Long distance contributions in $\overline{B}\to D\pi$ and $D^*\pi$ decays

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The amplitudes for the $\overline{B}\to D\pi$ and $D^*\pi$ decays are described in terms of a sum of a factorized amplitude as the short distance contribution and a hard pion amplitude as the long distance contribution. It is demonstrated that the long distance amplitude is rather small but still can efficiently interfere with the main amplitude. [S0556-2821(96)05417-3]

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A recent measurement of $\overline{B}\to D\pi$ decays [1] shows that the $\overline{B^0}\to D^0\pi^0$ decay is suppressed compared with the other decays into final states involving charged particle(s). It can be well understood in terms of the so-called color suppression based on the factorization in the Bauer-Stech-Wirbel (BSW) scheme [2] in which the effective hadronic weak Hamiltonian is given by

$$H_w^{(B)}=\frac{G_F}{\sqrt{2}}U_{cb}U_{ud}\{a_1O_1+a_2O_2+\text{H.c.}\}, \qquad (1)$$

where $O_1=(\overline{c}b)_L(\overline{d}u)_L$ and $O_2=(\overline{c}u)_L(\overline{d}b)_L$ with $(\overline{q'}q)_L=\overline{q'}\gamma_\mu(1-\gamma_5)q$. The Cabibbo-Kobayashi-Maskawa (CKM) matrix element U_{ij} is taken to be real since CP invariance is always assumed in this Brief Report.

The factorization prescription has been supported by two different kinds of arguments based on QCD: i.e., one in the color degree of freedom $N_c\to\infty$ limit [3] and the other in a specific kinematical limit [4]. However, phenomenological analyses [1,5] in two-body decays of B mesons based on the factorization suggest that the value of a_2 to reproduce the observed branching ratios for these decays [1,6] should be larger by about a factor of 2 than the one expected from perturbation theory [2,7] with $N_c=3$ and its sign should be opposite to the one in the $N_c\to\infty$ limit. It implies that the large N_c argument fails, at least, in nonleptonic decays of B mesons and that the factorization of the decay amplitude may hold only in the specific kinematical condition which may be realized in the $\overline{B}\to D\pi$ and $D^*\pi$ decays [4]. It also seems to suggest that, even in the decays of B mesons with high mass, short distance physics is not sufficient but long distance hadron dynamics still cannot be neglected. Therefore, we describe the amplitude for the $\overline{B}\to D\pi$ and $D^*\pi$ decays in terms of a sum of short and long distance contributions:

$$M(\overline{B}\to D[\text{or } D^*]\pi)=M_{\text{short}}(\overline{B}\to D[\text{or } D^*]\pi)+M_{\text{long}}(\overline{B}\to D[\text{or } D^*]\pi). \qquad (2)$$

We study the long distance amplitude for these decays using a hard pion technique. In our hard pion approximation, the amplitude for the $\overline{B}(\mathbf{p}_1)\to D(\mathbf{p}_2)[\text{or } D^*(\mathbf{p}_2)]\pi(\mathbf{k})$ decay can be given by [8]

$$M_{\text{long}}(\overline{B}\to D[\text{or } D^*]\pi)\simeq M_{\text{ETC}}(\overline{B}\to D[\text{or } D^*]\pi)+M_S(\overline{B}\to D[\text{or } D^*]\pi), \qquad (3)$$

which can be obtained by extrapolating $\mathbf{k}\to 0$ in the infinite momentum frame (IMF), i.e., $\mathbf{p}_1\to\infty$. The equal-time commutator (ETC) term

$$M_{\text{ETC}}(\overline{B}\to D[\text{or } D^*]\pi)=\frac{i}{f_\pi}\langle D[\text{or } D^*]|[V_{\overline{\pi}},H_w]|\overline{B}\rangle \qquad (4)$$

is of the same form as in the old soft pion approximation [9] but now has to be evaluated in the IMF. The surface term is given by a sum of all possible pole amplitudes:

$$M_S(\overline{B}\to D[\text{or } D^*]\pi)=-\frac{i}{f_\pi}\left\{\sum_n\left(\frac{m_{D[*]}^2-m_B^2}{m_n^2-m_B^2}\right)\langle D[\text{or } D^*]|A_{\overline{\pi}}|n\rangle\langle n|H_w|\overline{B}\rangle+\sum_\ell\left(\frac{m_{D[*]}^2-m_B^2}{m_\ell^2-m_{D[*]}^2}\right)\langle D[\text{or } D^*]|H_w|\ell\rangle\langle\ell|A_{\overline{\pi}}|\overline{B}\rangle\right\}, \qquad (5)$$

