K models and type IIB superstring backgrounds

Alok Kumar*
Institute of Physics, Bhubaneswar 751 005, India

Gautam Sengupta
Indodeep Housing Complex, 256/5, Dum Dum Road, Calcutta 700 074, India
(Received 29 February 1996)

A family of type IIB superstring backgrounds involving Ramond-Ramond fields are obtained in ten dimensions starting from a *K* model through a generalization of our recent results. The unbroken global SL$(2,R)$ symmetry of the type IIB equations of motion are implemented in this context as a solution generating transformation. A geometrical analysis, based on the tensor structure of the higher order α' terms in the equations of motion, is employed to show that these backgrounds are exact. [S0556-2821(96)03716-2]

PACS number(s): 11.25.Mj, 11.30.Ly

I. INTRODUCTION

Recent advances in string theory have provided a glimpse of their underlying nonperturbative structure. There now seems to be a strong indication that the several distinct string theories all arise from a yet unknown fundamental theory in 11 dimensions, which has been called the *M* theory [1]. The central guiding principle behind this unification has been the duality symmetries [2–5]. They include *T* duality [2] which is perturbative in the genus expansion and the conjectured nonperturbative *S* duality [3–5]. The latter relates electrical charged perturbative states to magnetic charged solitons and weak to strong coupling regimes. As an example, we have the type IIB string theory in ten dimensions [6,7]. These possess a global SL$(2,R)$ symmetry of the classical effective field theory which is broken to the discrete SL$(2,Z)$ *S*-duality group in the quantum theory which relates Neveu-Schwarz–Neveu-Schwarz (NS-NS) to the Ramond-Ramond (R-R) sector.

In the σ model approach to string theory [8], one considers the propagation of a string in the background of its massless excitations. The evolution is described by a two-dimensional σ model in which the background fields appear as couplings. Conformal invariance then requires the corresponding perturbative β functions for these couplings to vanish, leading to the background field equations of motion. The higher order terms [8] in these equations provide the stringy corrections to the tree level theory and may be computed perturbatively. However, for the type IIB string background, this remains a nontrivial exercise owing to the presence of R-R fields which arise from the solitonic sector [9]. These couple to the spin fields on the world sheet and make the corresponding perturbation theory intractable at higher orders [9,10]. There exists, however, a large class of string backgrounds for the bosonic and heterotic cases, where the higher order contributions are identically zero [11–15]. Hence the tree level equations of motion are exact to all orders in the σ model coupling α'. Among these are the class of backgrounds with a covariantly constant null Killing vector which are known as *K* models [11,13]. The simplest example in this class is the plane wave string backgrounds. For the bosonic and heterotic versions of these, it has been shown that they are exact (in α') through a purely geometrical analysis based on the existence of the covariantly constant null Killing vector [12,16–18].

Previously, in collaboration with Kar [19], we have shown that starting from such a plane wave background embedded trivially in a type IIB string theory (i.e., with vanishing R-R fields), it is possible to generate a nontrivial type IIB background with R-R fields in ten dimensions. We further showed, through a geometrical analysis, that these type IIB backgrounds were also exact to all orders in α'. This method avoids the complications arising from the world sheet couplings of the R-R fields. In this article, we extend our analysis to a more general class of *K* models with chiral couplings on the world sheet [15]. On compactification to lower dimensions, these couplings lead to background gauge fields. The bosonic and the heterotic versions of these *K* models describe strings propagating in a uniform magnetic field background [20] as one of the special cases. These can be formulated as exact confornal field theories and illustrate a phase transition at some critical value of the magnetic field where an infinite number of states becomes tachyonic. In lower dimensions they also describe charged black holes through the Kaluza-Klein mechanism [15]. We consider such a *K* model in ten dimensions trivially embedded (i.e., without R-R fields) in a type IIB background. Using the global SL$(2,R)$ symmetry of the equations of motion, we generate a type IIB background with a nontrivial R-R field background. Subsequently, assuming a specific structure for the field strengths, we show that the backgrounds obtained are exact (in α') through a geometrical analysis. In this context, unlike our earlier work, we show that the geometrical considerations based on the covariantly constant null Killing vector may be retranslated in the language of the index structure of the corresponding higher order tensors [17,19]. It is possible that the present approach is applicable to a wider class of models. This article is divided into four sections. In Sec. II we present a brief review of the backgrounds obtained from

* Electronic address: kumar@iopb.ernet.in

K models for the bosonic case and show that they are exact. Section III deals with K models embedded in a type IIB theory and their SL(2,R) transformations. In this section we also explicitly prove that these backgrounds are exact in the presence of R-R fields. We present the conclusions in Sec. IV.

