

## Comparative study of the hadronic production of $B_c$ mesons

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A comparative study of the full  $\alpha_s^4$  perturbative QCD calculation of the hadronic production of the  $B_c$  and  $B_c^*$  mesons and the fragmentation approximation is presented. We examined the various subprocesses in detail and the distribution  $C(z)$ , where  $z$  is twice the fraction of the  $B_c$  or  $B_c^*$  meson energy in the center of mass of the subprocess, which proved insightful and revealed the importance of certain nonfragmentation contributions. We concluded that the condition for the applicability of the fragmentation approximation in hadronic collisions is that the transverse momentum  $P_T$  of the produced  $B_c$  and  $B_c^*$  be much larger than the mass of the  $B_c$  meson: i.e.,  $P_T \gg M_{B_c}$ . Numerical results for the cross sections at the Fermilab Tevatron are also presented using updated parton distribution functions with various kinematic cuts. [S0556-2821(96)03619-3]

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The heavy flavored  $\bar{b}c$  meson states have attracted considerable interest due to their interesting properties. The experimental search for these mesons is now under way at high energy colliders such as the CERN  $e^+e^-$  collider LEP and the Fermilab Tevatron  $\bar{p}p$  collider. Results are expected in the near future. An important theoretical task, relevant to these experiments, is to calculate the production cross sections. Since they are weakly bound states hadronization is relatively simple and their production cross sections can be accurately predicted in the framework of perturbative QCD (PQCD). At LEP the production of the pseudoscalar ground state  $B_c$  and the vector meson state  $B_c^*$  is dominated by  $Z^0$  decay into a  $b\bar{b}$  pair, followed by the fragmentation of a  $\bar{b}$  quark into the  $B_c$  or  $B_c^*$  meson [1,2]. Hadronic production, as first pointed out by Chang and Chen [3], is dominated at high energies by the subprocess  $gg \rightarrow B_c(B_c^*)b\bar{c}$ . The interesting question we address here is how well the fragmentation approximation to the full order  $\alpha_s^4$  calculation works for the hadronic production of the  $B_c$  and  $B_c^*$  mesons and in which kinematic region.

The hadronic production of the  $B_c$  and  $B_c^*$  can be calculated fully to order  $\alpha_s^4$  in PQCD, which involves a difficult numerical calculation. Since Chang and Chen [3] first presented the numerical results for the hadronic production, calculations have also been done by several other authors [4–10], not all of which are in agreement. Slabospitsky [4] claimed an order of magnitude larger result than that by Chang and Chen [3]. Berezhnoy *et al.* [5] obtained a result larger than that by Chang and Chen [3] but smaller than the one by Slabospitsky [4]. Masetti and Sartogo [6] found a result similar to Berezhnoy *et al.* [5]. More recently, Berezhnoy *et al.* [8] found that a color factor 1/3 was overlooked in their previous work [5]. After including this factor, their revised result is in agreement with that of Chang and Chen [3]. Baranov [9] independently also obtained results similar to those of Chang and Chen [3]. Kołodziej *et al.* [10] presented

results using different parton distribution functions and energy scale from those used by Chang and Chen [3] so it is difficult to directly compare their final results. However, their results for the cross sections for the subprocess are similar to others [7,8]. Therefore, we are confident that the results of the full order  $\alpha_s^4$  PQCD calculations in Refs. [3,7–10] are in agreement.

An alternative way to calculate the hadronic production is to use the fragmentation approximation. From the general factorization theorem it is clear that, for large transverse momentum  $P_T$  of the  $B_c$  or  $B_c^*$ , hadronic production is dominated by fragmentation. The calculation can then be considerably simplified using this approximation, as was first done by Cheung [11]. Subsequently, the comparison between the full  $\alpha_s^4$  calculation and the fragmentation approximation has been discussed by several authors [7,8,10]. However, very different conclusions have been drawn, although their numerical results are similar. Chang *et al.* [7] found that when  $\sqrt{s}$  and  $p_T$  are small the difference between the fragmentation approximation and the  $\alpha_s^4$  calculation is large. Berezhnoy *et al.* [8] claimed that the fragmentation approximation breaks down even for very large  $P_T$  by examining the ratio of  $B_c^*$  to  $B_c$  production. Kołodziej *et al.* [10] claimed that the fragmentation approximation works well if  $P_T$  exceeds about 5–10 GeV, which is comparable to the  $B_c$  mass, by investigating the  $P_T$  distribution of the  $B_c$  meson.