where n and ℓ run over all possible single mesons, not only ordinary $\{q\overline{q}\}$, but also hybrid $\{q\overline{q}g\}$ and exotic $\{qq\overline{q}\overline{q}\}$ mesons. Because of large symmetry breaking in flavor SU$_f(4)$ and SU$_f(5)$ [10], we do not symmetrize (or antisymmatrize) the amplitude for the $\overline{B}\to D(\text{or } D^*)\pi$ decay under the exchange of the D and π in the final state (or the $\overline{B}$ and π in the crossed channel) in this Brief Report in contrast with our previous work [11]. In these decays, however, excited meson contributions will be neglected since the B meson mass m_B is much higher than those of charm mesons and since wave function overlappings between the ground-state $\{q\bar{q}\}_0$ and excited-state meson states are expected to be small.

Although the symmetry breaking of flavor $SU_f(4)$ and $SU_f(5)$ is not small, *asymptotic* matrix elements of V_π and A_π (matrix elements taken between single hadron states with infinite momentum) are assumed to satisfy

$$\langle \pi^0|V_{\pi^+}|\pi^-\rangle = \sqrt{2}\langle K^+|V_{\pi^+}|K^0\rangle = -\sqrt{2}\langle D^+|V_{\pi^+}|D^0\rangle = \sqrt{2}\langle B^+|V_{\pi^+}|B^0\rangle = \cdots = \sqrt{2},$$
$$\langle \rho^0|A_{\pi^+}|\pi^-\rangle = \sqrt{2}\langle K^{*+}|A_{\pi^+}|K^0\rangle = -\sqrt{2}\langle D^{*+}|A_{\pi^+}|D^0\rangle = \sqrt{2}\langle B^{*+}|A_{\pi^+}|B^0\rangle = \cdots = h, \tag{6}$$

which can be obtained by using the asymptotic $SU_f(5)$ symmetry [12], or an $SU_f(5)$ extension of the nonet symmetry in $SU_f(3)$. The $SU_f(4)$ part of the above parametrization reproduces well [13] the observed values of $\Gamma(D^*\to D\pi)$'s and $\Gamma(D^*\to D\gamma)$'s. Thus the hard pion amplitudes in the present approximation are controlled by the asymptotic ground-state meson matrix elements of H_w.

The amplitude, Eq. (3) with Eqs. (4) and (5), can be regarded as its decomposition [14] into (continuum contribution) + (Born term), which is a natural description of dynamical hadronic processes. The continuum contribution will develop a phase relative to the Born term. Since the $D\pi$ final state can have isospin $I=\frac{1}{2}$ and $\frac{3}{2}$, we decompose $M_{\rm ETC}$ as

$$M_{\rm ETC}(\overline{B^0}\to D^+\pi^-) = \sqrt{\tfrac{1}{3}}M_{\rm ETC}^{(3)}e^{i\delta_3} + \sqrt{\tfrac{2}{3}}M_{\rm ETC}^{(1)}e^{i\delta_1}, \tag{7}$$

$$M_{\rm ETC}(\overline{B^0}\to D^0\pi^0) = -\sqrt{\tfrac{2}{3}}M_{\rm ETC}^{(3)}e^{i\delta_3} + \sqrt{\tfrac{1}{3}}M_{\rm ETC}^{(1)}e^{i\delta_1}, \tag{8}$$

$$M_{\rm ETC}(B^-\to D^0\pi^-) = \sqrt{3}M_{\rm ETC}^{(3)}e^{i\delta_3}, \tag{9}$$

where $M_{\rm ETC}^{(2I)}$ is the isospin eigenamplitude with isospin I and δ_{2I} is the corresponding phase shift introduced. The corresponding decomposition of $M_{\rm ETC}(\bar{B}\to D^*\pi)$ can be obtained by replacing $M_{\rm ETC}^{(2I)}$ and δ_{2I} by $M_{\rm ETC}^{*(2I)}$ and δ_{2I}^* in Eqs. (7)–(9). In this way we can obtain the following approximate expressions of the long distance amplitudes for the $B\to D\pi$ decays:

$$M_{\rm long}(\overline{B^0}\to D^+\pi^-) \simeq -i\frac{\langle D^0|H_w|\overline{B_d^0}\rangle}{f_\pi}\left\{[\tfrac{1}{3}\,4e^{i\delta_1} - e^{i\delta_3}] - \frac{\langle D^{*0}|H_w|\overline{B_d^0}\rangle}{\langle D^0|H_w|\overline{B_d^0}\rangle}\left(\frac{m_B^2-m_D^2}{m_B^2-m_{D^*}^2}\right)\sqrt{\frac{1}{2}}h + \cdots\right\}, \tag{10}$$

