ and $\tau(B_s) = \tau(B_d)$, we have attempted a preliminary extraction of $\Delta\Gamma_{B_s}$, obtaining $(\Delta\Gamma/\Gamma)_{B_s} \geqslant 0.3 \pm 0.4$. This is still inconclusive but can be improved by better statistics in the future. We have also proposed an alternative route towards a measurement of $\Delta\Gamma_{B_s}$ that makes use of the $\phi\phi X$ and/or $D_s^\pm \phi X$ final states in B_s decay, which are expected to be dominantly CP even. The present experimental information may be complemented by the bound $(\Delta\Gamma/\Gamma)_{B_s} \leqslant 2B(b \to c\overline{cs})_{B_s} \approx 0.44 \pm 0.06$. This bound is not very strong, but it has the advantage of being valid independently of the heavy quark expansion and it is interesting for principal reasons.

In addition, we have briefly reviewed some phenomenological applications that could be opened up by further progress on the experimental as well as the theoretical side. These possibilities include new methods to study CP violation, complementary information on ΔM_{B_s} in case $\Delta\Gamma_{B_s}$ is measured first, and alternative constraints on $|V_{td}/V_{ts}|$, especially for small values of this ratio. Finally, the theory of inclusive B decays itself can be expected to profit from a confrontation of the heavy quark expansion for $\Delta\Gamma_{B_s}$ with experiment. In this respect $\Delta\Gamma_{B_s}$ provides an important special case that directly probes $O(1/m_b^3)$ contributions.

As we have seen, the topic of $\Delta\Gamma_{B_s}$ touches upon a rich variety of interesting physics issues and certainly merits the continued efforts needed to address the problems that are still unresolved.

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APPENDIX A: PENGUIN CONTRIBUTIONS TO $\Delta\Gamma_{B_s}$

In the following we discuss the impact of penguin operators on the width difference $\Delta\Gamma_{B_s}$. We will work to leading logarithmic accuracy in QCD and include the charm quark mass effects. For the purpose of this section we shall neglect $1/m_b$ corrections, CKM-suppressed modes, and light quark masses.

Taking gluonic penguin operators into account, the effective Hamiltonian in Eq. (10) is generalized to

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{cb}^{*}V_{cs}\sum_{r=1}^{6}C_rQ_r, \tag{A1}$$

where

$$Q_1=(\overline{b_i}s_i)_{V-A}(\overline{c_j}c_j)_{V-A},\quad Q_2=(\overline{b_i}s_j)_{V-A}(\overline{c_j}c_i)_{V-A}, \tag{A2}$$

$$Q_3=(\overline{b_i}s_i)_{V-A}(\overline{q_j}q_j)_{V-A},\quad Q_4=(\overline{b_i}s_j)_{V-A}(\overline{q_j}q_i)_{V-A}, \tag{A3}$$

$$Q_5=(\overline{b_i}s_i)_{V-A}(\overline{q_j}q_j)_{V+A},\quad Q_6=(\overline{b_i}s_j)_{V-A}(\overline{q_j}q_i)_{V+A}. \tag{A4}$$

A summation over $q=u, d, s, c$ is implied. $C_1,\ldots,C_6$ are the corresponding Wilson coefficient functions. $C_{1,2}$ have already been given in Eq. (11). For a recent review of this subject see [34], where further details may be found. Using our standard parameter set with $\mu=m_b$ the numerical values are

$$(C_1,\ldots,C_6)=(-0.272, 1.120, 0.012, -0.028, 0.008, -0.035). \tag{A5}$$

The calculation of the transition operator (9) using the extended operator basis is straightforward and leads to

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s}=16\pi^2B(B_s\to Xe\nu)\frac{f_{B_s}^2M_{B_s}}{m_b^3}\frac{V_{cs}^2}{g(z)\tilde{\eta}_{\rm QCD}}\Bigg\{\sqrt{1-4z}\Bigg[[2(1-z)(K_1+K_1'+K_1'')+(1-4z)(K_2+K_2'+K_2'')+6z(K_3'+K_3'')]\left(1+\frac{1}{N_c}\right)B+(1+2z)(K_2+K_2'+K_2''-K_1-K_1'-K_1'')\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S\Bigg]+3\Bigg[(2K_1''+K_2'')\left(1+\frac{1}{N_c}\right)B+(K_2''-K_1'')\frac{M_{B_s}^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right)B_S\Bigg]\Bigg\}. \tag{A6}$$

