Exact gravitational shock wave solution of higher order theories

M. Campanelli
Fakultät für Physik der Universität Konstanz, Postfach 5560 M 674, D-78434 Konstanz, Germany

C. O. Lousto*
Department of Physics, University of Utah, Salt Lake City, Utah 84112†
and IFAE–Grupo de Física Teórica, Universidad Autónoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain
(Received 2 January 1996; revised manuscript received 3 May 1996)

We find an *exact pp*-gravitational wave solution of the fourth-order gravity field equations. Outside the (δ-like) source this is *not* a vacuum solution of general relativity. It represents the contribution of the massive, $m=(-\beta)^{-1/2}$, spin-two field associated while the Ricci-squared term in the gravitational Lagrangian. The fourth-order terms tend to make the singularity of the curvature milder at the point where the particle is located. We generalize this analysis to D dimensions, extended sources, and higher than fourth-order theories. We also briefly discuss the scattering of fields by these kinds of plane gravitational waves. [S0556-2821(96)02018-8]

PACS number(s): 04.50.+h, 04.30.Nk

I. INTRODUCTION

Higher order theories of gravity are the generally covariant extension of general relativity when one considers in the Lagrangian nonlinear terms in the curvature. The field equations derived by variation of this Lagrangian contain derivatives of the metric of an order higher than the second (except for the Lovelock's Lagrangian constructed from the D-dimensional extension of four-dimensional invariants [1]). Historically, they have been introduced by Weyl right after general relativity [2]. These theories provided a framework of where to study the unification of gravity with other fundamental fields, and the possibility of classically avoiding cosmological singularities. More recently, higher order theories have been shown to lead to inflation [3] and dimensional reduction without the introduction of any additional scalar field. Nowadays, among the main motivations for their study are their appearance (as vacuum polarization terms) in the one-loop renormalization of fields in curved spacetimes [4] and in the low-energy limit of string theory [5].

In four dimensions, using the Gauss–Bonnet invariant, a general fourth-order (quadratic) covariant Lagrangian can be written as

$$I=I_G+I_m=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\{-2\Lambda+R+\alpha R^2+\beta R_{\mu\nu}R^{\mu\nu}+16\pi G\mathcal{L}_m\}, \tag{1.1}$$

where we have not considered surface terms since they will not contribute to the analysis of the field equations we will perform.

The field equations derived by extremizing the action I are given by

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}+\alpha H_{\mu\nu}+\beta I_{\mu\nu}=8\pi GT_{\mu\nu}\doteq-\frac{16\pi G}{\sqrt{-g}}\frac{\partial I_m}{\partial g^{\mu\nu}}, \tag{1.2}$$

where

$$H_{\mu\nu}=-2R_{;\mu\nu}+2g_{\mu\nu}\Box R-\frac{1}{2}g_{\mu\nu}R^2+2RR_{\mu\nu}, \tag{1.3}$$

and

$$I_{\mu\nu}=-2R^{\alpha}_{\mu;\nu\alpha}+\Box R_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\Box R+2R^{\alpha}_{\mu}R_{\alpha\nu}-\frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}. \tag{1.4}$$

Upon linearization of these equations one can see that in addition to the usual graviton field this theory contains a massive scalar field ϕ related to R and a massive spin-two field $\psi_{\mu\nu}$ related to $R_{\mu\nu}$ (see Ref. [6] for further details). Asking for both fields to have a real mass (to recover the Newtonian limit) leads to the, so-called, no-tachyon constraints

$$3\alpha+\beta\geqslant 0, \quad \beta\leqslant 0. \tag{1.5}$$

The value of these coupling constants can only be determined by experiments or could be computed from a fundamental theory that would unify gravity to the other forces in nature. It is then expected they will be of the order of the Planck scale.

The quantum properties of these theories have extensively been discussed [7]. However, several issues, such as the unitarity problem [8] and the semiclassical instabilities [9], remain to be solved. In addition, we have not even a complete

*Electronic address: lousto@mail.physics.utah.edu
†Present address.

understanding of the proper solutions of fourth-order theories. In [10] we have developed a perturbative method to find solutions of the field equations (1.2), given a solution to the general relativistic problem (see also Refs. [11,9]). The method essentially consists of writing the higher derivative terms as derivatives of the matter energy-momentum tensor $T_{\mu\nu}$. One obtains a series development around the general relativity metric in powers of the coupling constants α and β. We have thus studied the α and β corrections to the Reissner-Nordström and straight cosmic string metrics [12].

