Quark-lepton Yukawa unification at lower mass scales

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(Received 18 March 1996)

In the nonsupersymmetric two-Higgs-doublet standard model, we find t-b-τ Yukawa unification consistent with the CERN LEP data, experimental values of m_b, m_τ, and $m_t = 160$–190 GeV at lower mass scales: $M_C \simeq 3\times10^8$–3×10^9 GeV. We also show how such quark-lepton unification scales can be reconciled with $SU(2)_L\times SU(2)_R\times SU(4)_C$ intermediate breaking in SO(10) including threshold and gravitational corrections. [S0556-2821(96)05317-9]

PACS number(s): 12.10.Dm, 12.15.Ff, 12.60.Jv

Considerable attention has been paid to the study of implications of quark-lepton unification through the $SU(4)_C$-gauge hypothesis [1]. If $SU(4)_C$ occurs as a part of left-right gauge group such as $SU(2)_L\times SU(2)_R \times SU(4)_C(\equiv G_{224})$ with parity $(g_{2L}=g_{2R})$ or without it $(g_{2L}\neq g_{2R})$ [2], or even as $SU(2)_L\times U(1)_{I_{3R}}\times SU(4)_C$, it can undergo spontaneous symmetry breaking to the standard model (SM) gauge group, directly or by more than one step, such that weak interaction phenomenology at lower energies remains close to that of the SM. In addition, rare-kaon decays mediated by $SU(4)_C$-gauge bosons might be observable by low-energy experiments in the near future [3,4], if the relevant symmetry breaking scale is not too high. When embedded in a grand unified theory (GUT), the existence of a G_{224}-breaking scale near 10^6–10^7 might lead to observable proton decays [5] or neutron-antineutron oscillations [6]. If neutrinos are Majorana particles, a G_{224}-breaking scale near 10^9–10^{11} GeV yields small neutrino masses necessary to understand solar neutrino puzzle by Mikheyev-Smirnov-Wolfenstein (MSW) mechanism or dark matter of the universe [7].

Almost five years ago, an interesting possibility of relating the $SU(4)_C$-breaking scale to the top-quark mass was advanced by demanding unification of the three Yukawa couplings of the third generation fermions at the spontaneous symmetry breaking scale of the intermediate gauge symmetry $SU(2)_L\times SU(2)_R\times SU(4)_C(\equiv G_{224})$ in SO(10) grand unification, but by using [8],

$$\sin^2\theta_W = 0.226\pm 0.005,$$

$$\alpha_s = 0.100 - 0.117.$$

With the standard model (SM) gauge group containing one Higgs doublet below the intermediate scale, it was shown that either the running mass of bottom quark must be greater than 4.35 GeV, or the top-quark pole mass must be less than 80 GeV. However, with two Higgs doublets in SM, it was noted that for $\tan\beta \simeq 35$–45, the three Yukawa couplings meet in the allowed range of the intermediate scale $M_c = 10^{12.2}$–$10^{13.6}$ GeV yielding the bounds 80 GeV$<m_t<$180 GeV. Such a high scale of quark-lepton unification would predict branching ratios for rare kaon decays twelve orders smaller than the current experimental limit.

More recently, the question of Yukawa unification at the GUT scale has been analyzed in the context of nonsupersymmetric SO(10), where partial unification of gauge couplings is achieved through G_{224} intermediate symmetry [9].

The purpose of this Brief Report is to examine whether the idea of quark-lepton unification through $SU(4)_C$ is realizable at mass scales significantly lower than those investigated earlier but consistent with the most recent determinations of the input parameters [10].

$$\sin^2\theta_W(M_Z) = 0.2316\pm 0.0003,$$

$$\alpha_s(M_Z) = 0.118\pm 0.007,$$

$$\alpha^{-1}(M_Z) = 127.9\pm 0.1,$$

$$m_b = 4.25 \text{ GeV},$$

$$m_\tau = 1.785 \text{ GeV},$$

$$m_t = 174\pm 15 \text{ GeV}. \tag{1}$$

We also examine how such a scale is accommodated within SO(10) through G_{224}-intermediate breaking.

