Effects of the quantity σ_{TS} on the spin structure functions of nucleons in the resonance region

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In this paper, we investigate the effects of the quantity σ_{TS} on the spin structure functions of nucleons in the resonance region. The Schwinger sum rule for the spin structure function $g_2(x,Q^2)$ at the real photon limit is derived for the nucleon treated as a composite system, and it provides a crucial constraint on the longitudinal transition operator which has not been treated consistently in the literature. The longitudinal amplitude $S_{1/2}$ is evaluated in the quark model with the transition operator that generates the Schwinger sum rule. The numerical results of the quantity σ_{TS} are presented for both spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the resonance region. Our results show that this quantity plays an important role in the low Q^2 region, which can be tested in future experiments at CEBAF. [S0556-2821(96)01819-X]

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I. INTRODUCTION

The quantity σ_{TS}, defined in the spin structure functions of nucleons,

$$g_1(x,Q^2)=\frac{M_TK}{8\pi^2\alpha\left(1+\frac{Q^2}{\omega^2}\right)}\left[\sigma_{1/2}(\omega,Q^2)-\sigma_{3/2}(\omega,Q^2)+\frac{2\sqrt{Q^2}}{\omega}\sigma_{TS}(\omega,Q^2)\right] \tag{1}$$

and

$$g_2(x,Q^2)=\frac{M_TK}{8\pi^2\alpha\left(1+\frac{Q^2}{\omega^2}\right)}\left[\frac{2\omega}{\sqrt{Q^2}}\sigma_{TS}(\omega,Q^2)-[\sigma_{1/2}(\omega,Q^2)-\sigma_{3/2}(\omega,Q^2)]\right], \tag{2}$$

where K is the photon flux, x the scaling variable, and M_T the nucleon mass, was not fully investigated due to the fact that most studies were concentrated in the deep inelastic scattering region, where the quantity $(2\sqrt{Q^2}/\omega)\sigma_{TS}(\omega,Q^2)$ in $g_1(x,Q^2)$ vanishes. This is no longer the case now as there has been a growing interest in studying the spin structure functions in the small Q^2 region, where the resonance contributions are important. Consequently, the investigation of the effects of the quantity σ_{TS} in the small Q^2 region has become increasingly important.

Such a program began with the suggestion [1] that the Q^2 dependence of the spin-dependent sum rule should be taken into account in order to explain the data [2] for the spin structure function of the nucleon, which starts with a negative Drell-Hearn-Gerasimov (DHG) [3] sum rule in the real photon limit and ends with a positive sum rule [4] in the large Q^2 limit [5]:

$$\int_0^1 g_1(x,Q^2)dx=\begin{cases}-\frac{\omega_{\rm th}}{4M_T}\kappa^2, & Q^2=0,\\ \Gamma, & Q^2\to\infty,\end{cases} \tag{3}$$

where

$$\omega_{\rm th}=\frac{Q^2+2m_\pi M+m_\pi^2}{2M} \tag{4}$$

is the threshold energy of pion photoproductions, κ the anomalous magnetic moment, and Γ a positive quantity. Because the contributions from the quantity σ_{TS} to the sum rule in Eq. (3) also vanish in the real photon limit, most quantitative studies [5–7] of the Q^2 dependence of the sum rule in Eq. (3) were concentrated on the contributions from the quantity $\sigma_{1/2}-\sigma_{3/2}$ in $g_{1,2}(x,Q^2)$. Indeed, these investigations have shown a strong Q^2 dependence of the sum rule in the $Q^2\leqslant 2.5\ \mathrm{GeV}^2$ region. However, the study by Soffer and Teryvaev [8] suggested that the quantity σ_{TS} plays a significant role in the small Q^2 region, which is highlighted by another set of sum rules for the spin structure function $g_2(x,Q^2)$:

$$\int_0^1 g_2(x,Q^2)dx=\begin{cases}\frac{\omega_{\rm th}}{4M_T}\kappa(\kappa+e_T), & Q^2=0,\\ 0, & Q^2\to\infty,\end{cases} \tag{5}$$

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in which the same kinematics as that in Eq. (3) is used. The sum rules in Eq. (5) were first derived by Schwinger [9] in the real photon limit and by Burkhardt and Cottingham [10] in the large Q^2 limit. Combining Eqs. (3) and (5) leads to the sum rule for the quantity σ_{TS} in the real photon limit:

$$\lim_{Q^2\to 0}\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=\frac{4\pi^2\alpha}{4M_T^2}e_T\kappa. \tag{6}$$

The magnitude of the sum rule for the quantity σ_{TS} in the real photon limit is certainly comparable to the DHG sum rule. Thus, a more quantitative study of the contributions from the quantity σ_{TS} to the spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ is called for. Such a study not only enables us to give a more precise estimate of the Q^2 dependence of the sum rule for $g_1(x,Q^2)$, but also provides a quantitative calculation of the sum rule for $g_2(x,Q^2)$ for the first time in the quark model. The focus of this paper is to develop a framework in the quark model to evaluate the contributions from the quantity σ_{TS}, and to present the numerical results that can be tested in future Continuous Electron Beam Accelerator Facility (CEBAF) experiments.

The sum rules in Eqs. (3) and (5) are more general and model independent; therefore, they must be satisfied in the quark model in order to give a consistent evaluation of the spin structure functions in the resonance region. It has been shown [11,12] by many authors that the electromagnetic interaction for a many-body system which satisfies the DHG sum rule should be expanded to order v^2/c^2 and has the form

$$H_t=\sum_j\left\{e_j\vec{r}_j\cdot\vec{E}_j-\frac{e_j}{2m_j}\vec{\sigma}_j\cdot\vec{B}_j-\frac{e_j}{4m_j}\vec{\sigma}_j\cdot\left[\vec{E}_j\times\frac{\vec{p}_j}{2m_j}-\frac{\vec{p}_j}{2m_j}\times\vec{E}_j\right]+\sum_{j<l}\frac{1}{4M_T}\left[\frac{\vec{\sigma}_j}{m_j}-\frac{\vec{\sigma}_l}{m_l}\right]\cdot(e_l\vec{E}_l\times\vec{p}_j-e_j\vec{E}_j\times\vec{p}_l)\right\}, \tag{7}$$

where quark j at the position r_j has mass m_j and charge e_j, and M_T is the total mass of the system. The last two terms in Eq. (7) are the spin-orbit interaction and the nonadditive contribution associated with the Wigner rotation which transforms the quark spins from the frame of the recoiling quark to the frame of the recoiling hadron; they are the $O(v^2/c^2)$ corrections and essential to reproduce the DHG sum rule. In Ref. [6], we showed that the quantity Γ in Eq. (3) can also be generated from H_t in Eq. (7) and is related to the quark model matrix element