It is nontrivial to clarify these points since the full calculation is quite complicated, the dominant subprocess involving 36 Feynman diagrams; but the importance of investigating this issue is twofold: From the theoretical perspective, it provides an ideal example to quantitatively examine how well the fragmentation approximation works for calculating the hadronic production of heavy flavored mesons. In this process both the full  $\alpha_s^4$  contributions and the fragmentation approximation can each be calculated reliably. Experimentally, it is important to have a better understanding of the

production of the  $B_c$  mesons in the small  $P_T$  region where the  $B_c$  production cross section is the largest. The  $P_T$  distribution decreases very rapidly as  $P_T$  increases.

We carried out a detailed comparative study of the fragmentation approximation and the full  $\alpha_s^4$  QCD perturbative calculation. We first studied the subprocesses carefully. By analyzing the singularities appearing in the amplitudes and the  $P_T$  distributions of the subprocesses we gained very important insight into the processes. We then calculated the entire hadronic production cross section for  $p\bar{p}$  (or  $pp$ ) collisions at the Tevatron energy. To facilitate a quantitative comparison we found it very instructive to examine the distribution in  $z$ , twice the energy fraction carried by the  $B_c$  meson in the center of mass of the subprocess, which is, in principle, observable. By investigating the  $z$  distribution we found that the fragmentation contribution dominates when and only when  $P_T \gg M_{B_c}$ . We also found that it is insufficient to evaluate the validity of the fragmentation approximation by only examining the  $P_T$  distribution. In fact, we found that it is simply fortuitous that the  $P_T$  distribution in the fragmentation approximation for the case of the  $B_c$  meson is close to the results of the full order  $\alpha_s^4$  calculation when  $P_T$  does not satisfy the above condition; i.e.,  $P_T \gg M_{B_c}$ . It is not sufficient for  $P_T$  to simply be the same order as  $M_{B_c}$ , as we discuss below.

In order to obtain some insight into the process we first focus the discussion on the subprocess. At the lowest order  $\alpha_s^4$ , there are 36 Feynman diagrams responsible for the dominant gluon fusion subprocess  $g(k_1) + g(k_2) \rightarrow B_c(p) + b(q_2) + \bar{c}(q_1)$ , where  $k_1, k_2, p, q_1$ , and  $q_2$  are the respective momenta. When the energy in the center-of-mass system  $\sqrt{\hat{s}}$  is much larger than the heavy quark mass the main contributions to the cross section come from the kinematic region where certain of the amplitudes in the matrix element are nearly singular; i.e., some of the quark lines or gluon lines are nearly on-shell and the related propagators in the Feynman diagrams are nearly singular. This results in the cross section for the subprocess coming from the lowest twist contributions being proportional to  $(1/\hat{s})(f_{B_c}^2/M_{B_c}^2)$ , where  $f_{B_c}$  is the  $B_c$  decay constant, with some logarithmic correction terms such as  $\ln(\hat{s}/M_{B_c}^2)$ . Now, the possible singularities in the square of the matrix element for this subprocess must arise from the inverse power(s) of the following factors, or their products, which can appear in all the possible denominators of the quark and gluon propagators in the Feynman gauge:

$$q_i \cdot k_j, \quad p \cdot k_j, \quad (\alpha_i p + q_i)^2, \quad \text{and} \quad (k_j - \alpha_1 p - q_1)^2, \quad (1)$$