II. STRING BACKGROUNDS FROM K MODELS

We begin with a description of the string background fields obtained from the K models. The Lagrangian for the bosonic K model is given as [15]

$$\mathcal{L}=2\partial u\overline{\partial v}+K(u,x)\partial u\overline{\partial u}+2A_i(u,x)\partial u\overline{\partial x^i}+2\overline{A_i}(u,x)\overline{\partial u}\partial x^i+(G_{ij}+B_{ij})(u,x)\partial x^i\partial x^j+R^{(2)}\phi(u,x). \tag{1}$$

We specialize to the case where $G_{ij}=\delta_{ij}$, $B_{ij}=0$, and $\phi=\phi(u)$. For this case we have the metric

$$ds^2=2dudv+2A_i^+(u,x)dudx^i+K(u,x)du^2+dx^idx_i \tag{2}$$

and the antisymmetric tensor field

$$B_{\mu\nu}=\begin{pmatrix} 0 & 1 & A_i^- \\ -1 & 0 & 0 \\ -A_i^- & 0 & 0 \end{pmatrix}. \tag{3}$$

The greek indices (μ,ν) run over $(0,\ldots,9)$, (u,v) are the light-cone coordinates, and the latin indices $(i,j)=(2,\ldots,9)$ run over the transverse space coordinates x^i. We also have

$$A_i^{\pm}=A_i\pm\overline{A_i}. \tag{4}$$

The v independence of the metric leads to a Killing vector $l^\mu=(0,1,0,\ldots,0)$. It is possible to express the metric in a compact form in terms of l^μ which eases subsequent computations. Explicitly, we have

$$G_{\mu\nu}=M_{\mu\nu}+Kl_\mu l_\nu, \tag{5}$$

where $M_{\mu\nu}$ is a 10×10 symmetric matrix,

$$M_{\mu\nu}=\begin{pmatrix} 0 & 1 & A_i^+ \\ 1 & 0 & 0 \\ A_i^+ & 0 & I_8 \end{pmatrix}, \tag{6}$$

and I_8 is a 8×8 unit matrix. The inverse metric is obtained as

$$G^{\mu\nu}=\begin{pmatrix} 0 & 1 & 0 \\ 1 & -K+A_i^{+2} & -A_i^+ \\ 0 & -A_i^+ & I_8 \end{pmatrix}. \tag{7}$$

The only nonzero connections are Γ^i_{uu}, Γ^v_{uu}, Γ^v_{ui}, Γ^j_{ui}, and Γ^v_{ij}. Using these connections, it is easy to show that the null Killing vector is covariantly constant. We therefore have $D_\mu l^\nu=0$ and $D_\mu l_\nu=0$. The curvature tensor for the backgrounds (7) may be obtained as

$$R_{\lambda\mu\nu\kappa}=R^{(M)}_{\lambda\mu\nu\kappa}+2l_{[\lambda}\partial_{\mu]}\partial_{[\nu}Kl_{\kappa]}. \tag{8}$$

Notice that the only nonzero independent components of $R_{\lambda\mu\nu\kappa}$ are R_{uiuj} and R_{uijk}. These are

$$R_{uiuj}=\tfrac{1}{2}\partial_i\partial_jK-\tfrac{1}{2}\partial_u[\partial_iA_j^++\partial_jA_i^+]-\tfrac{1}{4}G^{mn}(\partial_jA_m^+-\partial_mA_j^+)\times(\partial_iA_n^+-\partial_nA_i^+) \tag{9}$$

and

$$R_{uijk}=\tfrac{1}{2}\partial_i[\partial_kA_j^+-\partial_jA_k^+]. \tag{10}$$

We get the expressions for the corresponding components of the Ricci tensor by appropriate contractions of the Riemann tensor as