$$\Gamma=\frac{1}{2}\left\langle i\left|\sum_j e_j^2\sigma_j^z\right|i\right\rangle_{P-A}, \tag{8}$$

where σ_j^z is the spin operator for the quark j, and $A(P)$ indicates that the directions of the polarization between photons and the target are antiparallel (parallel). On the other hand, the sum rule for the quantity σ_{TS} in Eq. (6) has not been previously investigated in the quark model. The derivation of Eq. (6) in the quark model is by no means trivial since it was proved [9] in QED by assuming the nucleon as an elementary particle. The similar transition of the DHG sum rule from an elementary particle to a many-body system led to extensive discussions in late sixties and early seventies [11]. Moreover, the proof of Eq. (6) requires evaluations of both helicity amplitude $A_{1/2}$ and the longitudinal amplitude $S_{1/2}$. While the helicity amplitude $A_{1/2}$ has been calculated [12] with the transition operator H_t in Eq. (7), the longitudinal amplitude $S_{1/2}$ has not been treated consistently in the literature. In particular, the problem of the current conservation was not fully understood [13], and an *ad hoc* current $J_3'=-(k_3J_3-k_0J_0)/k_3$ was introduced [14] to evaluate the longitudinal transitions of baryon resonances [15]. The sum rule for the quantity σ_{TS} provides an important test to the quark model; the consistency requires that the sum rules for both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ be generated by the same set of transition operators for a many-body system. Following the same approach as that in Ref. [11], we shall show in the next section that Eq. (6) is indeed generated by the electromagnetic interaction H_t in Eq. (7), which also satisfies the DHG sum rule. The longitudinal transition operator is obtained by requiring it satisfying the sum rule in Eq. (6), which is not only gauge invariant, but also consistent with H_t in Eq. (7). In particular, the spin-orbit interaction and the Wigner rotation that are crucial to the DHG sum rule for a many-body system should be present in the longitudinal transition operator.

In Sec. III, we show that the quantity σ_{TS} in $g_2(x,Q^2)$ cancels the transverse cross section $\sigma_{1/2}-\sigma_{3/2}$ in the large Q^2 limit, which leads to the well-known Burkhardt-Cottingham sum rule [10] for the spin structure function $g_2(x,Q^2)$. Thus, a consistent framework to evaluate the quantity σ_{TS} is established. In Sec. IV, we evaluate the longitudinal amplitude $S_{1/2}$ with the transition operator that generates the sum rule for the quantity σ_{TS}, which has not been done systematically in the literature. The numerical results for the spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the small Q^2 region are also shown in Sec. IV. Our results show that the effects of the quantity σ_{TS} on the spin structure functions are important in the resonance region. Finally, the conclusion is given in Sec. V.

II. THE SUM RULE FOR THE QUANTITY σ_{TS}

Because the spin structure functions of the nucleon are usually measured above the pion photoproduction threshold, the sum rule for the quantity σ_{TS} can be formulated as [5]

$$\int_0^1 [g_1(x,Q^2=0)+g_2(x,Q^2=0)]dx$$

$$= \lim_{Q^2\to 0} \frac{M\omega_{\rm th}}{4\pi^2\alpha}\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}. \tag{9}$$

The cross section σ_{TS} in Eq. (9) can be expressed in terms of the transverse and longitudinal helicity amplitudes, which is

$$\sigma_{TS}=\frac{\pi}{\sqrt{2}}\sum_{f>i}\{\langle i,\tfrac{1}{2}|H_l^*|f\rangle\langle f|H_t|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|H_t^*|f\rangle$$
$$\times\langle f,|H_l|i,\tfrac{1}{2}\rangle\}\delta(\omega-\omega_f), \tag{10}$$

where H_t is the transverse transition operator, and the longitudinal transition operator H_l is defined as

$$H_l=\epsilon_0 J_0-\epsilon_3 J_3. \tag{11}$$

Using the gauge-invariant condition, $k_\mu J^\mu=k_\mu\epsilon^\mu=0$, and choosing the longitudinal polarization vector ϵ_μ as

$$\epsilon_\mu^L=\{\epsilon_0,0,0,\epsilon_3\}=\left\{\frac{k_3}{\sqrt{Q^2}},0,0,\frac{\omega}{\sqrt{Q^2}}\right\}, \tag{12}$$

we have

$$\langle f|H_l|i\rangle=\frac{\sqrt{Q^2}}{\omega}\langle f|J_3|i\rangle, \tag{13}$$

or

$$\langle f|H_l|i\rangle=\frac{\sqrt{Q^2}}{k}\langle f|J_0|i\rangle. \tag{14}$$

Both Eqs. (13) and (14) can be used in the evaluation of the quantity σ_{TS} because of the current conservation. It can be shown that the quantity σ_{TS} is independent of which longitudinal current is used. Consequently, the sum rule for the quantity σ_{TS} does not depend on the choice of the longitudinal current as well, as long as it is gauge invariant.

By substituting Eq. (10) into Eq. (9), we have

$$\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=\frac{\pi}{\sqrt{2}\omega}\sum_{f>i}\{\langle i,\tfrac{1}{2}|J_3^*|f\rangle\langle f|H_t|i,-\tfrac{1}{2},\rangle$$
$$+\langle i,-\tfrac{1}{2}|H_t^*|f\rangle\langle f|J_3|i,\tfrac{1}{2}\rangle\}. \tag{15}$$

The operator H_t in Eq. (15) is also responsible to generate the DHG [3] sum rule for the transverse cross section $\sigma_{1/2}-\sigma_{3/2}$ in the real photon limit. Thus, the consistency requires that the same H_t in Eq. (7) should be also used in deriving Eq. (6). Following the same procedure as that in Refs. [6,11], we rewrite Eq. (7) by separating the center-of-mass motion from the internal motion:

$$H_t=\sqrt{\frac{\omega}{2}}(h^c+h^p), \tag{16}$$

where

$$h^c=i\sum_j\left\{e_j\vec{R}\cdot\vec{\epsilon}-\frac{1}{4M_T}\left(\frac{2e_j}{m_j}-\frac{e_T}{M_T}\right)\vec{\sigma}_j\cdot(\vec{\epsilon}\times\vec{P}_T)\right\}+\hat{\mu}^c \tag{17}$$

and

$$h^p=i\sum_j\left\{e_j\vec{\epsilon}\cdot(\vec{r}_j-\vec{R})-\frac{1}{4}\left(\frac{e_j}{m_j}-\frac{e_T}{M_T}\right)\right.$$
$$\left.\times\vec{\sigma}_j\cdot\left[\vec{\epsilon}\times\left(\frac{\vec{p}_j}{m_j}-\frac{\vec{P}_T}{M_T}\right)\right]\right\}+\hat{\mu}^p, \tag{18}$$

where $\epsilon=-(1/\sqrt{2})(1,i,0)$ is the transverse polarized vector of photons. The last terms $\hat{\mu}^c$ and $\hat{\mu}^p$ in Eqs. (17) and (18) correspond to the second term in Eq. (7). Their contributions to σ_{TS} are more complicated, and will be evaluated separately.