where  $i, j=1,2$  and  $\alpha_{1,2}=m_{c,b}/(m_c+m_b)$  is the ratio of quark masses. It is easy to see that when the  $P_T$  of the  $B_c$  meson is large only  $(\alpha_i p + q_i)^2$  can still be small ( $\sim m_i^2$ ). The fragmentation functions can then be extracted from the most singular part containing the inverse powers of this factor in the square of the matrix element. It then follows that in the large  $P_T$  region the subprocess is dominated by the fragmentation approximation. However, when the  $P_T$  of the  $B_c$  meson is small the produced  $B_c$ , as well as the  $b$  and the  $\bar{c}$  quarks, can be soft or collinear with the beam. In this

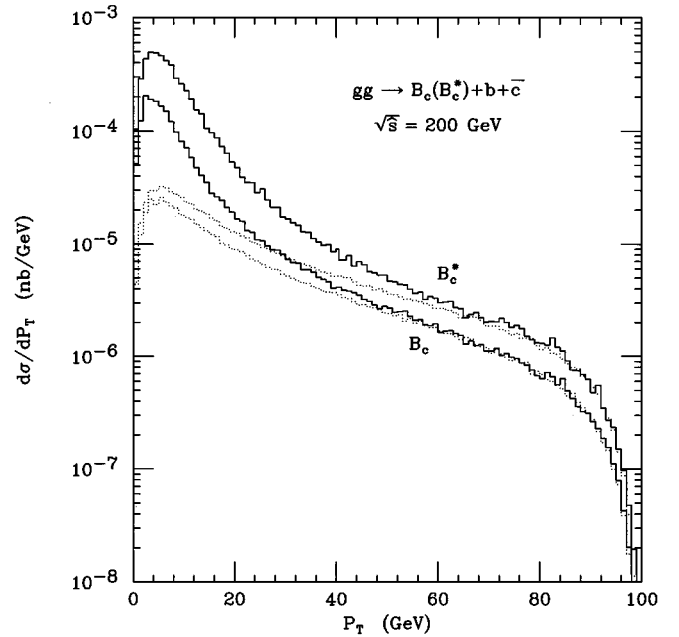


FIG. 1. The  $P_T$  distributions of the  $B_c$  and  $B_c^*$  meson for the subprocess with  $\sqrt{\hat{s}} = 200$  GeV. The solid and the dotted lines correspond to the full  $\alpha_s^4$  calculation and the fragmentation approximation, respectively.

region the square of the matrix element is highly singular because *two or more* of the internal quarks or gluons in certain Feynman diagrams can *simultaneously* be nearly on mass shell. Although this region is a smaller part of the phase space, these nearly singular Feynman diagrams, in fact, make a large contribution to the cross section and dominate the small  $P_T$  region.

Generally, in the square of the matrix element we can isolate all the terms which contribute to the lowest twist cross sections using singularity power counting rules [17]. When  $\sqrt{\hat{s}} \gg M_{B_c}$  the lowest twist contributions dominate, while the higher twist contributions are suppressed by a factor  $m^2/\hat{s}$  and are negligible. We can decompose the terms which contribute to the lowest twist cross sections into two components: One component is the fragmentation contribution, which dominates the large  $P_T$  region. The other is the nonfragmentation component, which comes from the other singular parts of the matrix element, those in which *two or more* quarks or gluons are nearly on shell, as discussed above. This nonfragmentation component dominates in the smaller  $P_T$  region. The contributions of these two components are quite clearly distinguishable in the  $P_T$  distribution of the subprocess, particularly at large  $\sqrt{\hat{s}}$ . In Fig. 1 we show the  $P_T$  distribution of the subprocess when  $\sqrt{\hat{s}} = 200$  GeV for both the full  $\alpha_s^4$  calculation and the fragmentation approximation with  $\alpha_s = 0.2$ ,  $m_b = 4.9$  GeV,  $m_c = 1.5$  GeV, and  $f_{B_c} = 480$  MeV. In the fragmentation calculation, to reduce the error caused by the phase space integrations, we directly used the squared matrix elements [1,14], from which the fragmentation functions are derived [1,14], rather than the fragmentation functions themselves. It is easy to see in Fig. 1 that when  $P_T$  is larger than about 30 GeV for the  $B_c$  and

