Entropy and temperature of black 3-branes

S. S. Gubser,* I. R. Klebanov,† and A. W. Peet‡
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544

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We consider slightly nonextremal black 3-branes of type IIB supergravity and show that their Bekenstein-Hawking entropy agrees, up to a mysterious factor, with an entropy derived by counting non-BPS excitations of the Dirichlet 3-brane. These excitations are described in terms of the statistical mechanics of a (3+1)-dimensional gas of massless open string states. This is essentially the classic problem of blackbody radiation. The blackbody temperature is related to the temperature of the Hawking radiation. We also construct a solution of type IIB supergravity describing a 3-brane with a finite density of longitudinal momentum. For extremal momentum-carrying 3-branes the horizon area vanishes. This is in agreement with the fact that the BPS entropy of the momentum-carrying Dirichlet 3-branes is not an extensive quantity. [S0556-2821(96)00218-4]

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I. INTRODUCTION

Apart from their intrinsic importance, black holes[1] provide a testing ground for the quantum theory of gravitation. Classical general relativity, together with quantum field theory, imply that a black hole should be assigned an entropy equal to one-fourth of its horizon area measured in Planck units [1,2]. In a fundamental theory of quantum gravity this Bekenstein-Hawking entropy should have a statistical interpretation. It has been argued [3–5] that string theory provides such an interpretation, because very massive fundamental string states should form black holes, and the number of such states exhibits the exponential Hagedorn growth.

Recently, a much improved understanding of the Ramond-Ramond charged string solitons has emerged through the Dirichlet brane description [6,7]. This has led to a rapid progress on the black hole entropy problem. In [8] a certain extremal five-dimensional black hole was constructed so that its horizon area is nonvanishing. It was shown that the logarithm of its ground state degeneracy, calculated with D-brane methods, precisely matches the Bekenstein-Hawking entropy. This remarkable finding has been extended in a number of directions. In [9] it was generalized to rotating black holes. In [10] a similar five-dimensional example was considered, and it was further shown that the entropy of slightly nonextremal black holes also matches the Bekenstein-Hawking result. This allowed for a D-brane calculation of the temperature of Hawking radiation. In [11] similar results were obtained for slightly nonextremal black strings in six dimensions (upon compactification these strings reduce in a certain limit to the five-dimensional black holes of [8]).

At this stage it is important to elucidate the criteria for agreement between the D-brane and the Bekenstein-Hawking entropy, and to find new successful examples. In this paper we provide a new and very simple example of a black p-brane whose D-brane entropy almost matches the Bekenstein-Hawking entropy. This is the self-dual 3-brane in ten dimensions. Since it couples to the self-dual five-form, it automatically carries equal electric and magnetic charge densities. A special property of this object, as well as of those in [8–11], is that the string coupling is independent of position. Control over the value of the string coupling at the horizon appears to be necessary for agreement between the two definitions of entropy. For p-branes with $p<3$ it is easy to check that the D-brane entropy is not proportional to the horizon area. This is likely due to the string coupling becoming strong near the p-brane.

The original 3-brane solution of type IIB supergravity was constructed in [12]. In Sec. II we observe that at extremality this solution has vanishing horizon area. We construct a new class of solutions describing 3-branes carrying finite-momentum density along one of its internal dimensions. Although the longitudinal momentum is known to stabilize the horizon area of extremal black strings [11], here we find that it does not. The fact that the classical entropy is zero agrees with the fact that the logarithm of the ground state degeneracy of the momentum-carrying Dirichlet 3-branes is not an extensive quantity. In order to address objects with nonvanishing horizon area, in Sec. III we consider slightly nonextremal 3-branes, whose masses satisfy $\delta M = M - M_0 \ll M_0$. To leading order in the parameter $\delta M/M_0$, which is a measure of deviation from extremality, we find agreement between the D-brane entropy and 1/4 of the horizon area. Amusingly, the statistical mechanics of a nonextremal 3-brane is that of a photon (and photino) gas in $3+1$ dimensions, which is the classic black-body radiation problem. The scaling of entropy with energy may be derived essentially from the well-known black-body scaling laws

$$M - M_0 \sim VT^4, \quad S \sim VT^3. \tag{1}$$

Working out the precise normalizations, we find that the Bekenstein-Hawking and statistical entropies are not identical, but are related by a mysterious proportionality factor. If, however, only the transverse excitation modes of the 3-brane are counted, then the statistical entropy becomes identical to