The longitudinal transition operator J_3 can be obtained by simply replacing the polarization vector $\vec{\epsilon}$ with the vector $\hat{\vec{k}}$, and the separation of the center of mass from the internal motions for the longitudinal transition is, therefore, easy to follow:

$$J_3=\sqrt{\frac{\omega}{2}}(j^c+j^p), \tag{19}$$

where

$$j^c=\sum_j\left\{ie_j\vec{R}\cdot\hat{\vec{k}}+\frac{i}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\right\} \tag{20}$$

and

$$j^p=\sum_j\left\{i\hat{\vec{k}}\cdot(\vec{r}_j-\vec{R})e_j+\frac{i}{4}\left(\frac{e_T}{M_T}-\frac{e_j}{m_j}\right)\right.$$
$$\left.\times\vec{\sigma}_j\cdot\left[\hat{\vec{k}}\times\left(\frac{\vec{p}_j}{m_j}-\frac{\vec{P}_T}{M_T}\right)\right]\right\}. \tag{21}$$

Now, we are in the position to derive Eq. (6) from the constituent quark model. Substituting Eqs. (16) and (19) into Eq. (15), we have

$$\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=\frac{4\pi^2\alpha}{2\sqrt{2}}\sum_{f>i}\{\langle i,\tfrac{1}{2}|j^{c*}|f\rangle\langle f|h^c|i,-\tfrac{1}{2}\rangle$$
$$+\langle i,-\tfrac{1}{2}|h^{c*}|f\rangle\langle f|j^c|i,\tfrac{1}{2}\rangle$$
$$+\langle i,\tfrac{1}{2}|j^{p*}|f\rangle\langle f|h^p|i,-\tfrac{1}{2}\rangle$$
$$+\langle i,-\tfrac{1}{2}|h^{p*}|f\rangle\langle f|j^p|i,\tfrac{1}{2}\rangle\}, \tag{22}$$

where the charge e^2 has been written as $4\pi\alpha$ explicitly so that the total charge for protons becomes 1 instead of e. Using the closure relation, Eq. (22) becomes

$$\sum_{f>i}\{\langle i,\tfrac{1}{2}|j^{c*}|f\rangle\langle f|h^c|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|h^{c*}|f\rangle\langle f|j^c|i,\tfrac{1}{2}\rangle$$
$$+\langle i,\tfrac{1}{2}|j^{p*}|f\rangle\langle f|h^p|i,-\tfrac{1}{2}\rangle$$
$$+\langle i,-\tfrac{1}{2}|h^{p*}|f\rangle\langle f|j^p|i,\tfrac{1}{2}\rangle\}$$
$$=\langle i,\tfrac{1}{2}|j^{c*}h^c+j^{p*}h^p|i,-\tfrac{1}{2}\rangle$$
$$+\langle i,-\tfrac{1}{2}|h^{c*}j^c+h^{p*}j^p|i,\tfrac{1}{2}\rangle$$
$$-\langle i,\tfrac{1}{2}|j^{c*}|i\rangle\langle i|h^c|i,-\tfrac{1}{2}\rangle-\langle i,-\tfrac{1}{2}|h^{c*}|i\rangle\langle i,|j^c|i,\tfrac{1}{2}\rangle. \tag{23}$$

We first consider the correlations between the longitudinal transition operators j^c $[j^p]$, and the first term $h^c-\mu^c(h^p-\mu^p)$ in Eq. (17) [Eq. (18)]. In Appendix we show that

$$\langle i,\tfrac{1}{2}|j^{c*}(h^c-\hat{\mu}^c)|i-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|(h^{c*}-\hat{\mu}^{c*})j^c|i,\tfrac{1}{2}\rangle$$
$$=-\left\langle i,\frac{1}{2}\left|\sum_j \sigma_j^+\frac{e_T}{2M_T}\left(\frac{2e_j}{m_j}-\frac{e_T}{M_T}\right)\right|i,-\frac{1}{2}\right\rangle\sqrt{2}, \tag{24}$$

and similarly

$$\langle i,\tfrac{1}{2}|j^{p*}(h^p-\hat{\mu}^p)|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|(h^{p*}-\hat{\mu}^{p*})j^p|i,\tfrac{1}{2}\rangle=\left\langle i,\frac{1}{2}\left|\sum_j \sigma_j^+\left(\frac{e_j}{m_j}-\frac{e_T}{M_T}\right)\left(\frac{e_T}{2M_T}-\frac{e_j}{2m_j}\right)\right|i,-\frac{1}{2}\right\rangle\sqrt{2}. \tag{25}$$

We turn to the correlation between the magnetic term $\hat{\mu}$ and the longitudinal transition operator j. The leading term for the operator μ is

$$\mu_0^c=\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}}) \tag{26}$$

and

$$\mu_0^p=0. \tag{27}$$

The correlation between μ_0^c and j^c gives

$$\langle i,\tfrac{1}{2}|j^{c*}\hat{\mu}_0^c|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|\hat{\mu}_0^{c*}j^c|i,\tfrac{1}{2}\rangle$$
$$=\sum_j\left\{\left\langle i,\frac{1}{2}\left|e_T\vec{R}\cdot\hat{\vec{k}}\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\right|i,-\frac{1}{2}\right\rangle\right.$$
$$\left.+\left\langle i,-\frac{1}{2}\left|\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}^*\times\hat{\vec{k}})e_T\vec{R}\cdot\hat{\vec{k}}\right|i,\frac{1}{2}\right\rangle\right\}. \tag{28}$$

By substituting the polarization $\epsilon=-(1/\sqrt{2})(1,i,0)$ and $\epsilon^*=-(1/\sqrt{2})(1,-i,0)$ into Eq. (28), we find that the two terms in Eq. (28) cancel each other so that they vanish. Thus, the nonzero contributions from the correlation between $\hat{\mu}$ and the longitudinal transition j should come from the next order. By expanding the photon wave function $e^{i\vec{k}\cdot\vec{r}_j}$, we have

$$\hat{\mu}=\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})(1+i\vec{k}\cdot\vec{r}_j+O(k)). \tag{29}$$