Our perturbative approach, evidently, gives no corrections when the matter energy-momentum tensor vanishes. In other words, the method confirms that vacuum solutions (including $\Lambda \neq 0$) of general relativity are also solutions of higher order theories. However, *the converse is in general not true.* Higher order theories have a richer set of vacuum solutions than general relativity. If we call this set Σ_{VHO} and the set of vacuum solutions of general relativity Σ_{VGR}, its difference, $\Sigma_{\Delta V}=\Sigma_{\mathrm{VHO}}-\Sigma_{\mathrm{VGR}}$, is in general a nonempty set. From the above-described inability of the perturbative approach to find solutions in $\Sigma_{\Delta V}$, we can deduce that such solutions, if they exist, have to be *nonperturbative* around α and β equal to zero.

For black hole [13] and de Sitter [14] solutions one can extend the no-hair theorems valid for general relativity to fourth-order theories in the case $\beta=0$. From there we can infer that $\Sigma_{\Delta V}$ black holes will have the form of the Kerr metric plus nonanalytic corrections in β only. The perturbative corrections in powers of β to the Reissner-Nordström metric have been given in Refs. [15,10].

In Sec. II we introduce the gravitational shock waves, a special case of a pp−wave solution of Einstein equations with a δ-like source term. We analyze its extension to fourth-order gravity and find the corresponding solution which is an exact proper solution of field equations (1.2). In Sec. III we generalize this exact solution to D-dimensional spacetime, extended sources, and theories of order higher than the fourth. We finally, in the Appendix, deal with the problem of the scattering of a scalar field by these shock wave geometries and compute the S matrix for the case of a source obtained by boosting to the speed of light the Kerr metric.

II. GRAVITATIONAL SHOCK WAVES

As pointed out by 't Hooft [16], at energies of the order or higher than the Planck scale the picture of particles propagating in flat spacetime ceases to be a good approximation. Interestingly enough the curved metric generated by such particles has a remarkable simple form:

$$ds^2=-dudv+H(u,x_\perp)du^2+dx_\perp^2, \tag{2.1}$$

where

$$H(u,x_\perp)=f(x_\perp)\,\delta(u), \quad u=t-z, \quad v=t+z. \tag{2.2}$$

This metric represents an impulsive gravitational wave localized in the plane $u=0$, i.e., along the motion of the particle. The shock wave is accompanying the particle, both traveling at the speed of light. The profile function $f(x_\perp)$ is the only quantity depending on the characteristic of the source. It is only a function of the coordinates perpendicular to the plane of motion, $x_\perp$. The geometry is just flat before and after the pass of the wave, i.e., for $u\neq 0$, and is a special case of plane fronted with parallel rays (pp) waves [17]. Geodesics are then just straight lines outside the wave front and have a finite discontinuity (or shift) in the v coordinate as they cross $u=0$ given by [see Ref. [18] for the case of the Aichelburg-Sexl metric, i.e., Eq. (2.10) below]

$$\Delta v=-f(\rho_i), \tag{2.3}$$

where ρ_i is the coordinate distance from the origin (where the source is located) to the point where the geodesic reaches $u=0$.

There is also an effect of spatial refraction of geodesics (see again Ref. [18])

$$\cot(\theta_{\mathrm{in}})+\cot(\theta_{\mathrm{re}})=\frac{1}{2}\partial_\rho f(\rho_i), \tag{2.4}$$

where θ_{in} and θ_{re} are the incident and refracted angles, respectively. Of course, locally physical measurable quantities involve the relative shift or refraction of nearby geodesics which involve one further derivative with respect to ρ, the cylindrical coordinate on the plane $u=0$. We shall come back to this point by the end of this section when we will compute the Riemann tensor components which give the geodesic deviation.

The only nonvanishing components of the Riemann tensor for metric (2.1) are (apart from the ones obtained by symmetry properties)

$$R_{iuju}=-\frac{1}{2}\partial_{ij}^2 H(u,x_\perp), \quad i,j=\{x_\perp\}. \tag{2.5}$$

And the only nonvanishing components of the Ricci tensor for metric (2.1) are (apart from the ones obtained by symmetry properties)

$$R_{uu}=-\frac{1}{2}\nabla_\perp^2 H(u,x_\perp), \tag{2.6}$$

where $\nabla_\perp^2$ stands for the Laplacian operator in the $x_\perp$ space.