We use the renormalization group equations (RGE's) for the top-quark, bottom-quark, and the τ-lepton Yukawa couplings in the two-Higgs-doublet SM in the range of mass scales, $m_t \leq \mu \leq M_c$,

$$16\pi^2 \frac{dh_t}{dt} = h_t\left[\frac{9}{2}h_t^2 + \frac{1}{2}h_b^2 - \sum_i c_i^{(t)} g_i^2\right],$$

$$16\pi^2 \frac{dh_b}{dt} = h_b\left[\frac{9}{2}h_b^2 + \frac{1}{2}h_t^2 + h_\tau^2 - \sum_i c_i^{(b)} g_i^2\right],$$

$$16\pi^2 \frac{dh_\tau}{dt} = h_\tau\left[\frac{5}{2}h_\tau^2 + 3h_b^2 - \sum_i c_i^{(\tau)} g_i^2\right], \tag{2}$$

where $t = \ln\mu$, h_j = Yukawa coupling of the jth fermion, and

$$c_i^{(t)} = \left(\frac{17}{20}, \frac{9}{4}, 8\right),$$

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$$c_i^{(b)}=\left(\frac{1}{4},\ \frac{9}{4},8\right),$$

$$c_i^{(\tau)}=\left(\frac{9}{4},\ \frac{9}{4},0\right),\quad i=Y,2L,3C.$$

For $\mu>m_t$, the integration of Eq. (2) gives

$$h_t(\mu)=\frac{h_t(m_t)}{A_t}\exp\left(\frac{9}{2}I_t+\frac{1}{2}I_b\right),$$

$$h_b(\mu)=\frac{h_b(m_t)}{A_b}\exp\left(\frac{1}{2}I_t+\frac{3}{2}I_b+I_\tau\right),$$

$$H_\tau(\mu)=\frac{h_\tau(m_t)}{A_\tau}\exp\left(3I_b+\frac{5}{2}I_\tau\right),$$

$$I_i=\int_{\ln m_t}^{\ln\mu}\frac{h_i^2(t)}{16\pi^2}dt,\quad i=t,b,\tau,$$

$$A_f=\prod_i\left(\frac{\alpha_i(\mu)}{\alpha_i(m_t)}\right)^{c_i^{(f)}/2a_i},\quad f=t,b,\tau. \tag{3}$$

In the earlier work [8] only h_t was included inside the square brackets, in the right-hand side of Eq. (2), but we take into account the contributions of all the three Yukawa couplings of the third generation.

The functions $A_f(f=t,b,\tau)$ are obtained by integrating out the gauge coupling contributions in Eq. (2) using terms up to one-loop out of the two-loop approximation,

$$\frac{1}{\alpha_i(\mu)}=\frac{1}{\alpha_i(m_t)}-\frac{a_i}{2\pi}\ln\frac{\mu}{m_t}-\frac{1}{4\pi}\sum_j\frac{b_{ij}}{a_j}\ln\frac{\alpha_j(\mu)}{\alpha_j(m_t)}, \tag{4}$$

where

$$a_i=\begin{pmatrix}\frac{21}{5}\\-3\\-7\end{pmatrix},\quad b_{ij}=\begin{pmatrix}\frac{104}{25}&\frac{18}{5}&\frac{144}{5}\\ \frac{8}{5}&8&12\\ \frac{11}{10}&\frac{9}{12}&-26\end{pmatrix}.$$

Using mass-operator renormalization of $SU(3)_C\times U(1)_{em}$ theory below M_Z, the QCD rescaling factors for $m_b(m_t)$ and $m_\tau(m_t)$ are obtained as [9]

$$\eta_b=1.53\pm{}^{0.06}_{0.07},$$

$$\eta_\tau=1.015,$$

where $m_i(m_t)=m_i/\eta_i$, $i=b,\ \tau$. The renormalization group equations for gauge couplings from M_Z-m_t gives