Because the leading term in Eq. (29) gives a zero contribution to the quantity σ_{TS}, we examine the second term in Eq. (29). In the real photon limit, we rewrite the second term in Eq. (29) as

$$\left\langle f\left|i\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\vec{k}\cdot\vec{r}_j\right|i\right\rangle$$
$$=i\left\langle f\left|\left[H,\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\vec{r}_j\right]\right|i\right\rangle$$
$$=\left\langle f\left|\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\frac{\vec{p}_j}{m_j}\right|i\right\rangle, \tag{30}$$

so that the closure relation could be used, because the transition operator has no explicit dependence on the transition energy ω. The operator H in Eq. (30) is the Hamiltonian of the system;

$$H=\sum_j \frac{\vec{p}_j^{\,2}}{2m_j}+\sum_{i,j} V_{ij}(\vec{r}_i-\vec{r}_j). \tag{31}$$

Therefore, $\hat{\mu}^c$ and $\hat{\mu}^p$ in Eqs. (17) and (18) are

$$\hat{\mu}_1^c=\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\frac{\vec{P}_T}{M_T} \tag{32}$$

and

$$\hat{\mu}_1^p=\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\left(\frac{\vec{p}_j}{m_j}-\frac{\vec{P}_T}{M_T}\right). \tag{33}$$

The correlation between $\hat{\mu}_1^{c,p}$ and $j^{c,p}$ gives

$$\langle i,\tfrac{1}{2}|j^{c*}\hat{\mu}_1^c+j^{p*}\hat{\mu}_1^p|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|\hat{\mu}_1^{c*}j^c+\hat{\mu}_1^{p*}j^p|i,\tfrac{1}{2}\rangle$$
$$=\left\langle i,\frac{1}{2}\left|\sum_j \sigma_j^+\left[\frac{e_T}{2M_T}\frac{e_j}{m_j}+\left(\frac{e_j}{m_j}-\frac{e_T}{M_T}\right)\frac{e_j}{2m_j}\right]\right|i,-\frac{1}{2}\right\rangle\sqrt{2}. \tag{34}$$

This shows that the nonzero contributions to the correlation between the magnetic transition $\hat{\mu}$ and the longitudinal transition operator j come from the higher order expansion in $\hat{\mu}$; this feature does not exist in the transverse correlations that leads to the DHG sum rule.

Therefore, by combining Eqs. (25), (28), and (34), we have

$$\langle i,\tfrac{1}{2}|j^{c*}h^c+j^{p*}h^p|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|h^{c*}j^c+h^{p*}j^p|i,\tfrac{1}{2}\rangle$$
$$=0. \tag{35}$$

That is, the sum rule for the quantity σ_{TS} is only determined by the static properties of ground states:

$$\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=-\frac{4\pi^2\alpha}{2\sqrt{2}}\{\langle i,\tfrac{1}{2}|j^{c*}|i\rangle\langle i|h^c|i,-\tfrac{1}{2}\rangle$$
$$+\langle i,-\tfrac{1}{2}|h^{c*}|i\rangle\langle i|j^c|i,\tfrac{1}{2}\rangle\}. \tag{36}$$

This is a general feature for the sum rules of both $g_1(x,Q^2)$ and $g_2(x,Q^2)$; the sum rules in real photon limit do not depend on the internal structure of the nucleon so that it behaves like an elementary particle in the low energy limit. Using the relation

$$\left\langle i\left|\sum_j \frac{e_j}{2m_j}\vec{\sigma}_j\right|i\right\rangle=\mu\langle i|\vec{\sigma}_T|i\rangle, \tag{37}$$

where $\vec{\sigma}_T$ is the total spin operator of a many-body system, we have

$$\langle i|h^c|i\rangle=\langle i|H^c|i\rangle \tag{38}$$

and

$$\langle i|j^c|i\rangle=\langle i|J^c|i\rangle, \tag{39}$$

where

$$H^c=i\left\{e_T\vec{R}\cdot\vec{\epsilon}+\mu\vec{\sigma}_T\cdot(\vec{\epsilon}\times\hat{\vec{k}})\frac{\vec{P}_T\cdot\hat{\vec{k}}}{M_T}-\frac{1}{2M_T}\right.$$
$$\left.\times\left(2\mu-\frac{e_T}{2M_T}\right)\vec{\sigma}_T\cdot(\vec{\epsilon}\times\vec{P}_T)\right\} \tag{40}$$

and

$$J^c=i\left\{e_T\vec{R}\cdot\vec{\epsilon}-\frac{1}{2M_T}\left(2\mu-\frac{e_T}{2M_T}\right)\vec{\sigma}_T\cdot(\vec{\epsilon}\times\vec{P}_T)\right\}. \tag{41}$$

Thus, the closure relation can be used because the operators H^c and J^c do not connect the ground state with the excited states:

$$\langle i,\tfrac{1}{2}|j^{c*}|i\rangle\langle i|h^c|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|h^{c*}|i\rangle\langle i|j^c|i,\tfrac{1}{2}\rangle$$
$$=\langle i,\tfrac{1}{2}|J^{c*}H^c|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|H^{c*}J^c|i,\tfrac{1}{2}\rangle$$
$$=\frac{\sqrt{2}}{2M_T^2}e_T\kappa, \tag{42}$$

which leads to the sum rule in Eq. (6). Consequently, the sum rule for the spin structure function g_2 in the real photon limit is just a linear combination of Eq. (6) and the DHG sum rule:

$$\lim_{Q^2\to 0}\int_0^1 dx g_2(x,Q^2)=\frac{\omega_{\rm th}}{M_T}\frac{\kappa(\kappa+e_T)}{4}. \tag{43}$$

This shows that the sum rules for both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the real photon limit can be derived consistently from the same set of the transition operators in the quark model. It also highlights the importance of the spin-orbit interaction and the nonadditive term in both transverse and longitudinal transition operators H_t and J_3. In the next section, we will give an intuitive proof that the same is also true for the sum rules in the large Q^2 limit.