$$R_{uu}=\tfrac{1}{2}\partial^i\partial_iK-\partial_u\partial^iA_i^+-\tfrac{1}{4}(F_{jm}+\overline{F_{jm}})(F^{jm}+\overline{F^{jm}}), \tag{11}$$

$$R_{uk}=-\tfrac{1}{2}\partial^m(F_{km}+\overline{F_{km}}), \tag{12}$$

where

$$F_{ij}=(\partial_iA_j-\partial_jA_i),\quad \overline{F_{ij}}=(\partial_i\overline{A_j}-\partial_j\overline{A_i}) \tag{13}$$

are the *field strengths* associated with the couplings A and $\overline{A}$. Next we consider the antisymmetric tensor field strength which is given in the standard form as

$$H_{\lambda\mu\nu}=(\partial_\lambda B_{\mu\nu}+\partial_\mu B_{\nu\lambda}+\partial_\nu B_{\lambda\mu}). \tag{14}$$

The only nonzero independent component of $H_{\lambda\mu\nu}$ is

$$H_{uij}=-(F_{ij}-\overline{F_{ij}}). \tag{15}$$

We now proceed to obtain the background field equations at the tree level. As mentioned earlier, these are provided by the one-loop β functions of the couplings $G_{\mu\nu}$, $B_{\mu\nu}$, and ϕ. Using standard expressions for these equations [8], we obtain three independent equations of motion [15]:

$$\partial^jF_{ij}=0,\quad \partial^j\overline{F_{ij}}=0, \tag{16}$$

and

$$-\tfrac{1}{2}\partial_i\partial^iK+\partial_i\partial^uA_i^++F_{ij}\overline{F^{ij}}+2\partial_u^2\phi=0. \tag{17}$$

After obtaining the background field equations of the general K models, we now specialize to the case where the field strengths F_{ij} and $\overline{F_{ij}}$ are constant in the transverse directions but arbitrary functions of u. A subclass of these, namely, the ones which are constant in u as well, describe the propagation of closed strings in uniform magnetic field backgrounds [20]. They have been shown to be represented by exact conformal field theory models. The heterotic generalizations of these models have also been found. The chiral couplings in this case are

$$A_i=-\tfrac{1}{2}F_{ij}(u)x^j,\quad \overline{A_i}=-\tfrac{1}{2}\overline{F_{ij}}(u)x^j. \tag{18}$$

As a consequence, we have, for the curvature tensor, $R_{uijk}=0$ and

$$R_{uiuj}=\frac{1}{4}\partial_i\partial_j K-\frac{1}{4}G^{mn}(F_{jm}+\overline{F_{jm}})(F_{in}+\overline{F_{in}})-\frac{1}{2}\partial_u(F_{ij}(u)+\overline{F_{ij}}(u)). \quad (19)$$

In this case, the first two background field equations in Eqs. (16) are trivially satisfied. For the metric equation we have,

$$-\frac{1}{2}\partial^i\partial_i K(u,x)+F_{ij}\overline{F^{ij}}(u)+2\partial_u^2\phi(u)-\frac{1}{2}\partial_u(F_{ii}(u)+\overline{F_{ii}}(u))=0. \quad (20)$$

Therefore K must be a quadratic function of the x^i in order to satisfy Eq. (20).

We now proceed to show that this background is exact to all orders in α'. For this, we first note that Eq. (20) is a second rank tensor equation. So we must consider all possible higher order second rank tensor contributions obtained from the background field configuration. The only possible covariant tensor components available for this purpose are $D_u\phi$, R_{uiuj}, and its covariant derivatives with respect to D_u and D_k, H_{uij}, and its covariant derivative with respect to D_u. One also has the corresponding contravariant components which are consistent with the form of the metric (2).