$K_{1,2}$ are defined in Eq. (15) and the remaining coefficients read

$$K_1'=2(N_cC_1C_3+C_1C_4+C_2C_3),\quad K_2'=2C_2C_4, \tag{A7}$$

$$K_3'=2(N_cC_1C_5+C_1C_6+C_2C_5+C_2C_6), \tag{A8}$$

$$K_1''=N_cC_3^2+N_cC_5^2+2C_3C_4+2C_5C_6,\quad K_2''=C_4^2+C_6^2, \tag{A9}$$

$$K_3''=2(N_cC_3C_5+C_3C_6+C_4C_5+C_4C_6). \tag{A10}$$

These expressions represent the interference of penguin operators with the leading operators $Q_{1,2}$ (coefficients K_i') and penguin-penguin contributions (coefficients K_i''). Numerically, they reduce $(\Delta\Gamma/\Gamma)_{B_s}$ by 0.0114, which is about 5% of the result without penguin contributions $(\Delta\Gamma/\Gamma)_{B_s}=0.22$, neglecting $1/m_b$ corrections. Note that since $C_3,\ldots,C_6$ are small, the effect of penguin contributions is dominated by the K_i', while the K_i'' are negligible.

APPENDIX B: CABIBBO-SUPPRESSED CONTRIBUTIONS TO $\Delta\Gamma_{B_s}$

In this appendix we briefly consider the CKM-suppressed contributions to $\Delta\Gamma_{B_s}$. They arise from $u\overline{c}$ $(\overline{u}c)$ or $u\overline{u}$ intermediate states in the diagram of Fig. 1. For our estimate we include again QCD corrections in the leading logarithmic approximation and keep charm quark mass effects. We neglect $1/m_b$ corrections and the small impact of penguin operators in the $u\overline{u}$ channel.

The contribution from $u\overline{c}$ and $\overline{u}c$ intermediate states is then found to be

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s,uc}=16\pi^2B(B_s\to Xe\nu)\frac{(1-z)^2}{g(z)\tilde{\eta}_{\rm QCD}}\frac{f_{B_s}^2M_{B_s}}{m_b^3}V_{cs}^2\times 2\,{\rm Re}\frac{\lambda_u}{\lambda_c}\Bigg[[(2+z)K_1+(1-z)K_2]\left(1+\frac{1}{N_c}\right)B+(1+2z)(K_2-K_1)\left(2-\frac{1}{N_c}\right)B_S\Bigg], \tag{B1}$$

where $\lambda_i = V_{ib}^* V_{is}$. Compared to the leading, CKM-allowed contribution with two charm quarks in the intermediate state, expression (B1) is suppressed by a factor

$$2\,\mathrm{Re}\frac{\lambda_u}{\lambda_c} = 2\lambda^2 \varrho \leq \pm 3\%. \tag{B2}$$

Here, we have used the Wolfenstein parametrization and the result that ϱ is restricted by $|\varrho| < 0.3$ in the standard model [34]. Since the difference between Eq. (B1) and the CKM-allowed contribution [see Eq. (30)] due to different charm quark mass dependences turns out to be negligible numerically, relation (B2) determines essentially the relative importance of Eq. (B1) for $(\Delta\Gamma/\Gamma)_{B_s}$. Note that the sign of Eq. (B1) is not yet fixed because both positive and negative values are still allowed for ϱ. Since ϱ could be close to zero, the CKM-suppressed contribution (B1) might also be well below the 3% given above. In any case, it can be safely neglected.

The contribution with two internal up quarks can be obtained from Eq. (B1) by replacing $2\,\mathrm{Re}(\lambda_u/\lambda_c) \to \mathrm{Re}(\lambda_u/\lambda_c)^2$ and setting $z \to 0$ everywhere except in the argument of $g(z)$. Since $|\mathrm{Re}(\lambda_u/\lambda_c)^2|$ can be estimated to be smaller than 4×10^{-4}, the resulting expression is still much more suppressed than Eq. (B1) and therefore completely irrelevant.

APPENDIX C: COMMENT ON τ_{B^+}/τ_{B_d}

Some of the issues in the calculation of lifetime differences among B_s and B_d mesons that we have discussed in this paper are also relevant for the prediction of τ_{B^+}/τ_{B_d}. We will therefore take the opportunity to also have a brief look at the question of the $B^+ - B_d$ lifetime difference. In the literature this quantity has been estimated to be [3]

$$\frac{\tau_{B^+}}{\tau_{B_d}} \simeq 1 + 0.05 \times \frac{f_B^2}{(200\ \mathrm{MeV})^2}, \tag{C1}$$

predicting the B^+ lifetime to exceed τ_{B_d} by several percent. In the following we would like to reexamine this estimate, emphasizing the theoretical uncertainties that are involved in its derivation. Assuming isospin symmetry, the mechanisms that produce a difference in τ_{B^+} and τ_{B_d} first enter at the level of dimension-six operators, or equivalently at $O(1/m_b^3)$, in the heavy quark expansion [3]. These effects are weak annihilation for the B_d and Pauli interference in the case of B^+. As we have seen in Sec. IV, the weak annihilation contribution to τ_{B_d} is very small and we shall neglect it. In this approximation the difference between τ_{B^+} and τ_{B_d} arises only through Pauli interference and one may write