The curvature scalars formed out of the Ricci squared, Kretschmann, and curvature scalar tensors all vanish identically since

$$R\equiv 0, \quad R_{\alpha\beta}R^{\alpha\beta}\equiv 0, \quad R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\equiv 0. \tag{2.7}$$

To determine the form of the profile function one considers a null source represented by the following energy-momentum tensor:

$$T_{uu}=\sigma(x_\perp)\,\delta(u), \tag{2.8}$$

with all other components vanishing.

Next, one has to impose the field equations to the metric. Einstein equations linearize and reduce to a Poisson equation in the $x_\perp$ space [19,18]:

$$-\frac{1}{2}\delta(u)\nabla_\perp^2 f(x_\perp)=8\pi G T_{uu}=8\pi G\sigma(x_\perp)\,\delta(u). \tag{2.9}$$

The simplest source one can consider is a spinless particle, with momentum p, represented by $\sigma_p(x_\perp)=p\,\delta(x_\perp)$. Then, the solution of Eq. (2.9) is readily found to be (in four dimensions for the sake of simplicity)

$$f_{\rm GR}(\rho)=-8Gp\ln\left(\frac{\rho}{\rho_0}\right), \tag{2.10}$$

where now the profile function only depends on the radial distance to the origin, ρ, in the plane $u=0$ (as expected for a cylindrically symmetric problem). ρ_0 is an integration constant having the units of ρ.

Metric (2.1) and (2.10) were originally obtained by Aichelburg and Sexl [20] by boosting the Schwarzschild metric along the z axis and taking simultaneously the limits $M\to 0$ and $v\to 1$.

The explicit form of the profile function have also been found for a variety of sources [21–25] by use of the above-sketched procedure.

As explained above one expects the shock wave metric to be relevant in processes involving energies of the order or higher than the Planck scale. At such huge energies, higher order corrections to the gravitational theory will also be relevant. It is of interest, then, to study the form of the profile function, f, in the fourth-order theory of gravity (1.1).

If one plugs the ansatz (2.1) for the shock wave into the fourth-order field equations (1.2), one obtains again a linearized equation for the profile function:

$$\begin{aligned}-\frac{1}{2}[\beta\nabla_\perp^4+\nabla_\perp^2]H(u,x_\perp)&=8\pi GT_{uu}\\&=-\frac{1}{2}\delta(u)[\beta\nabla_\perp^4+\nabla_\perp^2]f(x_\perp)\\&=8\pi G\sigma(x_\perp)\delta(u).\end{aligned} \tag{2.11}$$

where $\nabla_\perp^4=\nabla_\perp^2\nabla_\perp^2$ is the fourth-order Laplacian in the perpendicular space.

This equation can be integrated twice to give

$$[\beta\nabla_\perp^2+1]H(u,x_\perp)=H_{\rm GR}(u,x_\perp). \tag{2.12}$$

We have thus reduced the original fourth-order problem to a second-order one. Now, by making the decomposition

$$H(u,x_\perp)=H_Q(u,x_\perp)+H_{\rm GR}(u,x_\perp), \tag{2.13}$$

where the index Q refers to the purely quadratic part of the solution, and using the fact that $H_{\rm GR}$ satisfies general relativity equations, Eq. (2.12) can be rewritten as

$$\left[\nabla_\perp^2+\frac{1}{\beta}\right]H_Q(u,x_\perp)=16\pi GT_{uu}. \tag{2.14}$$

We first note here that the above form of the equation (but with different constant coefficients) has been found when analyzing shock waves on curved backgrounds in pure general relativity [18,26–28].

Let us now consider the simple example of the particle represented by the source term $\sigma_p(x_\perp)=p\,\delta(x_\perp)$ (in $D=4$). The left-hand side of Eq. (2.14) takes the form of a Bessel equation. Our problem can thus be solved in terms of the Green's function for a cylindrically symmetric problem. The solution can then be expressed as

$$H_Q(u,x_\perp)=f_Q(\rho)\,\delta(u),\quad f_Q(\rho)=-8GpI_0(0)K_0\left(\frac{\rho}{\sqrt{-\beta}}\right), \tag{2.15}$$

where I_0 and K_0 are the modified Bessel functions and where we have taken into account that $\beta<0$ from the nontachyon constraints (1.5).

Finally, the profile function generated by a massless uncharged particle is

$$f(\rho)=-8Gp\left[K_0\left(\frac{\rho}{\sqrt{-\beta}}\right)+\ln\left(\frac{\rho}{\rho_0}\right)\right]. \tag{2.16}$$

Equations (2.1) and (2.16) thus represent the exact solution to our problem in fourth-order gravity. Note that the coupling constant α does not appear in the solution. This can be traced back to the fact that the scalar curvature R, identically vanishes for metric (2.1), as can be readily verified.