We first examine the terms involving a single Riemann tensor with the structure $D^\lambda D^\nu R_{\lambda\mu\nu\kappa}$. An explicit evaluation provides the identities

$$D^uD^uR_{uiuj}=D^iD^jR_{uiju}=D^uD^iR_{ujiu}=D^iD^uR_{iuuj}=0, \quad \text{etc.}, \quad (21)$$

which implies

$$D^\lambda D^\nu R_{\lambda\mu\nu\kappa}=0. \quad (22)$$

It is apparent now that to construct second rank tensors with R_{uiuj} and its derivatives, it is required to contract at least two indices of R with another R or its appropriate derivatives. Potentially nonzero contributions may come from the contractions of covariant indices (u,i) and contravariant indices (v,i). However, this requires a covariant index v or contravariant index u, which are unavailable and contractions on derivatives have been shown to be zero. Hence we conclude that it is impossible to construct nonzero second rank tensors from contractions of $R_{\lambda\mu\nu\kappa}$ and its derivatives [17,19]. Thus all such higher order contributions are vanishing. Similarly notice that terms of the form $D_\lambda\phi R^{\lambda\mu\nu\kappa}$ require contraction of the covariant index u and hence it is also zero.

We proceed to consider higher order contributions from the field strength H and its derivatives. Notice that the only nonzero component of H is H_{uij} and the only nonzero covariant derivative is D_uH_{uij}. It is obvious that all terms involving only derivatives of H require contraction of the covariant index u which, as we showed earlier, was not possible. Hence these terms are all identically zero. Higher order contributions of the schematic form $(DR)H$ and $(D\phi)H$ may be proved to be identically zero from similar considerations. It is also possible to show that all scalars constructed from these covariant objects are also vanishing. Hence we conclude that the string background obtained from the K models are exact to all orders in α'. In the next section we show how these backgrounds may be considered to be trivially embedded in a type IIB string background and generate nontrivial type IIB backgrounds involving R-R fields.

III. TYPE IIB BACKGROUNDS AND RAMOND-RAMOND FIELDS

In this section we now proceed to first show how the backgrounds defined in Eqs. (2) and (3) and ϕ can be embedded in a type IIB string theory with vanishing R-R fields. We subsequently present the action of the global SL(2,R) transformations on a type IIB background. Utilizing these transfromations, we then generate a nontrivial type IIB background involving R-R fields. The field content of a type IIB string background consists of the following: the string frame metric $G_{\mu\nu}$, two three-form field strengths $H^{(k)}_{\lambda\mu\nu}$ where $k=(1,2)$, two scalars χ and ϕ from the NS-NS and R-R sectors, respectively, and a five-form field strength $F_{\lambda\mu\nu\kappa\rho}$. The two scalars χ and ϕ may be combined to form a complex scalar $\lambda=\chi+ie^{-\phi}$. So we may consider the background obtained from the K model defined by $\phi(u)$, and Eqs. (2) and (3) to be a special case of a type IIB background which has $H^{(2)}_{\lambda\mu\nu}=0$, $\chi=0$, and $F_5=0$. As shown in [6,7], type IIB strings in $D=10$ have a global SL$(2,R)$ symmetry at the level of the equations of motion [7,21]. This acts on the type IIB background fields as

$$G'_{\mu\nu}=|c\lambda+d|G_{\mu\nu}, \quad (23)$$

$$\lambda'=\frac{a\lambda+b}{c\lambda+d}, \quad (24)$$

and

$$H'^{(k)}_{\lambda\mu\nu}=\Lambda H^{(k)}_{\lambda\mu\nu}, \quad (25)$$

where Λ is an SL$(2,R)$ matrix such that

$$\Lambda=\begin{pmatrix} d & c \\ b & a \end{pmatrix}, \quad (26)$$

with $ad-bc=1$.

Implementing the transformations outlined in Eqs. (23)–(25), we generate a nontrivial type IIB background with R-R fields starting from the trivial type IIB configuration obtained from the K models [cf. Eqs. (2) and (3)] and $\phi(u)$. Explicitly, we have

$$G'_{\mu\nu}(u,x)=f(u)G_{\mu\nu}(u,x), \quad (27)$$

where $f(u)=[d^2+c^2e^{-2\phi(u)}]^{1/2}$ and

$$\lambda'=\frac{iae^{-\phi}+b}{ice^{-\phi}+d}, \quad (28)$$

with $\lambda'=\chi'+ie^{-\phi'}$ and $\lambda=ie^{-\phi}$. We then have the final expressions for the type IIB scalars as