III. THE EXTENSION TO THE LARGE Q^2 LIMIT

In the case of $Q^2\neq 0$, an extra term is generated from the transverse operator $h=h^c+h^p$ so that

$$h=h_0+h_1, \tag{44}$$

where h_0 represents the transition operator h at $Q^2=0$, and

$$h_1=\sum_j \frac{Q^2}{\omega^2}\frac{e_j}{2m_j}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\frac{\vec{p}_j}{m_j}, \tag{45}$$

while the longitudinal operator $j=j^c+j^p$ remains the same. Equation (35) shows that the correlation between h_0 and j is zero for the inclusive processes; thus only the correlation between h_1 and j needs to be investigated. Note that the Bjorken scaling variable x_j is related to the photon energy and the mass of partons [6]:

$$x_j=\frac{Q^2}{2M_T\omega}=\frac{m_j}{M_T} \tag{46}$$

in the large Q^2 limit. The operator h_1 can be written as

$$h_1=\sum_j \frac{2}{Q^2}e_j\vec{\sigma}_j\cdot(\vec{\epsilon}\times\hat{\vec{k}})\hat{\vec{k}}\cdot\vec{p}_j. \tag{47}$$

The correlation between h_1 and j gives

$$\langle i,\tfrac{1}{2}|j^*h_1|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|h_1^*j|i,\tfrac{1}{2}\rangle$$
$$=\frac{2\sqrt{2}}{Q^2}\left\langle i,\frac{1}{2}\left|\sum_j e_j^2\sigma_j^+\right|i,-\frac{1}{2}\right\rangle. \tag{48}$$

Therefore, we have

$$\lim_{Q^2\to\infty}\int_{\omega_{\rm th}}^{\infty}\sigma_{TS}\frac{d\omega}{\sqrt{Q^2}}=\frac{4\pi^2\alpha}{Q^2}\left\langle i,\frac{1}{2}\left|\sum_j e_j^2\sigma_j^+\right|i,-\frac{1}{2}\right\rangle. \tag{49}$$

A similar procedure [6] in the large Q^2 extension of the DHG sum rule gives

$$\int_{\omega_{\rm th}}^{\infty}(\sigma_{1/2}-\sigma_{3/2})\frac{d\omega}{\omega}=\frac{4\pi^2\alpha}{Q^2}\left\langle i\left|\sum_j e_j^2\sigma_j^z\right|i\right\rangle_{P-A}. \tag{50}$$

Combining Eqs. (49) and (50) gives the well-known Burkhardt-Cottingham (BC) sum rule [10] for the spin structure function g_2:

$$\int_0^1 g_2(x)dx=0. \tag{51}$$

Therefore, the sum rules for both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ can be obtained from the same set of electromagnetic interactions in Eqs. (7) and (19) for a many-body system. It shows that the transition of the spin-dependent sum rules [both DHG sum rule and Eq. (6)] from the real photon limit to the large Q^2 limit is an evolution from an exclusive, coherent elastic scattering to an inclusive, incoherent deep-inelastic scattering of a many-body system. Moreover, by reproducing the spin-dependent sum rules in the real photon and large Q^2 limits, we are able to establish a framework to evaluate the spin structure functions of nucleons in the finite Q^2 region, where the quark model has been very successful in describing the resonance contributions to the spin structure functions of the nucleon.

IV. THE EVALUATION OF THE SPIN-DEPENDENT SUM RULES IN THE LOW Q^2 REGION

The numerical studies of the quantity σ_{TS} require the evaluations of the transverse helicity amplitude $A_{1/2}$ and the longitudinal amplitude $S_{1/2}$. The helicity amplitude $A_{1/2}$ has been calculated [12] by using the transition operator in Eq. (7) that generates the DHG sum rule. Thus, only the longitudinal amplitude $S_{1/2}$ needs to be evaluated. Following Eq. (14), the longitudinal transition amplitude $S_{1/2}$ is

$$S_{1/2}=\langle f|J_0|i\rangle, \tag{52}$$

and we have the longitudinal transition operator [16]

$$J_0=\sqrt{\frac{1}{2\omega}}\left\{\sum_j\left(e_j+\frac{ie_j}{4m_j^2}\vec{k}\cdot(\vec{\sigma}_j\times\vec{p}_j)\right)e^{i\vec{k}\cdot\vec{r}_j}-\sum_{j<l}\frac{i}{4M_T}\left(\frac{\vec{\sigma}_j}{m_j}-\frac{\vec{\sigma}_l}{m_l}\right)\cdot(e_j\vec{k}\times\vec{p}_l e^{i\vec{k}\cdot\vec{r}_j}-e_l\vec{k}\times\vec{p}_j e^{i\vec{k}\cdot\vec{r}_l})\right\}, \tag{53}$$

where the second and third terms are the spin-orbit and nonadditive terms that represent the relativistic corrections to the leading charge operator. The study in previous sections clearly shows that the spin-orbit and the nonadditive terms are crucial in reproducing the sum rule for the quantity σ_{TS}.

Because the longitudinal amplitude $S_{1/2}$ of baryon resonances has not been systematically calculated with the transition operator J_0 in Eq. (53), we show the analytical expressions of the longitudinal amplitudes $S_{1/2}$ between the nucleon and baryon resonances in SU(6)$\otimes$O(3) symmetry limit in Table I. The evaluation of the Q^2 dependence of the longitudinal amplitudes $S_{1/2}$ follows the procedure of Foster and Hughes [17], and the longitudinal amplitudes $S_{1/2}$ as a function of Q^2 for the resonance $S_{11}(1535)$, $D_{13}(1520)$, and $F_{15}(1688)$ are shown in Fig. 1. These results are in better agreement with the analysis by Gerhardt [18] than the previous calculations [15], who extracted the longitudinal amplitudes from the electroproduction data. The numerical results in Fig. 1 show that the longitudinal amplitudes are quite large in the low Q^2 region, thus suggesting that they play a significant role in the spin structure functions of nucleon in the low Q^2 region.

