

## Implications of Abelian extended gauge structures from string models

Mirjam Cvetič\*

*Institute for Advanced Study, Olden Lane, Princeton, New Jersey 08540*

Paul Langacker

*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396,  
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030,*

(Received 22 November 1995)

Within a class of superstring vacua which have an additional nonanomalous  $U(1)'$  gauge factor, we address the scale of the  $U(1)'$  symmetry breaking and constraints on the exotic particle content and their masses. We also show that an extra gauge  $U(1)'$  provides a new mechanism for generating a naturally small effective  $\mu$  term. In general, existing models are not consistent with all phenomenological constraints; however, they do provide a testing ground to address the above issues, yielding a set of concrete scenarios. Under the assumptions that the spontaneous  $U(1)'$  breaking takes place in the observable sector and that the supersymmetry-breaking scalar mass square terms are positive at the string scale, the breaking of  $U(1)'$  symmetry is radiative. It can take place when the appropriate Yukawa couplings of exotic particles are of order 1, which occurs for  $Z_2 \times Z_2$  fermionic orbifold constructions at symmetric points of moduli space. The  $Z'$  mass is either of  $O(M_Z)$ , when the symmetry breaking is due to a single standard model singlet, or of a scale intermediate between the string and electroweak scales, determined by the radiative corrections (or by competing nonrenormalizable operators), when the breaking is due to two or more mirrorlike singlets. In the former case, the  $M_{Z'}/M_Z$  hierarchy achievable without excessive fine-tuning is within future experimental reach. [S0556-2821(96)01115-0]

PACS number(s): 12.60.Jv, 11.25.Sq, 12.10.Kt

### I. INTRODUCTION

One of the challenges of string theory is to make a successful connection to the observable world. Attempts to construct realistic models have not been completely successful at satisfying all of the phenomenological constraints. The two major problems of string models are the following.

Degeneracy of vacua: there are by now a large number of string models, which in general have a large gauge symmetry which is difficult to break to the standard model, a large number of families, and other exotic particles.

Supersymmetry breaking: presently, we do not have a fully satisfactory scenario for supersymmetry breaking, either at the level of world-sheet dynamics or at the level of the effective theory.

Both problems are believed to have an ultimate resolution in nonperturbative string dynamics. However, in spite of the absence of a unique string vacuum the string theory does provide certain generic and, for a certain class of string vacua, specific predictions.

One of the string predictions is the gauge coupling unification at  $M_{\text{string}} \sim g_U \times 5 \times 10^{17}$  GeV [1], where  $g_U$  is the gauge coupling at the string scale. The observed couplings are approximately consistent with this prediction. Taking the observed  $\alpha$  and weak angle  $\sin^2 \theta_W$  as inputs and extrapolating assuming the particle content of the minimal supersymmetric standard model (MSSM), one finds [2] that the running  $SU(2)$  and  $U(1)$  couplings meet at a scale  $M_U \sim 3 \times 10^{16}$  GeV. One can then predict  $\alpha_s(M_Z) \sim 0.130 \pm 0.010$  for the strong coupling.  $M_U$  is about

an order of magnitude below  $M_{\text{string}}$ . The actual value of  $\alpha_s(M_Z)$  is still controversial, with determinations generally in the range 0.11–0.125 [3]. In any case, when properly viewed as predictions for  $\ln(M_U/M_Z)$  and  $1/\alpha_s$ , the gauge unification works to within 10–15 %.

The gauge group at  $M_{\text{string}}$  is generically not a grand unified gauge group, but a product of semisimple group factors. There are frequently additional factors beyond the standard model group, such as extra  $U(1)$ 's.

There are a very large number of possible string models, which correspond to consistent perturbative vacua of string theory. It is not at present feasible to examine them all, but one can take a less ambitious attitude and consider only those vacua which have the potential to be realistic. There are no known fully realistic models, but nevertheless there are specific issues that can be addressed within semirealistic models, which may in turn provide a pattern of general string model predictions.

In particular, we study a class of specific string vacua, which at  $M_{\text{string}}$  possess  $N=1$  supersymmetry, the standard model (SM) gauge group as a part of the gauge structure, and a particle content that includes three SM families [5–7] and at least two SM Higgs doublets, i.e., the string vacua which have at least the ingredients of the MSSM. Specific models often contain additional matter multiplets as well, often with exotic standard model quantum numbers. In general, we do not want to give up the approximate unification of gauge couplings. This severely restricts the possibilities for new exotic matter [8]. However, we will sometimes relax the gauge unification constraint in discussing specific models which illustrate interesting scenarios.

A number of such models (not necessarily consistent with gauge unification) were constructed as orbifold models [9]

\*On sabbatical leave from the University of Pennsylvania.

with Wilson lines, as well as models based on the free (world-sheet) fermionic constructions [10,6,11]. The latter set of models are based on the  $Z_2 \times Z_2$  orbifold at special points of toroidal and Wilson line moduli space. As for the issue of supersymmetry breaking, we may parametrize our ignorance by introducing soft supersymmetry breaking terms in the observable sector.<sup>1</sup>

A set of models of that type constitutes a starting point to address specific phenomenological issues. Here, we would like to derive the consequences of an enhanced gauge symmetry in the observable sector of the above class of string vacua. For the sake of concreteness we shall concentrate on an additional nonanomalous  $U(1)'$  symmetry. A generalization to more than one  $U(1)$  symmetry factor is straightforward.

The phenomenology of heavy gauge bosons in gauge theories has been extensively studied in the past. There are stringent limits on the mass and mixings of such bosons from precision electroweak experiments [13] and from direct searches [15]. The limits vary significantly from model to model because of the different chiral couplings to the ordinary fermions, but typically the mass of a heavy  $Z'$  must exceed  $\sim 400$  GeV, while the  $Z$ - $Z'$  mixing angle must be smaller than a few times  $10^{-3}$ .

Furthermore, the identification and diagnostic study of heavy gauge bosons at future colliders has been investigated in detail [16]. There have also been studies of the present and future constraints on possible exotic matter [17]. For example, some models predict the existence of a heavy vector [ $SU(2)_L$  singlet] charge  $-1/3$  quark,  $D_L - D_R$ , which could be produced at a hadron collider by ordinary QCD processes and decay by  $D_L - b_L, s_L, d_L$  mixing into, e.g.,  $cW$ ,  $bZ$ , or  $bH$ , where  $H$  is a neutral Higgs boson. Currently,  $m_D > 85$  GeV if it mixes mainly with  $b$  [17]. However, heavy gauge bosons and exotic matter have rarely been addressed together [18]. Additionally, on purely phenomenological grounds there is no particular reason for the mass scale of new bosons or matter to be in the window (e.g., up to a few TeV) accessible to present or future experiments.

In contrast, a class of string models with the features mentioned above and an additional  $U(1)'$  symmetry provides a testing ground to address the following aspects of an enhanced Abelian gauge symmetry: (i) a scenario specifying the scale of  $U(1)'$  symmetry breaking; (ii) the mass scale and phenomenological implications of the exotic particles associated with an enhanced gauge symmetry; (iii) the implications of  $U(1)'$  symmetry for generating a naturally small effective  $\mu$  term.

We have identified several distinct scenarios, each of which is illustrated by a specific model. A thorough analysis within a large class of such models awaits further investigation.