*Electronic address: ssgubser@puhep1.princeton.edu
†Electronic address: klebanov@puhep1.princeton.edu
‡Electronic address: peet@viper.princeton.edu

[1]In this short article we will not attempt to refer to all of the developments in the recent black hole literature.

the Bekenstein-Hawking entropy. While this rule is suggestive, at the moment we do not know how to justify it.

Upon coupling of the 3-brane to the ten-dimensional world, waves colliding on the 3-brane may be converted to massless closed string states. This is Hawking radiation in the D-brane language [10]. The black-body temperature that one assigns to a nonextremal 3-brane acquires the interpretation of the Hawking temperature. In Sec. IV we conclude with a brief discussion.

II. ENTROPY OF 3-BRANES CARRYING LONGITUDINAL MOMENTUM

The 3-brane solution to the equations of type IIB supergravity was originally obtained by Horowitz and Strominger [12] and is given by

$$ds^2=-\Delta_+\Delta_-^{-1/2}dt^2+\Delta_-^{1/2}(dx_1^2+dx_2^2+dx_3^2)+\Delta_+^{-1}\Delta_-^{-1}dr^2+r^2d\Omega_5^2,$$

$$F_{(5)}=Q(\varepsilon_5+{}^*\varepsilon_5),$$

$$\Phi=\text{const.} \tag{2}$$

In these equations $F_{(5)}$ is the Ramond-Ramond self-dual five-form field strength coupling to the 3-brane, and the dilaton field has an arbitrary constant value for this solution. We have also defined

$$\Delta_\pm(r)=\left(1-\frac{r_\pm^4}{r^4}\right). \tag{3}$$

The charge density on the 3-brane is

$$Q=2r_+^2r_-^2\equiv 2r_0^4 \tag{4}$$

up to a convention-dependent proportionality constant. In this section we will ignore such constants since they are irrelevant to our calculations. For the solution to be well behaved, we need $r_+\geqslant r_-$. Extremality is achieved when the horizon radius r_+ becomes equal to r_-. The extremal Arnowitt-Deser-Misner (ADM) mass is proportional to Q, as required by supersymmetry. The extremal solution preserves one-half of the ten-dimensional type IIB supersymmetries, i.e., $N=1$. We also introduce an infrared cutoff by compactifying each internal coordinate x^i on a very large circle of radius L, i.e., imagine that the 3-brane is wrapped around a large 3-torus T^3.

The eight-dimensional area of the horizon is

$$A=\omega_5r_+^5L^3[\Delta_-(r_+)]^{3/4}, \tag{5}$$

where $\omega_5=\pi^3$ is the area of a unit five sphere. The classical black 3-brane entropy

$$S_{\text{BH}}=\frac{A}{4}, \tag{6}$$

therefore, vanishes in the extremal limit.

If we fix the charge and consider a slightly nonextremal black 3-brane then, as we will see in the next section, the entropy of the classical extremal black 3-brane scales as

$$S_{\text{ext}}\sim\omega_5L^3r_0^5\left[\frac{\delta M}{M_0}\right]^{3/4}. \tag{7}$$

In the case of the black string [12], which also had zero area at extremality, it was possible to perform a boost along the string to induce simultaneously finite ADM momentum and horizon area.