The resonance contributions to the sum rules of the spin structure functions can be expressed in terms of the helicity amplitudes, $A_{1/2}$ and $A_{3/2}$, and the longitudinal amplitudes $S_{1/2}$:

$$\int g_1(x,Q^2)dx=\sum_R E_K\left[|A_{1/2}^R|^2-|A_{3/2}^R|^2+\frac{Q^2}{\sqrt{2}\omega k}(S_{1/2}^R{}^*A_{1/2}^R+A_{1/2}^R{}^*S_{1/2}^R)\right] \tag{54}$$

and

$$\int g_2(x,Q^2)dx=\sum_R E_K\left[\frac{\omega}{\sqrt{2}k}(S_{1/2}^R{}^*A_{1/2}^R+A_{1/2}^R{}^*S_{1/2}^R)-(|A_{1/2}^R|^2-|A_{3/2}^R|^2)\right], \tag{55}$$

where the kinetic factor E_K is

$$E_K=\frac{M\omega_{\rm th}}{4\pi\alpha\left(1+\frac{Q^2}{\omega^2}\right)\omega} \tag{56}$$

and $\omega_{\rm th}$ is given in Eq. (4). The total width of each resonance is treated as zero so that the integration over the photon energy can be approximated by a summation over all the resonances. The background contributions from the nucleon-born terms in the single pion photoproductions are not included in Eqs. (54) and (55), and they can be easily included later in more detailed studies. Because these amplitudes are evaluated by using the transition operators that generate the the spin-dependent sum rules, the calculations of the spin structure function in the resonance region become straightforward. In Fig. 2, we show the resonance contributions to the sum rule for $g_1(x,Q^2)$, in which every resonance below 2 GeV is included. The resonances $P_{11}(1440)$ and $P_{33}(1600)$ are treated as the hybrid states [19], and the study shows [20] that the Q^2 dependence of the transition amplitudes of the hybrid $P_{11}(1440)$ and $P_{33}(1600)$ gives a better

TABLE I. Transition matrix elements between the nucleon and baryon resonances in the SU(6)$\otimes$O(3) symmetry limit. The full matrix elements are obtained by multiplying the entries in this table by a factor $\sqrt{(2\pi/k_0)}2\mu m_q \exp^{-(\mathbf{k}^2/6\alpha^2)}$, and $S^n_{1/2}=S^p_{1/2}$ for Δ states.

Multiplet	States	Proton	Neutron
$[70,1^-]_1$	$N(^2P_M)\frac{1}{2}^{-1}$	$\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1+\frac{\alpha^2}{6m_q^2}\right)$	$-\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1+\frac{\alpha^2}{6m_q^2}\right)$
	$N(^2P_M)\frac{3}{2}^{-1}$	$\frac{1}{3}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{12m_q^2}\right)$	$-\frac{1}{3}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{12m_q^2}\right)$
	$N(^4P_M)\frac{1}{2}^{-1}$	$\frac{1}{36\sqrt{2}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$	$-\frac{1}{108\sqrt{2}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$
	$N(^4P_M)\frac{3}{2}^{-1}$	$\frac{1}{9\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$	$-\frac{5}{27\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$
	$N(^4P_M)\frac{5}{2}^{-1}$	$\frac{1}{12\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$	$-\frac{5}{36\sqrt{10}}\frac{\alpha\lvert\mathbf{k}\rvert}{m_q^2}$
	$\Delta(^2P_M)\frac{1}{2}^{-1}$	$-\frac{1}{3\sqrt{2}}\frac{\lvert\mathbf{k}\rvert}{\alpha}\left(1-\frac{\alpha^2}{6m_q^2}\right)$	
	$\Delta(^2P_M)\frac{3}{2}^{-1}$	$\frac{1}{3}\lvert\mathbf{k}\rvert\alpha\left(1+\frac{\alpha^2}{12m_q^2}\right)$	
$[56,0^+]_2$	$N(^2S_{S'})\frac{1}{2}^+$	$-\frac{1}{3\sqrt{6}}\frac{\mathbf{k}^2}{\alpha^2}$	0
	$\Delta(^4S_{S'})\frac{3}{2}^+$	0	
$[56,2^+]_2$	$N(^2D_S)\frac{3}{2}^+$	$-\frac{1}{3\sqrt{15}}\frac{\mathbf{k}^2}{\alpha^2}\left(1+\frac{\alpha^2}{2m_q^2}\right)$	$-\frac{\mathbf{k}^2}{12\sqrt{15}m_q^2}$
	$N(^2D_S)\frac{5}{2}^+$	$-\frac{1}{3\sqrt{10}}\frac{\mathbf{k}^2}{\alpha^2}\left(1-\frac{\alpha^2}{3m_q^2}\right)$	$\frac{\mathbf{k}^2}{9\sqrt{10}m_q^2}$
	$\Delta(^4D_S)\frac{1}{2}^+$	$-\frac{5\mathbf{k}^2}{72\sqrt{15}m_q^2}$	
	$\Delta(^4D_S)\frac{3}{2}^+$	0	
	$\Delta(^4D_S)\frac{5}{2}^+$	$\frac{5\sqrt{5}\mathbf{k}^2}{216\sqrt{7}m_q^2}$	
	$\Delta(^4D_S)\frac{7}{2}^+$	$\frac{5\mathbf{k}^2}{36\sqrt{105}m_q^2}$	
$[70,0^+]_2$	$N(^2S_{M'})\frac{1}{2}^+$	$\frac{1}{18}\frac{\mathbf{k}^2}{\alpha^2}$	$-\frac{1}{18}\frac{\mathbf{k}^2}{\alpha^2}$

agreement with the existing data. The numerical result for the Q^2 dependence of the integral for the transverse cross section $\sigma_{1/2}-\sigma_{3/2}$ is also shown in Fig. 2, and it is in good agreement with a more sophisticated evaluation in Ref. [5]. The resonance contribution to the integral $\int g_1(x,Q^2=0)dx$ at the real photon limit is -0.121, which is in good agreement with the theoretical prediction $-(\omega_{\rm th}/4M_T)\kappa^2$ with $k_p=1.79$ for the proton target [5]. This result is consistent with the conclusions of our previous investigation [6]; the contributions from resonances, in particular the resonance $P_{33}(1232)$, dominate the DHG sum rule. The difference between the $g_1(x,Q^2)$ sum rule and the contribution from the quantity $\sigma_{1/2}-\sigma_{3/2}$ shows the importance of the quantity σ_{TS}. It is particularly significant in the small Q^2 region, and

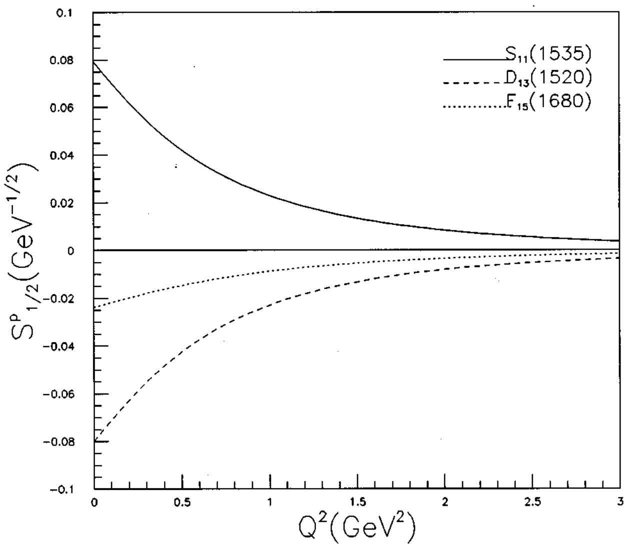

FIG. 1. The Q^2 dependence of the longitudinal amplitudes $S^P_{1/2}$ for the resonances $S_{11}(1535)$, $D_{13}(1520)$, and $F_{15}(1680)$.