A major conclusion of this paper is that a large class of string models considered here not only predict the existence of additional gauge bosons and exotic matter particles, but often imply the masses of the new gauge bosons and the exotic particles which necessarily accompany them to be in

the electroweak range. Each specific model leads to calculable predictions (which, however, depend on the assumed soft supersymmetry-breaking terms) for the masses, couplings, and mixing with the  $Z$  of the new boson(s), as well as for the masses and quantum numbers of the associated exotic matter.

The paper is organized as follows. In Sec. II we specify in more detail the features of the string models under investigation. In Sec. III we address specific  $U(1)'$  symmetry-breaking scenarios, achievable without excessive fine-tuning of the soft supersymmetry-breaking parameters. We show examples in which the additional  $Z'$  mass is (a) comparable to that of the  $Z$  (already excluded), (b) in the 300 GeV to 1 TeV range, which may be still barely allowed but easily within the range of future or present colliders, and (c) at an intermediate scale (e.g.,  $10^8 - 10^{14}$  GeV). It is argued that in case (b) it is difficult though not impossible to satisfy existing constraints on  $Z$ - $Z'$  mixing, especially for lower values of  $M_{Z'}$ , and that  $Z'$  masses above 1 TeV are not expected (given our assumptions) without excessive fine-tuning. In Sec. IV several issues related to  $Z'$  physics are briefly discussed. In particular, it is argued that in the case of  $U(1)'$  symmetry breaking at the 100 GeV to 1 TeV scale it may be possible to generate a naturally small effective  $\mu$  term by the vacuum expectation value of a standard model singlet field which is charged under  $U(1)'$ . On the other hand, in the absence of an extended gauge symmetry (and assuming positive soft supersymmetry-breaking scalar mass-square terms) standard model singlets generally either will not acquire vacuum expectation values, or will acquire vacuum expectation values (VEV's) at an intermediate scale. Such singlets would not be suitable for generating an effective  $\mu$  term, and would yield intermediate scale masses for the exotic matter to which they couple. Conclusions are given in Sec. V. In the Appendix the renormalization group equations and the analytic solutions for the soft supersymmetry-breaking mass-square terms are given for specific examples of Yukawa interactions under specific assumptions for the soft supersymmetry-breaking terms at  $M_{\text{string}}$ .

## II. GENERAL FEATURES OF A CLASS OF STRING MODELS

Let us first specify the generic features of  $N=1$  supersymmetric string models with the standard model (SM) gauge group  $SU(2)_L \times U(1)_Y \times SU(3)_C$ , three ordinary families, and at least two SM doublets, i.e., a set of models with at least the particle content of the minimal supersymmetric standard model (MSSM). Those models are primarily based on fermionic  $Z_2 \times Z_2$  orbifold constructions at a particular point in the toroidal and Wilson line moduli space.

Such models in general also contain a non-Abelian "shadow" sector group and a number of additional  $U(1)$ 's, one of them generically anomalous. The shadow sector is a set of SM singlets that transform nontrivially under the non-Abelian shadow gauge group. However, they communicate to the observable sector through additional  $U(1)$  factors. The shadow sector non-Abelian gauge group may play a role in dynamical supersymmetry breaking. In addition, there are a large number of additional matter multiplets, which transform nontrivially under the  $U(1)$ 's and/or the standard model symmetry.

<sup>1</sup>For different scenarios, in which supersymmetry is broken in the moduli-dilaton sector, see Ref. [12] and references therein.

Most of the available models of that type correspond to the level 1 Kač-Moody algebra for the non-Abelian gauge group factors, which in turn ensures that the chiral supermultiplets are all in the fundamental or singlet representations of the SM and the non-Abelian shadow sector gauge group. In addition, from a set of models we select only those with  $SU(3)_C$ ,  $SU(2)_L$ , and  $U(1)_Y$  all embedded into the  $SU(5)$  gauge group, since for other types of embedding the normalization of the  $U(1)_Y$  gauge group coupling is different from the one leading to the gauge coupling unification in the MSSM model.

The fact that at  $M_{\text{string}}$  the observable sector gauge group is not  $SU(5)$ , but the SM gauge group, implies [19] that the theory in general contains fractionally charged color singlets. A generic prediction for fractionally charged color singlets may have important phenomenological [20] consequences.

Due to an anomalous  $U(1)$  symmetry at genus 1, there is an additional contribution of  $O(M_{\text{string}}^2)$  [21,22(a)] to the corresponding  $D$  term.<sup>2</sup> The contribution of such a term is canceled [22(b),21,22(a)] by giving nonzero vacuum expectation values (VEV's) of  $O(M_{\text{string}})$  to certain multiplets in such a way that the  $D$  flatness and  $F$  flatness condition is maintained at genus-1 level of the effective string theory, thus providing a mechanism for “restabilizing” the vacuum at genus 1. At the same time, the nonzero VEV's can be chosen (at least in principle) in such a way that, while the SM gauge group remains intact, a number of additional nonanomalous  $U(1)$ 's are broken at  $M_{\text{string}}$  as well. In addition, a number of multiplets become massive. Thus the enhanced gauge symmetry and the exotic particle content of the observable sector are in general drastically reduced. Nevertheless, there are often one or more nonanomalous  $U(1)$ 's and associated exotic matter that are left unbroken. The study of symmetry-breaking scenarios of these leftover nonanomalous  $U(1)$ 's symmetries is the subject of this paper.

However, such models in general suffer from one or more of the following deficiencies: (i) It is not clear that for a desired gauge group choice there always exists a choice of VEV's which would ensure the  $F$  flatness to all orders in the nonrenormalizable terms;<sup>3</sup> (ii) it is also not clear how the supersymmetry breaking scenario can be implemented. Namely, the gaugino condensation in the “shadow” sector may not be possible, due to a large number of additional shadow sector matter multiplets which may render the shadow sector non-Abelian gauge group nonasymptotically free. We shall not address the dynamical origin of supersymmetry breaking—in particular, the difficulties with gaugino condensation or related issues of dynamical symmetry breaking in the shadow sector of the theory.<sup>4</sup>

There are also other, phenomenological, problems. (i) In general the models have additional color triplets in the spec-

trum which could mediate a too fast proton decay [26,27]. (ii) The detailed mass spectrum of the ordinary fermions [5,7] is not realistic. (iii) In the case of an enhanced symmetry in the observable sector a scenario for the enhanced symmetry breaking may not be consistent with phenomenological constraints on the exotic multiplets, such as gauge coupling unification. (iv) There is no  $\mu$  term in the superpotential; i.e., the coupling between the two SM Higgs doublets is absent, since these SM doublets are massless at  $M_{\text{string}}$ . An effective  $\mu$  term of the order of soft supersymmetry-breaking mass parameters is needed for the low energy phenomenology [28].

Here, we shall concentrate on phenomenological consequences of an additional  $U(1)'$  symmetry. We shall not attempt to solve all the problems of such a class of models, but rather address the specific aspects of  $U(1)'$  symmetry breaking.

### III. $U(1)'$ SYMMETRY-BREAKING SCENARIOS

We shall confine the analysis to the set of models whose features were specified in the previous section. In addition, we do not address dynamics associated with the shadow sector, and we parametrize the supersymmetry breaking by introducing soft supersymmetry-breaking terms. Within that context we are ignoring aspects of dynamical symmetry breaking due the formation of condensates in the non-Abelian shadow sector, which could at the same time break the additional  $U(1)'$  at a large scale.

Under these assumptions the  $U(1)'$  symmetry breaking must take place via the Higgs mechanism, in which the scalar component(s) of chiral supermultiplets  $S_i$ , which carry nonzero charges under the  $U(1)'$ , acquire nonzero vacuum expectation values (VEV's) and spontaneously break  $U(1)'$ . The low energy effective action, responsible for spontaneous symmetry breaking (SSB), is specified by the superpotential, Kähler potential, and soft supersymmetry-breaking terms.