about 40 GeV for the  $B_c^*$  the fragmentation approximation is close to the full  $\alpha_s^4$  calculation. However, when the value of the  $P_T$  is smaller than about 30 GeV for the  $B_c$  and about 40 GeV for the  $B_c^*$  the deviation between the fragmentation approximation and the full calculation becomes large and the nonfragmentation component clearly dominates their production. This critical value of  $P_T$  is certainly much larger than the heavy quark masses, or the  $B_c$  meson mass; we also found that it slowly increases with increasing  $\hat{s}$ , which may indicate that there is an additional enhancement due to logarithmic terms such as  $\ln\hat{s}/m^2$  in the nonfragmentation component compared to the fragmentation component. When  $\sqrt{\hat{s}}$  is not very large this two-component decomposition is less distinct, since the higher twist terms cannot be ignored [7,8,10]. In this case, the fragmentation approximation is not a very good approximation, there being quite a large discrepancy with the full  $\alpha_s^4$  calculation [7,8,10].

A similar process is the production of  $B_c$  and  $B_c^*$  mesons in photon-photon collisions, which is also instructive to examine more carefully. A comparative study of the full  $\alpha_s^2$  calculation with the fragmentation approximation in this process was presented in Refs. [12,13], where it was claimed that the fragmentation approximation is not valid. There are 20 Feynman diagrams which can be divided into four gauge-invariant subsets corresponding to various attachments of photons onto the quark lines; i.e., subsets I, II, III, and IV corresponding, respectively, to the attachment of both photons onto the  $b$  quark line, onto the  $\bar{c}$  quark line, one photon onto the  $b$  quark line and the other onto the  $\bar{c}$ , and the interchange of  $b$  and  $\bar{c}$ . Subset I is dominated by the  $\bar{b}$  quark fragmentation into the  $B_c$  when  $P_T$  is large, as discussed above. Subsets III and IV, called recombination diagrams, can only contribute to the nonfragmentation component and decrease rapidly when  $P_T \gg M_{B_c}$ , as also discussed above. However, subset II is somewhat unusual. Although this contribution is relatively suppressed by the smaller probability for subsequent  $b\bar{b}$  quark creation, it nevertheless gives quite a large contribution to the total cross section because of the enhancement of the  $c$  quark electric charge. For example, we found that when  $\sqrt{\hat{s}} = 100$  GeV the result of the full calculation is an order of magnitude larger than the fragmentation calculation, even for large  $P_T$ . However, when  $\sqrt{\hat{s}}$  becomes extremely large, e.g.,  $\sqrt{\hat{s}} = 800$  GeV, the contribution of this subset II is dominated by the  $c$  quark fragmentating into the  $B_c$  meson when  $P_T \gg M_{B_c}$ . This implies that the higher twist terms must have very large coefficients, which violates that naive power counting rules which imply that the higher twist terms are suppressed by  $M_{B_c}^2/\hat{s}$  and are, therefore, negligible when  $\sqrt{\hat{s}} \sim 100$  GeV  $\gg M_{B_c}$ . Fortunately, this type of contribution, which involves  $b\bar{b}$  quark pair creation, is not important in the hadronic production of the  $B_c$  and  $B_c^*$  simply because the factor of 16 enhancement due to the larger  $c$  quark electric charge is absent.