Since in the asymptotic regime, for both small and large values of ρ,

$$K_0\left(\frac{\rho}{\sqrt{-\beta}}\right)=\begin{cases}\sim-\ln\left(\dfrac{\rho}{2\sqrt{-\beta}}\right) & \text{for small } \rho/\sqrt{-\beta},\\ \sim\left(\dfrac{2\rho}{\pi\sqrt{-\beta}}\right)^{-1/2}\exp\left(\dfrac{-\rho}{\sqrt{-\beta}}\right) & \text{for large } \rho/\sqrt{-\beta},\end{cases} \tag{2.17}$$

we have then computed the contribution to the shock wave of the massive (with mass $m=1/\sqrt{-\beta}$) spin-two field. In the case of small ρ, the profile function assumes the form of an Aichelburg-Sexl profile for a pointlike source. Also note that the dependence on β is clearly nonperturbative, as expected from our comments in the Introduction. In fact, for $\rho\neq 0$, the profile (2.16) gives a vacuum solution to the fourth-order theory which is *not* a solution of general relativity, i.e., it is

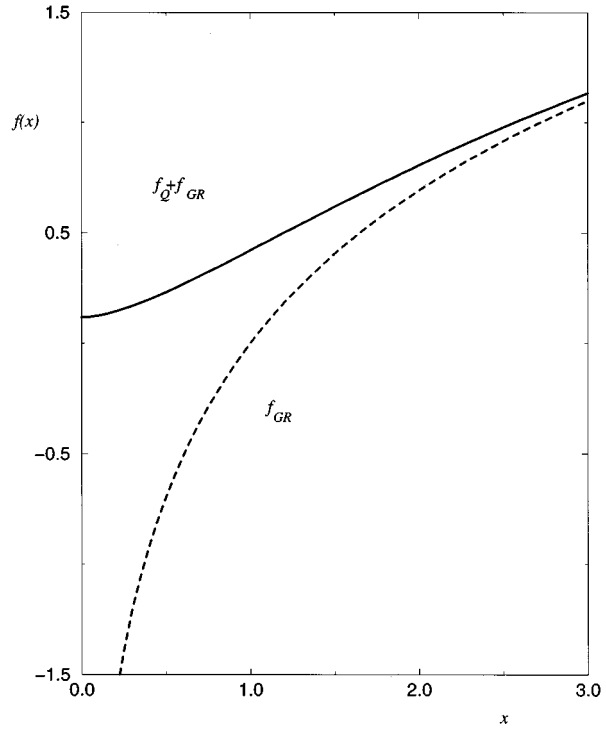

FIG. 1. We compare here the profile function for the general relativistic solution, given by the logarithmic behavior, with the profile solution of the quadratic theory. It is evident the regularizing effect of the quadratic terms near the particle, located at ($\rho=0$). We also observe that after a few mass distances ($m=1/\sqrt{-\beta}$) from the origin the two curves become undistinguishable. Here $x=\rho/\sqrt{-\beta}$, and we have substracted an irrelevant constant from Eq. (2.16), and set to one the factor $-8Gp$. We thus plot $K_0(x)+\ln(x)$.

in the set $\Sigma_{\Delta V}$. For $\beta\to 0$ we recover the general relativistic metric; while $p=0$ (no source) gives the flat space limit.[1]

In Fig. 1 we plot the resulting profile function and compare it to the general relativistic one. The first apparent difference is the nondivergence in the origin of coordinates, i.e., where the particle is located. Even if, as we have seen in Eq. (2.7), for plane gravitational waves the curvature scalars identically vanish, this does not necessarily imply the gravitational field will be nonsingular at the origin. In fact, for the cylindrically symmetric situation we are analyzing the only nonvanishing components of the Riemann tensor for metric (2.1) are (apart from the ones obtained by symmetry properties)

$$R_{\rho u\rho u}=-\frac{1}{2}\partial_\rho^2 H(u,\rho),\quad R_{\phi u\phi u}=-\frac{\rho}{2}\partial_\rho H(u,\rho), \tag{2.18}$$

and the only nonvanishing components of the Ricci tensor for metric (2.1) is

$$R_{uu}=G_{uu}=-\frac{1}{2\rho}\partial_\rho[\rho\partial_\rho H(u,\rho)]=\frac{1}{2\beta}H_Q(u,\rho). \tag{2.19}$$

It then follows that components of the Riemann tensor such as

$$R^{v}_{\rho u\rho}=\partial_\rho^2 H(u,\rho),$$

$$R^{\phi}_{\phi uu}=-\frac{1}{2\rho}\partial_\rho H(u,\rho),$$

$$R^{\rho}_{\rho uu}=\frac{1}{2}\partial_\rho^2 H(u,\rho) \tag{2.20}$$

will diverge logarithmically as $\rho\to 0$. This can be classified [29] as a ''parallelly propagated curvature singularity.'' However, we note here that the singularity is notably milder than the $1/\rho^2$ for general relativity.