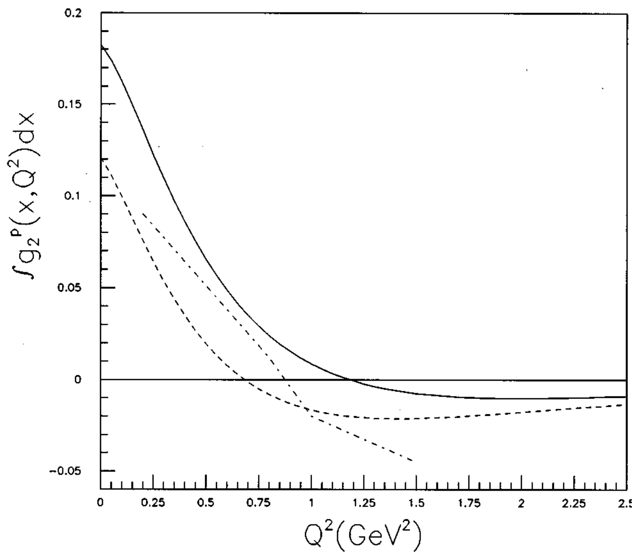

FIG. 3. The Q^2 dependence of the spin-dependent sum rule of $g_2(x,Q^2)$ in the resonance region. The solid and dash lines represent the calculations with and without the quantity σ_{TS}. The dot-dashed line comes from Ref. [21], see text.

the addition of the quantity σ_{TS} has pushed the crossing point that the sum rule is zero from 0.7 GeV^2 to around 0.5 GeV^2.

The sum rule for the spin structure function $g_2(x,Q^2)$ in the resonance region is shown in Fig. 3. The resonance contributions to the sum rule $\int g_2(x,Q^2)$ at the real photon limit is 0.182, while Eq. (43) gives 0.192 for the proton target. This shows that the resonance contributions dominate the sum rule for $g_2(x,Q^2)$ in the real photon limit as well. The $g_2(x,Q^2)$ is only significant in the $Q^2 \leqslant 1$ GeV^2 region, and decreases very quickly as Q^2 increases. There is also a sign change for the sum rule of $g_2(x,Q^2)$ at $Q^2 \approx 1$ GeV^2. A recent calculation [21] in the single pion channel of pion photoproduction has shown a similar behavior, in which only the nucleon-born term is considered. This behavior is not consistent with the Q^2 dependence of the sum rule of $g_2(x,Q^2)$ derived in Refs. [9,8]. It may represent the theoretical uncertainty of the quark model calculations. On the other hand, it would be very interesting to see if there is a sign change in the experimental data.

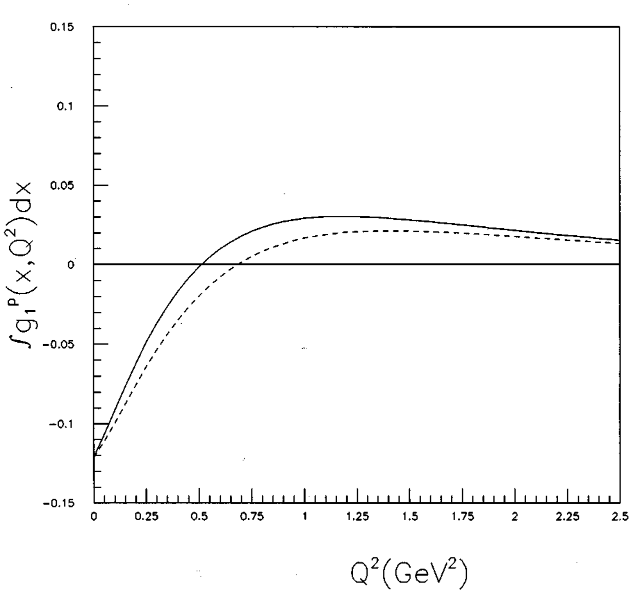

FIG. 2. The Q^2 dependence of the spin-dependent sum rule of $g_1(x,Q^2)$ in the resonance region. The solid and dash lines represent the calculations with and without the quantity σ_{TS}.

To highlight the importance of the quantity σ_{TS} in the resonance region, we present an estimate of the total sum rule for $g_1(x,Q^2)$ by including the contributions from outside the resonance region. Following the procedure in Ref. [5], the total spin-dependent sum rule should be written as

$$\int_0^1 g_1(x,Q^2) = \int_{x_r}^1 + \int_0^{x_r} g_1(x,Q^2)dx, \tag{57}$$

where

$$x_r = \frac{Q^2 + 2m_\pi M_T + m_\pi^2}{W_r^2 + Q^2 - M_T^2} \tag{58}$$

with $W_r = 2.0$ GeV. The first term in Eq. (57) represents the contributions from the resonance region; it shows that the contributions from the resonance region do not cover the whole kinetic region from $x=0$ to $x=1$. The second term in Eq. (57) comes from the outside resonance region, and we showed in Ref. [5] that this term becomes increasingly important as Q^2 increases. Because there is no experimental information on the quantity σ_{TS} outside the resonance region, one could only make a qualitative estimate on the second term in Eq. (57). We show the estimate of the Q^2 dependence of the spin-dependent sum rule $\int_0^1 g_1(x,Q^2)dx$ in Fig. 4. The contribution from the second term is obtained from the estimate of the nonresonant contribution in Ref. [5],

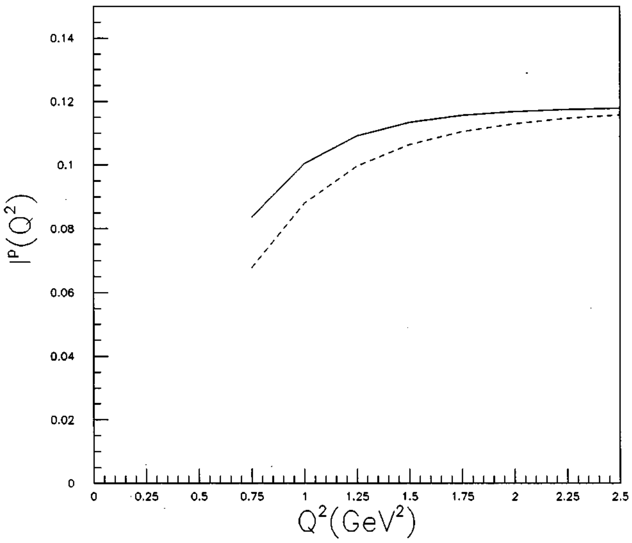

FIG. 4. The estimate of the sum rule $\int_0^1 g_1^p(x,Q^2)dx$. The nonresonant contribution comes from the result in Ref. [5]. The solid and dash lines correspond to the evaluations with and without the quantity σ_{TS}.