Having investigated the subprocess, we next calculated  $B_c$  and  $B_c^*$  production at  $p\bar{p}$  colliders, particularly the Fermilab Tevatron. From our above analysis when  $P_T \gg M_{B_c}$  and, therefore,  $\sqrt{\hat{s}} \gg M_{B_c}$  the process is  $\bar{b}$  quark fragmentation dominated. However, at smaller  $P_T$  the fragmentation

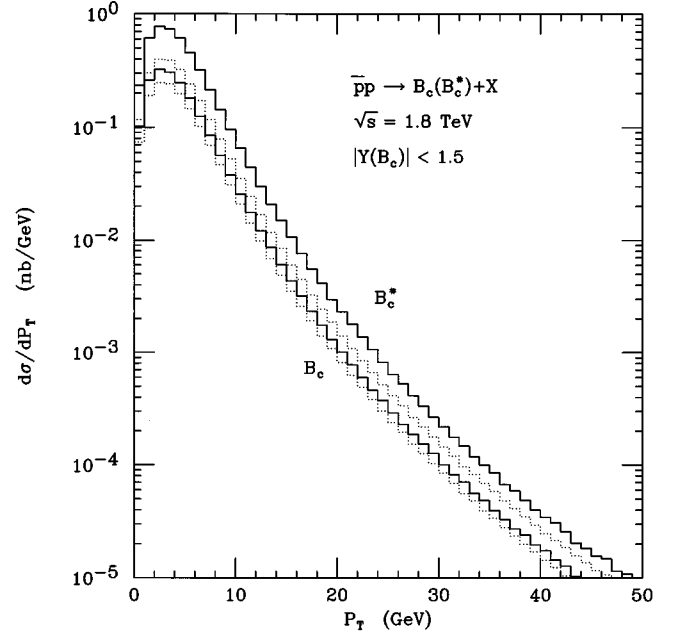


FIG. 2. The  $P_T$  distributions of  $B_c$  and  $B_c^*$  meson production at the Tevatron energy  $\sqrt{s} = 1.8$  TeV. The solid and the dotted lines correspond to the full  $\alpha_s^4$  calculation and the fragmentation approximation, respectively.

approximation breaks down. In Fig. 2, we compare the  $P_T$  distributions of the  $B_c$  and the  $B_c^*$  mesons coming from the full  $\alpha_s^4$  calculation with the fragmentation approximation. In these calculations we used the CTEQ3M parton distribution functions [15]. There is some ambiguity in the choice of the energy scale  $\mu$  in  $\alpha_s(\mu)$  and this is clearly dependent on the form of the factorization. For example, the component of the  $\bar{b}$  fragmentating into the  $B_c$  can be factorized with a  $b\bar{b}$  pair first being created at a very short distance, with a reasonable choice of this scale being  $\mu_1 = \sqrt{m_b^2 + P_{Tb}^2}$ , followed by the  $\bar{b}$  fragmentating into a  $B_c$  or  $B_c^*$  with a reasonable choice of this energy scale being  $\mu_2 = 2m_c$ . Factorization in the nonfragmentation component is more complicated and the choice of energy scales is not obvious. For simplicity, we will use a uniform choice of energy scales, choosing the same scale as set in the  $\bar{b}$  fragmentation component: i.e.,  $\alpha_s^2(\mu_1) \times \alpha_s^2(\mu_2)$ . From the numerical calculations, Fig. 2, we see that for the  $B_c$  meson the  $P_T$  distributions for  $P_T > 5$  GeV are very similar for the full  $\alpha_s^4$  calculations and the fragmentation approximation. This critical value of  $P_T$  is much smaller than that found above in the study of the subprocess. However, for the  $B_c^*$  meson, the result predicted by the full  $\alpha_s^4$  calculation in Fig. 2 is a factor 1.5 to 2.0 greater than the fragmentation calculation over a much larger range of  $P_T$ , more consistent with what was found in the study of the subprocess. Therefore, it is difficult to decide from only the  $P_T$  distributions where the fragmentation approximation is reliable for the hadronic production of the  $B_c$  and  $B_c^*$  mesons. In fact, we found that the use of  $P_T$  distribution alone can be misleading.