III. GENERALIZATIONS

Many of the present candidates to unify gravity with other interactions consider D, the dimensionality of the spacetime, bigger than four. Since Eq. (2.11) can be extended to any dimension, we find that the D-dimensional generalization of Eq. (2.16) is given by

$$f(\rho)=-\frac{16\pi Gp}{\Omega_{D-3}}\left[\frac{(-2\beta)^{2-D/2}}{\Gamma(D/2-1)}\left(\frac{\rho}{\sqrt{-\beta}}\right)^{2-D/2}K_{2-D/2}\left(\frac{\rho}{\sqrt{-\beta}}\right)+\frac{1}{(4-D)}\left(\frac{\rho}{\rho_0}\right)^{4-D}\right], \tag{3.1}$$

where $\Omega_{D-3}=2\pi^{D/2-1}/\Gamma(D/2-1)$ is the area unit in the $D-3$ sphere and Gp carry units of *length* $^{D-4}$.

In the case the source term is extended, but keeps its axial symmetry, i.e., $\sigma(x_\perp)=\sigma(\rho)$ we have

$$f(\rho)=f_{GR}(\rho)+16\pi G x^{2-D/2}\int_\infty^x\{K_{2-D/2}(x)I_{D/2-2}(r)-I_{D/2-2}(x)K_{2-D/2}(r)\}r^{D/2-1}\sigma(r)dr, \tag{3.2}$$

[1]Note that here we refer to the no source case as imposing a boundary condition on the solution such that it represents Minkowski space; while, when we refer to a vacuum solution for $\rho\neq 0$, it is the result of imposing other boundary conditions such that Eq. (2.16) is the solution. It is in the same sense that one refers to the Schwarzschild solution as a vacuum solution of Einstein theory, while for $M\to 0$ one finds Minkowski space.

where as before the index GR refers to the solution of the problem in Einstein theory and $x=\rho/\sqrt{-\beta}$. When the source has a θ dependence, we can Fourier transform it as well as the solution and it will look like Eq. (3.2) with the index of the Bessel functions being l now (in $D=4$ and where l refers to the corresponding Fourier mode in θ).

An interesting example of extended source easily solvable is the boosted straight string [22]. In four dimensions Eq. (2.11) becomes an ordinary fourth-order differential equation with constant coefficients and a $\delta(y)$-like source. The profile function then takes the form

$$f(y)=-8\pi Gp[\sqrt{-\beta}e^{-|y|/\sqrt{-\beta}}+|y|], \tag{3.3}$$

where y refers to the perpendicular distance to the string measured on the plane $u=0$. One can check that this exactly corresponds to a particle in $D=3$ dimensions from expression (3.1).

One can also see the problem of quadratic theories as giving a correction to the energy-momentum tensor (as in the semiclassical approach to quantum gravity) and think of the problem as one for Einstein gravity with an effective source. Then, from Eq. (2.12) we have

$$-\frac{1}{2}\nabla_\perp^2 f(x_\perp)=8\pi G\sigma_{\rm eff}(x_\perp)=\frac{1}{2\beta}(f-f_{\rm GR}). \tag{3.4}$$

Hence the explicit form of $\sigma_{\rm eff}$ in terms of integrals of σ can be read off of Eq. (3.2). For example, for the pointlike source considered in the generalization of the Aichelburg-Sexl metric, we obtain from Eq. (3.1)

$$\sigma_{\rm eff}(\rho)=\frac{2}{\Omega_{D-3}}\frac{(-2\beta)^{1-D/2}}{\Gamma(D/2-1)}\left(\frac{\rho}{\sqrt{-\beta}}\right)^{2-D/2}K_{2-D/2}\left(\frac{\rho}{\sqrt{-\beta}}\right), \tag{3.5}$$

which represents an *extended* effective source.