in which the quantity σ_{TS} is not included. Thus, this estimate could only be regarded as a lower limit of the spin-dependent sum rule for $g_1(x,Q^2)$. Nevertheless, the effects of the quantity σ_{TS} on the Q^2 dependence of the sum rule $\int_0^1 g_1(x,Q^2)dx$ are very important, and it could not be neglected if the high twist term that generates the leading $1/Q^2$ corrections to the spin structure function in the deep-inelastic scattering region is extracted from $Q^2 \approx 1.5 \sim 2.5$ GeV^2 region.

V. CONCLUSION

We have presented a consistent framework to investigate the spin structure functions of nucleon in the resonance region, in which the model-independent sum rules in the real photon limit and the large Q^2 limit are satisfied. We show that the same set of transition operators generates both DHG sum rule for the transverse cross section, $\sigma_{1/2}-\sigma_{3/2}$, and the sum rule for the quantity σ_{TS}. The sum rule for the quantity σ_{TS} also provides a crucial constraint on the longitudinal transition operator; it requires the longitudinal transition operator to be gauge invariant and to be expanded to order $O(v^2/c^2)$ consistently. The operator in Eq. (53) satisfies these requirements. This clarifies some of the problems in the literature on the longitudinal transitions, although the problem of the model space truncation discussed in Ref. [13] is not considered here.

A more quantitative calculation of the spin-dependent sum rules for both the spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ are presented for the first time in the quark model. Our numerical results indicate that the effects of the quantity σ_{TS} are very important in small Q^2 region, which certainly can be tested in future experiments at CEBAF [22].

ACKNOWLEDGMENTS

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APPENDIX

The terms that contribute to the spin flip in Eq. (24) are

$$\begin{aligned}\langle i,\tfrac{1}{2}|j^{c*}(h^c-\hat{\mu}^c)|i,-\tfrac{1}{2}\rangle+\langle i,-\tfrac{1}{2}|(h^c-\hat{\mu}^c)^*j^c|i,\tfrac{1}{2}\rangle=&\sum_j\left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\vec{R}\cdot\vec{\epsilon}\right|i,-\frac{1}{2}\right\rangle\\&+\sum_j\left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\vec{\epsilon}^*\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\right|i,\frac{1}{2}\right\rangle\\&+\sum_j\left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\hat{\vec{k}}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\vec{P}_T)\right|i,-\frac{1}{2}\right\rangle\\&+\sum_j\left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\vec{\epsilon}^*\times\vec{P}_T)\vec{R}\cdot\hat{\vec{k}}\right|i,\frac{1}{2}\right\rangle.\quad\text{(A1)}\end{aligned}$$

Let us consider the terms proportional to $\vec{R}\cdot\vec{\epsilon}$ in Eq. (A1). The product $\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)$ can be written as

$$\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)=-i\sigma_j^+(P_x-iP_y)+i\sigma_j^-(P_x+iP_y), \tag{A2}$$

where $\sigma^\pm=\frac{1}{2}(\sigma_x\pm i\sigma_y)$. By substituting $\epsilon=-(1/\sqrt{2})(1,i,0)$ into Eq. (A1), we have

$$\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\vec{R}\cdot\vec{\epsilon}\right|i,-\frac{1}{2}\right\rangle$$
$$+\sum_j \left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\vec{\epsilon}^*\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\right|i,\frac{1}{2}\right\rangle$$
$$=\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+\frac{i}{\sqrt{2}}P^-R^+\right|i,-\frac{1}{2}\right\rangle-\sum_j \left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^-\frac{i}{\sqrt{2}}R^-P^+\right|i,\frac{1}{2}\right\rangle, \quad \text{(A3)}$$

where $P^{\pm}=P_x\pm iP_y$ and $R^{\pm}=R_x\pm iR_y$. Notice that for the total 1/2 initial and final states

$$\langle i,\tfrac{1}{2}|\sigma_j^+|i,-\tfrac{1}{2}\rangle=\langle i,-\tfrac{1}{2}|\sigma_j^-|i,\tfrac{1}{2}\rangle \quad \text{(A4)}$$

in our convention for the Pauli matrix $\sigma^{\pm}$ and the spin-wave functions. Equation (A3) becomes

$$\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+\frac{i}{\sqrt{2}}[P^-R^+-R^-P^+]\right|i,-\frac{1}{2}\right\rangle$$
$$=\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+\sqrt{2}[1+iR_yP_x-iR_xP_y]\right|i,-\frac{1}{2}\right\rangle, \quad \text{(A5)}$$

where the term $R_yP_x-R_xP_y$ is an angular momentum operator for the center-of-mass motions of nucleons, which is zero in this process. Thus, we have

$$\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\vec{R}\cdot\vec{\epsilon}\right|i,-\frac{1}{2}\right\rangle+\sum_j \left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\vec{\epsilon}^*\vec{\sigma}_j\cdot(\hat{\vec{k}}\times\vec{P}_T)\right|i,\frac{1}{2}\right\rangle$$
$$=\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+\sqrt{2}\right|i,-\frac{1}{2}\right\rangle. \quad \text{(A6)}$$

Taking the same procedure, we have

$$\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{R}\cdot\hat{\vec{k}}\vec{\sigma}_j\cdot(\vec{\epsilon}\times\vec{P}_T)\right|i,-\frac{1}{2}\right\rangle+\sum_j \left\langle i,-\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\vec{\sigma}_j\cdot(\vec{\epsilon}^*\times\vec{P}_T)\vec{R}\cdot\hat{\vec{k}}\right|i,\frac{1}{2}\right\rangle$$
$$=\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+ i\sqrt{2}(P_zR_z-R_zP_z)\right|i,-\frac{1}{2}\right\rangle$$
$$=\sum_j \left\langle i,\frac{1}{2}\left|\frac{e_T}{4M_T}\left(\frac{e_T}{M_T}-\frac{2e_j}{m_j}\right)\sigma_j^+\sqrt{2}\right|i,-\frac{1}{2}\right\rangle. \quad \text{(A7)}$$

Combining Eqs. (A6) and (A7) gives Eq. (24).

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