To clarify this issue, we introduce the distribution

$$C(z) = \int dx_1 dx_2 g(x_1, \mu) g(x_2, \mu) \frac{d\hat{\sigma}(\sqrt{\hat{s}}, \mu)}{dz}, \quad (2)$$

where  $z \equiv [2(k_1 + k_2) \cdot p] / \hat{s}$  and  $g(x_i, \mu)$  are the gluon distribution functions. In the subprocess center-of-mass  $z$  is simply twice the fraction of the total energy carried by the  $B_c$  or  $B_c^*$  meson. The distribution  $C(z)$  provides a sensitive means to investigate the dynamics of the production process and the fragmentation approximation. Clearly, if the fragmentation approximation is valid,  $[d\hat{\sigma}(\sqrt{\hat{s}}, \mu)]/dz$  can be factorized as

$$\frac{d\hat{\sigma}(\sqrt{\hat{s}}, \mu)}{dz} = \sum_i \hat{\sigma}_{gg \rightarrow Q_i \bar{Q}_i} \otimes D_{Q_i \rightarrow B_c}(z, \mu), \quad (3)$$

where  $D_{Q_i \rightarrow B_c}(z, \mu)$  are the usual fragmentation functions and  $\hat{\sigma}_{gg \rightarrow Q_i \bar{Q}_i}$  is the gluon fusion subprocess cross section for production of the heavy quark pair  $Q_i \bar{Q}_i$ . In this approximation, the integrals over  $x_1$  and  $x_2$  can be performed, the fragmentation function can be factored out, and  $C(z)$  is simply proportional to a sum of the usual fragmentation functions which is insensitive to the parton distribution functions and to the kinematic cuts. However, if the distribution  $C(z)$  is quite different from the fragmentation functions, it is an indication that the fragmentation approximation is not valid. Therefore, comparing  $C(z)$  calculated in the fragmentation approximation with the full order  $\alpha_s^4$  calculation, provides a quantitative criterion to judge the validity of the fragmentation approximation. We emphasize that  $z$  is a very useful variable and is an experimentally measurable quantity, at least in principle.

In Fig. 3 we compare the  $z$  distribution  $C(z)$  calculated in the fragmentation approximation with the full order  $\alpha_s^4$  calculation for  $B_c$  and  $B_c^*$  meson production with a cut of  $P_T > 10$  GeV [Fig. 3(a)] and also with cuts of  $P_T > 20$  GeV and  $P_T > 30$  GeV [Fig. 3(b)]. The  $z$  distribution  $C(z)$  is sensitive to the smallest  $P_T$  region for a given  $P_T$  cut because the  $P_T$  distributions of the  $B_c$  and  $B_c^*$  mesons decrease very rapidly with increasing  $P_T$ . From Fig. 3 some general features are evident: For the  $B_c$  meson, in the fragmentation approximation, for smaller  $P_T$  cuts the  $z$  distributions  $C(z)$  in the higher  $z$  region are overestimated while it is underestimated in the lower  $z$  region; but, after integration over  $z$ , the result is similar to the full  $\alpha_s^4$  calculation. However, for the  $B_c^*$  meson, even for the largest  $P_T$  cut, the  $z$  distribution  $C(z)$  calculated in the fragmentation approximation is underestimated at all values of  $z$  and, after integration over  $z$ , the result is definitely smaller than the full  $\alpha_s^4$  calculation. This feature explains why the  $P_T$  distributions shown in Fig. 2 are similar for the  $B_c$  meson even down to  $P_T \sim M_{B_c}$  but are different for the  $B_c^*$  meson. This also shows that it is simply fortuitous that the  $P_T$  distribution of the  $B_c$  calculated in the fragmentation approximation is similar to that from the full  $\alpha_s^4$  calculation for  $P_T$  below this value, particularly down to  $P_T \sim M_{B_c}$ . It is also clear that when  $P_T$  is increased the distributions become closer. As shown in Fig. 3(b), when  $P_T$  is as large as 30 GeV, the curves calculated in the fragmentation approximation are quite close to the full  $\alpha_s^4$  calculation. This indicates that the fragmentation approximation is valid in the large  $P_T$  region, as expected. We emphasize here that the difference between the fragmentation approximation and full calculation is not universal, but is process dependent.