$$\alpha_1^{-1}(m_t)=58.51,\quad\alpha_2^{-1}(m_t)=30.15,$$

$$\alpha_3^{-1}(m_t)=9.30\pm0.5. \tag{5}$$

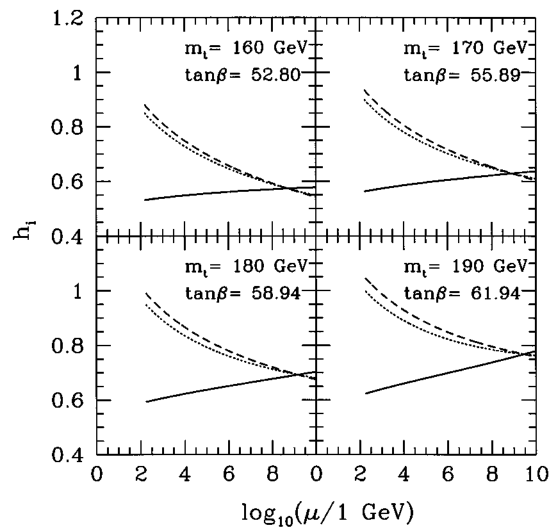

FIG. 1. Variation of Yukawa couplings for the top quark (dashed line), the bottom quark (dotted line), and the τ lepton (solid line) as a function of mass scale and tanβ exhibiting quark-lepton unification at 3×10^8–3×10^9 GeV.

For a given ratio of the vacuum expectation values of the two Higgs scalars, $v_u/v_d=\tan\beta$, we computed the three values of Yukawa couplings at $\mu=m_t$ using

$$h_t(m_t)=\frac{m_t(m_t)}{174\sin\beta},$$

$$h_b(m_t)=\frac{m_b(m_t)}{174\cos\beta},$$

$$h_\tau(m_t)=\frac{m_\tau(m_t)}{174\cos\beta}. \tag{6}$$

Using the initial values from Eqs. (5) and (6) as inputs in Eqs. (2) and (3), we have plotted $h_i(\mu)$ vs $\log_{10}\mu$, such that the solutions for $h_i(\mu)$ are consistent with Eqs. (2)–(4).

For smaller tanβ in the range of 5–10, the ratio h_b/h_τ is found to approach unity for $\mu=10^{8.3}$–10^9 GeV, but there is no possibility of the three Yukawa couplings having a common meeting point.

In the next step we searched the parameter space in the large tan β region taking the allowed values of the top quark mass from 160 to 190 GeV. Our results are shown in Fig. 1 where unification of the three Yukawa couplings is clearly exhibited for m_t=160, 170, 180, and 190 GeV with tanβ =52.80, 55.89, 58.94, and 61.94, respectively. The unification scale increases with top quark mass from $M_c=10^{8.5}$ GeV to $M_c=10^{9.5}$ GeV.

In Fig. 2 we have shown the variation of the unification scale $\mu(\equiv Mc)$ as a function of tanβ where in the larger (smaller) tanβ region, the values of μ and tanβ correspond to the unification of three (two) Yukawa couplings. The dotted, solid, and dashed lines correspond to m_t=160, 170, and 180 GeV, respectively. From the larger tanβ region of Fig. 2, it is clear that, for a fixed tanβ, the unification scale increases with m_t; and for a fixed m_t, the unification scale increases with tanβ, although the rate of increase is different in different regions of the curve. In Fig. 3 we show how, for smaller

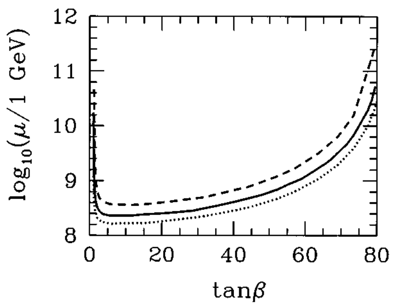

FIG. 2. Variation of the quark-lepton unification scale (μ) as a functional of $\tan\beta$ for $m_t = 160$ GeV (dotted line), $m_t = 170$ GeV (solid line), and $m_t = 180$ GeV (dashed line) where, for small $\tan\beta$, $h_b = h_\tau$ but for larger $\tan\beta \geqslant 52$, $h_b = h_\tau = h_t$ has been realized.

values of $\tan\beta$, the ratio h_b/h_τ approaches unity at $M_C = 10^{8.4}$ GeV ($m_t = 170$ GeV, solid line) and at $M_C = 10^9$ GeV ($m_t = 180$ GeV, dotted line).