Assuming that the soft supersymmetry-breaking mass-square terms for the scalar fields are positive at  $M_{\text{string}}$ , the only way of achieving SSB is via a radiative mechanism. Namely, since such SM singlets are massless at  $M_{\text{string}}$ , they have no bilinear term in the superpotential, and their soft supersymmetry-breaking mass-square terms need to be driven negative at lower energies to ensure a global minimum with nonzero VEV's for such fields. Since the soft supersymmetry-breaking mass-square terms are assumed to be positive (and often taken to be universal) at  $M_{\text{string}}$ , the radiative breaking scenario can be achieved if there are large Yukawa couplings of the  $S_i$ 's to a sufficient number of other fields. This is most easily achieved if there are couplings to SM doublets or color triplets. In this case the renormalization group equations ensure that the corresponding mass-square terms for  $S_i$  can be driven negative.<sup>5</sup>

Thus the scale of  $U(1)'$  symmetry breaking depends both on the type of SM singlets responsible for the  $U(1)'$  sym-

<sup>2</sup>In certain models there may be an additional numerical suppression factor of  $10^{-2}$ , causing the scale of the genus-1 contribution to be smaller than  $M_{\text{string}}$  by one to two orders of magnitude.

<sup>3</sup>In certain instances such constraints can be obtained by applying selection rules for the corresponding string amplitudes, as developed for orbifold [22(c)] and blown-up orbifold compactifications [23]. See also [5].

<sup>4</sup>See, however, [24,25].

<sup>5</sup>For the explicit form of the renormalization group equations which can be applied to this case, see, for example, Appendix A of Ref. [29]. See also [30].

metry breaking and on the Yukawa couplings of such multiplet(s) to other exotic particles. Interestingly, for the fermionic constructions based on  $Z_2 \times Z_2$  orbifolds at special points of toroidal and Wilson line moduli space, the corresponding Yukawa couplings are either<sup>6</sup> of  $O(g)$  or zero.<sup>7</sup> Thus, if the relevant coupling is nonzero it may be sufficiently large to ensure radiative breaking of the  $U(1)'$  symmetry.

For each of these possibilities the pattern of  $U(1)'$  symmetry breaking and the running of the gauge couplings still depend on the specific exotic particle content and their couplings. We shall now discuss the possible scenarios for observable sector  $U(1)'$  symmetry breaking. For the sake of simplicity we shall address scenarios in which the electroweak symmetry is broken due to the nonzero VEV of the Higgs doublet that couples to the top quark, i.e., a large  $\tan\beta$  scenario of the MSSM. A generalization to scenarios that accommodate other ranges of  $\tan\beta$  is straightforward. We will emphasize the general features which hold in each scenario. However, we emphasize that in each specific model the  $Z'$  mass, mixing, and couplings, as well as the properties of the exotic matter, are in principle calculable, though in practice they depend on the details of the soft supersymmetry breaking.

#### A. $U(1)'$ breaking without $U(1)'$ charged standard model singlets

This is the case in which the low energy spectrum of the theory contains no SM model singlets that are charged under  $U(1)'$ , or in which their effective mass-square terms remain positive. In this case radiative breaking of the electroweak symmetry, due to nonzero VEV of the SM Higgs doublet(s), would ensure the SSB of the additional  $U(1)$ 's as well, provided that they are charged under  $U(1)'$ .

The  $Z$ - $Z'$  mass-square matrix is then of the form

$$M_{Z-Z'}^2 = \begin{pmatrix} \frac{1}{2} G^2 H^2 & G g' Q'_H H^2 \\ G g' Q'_H H^2 & 2 g'^2 Q'^2_H H^2 \end{pmatrix}, \quad (1)$$

where  $G \equiv \sqrt{g^2 + g_Y^2}$ . Here  $g, g_Y$ , and  $g'$  are the gauge couplings at the SSB scale for  $SU(2)_L$ ,  $U(1)_Y$ , and  $U(1)'$ , respectively,  $H$  is the VEV of the SM Higgs doublet(s), and  $Q'_H$  is its  $U(1)'$  charge. [There is an obvious generalization for the case of two Higgs doublets with different  $U(1)'$  charges.]

In this case, the  $Z'$  mass is necessarily of  $O(M_Z)$  and the  $Z$ - $Z'$  mixing is of order (1). Such a scenario is likely not to be compatible with the current bounds on  $M_{Z'}$  [13]. An exception would be the (highly unlikely) possibility that the SM Higgs doublet(s) have  $U(1)'$  charges  $|Q'_H| \gg 1$ . In this case the mass of  $M_{Z'}$  may be large enough and the  $Z$ - $Z'$  mixing sufficiently suppressed to evade the current bounds.

<sup>6</sup>There is still uncertainty about the precise value of the Yukawa coupling, since the latter is related to the corresponding three-linear term in the superpotential by a factor due to the Kähler potential contribution.

<sup>7</sup>The same is true for the Yukawa coupling of the ordinary families to the Higgs doublets.

An illustration of this scenario is provided by a particular version of the  $Z_2 \times Z_2$  orbifold model specified in Ref. [7], in which the vacuum at genus 1 is restabilized by giving non-zero VEV's to the SM singlets  $\Phi_{45,13}$ ,  $\Phi_{1,2}^\pm$ ,  $\bar{\Phi}_{1,2}^\pm$ ,  $\bar{\Phi}_{2,3}^-$ ,  $\bar{\Phi}_{23,13}$ ,  $\xi_{1,2}$ .<sup>8</sup> In this case there are no massless SM singlets that are charged under the surviving  $U(1)'$ . The SM Higgs doublet(s) responsible for the electroweak SSB have  $Q'_H = -1$ . The model contains the relevant Yukawa coupling between the third family quarks and leptons to the SM Higgs doublet, thus allowing for radiative symmetry breaking. Since the  $|Q'_H|$  charge is not large enough, this SSB scenario is incompatible with current experimental bounds. In addition, the model has a number of additional light SM doublets and color triplets as well as fractionally electrically charged color singlets, which affect the running of the gauge couplings.

#### B. Symmetry breaking due to one $U(1)'$ charged standard model singlet

Now suppose that the radiative breaking of  $U(1)'$  is due to one SM singlet  $S$ . Namely, only  $S$  has its effective mass square driven to a negative value in the infrared regime, thus allowing for a nonzero VEV. The  $Z$ - $Z'$  mass-square matrix is then

$$M_{Z-Z'}^2 = \begin{pmatrix} \frac{1}{2} G^2 H^2 & G g' Q'_H H^2 \\ G g' Q'_H H^2 & 2 g'^2 (Q'^2_H H^2 + Q'^2_S S^2) \end{pmatrix}, \quad (2)$$

where  $H$  and  $S$  now denote the VEV's for the SM Higgs doublet and singlet, respectively, and  $Q'_{H,S}$  are the corresponding  $U(1)'$  charges.

The exotic matter to which  $S$  couples acquires a mass of order  $\mathcal{H}S$ , where  $\mathcal{H}$  is the relevant Yukawa coupling between the particular exotic matter and  $S$ . In general, there will be an additional soft supersymmetry-breaking mass term contributing to the mass of the exotic matter even in the absence of the relevant Yukawa coupling(s).