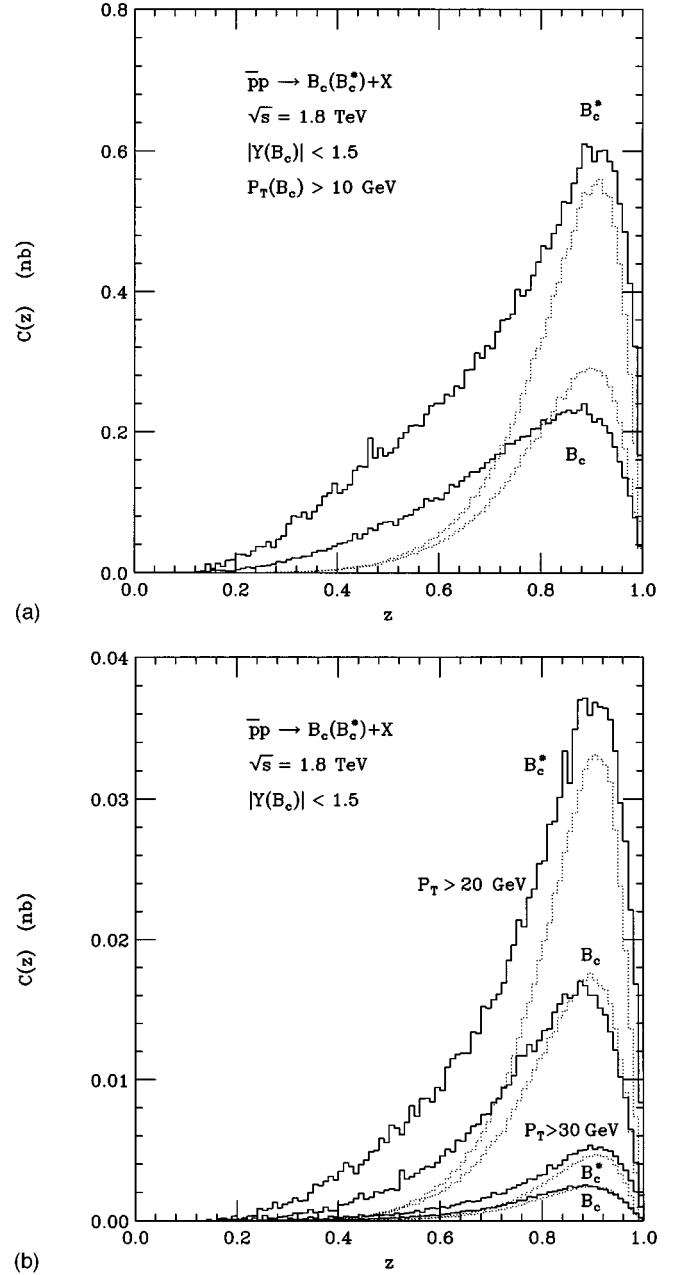


FIG. 3. The  $z$  distributions  $C(z)$  of the  $B_c$  and  $B_c^*$  at the Tevatron energy  $\sqrt{s} = 1.8$  TeV. The solid lines are the full  $\alpha_s^4$  calculation and the dotted lines are the fragmentation approximation (a) with the cut  $P_T > 10$  GeV and (b) with the cuts  $P_T > 20$  GeV and  $P_T > 30$  GeV.