$$M_{\rm long}(\overline{B^0}\to D^0\pi^0) \simeq -i\frac{\langle D^0|H_w|\overline{B_d^0}\rangle}{f_\pi}\left\{\frac{\sqrt{2}}{3}[2e^{i\delta_1}+e^{i\delta_3}] - \sqrt{\frac{1}{2}}\frac{\langle D^{*0}|H_w|\overline{B_d^0}\rangle}{\langle D^0|H_w|\overline{B_d^0}\rangle}\left[\left(\frac{m_B^2-m_D^2}{m_B^2-m_{D^*}^2}\right) + \frac{\langle D^0|H_w|\overline{B_d^{*0}}\rangle}{\langle D^0|H_w|\overline{B_d^0}\rangle}\left(\frac{m_B^2-m_D^2}{m_{B^*}^2-m_D^2}\right)\right]\sqrt{\frac{1}{2}}h + \cdots\right\}, \tag{11}$$

$$M_{\rm long}(B^-\to D^0\pi^-) \simeq -i\frac{\langle D^0|H_w|\overline{B_d^0}\rangle}{f_\pi}\left\{-e^{i\delta_3} + \frac{\langle D^0|H_w|\overline{B_d^{*0}}\rangle}{\langle D^0|H_w|\overline{B_d^0}\rangle}\left(\frac{m_B^2-m_D^2}{m_{B^*}^2-m_D^2}\right)\sqrt{\frac{1}{2}}h + \cdots\right\}, \tag{12}$$

where the ellipses denote the neglected excited-state meson contributions. The corresponding amplitudes for the $\bar{B}\to D^*\pi$ decays can be obtained by replacing $D^0\leftrightarrow D^{*0}$ and $\delta_{2I}\to\delta_{2I}^*$ in Eqs. (10)–(12).

To evaluate the above amplitudes, we need to know the sizes of the asymptotic matrix elements of H_w and axial charges A_π's taken between heavy meson states. To estimate the former, we use the factorization prescription since the heavy mesons will annihilate at the weak vertex in the weak boson mass $m_W\to\infty$ limit. Then they can be described in terms of the decay constants f_D, f_{D^*}, f_B, and f_{B^*}. Using $f_{D^*}\simeq f_D\simeq 0.22$ GeV and $f_{B^*}\simeq f_B\simeq\sqrt{m_D/m_B}f_D\simeq 0.13$ GeV as in [2,5], we obtain $\langle D^0|H_w|B^0\rangle\simeq V_{cb}V_{ud}\{-0.37a_2\}\times 10^{-5}$ GeV^2, $\langle D^{*0}|H_w|B^0\rangle\simeq V_{cb}V_{ud}\{0.28a_2\}\times 10^{-5}$ GeV^2, $\langle D^0|H_w|B^{*0}\rangle\simeq V_{cb}V_{ud}\{-0.28a_2\}\times 10^{-5}$ GeV^2, and $\langle D^{*0}|H_w|B^{*0}\rangle\simeq V_{cb}V_{ud}\{0.38a_2\}\times 10^{-5}$ GeV^2. From the observed rate [6] $\Gamma(\rho\to\pi\pi)_{\rm expt}\simeq 150$ MeV, the asymptotic matrix element of A_π is estimated to be $|h|\simeq 1.0$ [8,15] by using partially conserved axial-vector currents (PCAC's). The above result on the asymptotic matrix elements of H_w and A_π's leads us to the long distance amplitudes listed in the second column of Table I, where the CKM matrix elements are factored out.

We now review the factorized amplitudes as the short distance contributions to the $\bar{B}\to D\pi$ and $D^*\pi$ decays which have been studied by using the factorization prescription. The calculated branching ratios for the $\bar{B}\to D\pi$ and $D^*\pi$ decays have been listed in [1,2]. In the third column of Table I, we list the factorized amplitudes which are compatible with these expressions of branching ratios. In [1,5], the values of a_1 and a_2 to reproduce extensively the observed branching ratios for two-body decays of B mesons have been searched for. Since the long distance contributions were not

TABLE I. The amplitudes for the $\overline{B}\to D\pi$ and $D^*\pi$ decays.