$$\frac{\tau_{B^+}}{\tau_{B_d}} = 1 + 24\pi^2 B(B\to X e\nu)\frac{f_B^2}{m_b^2} V_{ud}^2 \frac{(1-z)^2}{g(z)\,\tilde{\eta}_{\mathrm{QCD}}} \times\left[(C_-^2 - C_+^2)B_1^{(u)} - \frac{1}{N_c}(C_+^2 + C_-^2)B_2^{(u)}\right], \tag{C2}$$

where

$$\langle B^+|(\overline{b_i}u_i)_{V-A}(\overline{u_j}b_j)_{V-A}|B^+\rangle = f_B^2 m_b^2 B_1^{(u)},$$

$$\langle B^+|(\overline{b_i}u_j)_{V-A}(\overline{u_j}b_i)_{V-A}|B^+\rangle = \frac{1}{N_c} f_B^2 m_b^2 B_2^{(u)} \tag{C3}$$

define the bag parameters $B_{1,2}^{(u)}$. The Wilson coefficients $C_\pm$ have been given in Eq. (11).

With $m_b = 4.8$ GeV, $m_c = 1.4$ GeV, $\Lambda_{LO} = 0.2$ GeV, and taking $B_{1,2}^{(u)} = 1$, $f_B = 0.2$ GeV, one finds $\tau_{B^+}/\tau_{B_d} = 1.02$, indicating a slightly longer lifetime for B^+ than that for B_d. This number can, however, not be viewed as a very accurate prediction. In fact, the two contributions proportional to $B_1^{(u)}$ and $B_2^{(u)}$ in Eq. (C2) enter with different sign. This leads to a partial cancellation that has the tendency to make the result unstable. For instance, allowing the unphysical scale $\mu = O(m_b)$ in the coefficients $C_\pm$ to vary from $m_b/2$ to $2m_b$ gives a range of 1.00–1.06 for the B^+ to B_d lifetime ratio. Switching off short-distance QCD corrections completely $(C_\pm \to 1)$, the hierarchy of lifetimes would even be reversed to $\tau_{B^+}/\tau_{B_d} = 0.95$, which is another aspect of the large sensitivity to QCD effects. An alternative way of estimating the present uncertainty is to allow a variation in the bag parameters (keeping $\mu = m_b$ fixed). A range of $B_{1,2}^{(u)} = 1.0 \pm 0.3$ is certainly conceivable, considering the uncertainties in the nonperturbative dynamics and from the scale and scheme dependence in the long-distance to short-distance matching. Assuming this, we obtain for $f_B = 0.2$ GeV, $\tau_{B^+}/\tau_{B_d} = 1.02 \pm 0.04$. A combination of both variations, of scale and bag parameters, would even allow us to obtain a lifetime difference of up to 20%, $\tau_{B^+}/\tau_{B_d} \sim 1.2$. Although we consider this case highly unlikely, the point to note is that a lifetime that large could be tolerated by QCD as well as equal lifetimes, or even a marginally shorter lifetime for the B^+. A decisive improvement of this situation could only be achieved by a reliable lattice calculation of $B_{1,2}^{(u)}$ in conjunction with a next-to-leading order computation of short-distance QCD corrections to ensure a proper matching in renormalization scheme and scale between Wilson coefficients and hadronic matrix elements. Alternatively, one could use the present measurement $\tau_{B^+}/\tau_{B_d} = 1.06 \pm 0.04$ [28] to constrain the bag parameters. At present, such constraints appear to be of limited use, because of the large renormalization scale dependence of Pauli interference at leading order. Similar conclusions have been reached in the recent paper by Neubert and Sachrajda [38].

The authors of [3] have modeled the bag parameters in their estimate of τ_{B^+}/τ_{B_d} by factorizing at a low scale $\mu_h < m_b$ and explicitly including the leading logarithms of HQET. This yields

$$B_1^{(u)}(m_b) = \frac{8}{9}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_h)}\right]^{-3/50} + \frac{1}{9}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_h)}\right]^{12/25},$$

$$B_2^{(u)}(m_b) = \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_h)}\right]^{12/25}. \tag{C4}$$

Taking $\mu_h = 1$ GeV this gives $B_1^{(u)}(m_b) = 1.01$, $B_2^{(u)}(m_b) = 0.72$, and $\tau_{B^+}/\tau_{B_d} = 1.04$ (for $f_B = 0.2$ GeV), favoring $\tau_{B^+} > \tau_{B_d}$. However, as discussed at the end of Sec. II, the quantitative reliability of an estimate based on hybrid logarithms is not entirely clear.