It is also easy to inject momentum P along one[2] of the three spatial world-brane directions, which we take to be x^1. The appropriate solution may be found by performing a (now standard) boost on the solution Eq. (2). In this way we obtain

$$\begin{aligned}ds^2=&-(\cosh^2\alpha\Delta_+\Delta_-^{-1/2}-\sinh^2\alpha\Delta_-^{1/2})dt^2\\&+(\cosh^2\alpha\Delta_-^{1/2}-\sinh^2\alpha\Delta_+\Delta_-^{-1/2})dx_1^2\\&+\sinh(2\alpha)(\Delta_-^{1/2}-\Delta_+\Delta_-^{-1/2})dt\,dx_1\\&+\Delta_-^{1/2}(dx_2^2+dx_3^2)+\Delta_+^{-1}\Delta_-^{-1}dr^2+r^2d\Omega_5^2.\end{aligned} \tag{8}$$

If we imagine that the T^3 is small, then we can think of the configuration Eq. (8) as a seven-dimensional black hole. The black hole has a gauge charge corresponding to the gauge field which comes from the (t,x^1) cross term in the metric. Note that this extremal solution is still Bogomol'ni-Prasad-Sommerfield- (BPS-) saturated, as it preserves one supersymmetry of a possible four (type IIB compactified on T^3 to $d=7$ has $N=4$ supersymmetry). In ten-dimensional language this ''charge'' is just the total ADM momentum, which is given by

$$P_{\text{ADM}}=\frac{L^3\omega_5}{8\pi}\sinh(2\alpha)(r_+^4-r_-^4),$$

$$\equiv\frac{2\pi n}{L}, \tag{9}$$

where n is an integer and we are keeping the ten-dimensional Newton constant fixed.

If we let the deviation from extremality go to zero, but also take the limit of infinite boost parameter, then for finite ADM momentum

$$P_{\text{ADM}}\sim L^3\omega_5Q\left[e^{2\alpha}\frac{\delta M}{M_0}\right], \tag{10}$$

we need the scaling $\delta M/M_0\sim e^{-2\alpha}$.

Then, the entropy of a BPS-saturated state with this momentum number n is finite and given by

[2]Note that our conclusions would be unchanged if we performed additional boosts involving any of the other spatial world-brane directions.

$$S_{\rm BPS} \sim 2\pi\sqrt{2n},$$
$$\sim L^2[\omega_5 Q]^{1/2}\left[\frac{\delta M}{M_0}e^{2\alpha}\right]^{1/2}. \quad (11)$$

This quantity is not extensive in the spatial world volume of the 3-brane. The entropy density, measured per unit spatial world volume, goes as

$$s_{\rm BPS} \equiv \frac{S_{\rm BPS}}{L^3} \to 0. \quad (12)$$

For a Dirichlet p-brane, this zero BPS entropy will actually happen for any value of $p>1$, as follows. A BPS-saturated excitation on the world volume is effectively restricted to live in a single dimension, because if there were two finite orthogonal momenta, then the state would no longer be BPS saturated. Therefore, the scaling goes as $S_{\rm BPS} \sim \sqrt{n}$, while $P_{\rm ADM} \sim L^p$, so that $n \sim L^{p+1}$, and, therefore,

$$s_{\rm BPS} \sim L^{(p+1)/(2p)} \to 0. \quad (13)$$

So we see that in order to have finite, nonzero ADM momentum and finite, nonzero entropy, both measured per unit spatial world volume, we need $p=1$, i.e., the string.