Decay	$A_{\rm long}(10^{-5}$ GeV)	$A_{\rm short}(10^{-5}$ GeV)
$\overline{B^0}\to D^+\pi^-$	$-2.8a_2\left\{\frac{1}{3}[4e^{i\delta_1}-e^{i\delta_3}]+0.55\right\}$	$1.54a_1\left\{1+0.1\left(\frac{a_2}{a_1}\right)\right\}$
$\overline{B^0}\to D^0\pi^0$	$-2.8a_2\left\{\frac{\sqrt{2}}{3}[2e^{i\delta_1}+e^{i\delta_3}]+0.06\right\}$	$-1.34a_2\left\{\frac{f_D}{0.22\ \text{GeV}}\right\}$
$B^-\to D^0\pi^-$	$2.82a_2\{e^{i\delta_3}-0.47\}$	$1.54a_1\left\{1+1.22\left(\frac{a_2}{a_1}\right)\right\}$
$\overline{B^0}\to D^{*+}\pi^-$	$2.2a_2\left\{\frac{1}{3}[4e^{i\delta_1^*}-e^{i\delta_3^*}]+0.91\right\}$	$-1.53a_1\left\{1+0.3\left(\frac{a_2}{a_1}\right)\right\}$
$\overline{B^0}\to D^{*0}\pi^0$	$2.2a_2\left\{\frac{\sqrt{2}}{3}[2e^{i\delta_1^*}+e^{i\delta_3^*}]-0.04\right\}$	$1.34a_2\left\{\frac{f_{D^*}}{0.22\ \text{GeV}}\right\}$
$B^-\to D^{*0}\pi^-$	$-2.2a_2\{e^{i\delta_3^*}-0.97\}$	$-1.53a_1\left\{1+1.29\left(\frac{a_2}{a_1}\right)\right\}$

sufficiently considered, however, the value of a_2 to give the best fit was far from the one expected by the perturbation theory as discussed earlier.

We are now ready to estimate the rates for the $\overline{B}\to D\pi$ and $D^*\pi$ decays by taking a sum of the hard pion amplitude (the second column in Table I) and the factorized amplitude (the third column in Table I) as the total amplitude. However, the value of U_{cb} still contains large ambiguities [6], $U_{cb}=0.032-0.048$. To get rid of this uncertainty, we first consider the following ratios of decay rates:

$$R_1=\frac{\Gamma(B^-\to D^0\pi^-)}{\Gamma(B^0\to D^+\pi^-)},\quad R_2=\frac{\Gamma(\overline{B^0}\to D^0\pi^0)}{\Gamma(B^-\to D^0\pi^-)},$$

$$R_3=\frac{\Gamma(B^-\to D^{*0}\pi^-)}{\Gamma(\overline{B^0}\to D^{*+}\pi^-)},\quad R_4=\frac{\Gamma(\overline{B^0}\to D^{*0}\pi^0)}{\Gamma(B^-\to D^{*0}\pi^-)},$$

$$R_5=\frac{\Gamma(\overline{B^0}\to D^{*+}\pi^-)}{\Gamma(\overline{B^0}\to D^+\pi^-)}. \tag{13}$$

We here take the values of the Wilson coefficients $a_1=1.03$ and $a_2=0.11$ which are expected from perturbation theory with $N_c=3$ [2,7]. The phases $\delta_1^{[*]}$ and $\delta_3^{[*]}$ arising from contributions of nonresonant multihadron intermediate states with isospin $I=\frac{1}{2}$ and $\frac{3}{2}$ have not been measured by experiments. (Resonant contributions have already been extracted as pole amplitudes in M_S, although they were not very important and neglected since m_B is so high and far beyond the charm resonance region.) However, the nonresonant phase shifts will be restricted in the region $|\delta_{2I}^{[*]}|<90°$. We here assume $\delta_1\simeq\delta_1^*$ and $\delta_3\simeq\delta_3^*$ since D and D^* are degenerate under the strong interactions in heavy quark effective theory [16]. Then our calculated ratios of rates, Eqs. (13), for $40°<|\delta_1^{[*]}|<80°$ and $|\delta_3^{[*]}|<80°$ reproduce well the observed ones. As an example, we list our result for $\delta_1=\delta_1^*=60°$ and $\delta_3=\delta_3^*=-35°$ in Table II. As discussed earlier, $(R_i)_{\rm short}, i=1$–5, which include only the factorized amplitudes, do not reproduce the observed ones as long as the values of the Wilson coefficients expected from the perturbation theory with $N_c=3$ are taken.