$$\chi'(u)=\frac{1}{f(u)^2}[db+ace^{-2\phi}], \quad (29)$$

$$\phi'(u)=\phi(u)+2\ln f(u). \tag{30}$$

For the three-form field strength $H^{(k)}$, $k=1,2$, we have

$$H'^{(1)}_{\lambda\mu\nu}=dH^{(1)}_{\lambda\mu\nu} \tag{31}$$

and

$$H'^{(2)}_{\lambda\mu\nu}=bH^{(1)}_{\lambda\mu\nu}. \tag{32}$$

The new metric is now given as

$$ds^2=2f(u)dudv+f(u)dx^idx_i+2f(u)A_i^+dudx^i + f(u)K(u,x)du^2, \tag{33}$$

where $K(u,x)=f(u)F(u,x)$. A rescaling $f(u)du=dU$ of the metric leads to the general form

$$ds^2=-2dUdv+\widetilde{f}(U)dx^idx_i+2\widetilde{A}_i^+(U,x)dUdx^i + \frac{\widetilde{K}(U,x)}{\widetilde{f}(U)}dU^2. \tag{34}$$

Dropping the tildes and rewriting U as u in Eq. (34) we have

$$ds^2=2dudv+2f(u)dx^idx_i+2A_i^+(u,x)dudx^i + \hat{K}(u,x)du^2. \tag{35}$$

In subsequent discussions we drop the primes on the nontrivial type IIB background fields generated by the SL(2,R) transformations from the K model backgrounds. It can be seen from the definitions that $\hat{K}$ in Eq. (35) is also a quadratic function of x^i's. This fact becomes important in proving that these backgrounds are all-order solutions of the type IIB equations of motion.

As earlier, the v independence leads to a null Killing vector l^μ. We reexpress the metric in Eq. (35) in terms of l^μ as

$$G_{\mu\nu}=M_{\mu\nu}+\hat{K}l_\mu l_\nu, \tag{36}$$

where $M_{\mu\nu}$ is once again a 10×10 matrix given as

$$M_{\mu\nu}=\begin{pmatrix} 0 & 1 & A_i^+ \\ 1 & 0 & 0 \\ A_i^+ & 0 & f(u)I_8 \end{pmatrix} \tag{37}$$

and I_8 is a 8×8 unit matrix. The inverse metric may be easily computed to obtain

$$G^{\mu\nu}=\begin{pmatrix} 0 & 1 & 0 \\ 1 & -\hat{K}+\dfrac{A_i^{+2}}{f} & -\dfrac{A_i^+}{f} \\ 0 & -\dfrac{A_i^+}{f} & \dfrac{I_8}{f} \end{pmatrix}. \tag{38}$$

Using these, it may be shown that the only nonzero components of the Christoffel connections are, once again, Γ^v_{uu}, Γ^i_{uu}, Γ^v_{ui}, Γ^j_{ui}, and Γ^v_{ij}. This leads to the null Killing vector being covariantly constant, i.e., $D_\mu l^\nu=0$ and $D_\mu l_\nu=0$.

We now proceed to compute the Riemann curvature tensor for the metric (35) of the type IIB background generated by us. Employing the closed form expression for the new metric, we once again get the the Riemann tensor to be of the form in Eq. (8) with $R^{(M)}_{\lambda\mu\nu\kappa}$ now being the Riemann tensor for the metric $M_{\mu\nu}$ in Eq. (37). Once again the only nonzero independent component of the Riemann tensor is R_{uiuj} when we specialize to the background described by Eqs. (18). Explicitly, the expression for the Riemann tensor is

$$\begin{aligned} R_{uiuj}=&\tfrac{1}{2}\partial_i\partial_jK-\tfrac{1}{2}\partial_u[\partial^iA_j^++\partial_jA_i^+]+\tfrac{1}{2}\partial_u^2f\delta_{ij} \\ &-\tfrac{1}{4}f^{-1}(u)[-\partial_uf\delta_{ik}+(\partial_iA_k^+-\partial_kA_i^+)] \\ &\times[-\partial_uf\delta_{jk}+(\partial_jA_k^+-\partial_kA_j^+)]. \end{aligned} \tag{39}$$