Finally, we examined the ratio of cross sections for  $B_c^*$  production calculated in the fragmentation approximation with the results of the full  $\alpha_s^4$  calculation. As discussed above, for  $B_c^*$  meson production the fragmentation approximation always underestimates the full  $\alpha_s^4$  result. The deviation from the full calculation for the  $B_c^*$  meson can be used as a criterion to test the validity of the fragmentation approximation. The results for the total cross section  $\sigma(P_T > P_{T\min})$  for various  $P_T$  cuts are listed in Table I. We also list the ratio of  $[\sigma_{B_c}(P_T > P_{T\min})]/[\sigma_{B_c^*}(P_T > P_{T\min})]$  from the full calculation and the fragmentation approximation for various  $P_{T\min}$ . Taking agreement within 30% as the criterion

TABLE I. Total cross sections  $\sigma(P_{TB_c} > P_{T\min})$  in  $nb$  for hadronic production of the  $B_c$  and the  $B_c^*$  mesons predicted by the  $\alpha_s^4$  calculation and the fragmentation approximation assuming various  $P_T$  cuts and  $|Y| < 1.5$ . The CTEQ3M parton distribution functions were used and the values  $f_{B_c} = 480$  MeV,  $m_c = 1.5$  GeV,  $m_b = 4.9$  GeV, and  $M_{B_c} = 6.4$  GeV.

$P_{T\min}$ (GeV)	0	5	10	15	20	25	30
$\sigma_{B_c}(\alpha_s^4)$	1.8	0.57	0.087	0.018	$4.8 \times 10^{-3}$	$1.6 \times 10^{-3}$	$6.3 \times 10^{-4}$
$\sigma_{B_c}(\text{frag})$	1.4	0.47	0.071	0.014	$4.0 \times 10^{-3}$	$1.3 \times 10^{-3}$	$5.3 \times 10^{-4}$
$\sigma_{B_c^*}(\alpha_s^4)$	4.4	1.4	0.22	0.041	$1.1 \times 10^{-2}$	$3.4 \times 10^{-3}$	$1.3 \times 10^{-3}$
$\sigma_{B_c^*}(\text{frag})$	2.3	0.78	0.12	0.025	$6.8 \times 10^{-3}$	$2.3 \times 10^{-3}$	$9.2 \times 10^{-4}$
$\sigma_{B_c^*}(\text{frag})$	0.52	0.55	0.56	0.61	0.63	0.67	0.70
$\sigma_{B_c^*}(\alpha_s^4)$							

for the validity of the fragmentation approximation we also see that  $P_T$  should exceed about 30 GeV, a value considerably larger than the heavy quark masses. We note that this conclusion is rather insensitive to the choice of the energy scale  $\mu$  and the parton distribution functions.

Both the spectroscopy and the decays of the  $B$  mesons have been widely studied [16,18–20]. The excited states below the threshold will decay to the ground state  $B_c$  by emitting the photon(s) or  $\pi$  mesons. The golden channel to detect the  $B_c$  meson is  $B_c \rightarrow J/\psi + \pi(\rho)$ . However, the branching ratio is quite small, around 0.2%. The exclusive semileptonic decay mode  $B_c \rightarrow J/\psi + l + \nu_l$  has a relatively larger branch ratio, but there is “missing energy.” Our calculations should be helpful in searching for the  $B_c$  since the  $P_T$  distribution falls so rapidly that the small  $P_T$  region is quite important.

In summary, we carried out a detailed comparative study of the fragmentation approximation and the full order  $\alpha_s^4$  QCD perturbative calculation. We first studied the subprocess and obtained some insight into the process by analyzing the singularities appearing in the amplitude and the  $P_T$  dis-

tribution of the subprocess. This revealed that there are Feynman diagrams present in the full order  $\alpha_s^4$  matrix element in which *two or more* quarks or gluons can be *simultaneously* nearly on-mass shell and that these can dominate the subprocess over the fragmentation approximation in certain regions of the phase space. We then calculated the hadronic production process and investigated the  $z$  distribution  $C(z)$ , which was used to test the fragmentation approximation. From the study of both the subprocess and the hadronic process we conclude that the fragmentation mechanism dominates when and only when  $P_T \gg M_{B_c}$ . This conclusion is independent of the choice of the energy scales and the parton distribution functions. It is only fortuitous that the  $P_T$  distribution of the  $B_c$  in the fragmentation approximation is similar to that of the full calculation for  $P_T$  as low as  $M_{B_c}$ .

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