Let us now compare this conclusion about the Dirichlet 3-brane entropy with results for the classical black 3-brane configuration. Because of the boost, we find that the Bekenstein-Hawking entropy of the classical configuration Eq. (2) is altered from its previous value to

$$S_{\rm BH} = \frac{\omega_5}{4} r_+^5 L^3 [\Delta_-(r_+)]^{3/4} \cosh\alpha,$$
$$\sim \omega_5 L^3 r_0^5 \left[\frac{\delta M}{M_0}\right]^{3/4} e^{\alpha}, \quad (14)$$

as $\alpha \to \infty$ and $\delta M/M_0 \to 0$. Let us now take the limit such that the ADM momentum remains finite. Then, we need the scaling $\delta M/M_0 \sim e^{-2\alpha}$ and so the classical 3-brane area goes as

$$A \sim e^{-3\alpha/2} e^{\alpha} \to 0. \quad (15)$$

This tells us that the BPS-saturated 3-brane with finite nonzero momentum still has zero area. Note that if we consider a modified area given by the classical horizon area divided by $\sqrt{g_{22}(r_+)g_{33}(r_+)}$, this scales similarly to the quantity (11); however, it is difficult to give this modified area an enlightening physical interpretation.[3]

Therefore, we see that the entropy of the BPS-saturated classical 3-brane with momentum, which by definition is extensive in the horizon area, is also zero. It is satisfying that the entropies on the classical black 3-brane and Dirichlet 3-brane sides agree, as expected.

[3]Note also that in the above scaling limit, g_{tt} diverges on the horizon. We thank Gary Horowitz for pointing this out to us.

III. STATISTICAL MECHANICS OF NONEXTREMAL 3-BRANES

In this section we will consider non-BPS excitations of the 3-brane. In the D-brane picture the excitations we have in mind are described by a dilute gas of massless open string states running along the brane in arbitrary directions. The average total momentum is zero. The momenta of the massless string states are quantized:

$$\vec{p} = \frac{2\pi}{L}\vec{n}, \quad (16)$$

where $\vec{n} \in \mathbf{Z}^3$. The mass of the excited 3-brane is

$$M = M_0 + \delta M = \frac{\sqrt{\pi}}{\kappa}L^3 + \sum_{i=1}^{k} \frac{2\pi}{L}|\vec{n_i}| + O(g). \quad (17)$$

Here, M_0 is the mass of the extremal 3-brane [13], k is the number of open strings, and

$$\kappa = \sqrt{8\pi G_N} = g\alpha'^2. \quad (18)$$

The $O(g)$ term in (17) accounts for interactions among the strings. The validity of counting these states and no others to obtain the entropy of a nonextremal p-brane was discussed in [14] for the case $p=1$, and the same arguments apply here. In particular, our ability to control the decay rate of the non-BPS states by making L large allows us to count these states reliably with g and hence G_N finite.

Rather than calculating the degeneracy of excited 3-brane states at a given δM directly, let us instead consider the statistical mechanics of massless open string states in the grand canonical ensemble. The temperature T will later be identified as the Hawking temperature, but for now one can regard our ensemble calculations as a trick to figure out the degeneracies of brane excitation levels.

For a system with N massless boson and fermion physical degrees of freedom, the correct partition function is

$$Z = \prod_{\vec{n} \in \mathbf{Z}^3} \left(\frac{1+q^{|\vec{n}|}}{1-q^{|\vec{n}|}}\right)^N, \quad (19)$$

where we have defined

$$q = e^{-2\pi/LT}. \quad (20)$$

One expects $N=8$, but for now we leave it arbitrary. The dynamics of these modes on the brane is given by $\mathcal{N}=4$ supersymmetric pure Yang-Mills theory with gauge group U(1) [15–17]. For our purposes, however, it is more revealing to view this theory as $\mathcal{N}=1$ Yang-Mills plus six chiral multiplets. The chiral multiplets are associated with transverse oscillations of the brane, while the gauge multiplet describes internal degrees of freedom. We will find that, to obtain perfect agreement with the Bekenstein-Hawking entropy, it is necessary to count only the modes of transverse oscillation, hence setting $N=6$ in Eq. (19).