Another kind of generalization is to consider a gravitational Lagrangian containing terms in the curvature and its derivatives that will generate field equations with derivatives higher than the fourth, for example, terms like R^3, $RR_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu;\lambda}R^{\mu\nu;\lambda}$, etc. When we consider solutions to the field equations of the form Eq. (2.1) there will not be any contribution coming from the terms R^n with $n\geqslant 2$ since $R\equiv 0$ for this metric. Neither terms with contractions of the curvature tensors involving powers higher than the second, since the only nonvanishing component of the Ricci tensor is R_{uu}. Terms involving covariant derivatives of the curvature tensors (like $R_{\mu\nu;\lambda}R^{\mu\nu;\lambda}$) will, however, give a contribution in the form of higher order Laplacian operators (like $\nabla_\perp^6$ in the above example). For a theory containing terms of up to power N in the curvature tensor and its derivatives we expect to have the following form of the field equations for a metric of the type (2.1):

$$-\frac{1}{2}\sum_{n=1}^{N}\gamma_n\nabla_\perp^{2n}H(u,x_\perp)=-\frac{1}{2}\left[\prod_{n=1}^{N}(\nabla^2-m_n^2)\right]H(u,x_\perp)=8\pi GT_{uu}, \tag{3.6}$$

where the m_n's are constants related algebraically to the γ_n's and can be seen as the masses of some of the particles of the theory, for example, $m_1=0$ corresponds to the graviton, $m_2=(-\beta)^{-1/2}$ to the massive spin-two field of quadratic theories, and so on.

Let H_n be a solution of the equation $(\nabla^2-m_n^2)H_n=0$. Then $H_{\rm hom}=\Sigma_{n=1}^{N}C_nH_n$ (with C_n constants to be determined by imposing the boundary conditions of our problem), will be a general solution to the homogeneous problem associated to Eq. (3.6) as can be easily verified. We thus construct the general solution to the inhomogeneous problem by adding a particular solution that can be found by use of the Wronskians method: i.e.,

$$H_p(u,\rho)=\sum_{n=1}^{N}H_n(u,\rho)\int^{\rho}\frac{W_n(u,r)}{W(u,r)}dr. \tag{3.7}$$

Still a further generalization of the solution can be done by considering the wave traveling on a curved background instead of a flat one. This can be performed following the steps Dray and 't Hooft made in Ref. [18] for Einstein theory and should present no further difficulties than obtaining the coefficients in the resulting field equation at $u=0$ [generalization of Eq. (2.11)].

IV. DISCUSSION

We have seen that a nice feature of the exact solution Eq. (2.16) is the fact that it makes milder the curvature singularity at the location of the particle generating the gravitational field. It is in fact one of the historical classical motivations to introduce higher order theories in cosmological scenaria as we recalled in the Introduction. It is also a desirable feature when one thinks of this quadratic theory as an effective one being the by-product of the low-energy limit of a finite quantum theory of gravity. We have studied an exact solution of the theory given by the action (1.1). Among the motivations for studying this kind of theory we gave the argument of its similarity with what one finds renormalizing to one loop a field theory in curved backgrounds. In this case one has also to take into account nonlocal corrections in the renormalized energy-momentum tensor. That, in fact, can be done taking into account the results of [30]. Let us recall that the final result in this case would be only valid to order $\hbar$ since the renormalized energy-momentum tensor was computed to that order. This point brings us to the question of ''self-consistency'' of the perturbative approach discussed in Ref. [9]. There it is in general considered only solutions linear in $\hbar$ (and in the coupling constants α and β) for consistency with the field equations which are considered to be only precise to one loop. This point of view has the advantage of avoiding the, so-called, runaway solutions; rendering thus, Minkowski space stable. In our view this procedure is too restrictive and precludes some well-behaved physical solutions (see also Ref. [31]). In the case of the plane gravitational wave we have studied in this paper we note that the solution is clearly nonperturbative in the coupling constant β and still the full solution physically makes sense. The ''runaway'' or ''unphysical'' solution [I_0 does not appear in Eq. (2.16)] is naturally discarded here. In addition, we have seen that this kind of gravitational wave will be a solution of

more general Lagrangian than Eq. (1.1) with field equations containing derivatives of order higher than the fourth.

Note added. After completion of this work we found Ref. [35] which deals with sourceless *pp* waves in higher order theories. Our results appear to be completely compatible with those of Ref. [35].

ACKNOWLEDGMENTS

C.O.L. was partially supported by the Dirección General de Investigación Científica y Técnica of the Ministerio de Educación y Ciencia de España, CICYT AEN 93-0474, by the NSF Grant No. PHY-95-07719 and by research funds of the University of Utah. M.C. thanks the Deutscher Akademischer Austauschdienst for financial support.