The nature of the  $Z$ - $Z'$  hierarchy now crucially depends on the allowed VEV's  $S$  and  $H$ , which are constrained by the form of the potential, and can be written for the particular direction with nonzero VEV's as

$$V = -|m_H^2|H^2 - |m_S^2|S^2 + \frac{1}{8}G^2H^4 + \frac{g'^2}{2}(Q'^2_H H^2 + Q'^2_S S^2)^2, \quad (3)$$

where we have extracted explicit minus signs from the negative mass-square terms.

One encounters the following two scenarios.

(i) The relative signs of  $Q'_H$  and  $Q'_S$  are opposite. In this case the minimum of the potential is for

$$H^2 = \frac{4(|m_H^2| + (|Q'_H|/|Q'_S|)|m_S^2|)}{G^2},$$

$$S^2 = \frac{|m_S^2|}{|Q'_S|^2 g'^2} + \frac{|Q'_H|H^2}{|Q'_S|}, \quad (4)$$

and the  $Z$ - $Z'$  mass-square matrix is

<sup>8</sup>For notation and quantum number assignments, see Ref. [7].

$$M_{Z-Z'}^2 = 2 \begin{pmatrix} \left( |m_H^2| + \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) & \frac{2g'Q'_H}{G} \left( |m_H^2| + \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) \\ \frac{2g'Q'_H}{G} \left( |m_H^2| + \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) & \frac{4g'^2Q'^2_H}{G^2} \left( 1 + \frac{|Q'_S|}{|Q'_H|} \right) \left( |m_H^2| + \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) + |m_S^2| \end{pmatrix}. \quad (5)$$

It is difficult to achieve the needed hierarchy between  $M_Z$  and  $M_{Z'}$ , unless  $|Q'_S| \gg |Q'_H|$  and  $|m_S^2| \gg |m_H^2|$ , in such a way that  $|Q'_H/Q'_S| = O(|m_H^2|/|m_S^2|)$ . The first condition is not normally expected to hold, except in the limiting case  $Q'_H = 0$ .

(ii) The relative signs of  $Q'_H$  and  $Q'_S$  are the same. The minimum of the potential (3) now occurs for

$$H^2 = \frac{4(|m_H^2| - (|Q'_H|/|Q'_S|)|m_S^2|)}{G^2}, \quad S^2 = \frac{|m_S^2|}{|Q'_S|^2 g'^2} - \frac{|Q'_H|H^2}{|Q'_S|}, \quad (6)$$

and the  $Z$ - $Z'$  mass-square matrix becomes

$$M_{Z-Z'}^2 = 2 \begin{pmatrix} \left( |m_H^2| - \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) & \frac{2g'Q'_H}{G} \left( |m_H^2| - \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) \\ \frac{2g'Q'_H}{G} \left( |m_H^2| - \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) & \frac{4g'^2Q'^2_H}{G^2} \left( 1 - \frac{|Q'_S|}{|Q'_H|} \right) \left( |m_H^2| - \frac{|Q'_H|}{|Q'_S|} |m_S^2| \right) + |m_S^2| \end{pmatrix}. \quad (7)$$

In this case one encounters an interesting possibility for achieving a hierarchy without an unusually small ratio of  $|Q'_H/Q'_S|$ , provided  $0 < |m_H^2| - (|Q'_H|/|Q'_S|)|m_S^2| \ll |m_S^2|$ . In this limit, the  $Z$ - $Z'$  mixing angle is

$$\theta_{Z-Z'} \sim \frac{2g'Q'_H}{G} \frac{M_Z^2}{M_{Z'}^2}. \quad (8)$$

For small  $g'Q'_H/G$  the  $Z$ - $Z'$  mixing could be sufficiently suppressed to be consistent with the experimental bounds for  $M_{Z'} \lesssim 1$  TeV.

One can also illustrate the above scenarios in a particular class of  $Z_2 \times Z_2$  models discussed in Ref. [7].

Case (i). In this case the vacuum at genus 1 is restabilized by nonzero VEV's of the following SM singlets:  $\Phi_{23}, \bar{\Phi}_{23}$ , while the following SM model singlets should necessarily have zero VEV's:  $\xi_1, \xi_2, \xi_3$ .<sup>9</sup> The relevant SM singlet  $S$  is identified with the field  $H_{18}$  with  $Q'_S = 5/4$ , and thus has the *opposite* sign from  $Q'_H = -1$ . In addition,  $H_{18}$  has a Yukawa coupling of order 1 to two color triplets (in the notation of Ref. [7], the coupling is of the type  $D_{45}H_{18}H_{21}$ ), and thus its mass square can become negative in the infrared regime (see the Appendix). The additional  $U(1)'$  charged SM singlets, e.g.,  $H_{17}$ , have mass-square terms which remain positive in the infrared regime. However, in this case the magnitude of charges  $Q'_{H,S}$  is such that *no hierarchy* is possible, and thus the model is excluded by experiment.

Case (ii). The vacuum at genus 1 is restabilized with nonzero VEV's for the SM singlets  $\xi_1, \xi_3, \Phi_{23}$ , while  $\xi_2, \bar{\Phi}_{12}, \bar{\Phi}_{23}$  should have zero VEV's.  $S$  is identified with the field  $H_{17}$ , which has  $Q'_S = -5/4$  and has a Yukawa coupling of order 1 to two SM doublets (in the notation of Ref. [7], the

coupling is of the type  $h_2 H_{16} H_{17}$ ). Thus, its mass square can become negative in the infrared regime (see the Appendix). The additional  $U(1)'$  charged SM singlets, e.g.,  $H_{18}$ , again have mass-square terms which remain positive in the infrared regime.

This is the case in which a reasonable hierarchy can be achieved without fine tuning of the soft supersymmetry-breaking parameters. In the case of simplified assumptions for the soft supersymmetry-breaking parameters (see the Appendix), i.e., only universal soft supersymmetry-breaking mass-square terms are assumed to be nonzero, the model yields the mass parameters  $|m_H^2| = 0.40m_{3/2}^2$ ,  $|m_S^2| = 0.25m_{3/2}^2$ , and  $|m_H^2| - |m_S^2||Q'_H|/|Q'_S| = 0.20m_{3/2}^2$ , which unfortunately do not yield the necessary hierarchy. However, a minor deviation from the above assumptions for the soft supersymmetry-breaking parameters, or just an example of a model with a slightly different ratio of  $|Q'_H/Q'_S|$ , can provide for a hierarchy  $|m_H^2| - |m_S^2||Q'_H|/|Q'_S| \ll |m_S^2|$ , say,  $|m_H^2| - |m_S^2||Q'_H|/|Q'_S| \sim |m_S^2|/10$ . If, in addition, one takes,<sup>10</sup> e.g.,  $g'|Q'_H|/G \sim 1/4$ , one obtains  $M_{Z'}^2 \sim 10M_Z^2$  and the mixing angle  $\theta_{Z-Z'} \sim 0.05$ . In this example,  $M_{Z'}$  is barely within the current experimental bounds, while  $\theta_{Z-Z'}$  is too large for most choices of  $Z'$  couplings [13,15]. Somewhat larger values of  $M_{Z'}$  and smaller values of  $g'|Q'_H|/G$  may be consistent with observations.

<sup>9</sup>We have checked the  $F$  flatness of these and subsequent choices of VEV's only at the level of renormalizable terms.