So far no specific intermediate gauge group embedding $SU(4)_C$ has been assumed. The unification of the three Yukawa couplings is a consequence of low-energy data and the two-Higgs-doublet standard model. In order to embed such a scenario in SO(10), the most attractive candidate is the intermediate gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C$ with parity broken at the GUT scale [2]:

$$\mathrm{SO}(10) \underset{\mathbf{210}}{\overset{M_U}{\longrightarrow}} G_{224} \underset{\mathbf{126}}{\overset{M_C}{\longrightarrow}} \mathrm{SM} \underset{\mathbf{10}}{\overset{M_Z}{\longrightarrow}} \mathrm{U}(1)_{\mathrm{em}} \times \mathrm{SU}(3)_C,$$

where from M_Z to M_c we have now two weak doublets ϕ_u and ϕ_d instead of one as in earlier analyses [11–13]. The Yukawa coupling for every fermion of the third generation in the presence of G_{224} symmetry has the form [9]

$$16\pi^2 \frac{dh_f}{dt} = h_f\left(6h_f^2 - \sum_i c_i^{(f)} g_i^2\right),$$

$$c_i^f = \left(\frac{9}{4}, \frac{9}{4}, \frac{45}{4}\right), \tag{7}$$

and the renormalization group equations (RGE's) for the gauge couplings for $\mu > M_C$ are

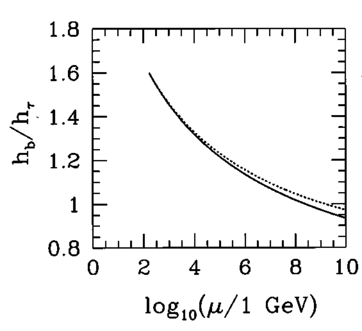

FIG. 3. Variation of the ratio h_b/h_τ as a function of mass scale for smaller values of $\tan\beta$ and $m_t = 170$ GeV (solid line), $m_t = 180$ GeV (dotted line).

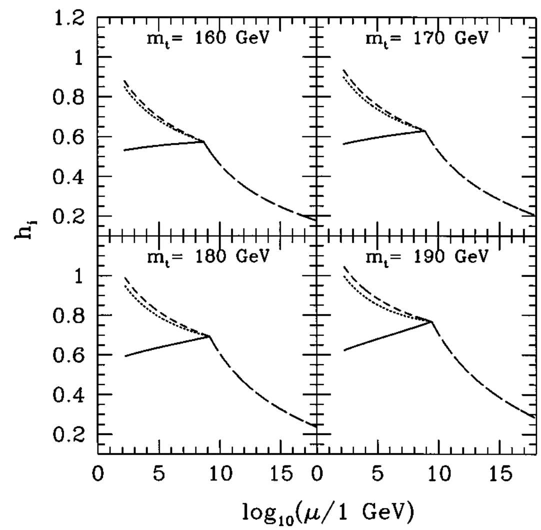

FIG. 4. Extrapolation of Yukawa couplings beyond the intermediate scale for $m_t = 160$–190 GeV.

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_C)} - \frac{a_i'}{2\pi} \ln \frac{\mu}{M_C} - \frac{1}{4\pi} \sum_j \frac{b_{ij}'}{a_j'} \ln \frac{\alpha_j(\mu)}{\alpha_j(M_C)}, \tag{8}$$

where

$$a_i' = \begin{pmatrix} -3 \\ \frac{11}{3} \\ \frac{-23}{3} \end{pmatrix}, \quad b_{ij}' = \begin{pmatrix} 8 & 3 & \frac{45}{2} \\ 3 & \frac{584}{3} & \frac{765}{2} \\ \frac{9}{2} & \frac{153}{2} & \frac{643}{6} \end{pmatrix}.$$

Integrating the fermion Yukawa coupling in Eq. (7) with the help of one-loop approximation from Eq. (8) and using the unification value at M_C, for $h_f(M_C)$, we have plotted $h_f(\mu)$ as a function of μ as shown in Fig. 4 by the dashed line for $\mu = M_C - 10^{18}$ GeV. It is clear that h_f uniformly decreases and attains the value of 0.35 and 0.27 at $\mu = M_U = 10^{15.9}$ GeV and $\mu = 10^{18}$ GeV, respectively.