We have seen above that our ratios of the decay rates can reproduce well the observed ones when we take the reasonable values of the Wilson coefficients in the effective weak Hamiltonian and the strong interaction phases. Therefore we are now interested in the decay rates for the $\overline{B}\to D\pi$ and $D^*\pi$ decays. However, the value of U_{cb} is still ambiguous as shown before. We here take $U_{cb}=0.045$ as in [2,5]. Using the observed lifetime [6] $\tau(B^-)\simeq\tau(\overline{B^0})\simeq1.5\times10^{-12}$ s, we obtain the branching ratios listed in Table III, although they still contain large ambiguities arising from the uncertainty of U_{cb}. We can see that the long distance contribution is truly small but still can interfere efficiently with the main amplitude arising from the short distance contribution. If the short distance contributions were not taken into account, the calculated branching ratios for the $\overline{B}\to D\pi$ and $D^*\pi$ decays would be much smaller than the observed ones as seen in Table III. [In our previous work [11], the $\overline{B^0}\to D^0\pi^0$ decay could not be suppressed because of the assumed dominance of long distance physics. Since we now do not assume its dominance in this Brief Report, however, the present value of $B(\overline{B^0}\to D^0\pi^0)$ is compatible with the experiment.] The calculated branching ratios $B(\overline{B}\to D\pi)_{\rm long+short}$ and $B(\overline{B}\to D^*\pi)_{\rm long+short}$, although still involving large ambigu-

TABLE II. Ratios of the rates for the $\overline{B}\to D\pi$ and $D^*\pi$ decays. The ratios R_1–R_5 are defined in the text. The values of the Wilson coefficients $a_1=1.03$ and $a_2=0.11$ are taken. The values of the strong interaction phases $\delta_1=\delta_1^*=60°$ and $\delta_3=\delta_3^*=-35°$ are tentatively chosen. The data values are taken from Ref. [1].

Ratio	Short	Short+long	Experiment
R_1	1.25	1.94	$1.89\pm0.26\pm0.32$
R_2	0.01	0.06	<0.09
R_3	1.22	1.70	$2.00\pm0.37\pm0.28$
R_4	0.01	0.04	<0.2
R_5	0.97	1.03	$1.12\pm0.19\pm0.24$

TABLE III. The branching ratios (%) for the $\overline{B}\to D\pi$ and $D^*\pi$ decays. The values of the CKM matrix elements $U_{cb}=0.045$ and $U_{ud}=0.98$ are taken. The values of the other parameters involved are the same as in Table II. The observed life time [6] $\tau(B^-)\simeq\tau(\overline{B^0})\simeq 1.5\times 10^{-12}$ s is used. The data values are taken from Ref. [1].

Decay	$B_{\rm long}$	$B_{\rm short}$	$B_{\rm short+long}$	$B_{\rm expt}$
$\overline{B^0}\to D^+\pi^-$	0.040	0.49	0.28	0.29 ± 0.12
$\overline{B^0}\to D^0\pi^0$	0.017	0.02	0.03	<0.048
$B^-\to D^0\pi^-$	0.007	0.74	0.55	0.55 ± 0.11
$\overline{B^0}\to D^{*+}\pi^-$	0.030	0.52	0.27	0.26 ± 0.08
$\overline{B^0}\to D^{*0}\pi^0$	0.008	0.02	0.02	<0.055
$B^-\to D^{*0}\pi^-$	0.005	0.75	0.47	0.52 ± 0.17

ities arising from the uncertainty of U_{cb}, reproduce the observed ones. It implies that the long distance hadron dynamics improves remarkably $B(\overline{B}\to D\pi)_{\rm short}$ and $B(\overline{B}\to D^*\pi)_{\rm short}$, which include only the factorized amplitudes.

In summary, we have investigated the $\overline{B}\to D\pi$ and $D^*\pi$ decays by assuming that the amplitudes are given by a sum of long and short distance contributions. The long distance amplitudes have been calculated by using a hard pion approximation and, as the short distance amplitudes, the usual factorized amplitudes have been taken. The size of the long distance contribution to the $\overline{B}\to D\pi$ and $D^*\pi$ decays has been estimated to be rather small in these decays but still not negligible. By taking reasonable values of the phase shifts arising from the final state interactions, the observed branching ratios for these decays have been reproduced in terms of a sum of the hard pion amplitude and the factorized amplitude by taking the values of the Wilson coefficients a_1 and a_2 from perturbation theory with $N_c=3$. This implies that, in hadronic weak interactions of B mesons, short distance physics is not sufficient but long distance hadron dynamics should be carefully taken into account.

To determine the size of long distance effects on the $\overline{B}\to D\pi$ decays, measurements of the decay rate $\Gamma(\overline{B^0}\to D^0\pi^0)$ are inevitably important.

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