The other background fields are the two antisymmetric tensor field strengths from the NS-NS and R-R sectors, respectively, which are given by $H^{(k)}_{\lambda\mu\nu}$ and $(k=1,2)$ in Eqs. (31) and (32). Having obtained the type IIB background with nontrivial R-R fields, described by Eqs. (29)–(32) and (35), we now proceed to show that these are exact to all orders (in α'). We once again adopt the geometrical approach [17,19] outlined in Sec. II for this purpose. The background field equations for the type IIB superstring are, to the lowest order, those of $N=2$, $D=10$ supergravity in [21]. They are all tensor equations of a definite rank. For the type IIB background under consideration, we have second rank tensor equations for the string form metric $G_{\mu\nu}$ and the antisymmetric tensor field $B_{\mu\nu}$. We also have scalar equations for the NS-NS and R-R scalars ϕ and χ and a fifth rank completely antisymmetric tensor equation for the five-form field strength $F_{\mu\nu\rho\sigma\kappa}$. The last equation expresses the self-duality condition on the five-form field strength.

To study the all order contributions to the background field equations of motion, the possible corrections to all these tensor equations must be considered. Notice that the contributions from the background gauge fields $B_{\mu\nu}$ and $D_{\mu\nu\rho\sigma}$ appear in the higher order terms as the corresponding gauge-invariant field strengths. As a consequence, we need to consider the higher order terms in these equations obtained from the quantities $R_{\mu\nu\rho\sigma}$, $H^{(k)}_{\mu\nu\rho}$, $D_\mu\phi$, $D_\mu\chi$, $F_{\mu\nu\rho\sigma\kappa}$, and their covariant derivatives. In our case, we choose F to be zero, which is obtained by setting the four-form field $D_{\mu\nu\rho\sigma}=0$ in its definition, together with the form of $H^{(k)}$ in Eqs. (31) and (32).

As in the bosonic case, possible nonzero independent tensor components for the background defined by Eqs. (29)–(32) and (35) are $D_u\phi$, $D_u\chi$, R_{uiuj}, and its covariant derivatives with respect to D_u and D_j, H_{uij}, and its covariant derivatives with respect to D_u, and the corresponding covariant components. Notice that the index structures of the appropriate nonzero tensor components are exactly as earlier in the trivial case. The only additions for the field content in the nontrivial type IIB case are the nonzero R-R fields $\chi(u)$ and $H^{(2)}$ as the five-form field strength is zero. Hence similar arguments show that all such higher order contributions, as earlier, are vanishing. For the two additional equations also, namely, scalar and the five-form R-R fields, similar geometrical arguments show that the higher order constributions vanish. Hence the background field equations, which to the lowest order are those of $N=2$, $D=10$ supergravity, are exact, to all orders in α', also in presence of R-R fields in ten dimensions.

IV. CONCLUSIONS

To conclude, we have obtained a class of type IIB superstring backgrounds involving R-R fields from the bosonic K models embedded in a type IIB background with vanishing R-R fields. The complications involving the σ model couplings of the R-R fields have been obviated by adopting a purely geometrical approach to compute the higher order terms in the equations of motion. This approach, which is based on the analysis of the tensorial index structure of higher order contributions, seems to be applicable to a wider class of backgrounds than those obtained from the K models. We mention in passing that our analysis has been restricted to K model backgrounds with vanishing five-form self-dual field strength. Furthermore, we have focused on strictly u-dependent antisymmetric tensor field strengths and dilaton. It would be an interesting exercise to obtain type IIB backgrounds from more general K models and show that they are also exact following the geometrical approach which has been elucidated in this article. In particular, for K models in the bosonic case, $A=0$ or $\bar{A}=0$ conditions provide another class of all-order solutions. It will be interesting to show that they are exact for type IIB as well.

Our results also indicate that type IIB strings in a constant magnetic field background may be formulated as an exact conformal field theory. It will be interesting to study the phase transition, discussed in [20], for this case. The status of unbroken space-time supersymmetries, like the K models with trivial embeddings in superstrings, is also of interest to investigate in the presence of R-R fields. These will have implications for the present backgrounds to be solutions in the presence of local string-loop corrections as well.