<sup>10</sup>This ratio depends on the exotic particle content, which, in addition to the particles contributing to the radiative breaking, may include additional exotic particles. The full exotic particle content depends on the nonzero VEV's of the SM singlets restabilizing the vacuum at genus 1. In the above analysis we specified the minimal choice of nonzero and zero VEV's of the corresponding SM singlets, in order to ensure radiative breaking of  $U(1)'$ . In general, one also has to ensure that the exotic particle content is compatible with the unification of the SM gauge coupling constants. This imposes another stringent constraint on the allowed exotic particle content.

This particular scenario is most interesting, since it in principle allows, without excessive fine-tuning of the soft supersymmetry breaking parameters, for prediction of  $M_{Z'}$  within the experimental reach of present or future colliders. However, when the experimental bounds on  $M_{Z'}$  exceed the 1 TeV region, this scenario *cannot* be implemented without excessive fine-tuning of the soft supersymmetry-breaking parameters or unusual choices of the  $U(1)'$  charge assignments.

### C. Symmetry breaking due to mirrorlike pairs of $U(1)'$ charged standard model singlets

In this case, negative mass-square terms are induced for two (or more)  $U(1)'$  charged SM singlets  $S_{1,2}$ , whose  $Q'_{S_1, S_2}$  charges have opposite sign. One has flatness of the  $D$  term along the direction  $Q'_{S_1} S_1^2 = -Q'_{S_2} S_2^2 \equiv S^2$ . One now has to include the renormalization group improved potential, which along the flat direction is of the form

$$V = m_S^2(\mu = S)S^2. \quad (9)$$

Thus the minimum occurs near the scale  $\mu_{\text{crit}}$  at which  $m_S^2$  turns negative. In the case of radiative breaking with Yukawa couplings of order 1, it turns out that  $m_S^2(\mu_{\text{crit}})$  is much larger than the soft supersymmetry-breaking mass terms. For the examples in the Appendix,  $\mu_{\text{crit}}$  is typically four to ten orders of magnitude below  $M_{\text{string}}$ . Therefore, in the case of flat directions the scale of symmetry breaking, i.e., the VEV of  $S$ , is  $\sim (10^{-10} - 10^{-4})M_{\text{string}} \sim 10^8 - 10^{14}$  GeV.

Nonrenormalizable terms in the superpotential of the form  $S^{K+3}/M_{\text{Planck}}^K$  ( $K \geq 1$ ) could in principle compete with the radiative corrections included in (9) and determine the scale to be of the order <sup>11</sup> of  $O([M_Z M_{\text{Planck}}^K]^{1/(K+1)})$ . For example, for  $K=1$  the symmetry breaking scale is of the order  $10^{11}$  GeV.

In either case the  $Z'$  acquires mass of the order of the intermediate scale. On the other hand, it is straightforward to show that the mass of the physical Higgs boson associated with  $S$  is of the order of the soft supersymmetry-breaking mass terms. The exotic matter to which  $S$  couples via the Yukawa couplings of magnitude  $\mathcal{H}$  acquires mass of order  $\mathcal{H}S$ , i.e., that of the intermediate scale. In the absence of the relevant Yukawa couplings, the exotic matter would have a mass set by soft supersymmetry-breaking mass terms, and thus of the weak scale.

An illustration of this scenario of symmetry breaking is again provided by a version of the  $Z_2 \times Z_2$  orbifold model [7], in which the vacuum at genus 1 is restabilized by giving nonzero VEV's to the SM singlets  $\Phi_{23}$  and  $\bar{\Phi}_{45}$ , while the SM model singlets  $\xi_1, \xi_2, \xi_3$  should necessarily have zero VEV's. The relevant mirror-pair SM singlets  $S_{1,2}$  are identified with the fields  $H_{17,18}$ , respectively, with charges  $Q'_S = \mp 5/4$ . In addition,  $H_{17}$  has a Yukawa coupling of order 1 to two SM doublets (of the type  $h_2 H_{16} H_{17}$ ), while  $H_{18}$  has

a coupling of order 1 to two color triplets (of the type  $D_{45} H_{18} H_{21}$ ), and thus its mass-square term can become negative for both fields in the infrared regime (see the Appendix). Again, additional  $U(1)'$  charged SM singlets, e.g.,  $H_{25}$ , have mass-square terms which remain positive in the infrared regime.

## IV. OTHER IMPLICATIONS OF $U(1)'$

In this section we briefly discuss other consequences of an extended  $U(1)'$  gauge symmetry.

One of the generic problems of the MSSM is the so-called  $\mu$  problem [32]. In the usual scenario for radiative electroweak breaking, the renormalizable terms in the superpotential, relevant to the Higgs mechanism, are

$$W = \mu h_1 h_2 + \mathcal{H}_t Q_L Q_R h_1, \quad (10)$$

where  $h_1$  and  $h_2$  are the two SM Higgs doublets which give mass to the  $t$  and  $b$  quarks, respectively. The quark doublet  $Q_L$  and singlet  $Q_R$  are identified with the third family, and  $\mathcal{H}_t$  is the [large-order-1] Yukawa coupling.

$h_1$  and  $h_2$  are assumed to acquire positive mass-square terms of  $O(m_{3/2}^2)$  at a large scale. The mass-square term for  $h_1$  is driven negative at low energies due to the large Yukawa coupling  $\mathcal{H}_t$ . However, to achieve a realistic mass spectrum, the bilinear  $\mu$  term is needed, which yields supersymmetric mass contributions for  $h_i$  and contributes to the chargino and neutralino masses. Also, the soft supersymmetry-breaking term of the type  $B\mu h_1 h_2$ , where  $B$  is a soft supersymmetry-breaking parameter, leads to  $h_1$ - $h_2$  mixing and to a nonzero VEV for  $h_2$ , yielding the  $b$  quark mass.

For a realistic mass spectrum  $\mu$  in Eq. (10) should be  $O(m_{3/2}^2)$ . However, since it is a coupling in the supersymmetric Lagrangian, then, at least in the context of the MSSM, there is no reason for it not to be much larger. This problem is referred to as the  $\mu$  problem

In the next to minimal supersymmetric model (NMSSM) [33], the  $\mu$  problem is addressed in the following way. One assumes that the  $\mu$  term is absent due to a symmetry, and that it is replaced by an effective  $\mu$  given by the VEV of a standard model singlet  $S$ . That is, one replaces the superpotential (10) by

$$W = S h_1 h_2 + \mathcal{H}_t Q_L Q_R h_1 - \frac{\kappa}{3} S^3. \quad (11)$$

It is assumed that the scalar component of  $S$  acquires a nonzero VEV due to a negative soft supersymmetry-breaking mass-square term of  $O(m_{3/2}^2)$  at the electroweak scale. The cubic  $\kappa$  term in Eq. (11) yields a quartic term for  $S$  in the scalar potential, so that for  $\kappa$  of order 1, the VEV of  $S$  is of  $O(m_{3/2}^2)$ , thus yielding an effective  $\mu$  parameter of  $O(m_{3/2}^2)$ .

Another possibility is that  $\mu$  is absent in the superpotential due to a symmetry, but that an effective  $\mu$  term is generated by nonrenormalizable operators [34,35]. In particular, terms in the Kähler potential, which are proportional to  $h_1 h_2$ , are transmitted to the observable sector as effective  $\mu$  terms, due to gravitational effects after the spontaneous

<sup>11</sup>Such a scenario, with interplay of the nonrenormalizable superpotential terms and the negative mass-square terms, has been implemented in a related context of breaking of  $E_6$  symmetry for Calabi-Yau compactifications [31].

supersymmetry breaking, and are therefore of the same order of magnitude as the soft supersymmetry-breaking terms [34].