What subtlety of the gauge dynamics might prevent the gauge degrees of freedom from being enumerated along with the transverse oscillations? Tseytlin has suggested to us the following interesting mechanism [18]. If one imposes periodic boundary conditions on the gauginos along the Euclidean time direction rather than the standard antiperiodic boundary conditions, then the two physical gaugino degrees of freedom introduce a factor $(1-q^{|\vec{n}|})^2$ into the partition function, exactly canceling the gauge boson contribution, $(1-q^{|\vec{n}|})^{-2}$. Thus, the gauge dynamics becomes, in effect, topological. We look forward to exploring possible justifications and consequences of this insightful guess for the gaugino boundary conditions.

Equation (19) includes N bosonic and N physical fermionic modes, and in $3+1$ dimensions each fermion mode makes 7/8 of the contribution of a boson mode to the entropy and energy (the corresponding ratio in $1+1$ dimensions is 1/2). Using the relations

$$F=-T\ln Z,$$

$$E=T^2\frac{\partial}{\partial T}\ln Z,$$

$$S=(E-F)/T, \tag{21}$$

we find

$$E=\frac{\pi^2}{16}NL^3T^4,$$

$$S=\frac{\pi^2}{12}NL^3T^3. \tag{22}$$

At this point it is easy to see how things change when n_w 3-branes are stacked on top of one another. The massless open strings can now connect any two of the branes, so there are n_w^2 states for every one state we had before. In this context it is important to recall that there is no binding energy among the 3-branes [15], so strings running between different branes really are massless. Furthermore, when L is large, it makes no difference whether we consider n_w singly wound branes or one brane wrapped n_w times around T^3: the asymptotic density of massless string states per unit volume is unaffected by such changes in boundary conditions.

To recapitulate, the prescription for $n_w>1$ is to consider n_w^2 (very weakly) coupled thermodynamic systems, each identical to the $n_w=1$ system treated above. Thus, Eq. (22) becomes

$$E=\frac{\pi^2}{16}Nn_w^2L^3T^4,$$

$$S=\frac{\pi^2}{12}Nn_w^2L^3T^3. \tag{23}$$

The relation between E and S in the microcanonical ensemble is determined by eliminating T from Eq. (23):

$$S=\tfrac{2}{3}N^{1/4}\sqrt{\pi n_w}L^{3/4}E^{3/4}. \tag{24}$$

Setting $E=\delta M$ in Eq. (24), one obtains the entropy of nonextremal 3-branes with mass $M_0+\delta M$. Using the formula [13]

$$M_0=\frac{\sqrt{\pi}}{\kappa}n_wL^3, \tag{25}$$

one can show finally that

$$S=\tfrac{2}{3}N^{1/4}\pi^{7/8}n_w^{5/4}\kappa^{-3/4}L^3(\delta M/M_0)^{3/4}. \tag{26}$$

This expression for S should be comparable to the Bekenstein-Hawking entropy. Let us, therefore, turn to the calculation of the horizon area in the low-energy supergravity theory.

The ADM mass formula for the black 3-brane described by the metric (2) is [19]

$$M_{\rm ADM}=\frac{\omega_5L^3}{2\kappa^2}(5r_+^4-r_-^4). \tag{27}$$

Applying this formula to the extremal case $r_+=r_-=r_0$ and comparing with Eq. (25), one finds

$$r_0^4=\frac{\sqrt{\pi}}{2\omega_5}n_w\kappa. \tag{28}$$

The RR charge remains unchanged as we perturb away from extremality, so $r_-=r_0^2/r_+$. Writing $r_+=r_0+\varepsilon$, one finds from Eq. (27) that

$$\frac{\delta M}{M_0}=6\frac{\varepsilon}{r_0} \tag{29}$$

to lowest order in ε. Thus the horizon area of the metric (2) is

$$A=\omega_5r_+^5L^3\left(1-\frac{r_-^4}{r_+^4}\right)^{3/4}$$

$$=2^{9/4}\omega_5r_0^5L^3\left(\frac{\varepsilon}{r_0}\right)^{3/4}$$

$$=2^{1/4}3^{-3/4}\pi^{-1/8}(n_w\kappa)^{5/4}L^3(\delta M/M_0)^{3/4}, \tag{30}$$

and the Bekenstein-Hawking entropy is

$$S_{\rm BH}=\frac{2\pi A}{\kappa^2}=2^{5/4}3^{-3/4}\pi^{7/8}n_w^{5/4}\kappa^{-3/4}L^3(\delta M/M_0)^{3/4}. \tag{31}$$