ACKNOWLEDGMENTS

G.S. would like to thank Institute of Physics, Bhubaneswar where this work was partially completed and the High Energy Theory Group there for the warm hospitality and the stimulating research environment.

[1] E. Witten, Nucl. Phys. **B443**, 85 (1995); J. H. Schwarz, Phys. Lett. B **367**, 97 (1996); J. Maharana, *ibid.* **372**, 53 (1966); K. Dasgupta and S. Mukhi, ''Orbifolds of *M* Theory,'' Tata Institute Report No. hep-th /9512196 (unpublished); E. Witten, Nucl. Phys. **B463**, 383 (1996); J. H. Schwarz, ''*M* Theory Extensions of T-Duality,'' Caltech Report No. hep-th/9601077 (unpublished); A. Sen, Phys. Rev. D **53**, 6725 (1996).

[2] For a review see A. Giveon, M. Porrati, and E. Rabinovici, Phys. Rep. **244**, 77 (1994), and references therein. See also E. Alvarez, L. Alvarez-Gaume, and Y. Lozano, *An Introduction to T duality in String Theory*, Report No. hep-th/9410237 (unpublished).

[3] A. Font, L. Ibanez, D. Luest, and F. Quevedo, Phys. Lett. B **249**, 35 (1990).

[4] A. Sen, Int. J. Mod. Phys. A **9**, 3707 (1994).

[5] J.H. Schwarz, ''String Theory Symmetries,'' Report No. hep-th/9503127 (unpublished); Lett. Math. Phys. **34**, 309 (1995); E. Witten, Nucl. Phys. **B443**, 85 (1995).

[6] C. M. Hull and P. K. Townsend, Nucl. Phys. **B438**, 109 (1995); **B451**, 525 (1995); P. Aspinwall, ''Some Relationships Between Dualities in String Theory,'' Report No. hep-th/9508154 (unpublished).

[7] E. Bergshoeff, C. M. Hull, and T. Ortin, Nucl. Phys. **B451**, 547 (1995); E. Bergshoeff ''Duality Symmetries and Type II String Effective Actions,'' Groningen Report No. UG-11/95, hep-th/ 9509145 (unpublished).

[8] For a review see A. Tseytlin, Int. J. Mod. Phys. A **4** 1257 (1989), and references therein.

[9] D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. **B271**, 93 (1986).

[10] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); D. Polyakov, ''R-R Dilaton Interaction in a Type IIB Superstring,'' Rutgers Univ. Report No. RU-95-85, hep-th/9512028 (unpublished); A. Tseytlin, ''On dilaton dependence of type II superstring action,'' Report No. hep-th/9601109 (unpublished).

[11] A. Tseytlin, Nucl. Phys. **B390**, 193 (1993).

[12] E. Bergshoeff, R. Kallosh, and T. Ortin, Phys. Rev. D **47**, 5444 (1993); E. Bergshoeff, I. Entrop, and R. Kallosh, *ibid.* **49**, 6663 (1994).

[13] G. Horowitz and A. Tseytlin, Phys. Rev. D **50**, 5204 (1994); A. Tseytlin, ''Exact String Solutions and Duality,'' Imperial College Report No. Imperial/TP/93-94/46, hep-th/9407099 (unpublished).

[14] G. Horowitz and A. Tseytlin, Phys. Rev. D **51,** 2896 (1995).

[15] A. Tseytlin, Class. Quantum Grav. **12**, 2365 (1995).

[16] G. Horowitz, and A. R. Steif, Phys. Rev. Lett. **64**, 260 (1990); Phys. Rev. D **42**, 1950 (1990).

[17] C. Duval, Z. Horvath, and P. A. Horvathy, Phys. Lett. B **313**, 10 (1993).

[18] R. Guven, Phys. Lett. B **191**, 275 (1987).

[19] S. Kar, A. Kumar, and G. Sengupta, Phys. Lett. B (to be published).

[20] J. G. Russo and A. A. Tseytlin, Nucl. Phys. **B448**, 293 (1995); **B454**, 164 (1995).

[21] J. H. Schwarz, Nucl. Phys. **B226**, 269 (1983).