Remarkably, in string models  $\mu=0$  at  $M_{\text{string}}$ , since by definition  $h_1$  and  $h_2$  belong to the massless sector and do not have bilinears in the superpotential. It was shown in Refs. [36] that a number of string models possess nonrenormalizable terms in the Kähler potential, which are proportional to  $h_1 h_2$ , and may thus provide a resolution to the  $\mu$  problem in the manner of [34].

On the other hand, the NMSSM mechanism is difficult to implement directly in string models. There may be an additional singlet (or singlets)  $S$  which has the appropriate coupling  $Sh_1 h_2$ , and  $S$  may acquire a negative mass square at the electroweak scale if it has additional Yukawa couplings to exotic matter. However, the underlying symmetries generally forbid the appearance of the needed  $(\kappa/3)S^3$  term in the superpotential, so there is no quartic term in  $S$  in the potential. The situation is analogous to that of a  $D$ -flat direction discussed in Sec. III C. In particular, one expects  $S$  to acquire an intermediate scale VEV either due to higher order terms in the effective potential (when  $m_S^2$  goes through 0), or by nonrenormalizable terms in the superpotential. In either case, the VEV of  $S$ , and thus the scale of electroweak breaking, would be many orders of magnitude above  $m_{3/2}^2$ , which is clearly not satisfactory.

The situation changes in the presence of an additional  $U(1)'$  gauge symmetry,<sup>12</sup> provided the term  $h_1 h_2$  is not a gauge singlet.<sup>13</sup> Then an  $Sh_1 h_2$  term can generate an effective  $\mu$  that is naturally of order  $m_{3/2}$ , provided  $S$  develops a VEV by the radiative mechanism. The soft supersymmetry-breaking term  $ASH_1 h_2$  generates the needed  $h_1$ - $h_2$  mixing.

For example, suppose the superpotential contains

$$W \sim Sh_1 h_2 + \mathcal{H}_i Q_L Q_R h_1 + \mathcal{H}_E E_1 E_2 S, \quad (12)$$

where  $E_{1,2}$  represent additional exotic matter. Assuming that the corresponding soft supersymmetry-breaking mass-square terms  $m_S^2$ ,  $m_{h_1}^2$ , and  $m_{h_2}^2$  are positive at  $M_{\text{string}}$ , then for sufficiently large Yukawa couplings  $\mathcal{H}_{i,E}$  and a sufficiently large representation of  $E_i$  under the SM gauge group, both  $m_S^2$  and  $m_{h_1}^2$  are driven negative at low energies. Two examples, in which the  $E_i$  are respectively  $SU(2)$  doublets and  $SU(3)$  singlets, are described in the Appendix. As discussed above, without  $U(1)'$  symmetry the VEV of  $S$  will be at an unacceptably large intermediate scale. However, if  $S$  carries a nonzero charge under an extended gauge symmetry, then the  $U(1)'$   $D$  term will provide, in the absence of flat directions (scenarios discussed in Sec. III A), a quartic potential for  $S$ , ensuring that the VEV  $S$  will be of order  $m_{3/2}$ .

Thus, the existence of an additional gauge  $U(1)'$  provides a scenario which leads to an effective  $\mu$  term of  $O(m_{3/2})$ , thus providing a resolution to the  $\mu$  problem within string

models. This mechanism is complementary to the mechanism [36] based on nonrenormalizable terms [34,35]. The latter mechanisms can occur if there is no additional  $U(1)'$  symmetry. However, if such a  $U(1)'$  is present it will forbid the needed terms proportional to  $h_1 h_2$  in the Kähler potential or higher order terms in the superpotential.

Let us briefly comment on a few related topics. Even in the absence of a  $U(1)'$  gauge symmetry it is possible for SM singlet scalar fields to acquire negative mass-square values at low energies due to the radiative mechanism if they have sufficiently large Yukawa couplings to other fields. As was discussed above in connection with the  $\mu$  problem, such scalars generally will not have quartic terms in the potential, and thus they would acquire intermediate scale VEV's.

We have seen that under a certain set of assumptions the VEV's of standard model scalars will typically be either at the electroweak scale, if there is an additional  $U(1)'$  with no dangerous flat directions, or at an intermediate scale. Intermediate scales are of interest in implementing the seesaw model of neutrino mass. However, one may still need nonrenormalizable terms in the superpotential to implement realistic neutrino mass scenarios [38]. Also, one promising scenario for baryogenesis [39] is that a large lepton asymmetry is generated by the decays of the heavy Majorana neutrino associated with the seesaw, and then converted to a baryon asymmetry during the electroweak transition. Such scenarios, while very attractive, cannot occur if there is a  $U(1)'$  which is only broken at the electroweak to TeV scale [40], unless, of course, the heavy Majorana neutrino is a  $U(1)'$  singlet.

The radiative mechanisms discussed in this paper require the existence of sufficiently large Yukawa couplings to drive the mass-square values of the SM singlet  $S$  negative. This is most easily implemented if there exists exotic matter which transforms nontrivially under the SM gauge group. The exotic matter will then acquire mass terms given by the relevant Yukawa coupling times the VEV of  $S$ , as well as contributions from soft supersymmetry breaking. Such exotic matter typically exists in string models. However, if it carries SM quantum numbers it can destroy the success of the gauge coupling unification [8]. Such effects largely cancel if the light exotic matter corresponds to complete  $SU(5)$  multiplets, but that is not typically expected in the types of semirealistic models we are discussing. There are many ways in which cancellations between different multiplets or with other effects, such as higher Kač-Moody levels, can occur. However, preserving gauge unification without fine-tuning is a stringent constraint on string model building, with or without an additional  $U(1)'$ .

## V. CONCLUSIONS

We have explored the possible scenarios for nonanomalous  $U(1)'$  symmetry breaking, as is expected for a class of string compactifications with the standard model gauge group and additional  $U(1)$  gauge factors. This is the case for a large number of string vacua based on orbifold models [9] or based on the free fermionic constructions [10,6,11]. Under the assumption that the symmetry-breaking takes place in the observable sector and that the soft supersymmetry-breaking scalar mass-square terms are positive, the breaking is neces-

<sup>12</sup>For scenarios addressing the  $\mu$  problem within anomalous  $U(1)'$ , see Ref. [37].

<sup>13</sup>This is not the case for the models in [7], but is expected in, e.g., models for which the  $U(1)'$  is associated with the embedding of  $SO(10)$  into  $E_6$ .

sarily radiative and requires the existence of additional matter, most easily associated with standard model nonsinglets. Then, within a particular model with definite soft supersymmetry-breaking terms, the symmetry-breaking pattern, the couplings and the masses of the new gauge bosons, and those of the accompanying exotic particles are calculable. In that sense the string models yield predictions for the new physics associated with the new gauge bosons.

It turns out that for the class of string models considered the radiative  $U(1)'$  symmetry breaking is either of  $O(M_Z)$  or the intermediate scale of order  $\geq 10^{8-14}$  GeV. However, in both cases the mass of the associated physical Higgs bosons is in the electroweak region. Our major conclusion, therefore, is that a large class of string models considered here not only predict the existence of additional gauge bosons and exotic matter, but often imply that their masses should be in the electroweak range. Many such models are already excluded by indirect or direct constraints on heavy  $Z'$  bosons, and the  $Z$ - $Z'$  mixing is often too large, especially for lower values of  $M_{Z'}$ . The scenario in which  $M_{Z'}$  is in the electroweak range allows, without excessive fine-tuning of the soft supersymmetry-breaking parameters, for predictions of  $M_{Z'}$  within experimental reach of present or future colliders. On the other hand, when the experimental bounds on  $M_{Z'}$  exceed the 1 TeV region, this scenario *cannot* be implemented without excessive fine-tuning of soft supersymmetry-breaking parameters, or unusual choices of  $U(1)'$  charge assignments.