If we include all eight bosonic and fermionic modes in the statistical mechanics treatment of D-brane excitations, we obtain the following relation between the entropies [20]

$$S=(\tfrac{4}{3})^{1/4}S_{\rm BH}. \tag{32}$$

While the scaling exponents agree perfectly, a mysterious numerical factor appears. We do not understand why the statistical counting gets so close, yet fails to reproduce the Bekenstein-Hawking entropy. Note, however, that if we set $N=6$ then $S=S_{\rm BH}$. It is tempting to conjecture that a subtle modification of the world-volume dynamics, such as the twisted boundary conditions proposed by Tseytlin, is responsible for this. The bottom line is that an ideal gas on $6n_w^2$ massless bosons and fermions on the world volume reproduces the Bekenstein-Hawking entropy. The fact that this number is $\sim n_w^2$ agrees with the enhanced symmetry of coincident 3-branes. The necessary number is *smaller* than the $8n_w^2$ massless modes of the weakly coupled $\mathcal{N}=4$ $U(n)$ gauge theory. A resolution of this puzzle may be related to the question of binding of the 3-branes. If the n_w parallel 3-branes form a marginal bound state, then the number of massless modes is indeed reduced compared to what is expected for unbound 3-branes. Although we do not know what produces this bound state, we may speculate that it is related to confinement.

A bonus we get for computing the entropy in the grand canonical ensemble is that the blackbody temperature T used in Eqs. (19)–(23) is related to the Hawking temperature. This is a trivial consequence of the relation $M=M_0+E$ where E is the energy of the gas of massless open strings. We know from ordinary statistical mechanics that $dE=TdS$ when L is held fixed. But $dE=dM$, so the relation $dM=T_H dS_{\rm BH}$ from black hole thermodynamics leads immediately to

$$T_H=\left(\frac{8}{3\pi^2 n_w}\frac{\delta M}{L^3}\right)^{1/4}=\left(\frac{N}{6}\right)^{1/4}T. \tag{33}$$

At first, it seems surprising that the Hawking temperature should be independent of the string coupling g. But it becomes inevitable when one realizes that $T_H\sim T$, since the properties of the dilute gas of open string states characterizing the excitation of the D-brane depend in no way on g. The string coupling determines only the degree of diluteness necessary to make our arguments valid. It remains a fascinating problem to derive this g-independent temperature from a string perturbative calculation of the amplitudes for decay processes of the excited 3-brane, similar to the scattering amplitudes computed in [21].

IV. DISCUSSION

In this paper we have presented a very simple Dirichlet brane system whose entropy is almost identical to the Bekenstein-Hawking entropy of the corresponding low-energy supergravity solution. This relation is so miraculous that it clearly requires a deeper understanding. How does classical type IIB supergravity ''know'' the Planck formula for blackbody spectrum? Apparently it does. The numerical factor relating the statistical and Bekenstein-Hawking entropies poses a puzzle, however. We are inclined to regard this factor as a hint that we have yet to learn everything about the dynamics of coincident 3-branes. The suggestion [18] to make the gaugino fields periodic in Euclidean time is a simple way to obtain perfect agreement with the Bekenstein-Hawking formula, but justification for this guess awaits a more thorough understanding of the world-volume gauge field.

Motivated by [10] we would also like to show precisely how the 3-brane blackbody temperature translates into the Hawking temperature of the outgoing closed string radiation. We hope to report on these issues in the near future.

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