APPENDIX: SCATTERING OF FIELDS BY SHOCK WAVES

A further interesting application of the gravitational shock waves was stressed by 't Hooft [16]. These geometries are relevant for ultra-high-energy scattering processes (i.e., Planck scale scattering). Let us consider a frame of reference where particle one is practically at rest and can be described, in a semiclassical approach, by a scalar field satisfying the Klein-Gordon equation in the curved background generated by particle two, which carries Planckian energies in our chosen system of reference. In this case one can compute the form of the scattering matrix [32] and prove that there will not be particle production of the scalar (or other spin) fields. For the axial symmetric case the S matrix takes the form

$$S(\vec{p}_{\perp,1},\vec{p}_{\perp,2},\omega)=\frac{1}{2\pi}\int_0^\infty J_0(|\vec{p}_{\perp,1}-\vec{p}_{\perp,2}|\rho)e^{i\omega f(\rho)/4}\rho\, d\rho. \tag{A1}$$

In Ref. [22] the S matrix has been computed and studied for several sources. It is evident from the form of the profile function (2.16), that it is difficult to find an exact analytic expression for the corresponding S matrix. To illustrate the use of the above expression we shall consider a recent interesting result [25] where the profile function for the ultrarelativistic Kerr geometry was obtained

$$f(\rho)=-8Gp\Theta(\rho-a)\ln\rho+8Gp\Theta(a-\rho)\times\left[-\ln(a+\sqrt{a^2-\rho^2})+\frac{1}{2a}\sqrt{a^2-\rho^2}\right] \tag{A2}$$

for the boost along the axis of symmetry and where Θ is the step function. It is still difficult to perform the integration in Eq. (A1). We thus approximate the profile function to be constant inside the ''ring'' of radius a and leave the exact logarithmic dependence outside it, as shown in Fig. 2. One can easily check that this approximation is good up to order ρ^4 for small ρ while is also good near $\rho=a$ where $df/d\rho$ diverges. We thus decompose the integration interval into two pieces for the dimensionless variable $\tilde{\rho}=\rho/a$, form 0 to 1 and from 1 to infinity. The final result is given by [33]

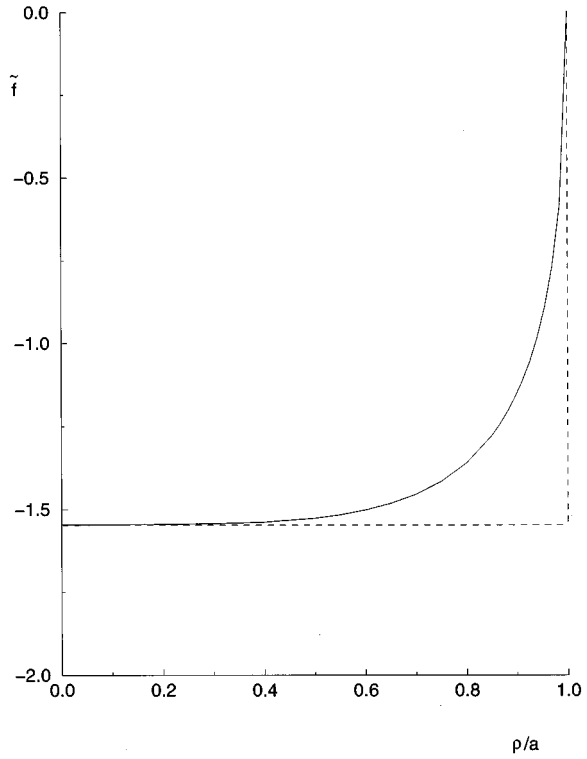

FIG. 2. The profile function $\tilde{f}(\rho)=f(\rho)+8Gp\ln(a/\rho_0)$ for the ultrarelativistic Kerr geometry and our approximation for $\rho\leqslant a$ to compute the scattering matrix, S. For $\rho\geqslant a$ we take the exact $(\ln\rho)$ behavior. The approximation is very good near $\rho=0$ and $\rho=a$, and allows us to get the relevant features of S.

$$S(q,s)=\frac{(a)^{2-is}}{2\pi q}\{2^{-is}e^{is/2}J_1(q)+q^{is}[isJ_0(q)S_{-is,-1}(q)-J_1(q)S_{1-is,0}(q)]\}, \tag{A3}$$

where $q=(|\vec{p}_{\perp,1}-\vec{p}_{\perp,2}|)/a$, $s=2Gp\omega$, the Mandelstam variable, and $S_{\mu,\nu}(z)$ is a Lommel function. A relevant point to see in the structure of the S matrix is whether it has poles that would eventually correspond to bound states of the system. One can see [24] that the above expression has *no* poles in the s variable. This has to do with the fact the source has an extended nature, as opposed to the boosted Schwarzschild geometry (i.e., Aichelburg-Sexl metric), for which there are poles [34] at is a natural number.