Now we include threshold [11] and gravitational corrections to the two-loop approximation for the intermediate scale M_C and the GUT scale M_U, which are predicted up to one loop as

$$\ln \frac{M_C}{M_Z} = \frac{36\pi}{47\alpha}\left(\frac{7}{24} - \sin^2\theta_W + \frac{2}{9}\frac{\alpha}{\alpha_s}\right),$$

$$\ln \frac{M_U}{M_Z} = \frac{15\pi}{47\alpha}\left(\frac{1}{10} + \sin^2\theta_W - \frac{19}{15}\frac{\alpha}{\alpha_s}\right). \tag{9}$$

With the input parameters given in Eq. (1), the model predictions in two-loop approximations are $M_U^0 = 10^{15.9\pm0.2}$ GeV, $M_C^0 = 10^{10.9\pm0.2}$ GeV, where the uncertainties are due to the input parameters. Threshold corrections are very close to the values obtained in Ref. [11]. We now evaluate gravitational corrections due to the five-dimensional operators [14] contributing to the nonrenormalizable term [15]

TABLE I. Gravitational corrections to mass-scale predictions by five-dimensional operator in SO(10) with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate symmetry.

η	$\delta_C^{\rm NRO}$	$\delta_U^{\rm NRO}$	$\frac{M_C}{M_C^0}$	$\frac{M_U}{M_U^0}$
± 5	± 3.2	∓ 1.3	$10^{\pm 1.3}$	$10^{\pm 0.6}$
± 10	± 6.2	∓ 2.6	$10^{\pm 2.6}$	$10^{\mp 1.3}$

$$\alpha_{\rm NRO} = -\frac{\eta}{2M_{\rm Pl}} \mathrm{Tr}(F_{\mu\nu}\phi_{(210)}F^{\mu\nu}). \tag{10}$$

When the 210-dimensional Higgs field acquires vacuum expectation values, the added presence of Eq. (10) in the renormalizable Lagrangian leads to the modification of the boundary conditions on gauge couplings and consequently the gravitational corrections on the mass scales are

$$\delta_C^{\rm NRO} \equiv \delta \ln \frac{M_C}{M_Z} = \frac{9}{94}\left(\frac{3\pi}{2\alpha_G^3}\right)^{1/2} \eta \frac{M_U^0}{M_{\rm Pl}},$$

$$\delta_U^{\rm NRO} \equiv \delta \ln \frac{M_U}{M_Z} = -\frac{15}{376}\left(\frac{3\pi}{2\alpha_G^3}\right) \eta \frac{M_U^0}{M_{\rm Pl}}. \tag{11}$$

Using GUT coupling constant, $\alpha_G^{-1} = 46.3 \pm 0.3$, $M_{\rm pl} = 10^{19}$ GeV/$2\pi = 1.6 \times 10^{18}$ GeV, and M_U^0 and M_C^0 at their two-loop values, we present $\delta_C^{\rm NRO}$ and $\delta_U^{\rm NRO}$ in Table I for $\eta = -10$ to 10. We note that while positive (negative) values of η increase (decrease) M_C, the same values tend to decrease (increase) M_U. Using $\eta = \pm 5$ and the threshold corrections of Ref. [11], our estimations on the mass scales are

$$M_C = 10^{10.9^{+2.7}_{-1.4} \pm 1.3 \pm 0.2},$$

$$M_U = 10^{15.9^{+0.8}_{-1.7} \pm 0.6 \pm 0.2},$$

where the first (second) uncertainty represents threshold (gravitational) corrections. It is clear that when threshold and five-dimensional operator effects are taken into account, the intermediate scale encompasses the quark-lepton-Yukawa unification scales for the third generation fermions. Our analysis includes the most recent global fits to the CERN e^+e^- collider LEP data encompassing $\alpha_s(M_z) = 0.111 - 0.125$ and effects of Yukawa couplings of all the three fermions of third generation. The quark-lepton unification at $M_C \simeq 3 \times 10^8 - 3 \times 10^9$ GeV has been demonstrated as a consequence of low-energy data and two-Higgs-doublet standard model below the G_{224}-breaking scale. In this work we have not considered large threshold effects on Yukawa couplings and possibility of unification at still lower scales $M_C \simeq 10^6$ Gev, which might be more model dependent [16].

The allowed values of intermediate scales for Yukawa unification predict the branching ratio for rare-kaon decays to be $B(K_L \to \overline{\mu e}) \simeq 10^{-16} - 10^{-18}$ and ν_τ as a candidate for hot and cold dark matter of the universe.

The authors thank the Theory Group, ICTP for hospitality. One of the authors (M.K.P.) thanks Dr. K. S. Babu for discussions and acknowledges financial support through Research Project No. SP/S2/K-09/91.

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