In addition,  $U(1)'$  symmetry, broken at the electroweak scale, provides a simple mechanism for generating an effective  $\mu$  term of the order of the electroweak scale.

The analysis has set the stage for detailed investigation of the potentially phenomenologically viable string models with additional  $U(1)'$  gauge symmetry.

#### ACKNOWLEDGMENTS

The work was supported in part by U.S. Department of Energy Grant No. DOE-EY-76-02-3071, the National Science Foundation Grant No. PHY94-07194, the Institute for Advanced Study funds and J. Seward Johnson foundation (M.C.), and the National Science Foundation Career Advancement Grant No. PHY95-12732 (M.C.). We would like to thank P. Binetruy, M. K. Gaillard, A. Faraggi, G. Kane, V. Kaplunovsky, J. Louis, J. Polchinski, and especially S. Chaudhuri for discussions. M.C. acknowledges hospitality of the Institute for Theoretical Physics, Santa Barbara, where the work was initiated.

#### APPENDIX

We provide analytic expressions for the renormalization group equations (RGE's) for the running of the soft supersymmetry-breaking mass-square terms in the presence of sufficiently large Yukawa couplings.

The analysis is based on rough approximations, which, however, illustrate the mechanism of radiative supersymmetry breaking for different types of Yukawa couplings of the  $U(1)'$  charged SM singlet  $S$  to the SM  $SU(2)$  doublets  $h_{1,2}$  and/or  $SU(3)$  triplets  $D_{1,2}$ . In the RGE's for the soft mass-square terms the following approximations are used:

we replace the Yukawa couplings with a constant value close to the infrared fixed point, and the three-linear soft supersymmetry-breaking terms as well as the gaugino masses are assumed to be smaller than the soft supersymmetry-breaking mass-square terms. The first approximation is in general a good one, since the Yukawa couplings, which are of order 1 at  $M_{\text{string}}$ , reach the infrared fixed point, which is also of order 1, quickly. On the other hand, neglecting the three-linear soft supersymmetry-breaking terms and gaugino masses may not be a good approximation in general. However, the qualitative picture is not drastically changed with the inclusion of such terms.<sup>13</sup>

For simplicity we shall also assume universal positive mass-square terms  $m_{3/2}^2$  at  $M_{\text{string}}$  and take the infrared fixed point value of the corresponding Yukawa coupling to be 1. For different infrared fixed point values  $\mathcal{H}_Y$  of the corresponding Yukawa couplings, the parameter  $t$  in the analytic expressions is replaced by  $\mathcal{H}_Y^2 t$ .

We discuss the results for the following types of Yukawa interactions.

Yukawa coupling  $h_1 h_2 S$ . In this case the RGE's assume the form

$$2 \frac{dm_{h_{1,2}}^2}{dt} = \frac{dm_S^2}{dt} = 4\Sigma, \quad (\text{A1})$$

where  $t = (1/16\pi^2) \ln(\mu/M_{\text{string}})$  and  $\Sigma = m_{h_1}^2 + m_{h_2}^2 + m_S^2$ . At scale  $\mu$  the solution is

$$m_{h_1, h_2}^2 = \frac{1}{4} m_{3/2}^2 + \frac{3}{4} m_{3/2}^2 e^{8t}, \quad m_S^2 = -\frac{1}{2} m_{3/2}^2 + \frac{3}{2} m_{3/2}^2 e^{8t}. \quad (\text{A2})$$

For  $\mu \sim 100$  GeV,  $e^{8t} \sim 0.17$ , and thus  $m_S^2$  is negative.  $m_S^2$  goes through zero at  $\mu_{\text{crit}} \sim 2 \times 10^8$  GeV.

Yukawa couplings  $D_1 D_2 S$ . The RGE's are

$$3 \frac{dm_{D_{1,2}}^2}{dt} = \frac{dm_S^2}{dt} = 6\Sigma, \quad (\text{A3})$$

where  $\Sigma = m_{D_1}^2 + m_{D_2}^2 + m_S^2$ . At scale  $\mu$

$$m_{D_1, D_2}^2 = \frac{2}{3} m_{3/2}^2 + \frac{3}{5} m_{3/2}^2 e^{10t}, \quad m_S^2 = -\frac{4}{5} m_{3/2}^2 + \frac{9}{5} m_{3/2}^2 e^{10t}. \quad (\text{A4})$$

For  $\mu \sim 100$  GeV,  $e^{10t} \sim 0.11$  and thus  $m_S^2$  approaches a negative value, a factor  $\sim 2$  more negative than the one in the previous case. In this case,  $\mu_{\text{crit}} \sim 10^{12}$  GeV.

Yukawa coupling  $Q_L Q_R h_1$ . This coupling is relevant to the breaking of the electroweak gauge symmetry by the Yukawa coupling of the SM Higgs doublet  $h_1$  to the left-handed quark doublet  $Q_L$  and the right-handed quark  $Q_R$ , which are of course to be identified with the third family. The RGE's are

<sup>13</sup>See Appendix B of Ref. [29] for a related discussion of analytic expressions within the MSSM with a heavy fourth family, when such terms are included.

$$3 \frac{dm_{Q_L}^2}{dt} = \frac{3}{2} \frac{dm_{Q_R}^2}{dt} = \frac{dm_{h_1}^2}{dt} = 6\Sigma_1, \quad (\text{A5})$$

where  $\Sigma_1 = m_{Q_L}^2 + m_{Q_R}^2 + m_{h_1}^2$ . One obtains

$$m_{h_1}^2 = -\frac{1}{2} m_{3/2}^2 + \frac{3}{2} m_{3/2}^2 e^{12t}. \quad (\text{A6})$$

Yukawa coupling  $Q_L Q_R h_1 + h_1 h_2 S$ . This coupling is relevant when the same SM Higgs doublet couples to both the SM singlet  $S$  as well as through another Yukawa coupling to the left-handed quark doublet  $Q_L$  and the right-handed quark  $Q_R$ . In this case the RGE's are

$$\begin{aligned} 2 \frac{dm_{Q_L}^2}{dt} &= \frac{dm_{Q_R}^2}{dt} = 4\Sigma_1, \\ 2 \frac{dm_{h_2}^2}{dt} &= \frac{dm_S^2}{dt} = 4\Sigma_2, \\ \frac{dm_{h_1}^2}{dt} &= 6\Sigma_1 + 2\Sigma_2, \end{aligned} \quad (\text{A7})$$

where  $\Sigma_1 = m_{Q_L}^2 + m_{Q_R}^2 + m_{h_1}^2$  and  $\Sigma_2 = m_{h_1}^2 + m_{h_2}^2 + m_S^2$ . The solution for  $m_{h_1}^2$  and  $m_S^2$  is

$$m_{h_1}^2 = -\frac{5}{7} m_{3/2}^2 + \frac{12}{7} m_{3/2}^2 e^{14t}, \quad m_S^2 = \frac{1}{7} m_{3/2}^2 + \frac{6}{7} m_{3/2}^2 e^{14t}, \quad (\text{A8})$$

indicating that at  $\mu \sim 100$  GeV the Higgs doublet  $h_1$  acquires negative mass-square terms, while the SM singlet  $S$  has a mass-square term that remains positive.