The same conclusions apply to the wave (2.16). In fact, now the corrections due to the higher derivative theory produces a profile function (that is what matters to the scattering of scalar fields) which is finite at $\rho=0$, as we see in Fig. 1. The same approximation procedure can be also applied to the profile (2.16). This time we approximate the profile for, let us say, $x<3$ by a straight line given by $f(x)\cong 0.43x+0.116$, and the match to the general relativistic logarithmic behavior for $x>3$. The nonappearance of poles in the S matrix, again can be traced back to the fact that the β-dependent term can be seen as an extended source [see Eq. (3.5)].

[1] D. Lovelock, J. Math. Phys. **12**, 498 (1971); **13**, 874 (1972).
[2] H. Weyl, *Raum-Zeit-Materie*, 4th ed. (Springer-Verlag, Berlin, 1921).
[3] A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[4] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[5] B. Zwiebach, Phys. Lett. **156B**, 315 (1985); A. Tseytlin, *ibid.* **176**, 92 (1986).
[6] J. Audretsch, A. Economou, and C. O. Lousto, Phys. Rev. D **47**, 3303 (1993).
[7] I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (IOP, Bristol, 1992).
[8] S. W. Hawking, in *Quantum Field Theory and Quantum Statistics*, edited by I. A. Batalin, C. J. Isham, and G. A. Vilkovisky (Hilger, Bristol, 1987), p 129.
[9] L. Parker and J. Z. Simon, Phys. Rev. D **47**, 1339 (1993).
[10] M. Campanelli, C. O. Lousto, and J. Audretsch, Phys. Rev. D **49**, 5188 (1994); **51**, 6810 (1995).
[11] L. Bel and H. Sirousse Zia, Phys. Rev. D **32**, 3128 (1985).
[12] M. Campanelli and C. O. Lousto (in preparation).
[13] B. Whitt, Phys. Lett. **145B**, 176 (1984).
[14] S. Kluske and H. -J. Schmidt, Report No. UNIP-MATH-94/14, gr-qc/9503021 (unpublished).
[15] A. Economou and C. O. Lousto, Phys. Rev. D **49**, 5278 (1994).
[16] G. 't Hooft, Phys. Lett. B **198**, 61 (1987).
[17] P. C. Aichelburg and H. Balasin, Class. Quantum Grav. **11**, L121 (1994); **13**, 723 (1996).
[18] T. Dray and G. 't Hooft, Nucl. Phys. **B253**, 173 (1985).
[19] P. Jordan, J. Ehlers, and W. Kundt, Akad. Wiss. Lit. Mainz Abh. Math. Naturwiss. KL. **2**, 21 (1960).
[20] P. C. Aichelburg and R. U. Sexl, Gen. Relativ. Gravit. **2**, 303 (1971).
[21] C. O. Lousto and N. Sánchez, Int. J. Mod. Phys. A **5**, 915 (1990).
[22] C. O. Lousto and N. Sánchez, Nucl. Phys. **B355**, 231 (1991).
[23] C. O. Lousto and N. Sánchez, Nucl. Phys. **B383**, 377 (1992).
[24] C. O. Lousto and N. Sánchez, Phys. Rev. D **46**, 4520 (1992).
[25] H. Balasin and H. Nachbagauer, Class. Quantum Grav. **12**, 707 (1995); **13**, 731 (1996).
[26] C. O. Lousto and N. Sánchez, Phys. Lett. B **220**, 55 (1989).
[27] K. Sfetsos, Nucl. Phys. **B436**, 721 (1995).
[28] C. O. Lousto, Phys. Rev. D **51**, 1733 (1995).
[29] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984), p 215.
[30] J. Garriga and E. Verdaguer, Phys. Rev. D **43**, 391 (1991); C. O. Lousto and F. D. Mazzitelli (in preparation).
[31] E. E. Flanagan and R. M. Wald, Report No. gr-qc/9602052 (unpublished).
[32] H. J. de Vega and N. Sánchez, Nucl. Phys. **B317**, 731 (1989).
[33] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic Press, New York, 1965).
[34] G. 't Hooft, Nucl. Phys. **B304**, 867 (1988).
[35] H. A. Buchdahl, J. Phys. A **16**, 1441 (1983).