[1] I. Dunietz, Phys. Rev. D **52**, 3048 (1995).
[2] T. E. Browder and S. Pakvasa, Phys. Rev. D **52**, 3123 (1995).
[3] I. Bigi *et al.*, in *B Decays,* 2nd ed., edited by S. Stone (World Scientific, Singapore, 1994).
[4] D. Atwood, I. Dunietz, and A. Soni, Report No. FERMILAB-PUB-94/388-T (unpublished); R. Fleischer and I. Dunietz, Report No. FERMILAB-PUB-96/079-T, hep-ph/9605220 (unpublished); Report No. FERMILAB-PUB-96/080-T, hep-ph/9605221 (unpublished).
[5] A.S. Dighe, I. Dunietz, and R. Fleischer, Report No. FERMILAB-PUB-96/081-T (unpublished).
[6] J. S. Hagelin, Nucl. Phys. **B193**, 123 (1981).
[7] E. Franco, M. Lusignoli, and A. Pugliese, Nucl. Phys. **B194**, 403 (1982).
[8] L. L. Chau, Phys. Rep. **95**, 1 (1983).
[9] A. J. Buras, W. Słominski, and H. Steger, Nucl. Phys. **B245**, 369 (1984).
[10] M. B. Voloshin, N. G. Uraltsev, V. A. Khoze, and M. A. Shifman, Sov. J. Nucl. Phys. **46**, 112 (1987).
[11] A. Datta, E. A. Paschos, and U. Türke, Phys. Lett. B **196**, 382 (1987); A. Datta, E. A. Paschos, and Y. L. Wu, Nucl. Phys. **B311**, 35 (1988).
[12] R. Aleksan *et al.*, Phys. Lett. B **316**, 567 (1993).
[13] Y. Grossman, Weizmann Institute Report No. WIS-96/13/Mar-PH, hep-ph/9603244 (unpublished).
[14] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[15] A. J. Buras, M. Jamin, and P. H. Weisz, Nucl. Phys. **B347**, 491 (1990).
[16] V. Novikov, M. Shifman, A. Vainshtein, and V. Zakharov, Fortschr. Phys. **32**, 585 (1984); in the context of heavy quark expansions, see also M. Shifman, TASI Lectures 1995, hep-ph/9510377 (unpublished).
[17] W. Kilian and T. Mannel, Phys. Lett. B **301**, 382 (1993).
[18] Y. Nir, Phys. Lett. B **221**, 184 (1989).
[19] M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987).
[20] H. Politzer and M. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988).
[21] J. Flynn, O. Hernández, and B. Hill, Phys. Rev. D **43**, 3709 (1991).
[22] I. I. Bigi and N. G. Uraltsev, Z. Phys. C **62**, 623 (1994).
[23] We are grateful to David Atwood for pointing out to us that the right-hand side must have a factor of 2, which is missing in [1]. See also D. Atwood, I. Dunietz, and J. Incandela, Report No. FERMILAB-PUB-95/402-T (unpublished).
[24] Y. Kwon, Talk given at Moriond, 1996 (unpublished).
[25] G. Buchalla, I. Dunietz, and H. Yamamoto, Phys. Lett. B **364**, 188 (1995).
[26] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).
[27] E. Meschi, Report No. FERMILAB-CONF-96/013-E (unpublished).
[28] We thank Slawek Tkaczyk, who averaged the available CDF measurements with the most recent LEP average as reported by M.-H. Schune, Talk given at Moriond, 1996 (unpublished).
[29] We thank G. Eigen for a discussion on this point.
[30] I. Dunietz, Report No. FERMILAB-PUB-94/163-T, hep-ph/9409355 (unpublished).
[31] Hitoshi Yamamoto (private communication).
[32] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986); I. Bigi *et al.*, in *CP violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989), p. 175.
[33] ALEPH Collaboration, D. Buskulic *et al.*, Report No. CERN-PPE/96-30 (unpublished).
[34] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. (to be published).
[35] A. Ali and C. Greub, Phys. Lett. B **293**, 226 (1992); Z. Phys. C **60**, 433 (1993); J. M. Soares, Phys. Rev. D **49**, 283 (1994).
[36] Y. Nir, Phys. Lett. B **327**, 85 (1994).
[37] G. Buchalla and A. J. Buras, Nucl. Phys. **B412**, 106 (1994).
[38] M. Neubert and C.T. Sachrajda, Report No. CERN-TH/96-19, hep-ph/9603202 (unpublished).
[39] J. Lee-Franzini *et al.*, Phys. Rev. Lett. **65**, 2947 (1990); DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **68**, 353 (1995).