Yukawa coupling  $Q_L Q_R h_1 + h_1 h_2 S + E_1 E_2 S$ . This example is relevant to the generation of an effective  $\mu$  term.  $h_1$  has the normal Yukawa coupling to the third generation of quarks, the second term will play the role of a  $\mu$  term when  $S$  acquires a VEV, and the  $E_1 E_2 S$  coupling to exotic particles  $E_1$  and  $E_2$  can drive  $m_S^2$  negative at an intermediate scale. The resulting set of coupled RGE's can easily be solved numerically. We find that in the case that  $E_i$  represent a second pair of SU(2) doublets both  $m_S^2$  and  $m_{h_1}^2$  are indeed negative (and are equal within the approximations), and that at  $\mu \sim 100$  GeV

$$m_S^2 = m_{h_1}^2 = -0.52 m_{3/2}^2, \quad m_{h_2}^2 = +0.77 m_{3/2}^2, \quad (\text{A9})$$

thus allowing the generation of an effective  $\mu$  term. Similarly, when the  $E_i$  are SU(3) triplets,

$$m_S^2 = -0.74 m_{3/2}^2, \quad m_{h_1}^2 = -0.48 m_{3/2}^2, \quad m_{h_2}^2 = +0.84 m_{3/2}^2 \quad (\text{A10})$$

at  $\mu \sim 100$  GeV.

- 
- [1] V. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988); **B383**, 436(E) (1992).
- [2] For recent discussions, see P. Langacker and N. Polonsky, Phys. Rev. D **52**, 3081 (1995); P. Langacker, in Proceedings of SUSY-95 (unpublished), Report No. hep-ph/9511207 (unpublished), and references therein.
- [3] For a review, see the article by I. Hinchliffe in [4]; M. Shifman, Mod. Phys. Lett. A **10**, 605 (1995); J. Erler and P. Langacker, Phys. Rev. D **52**, 441 (1995).
- [4] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994), and 1995 off-year partial update for the 1996 edition available on the PDG WWW pages (URL: <http://pdg.lbl.gov/>).
- [5] A. Font, L. E. Ibáñez, F. Quevedo, and A. Sierra, Nucl. Phys. **B331**, 421 (1991).
- [6] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos, Phys. Lett. B **231**, 65 (1989).
- [7] A. Faraggi, Nucl. Phys. **B403**, 102 (1993); Phys. Lett. B **274**, 47 (1992).
- [8] See K. Dienes and A. Faraggi, Nucl. Phys. **B457**, 409 (1995), and references therein; P. Binetruy and P. Langacker (in preparation).
- [9] L. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B261**, 678 (1985); **B274**, 285 (1986).
- [10] I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. **B289**, 87 (1987); H. Kawai, D. Lewellen, and S.H.-H. Tye, Phys. Rev. Lett. **57**, 1832 (1986); Phys. Rev. D **34**, 3794 (1986).
- [11] S. Chaudhuri, S.-W. Chung, G. Hockney, and J. Lykken, Nucl. Phys. **B456**, 89 (1995); S. Chaudhuri, G. Hockney, and J. Lykken, Report No. Fermilab-PUB-95-349-T, hep-ph/9510241 (unpublished).
- [12] A. Brignole, L. Ibáñez, and C. Muñoz, Nucl. Phys. **B422**, 125 (1994); **B436**, 747(E) (1995); A. Brignole, L. Ibáñez, C. Muñoz, and C. Scheich, Report No. FTUAM-95-26, hep-ph/9508258 (unpublished).
- [13] For a recent review, see Langacker [14], p. 883.
- [14] *Precision Tests of the Standard Electroweak Model*, edited by P. Langacker (World Scientific, Singapore, 1995).
- [15] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **51**, R949 (1995); Phys. Rev. Lett. **74**, 2900 (1995); S. Park, Report No. CDF/PUB/EXOTIC/PUBLIC/3199 (unpublished).
- [16] For a review see, e.g., M. Cvetič and S. Godfrey, in Proceedings of *Electro-weak Symmetry Breaking and beyond the Standard model*, edited by T. Barklow, S. Dawson, H. Haber, and J. Siegrist (World Scientific, Singapore, 1995); Report No. hep-ph/9504216 (unpublished) and references therein.
- [17] For indirect constraints, see P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988); D. London [14], p. 951. For a recent discussion, including direct collider constraints, see V. Barger, M. S. Berger, and R. J. N. Phillips, *ibid.* **52**, 1663 (1995).
- [18] One exception is E. Nardi, E. Roulet, and D. Tommasini, Nucl. Phys. **B386**, 239 (1992); Phys. Rev. D **46**, 3040 (1992).
- [19] A. N. Schellekens, Phys. Lett. B **237**, 363 (1990).
- [20] G. Athanasiu, J. Atick, M. Dine, and W. Fishler, Phys. Lett. B **214**, 55 (1988).
- [21] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. **B289**, 585 (1986).
- [22] (a) J. Atick, L. Dixon, and A. Sen, Nucl. Phys. **B292**, 109 (1987); L. Dixon and V. Kaplunovsky (unpublished); (c) L. Dixon, E. Martinez, D. Friedan, and S. Shenker, Nucl. Phys. **B282**, 13 (1987).

- [23] L. Dixon and M. Cvetič (unpublished); L. Dixon and M. Cvetič, *Superstrings, Cosmology, and Composite Structures*, Proceedings of the Conference, College Park, Maryland, 1987, edited by S. J. Gates and R. Mohapatra (World Scientific, Singapore, 1987); M. Cvetič, Phys. Rev. Lett. **59**, 2829 (1987).
- [24] J. Lopez and D. Nanopoulos, Nucl. Phys. **B338**, 73 (1990).
- [25] A. Faraggi and E. Halyo, Nucl. Phys. **B416**, 63 (1994).
- [26] L. E. Ibáñez, H. P. Nilles, and F. Quevedo, Phys. Lett. B **192**, 332 (1987).
- [27] A. Faraggi, Nucl. Phys. **B428**, 111 (1994).
- [28] The possibility of a very small ( $\sim 1$  GeV)  $\mu$  is discussed by J. L. Feng, N. Polonsky, and S. Thomas, Phys. Lett. B **370**, 95 (1996).
- [29] M. Cvetič and C. Preitschopf, Nucl. Phys. **B272**, 490 (1986).
- [30] For a recent discussion, including extensive references to earlier works, see V. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D **49**, 4908 (1994), and references therein.
- [31] M. Dine, V. Kaplunovsky, M. Mangano, and C. Nappi, Nucl. Phys. **B259**, 549 (1985).
- [32] J. E. Kim and H. P. Nilles, Phys. Lett. **138B**, 150 (1984).
- [33] S. F. King and P. L. White, Phys. Rev. D **52**, 4183 (1995), and references therein.
- [34] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).
- [35] J. A. Casas and C. Muñoz, Phys. Lett. B **306**, 288 (1993).
- [36] V. Kaplunovsky and J. Louis, in *From Superstrings to Supergravity*, Proceedings of the Erice Conference, 1992, edited by M. Duff *et al.* (World Scientific, Singapore, 1994), p. 234; I. Antoniadis, K. S. Narain, and T. Taylor, Nucl. Phys. **B432**, 187 (1994); G. Lopes-Cardoso, D. Lüst, and T. Mohaupt, *ibid.* **B432**, 68 (1994).
- [37] V. Jain and R. Shrock, Phys. Lett. B **352**, 83 (1995); Report No. ITP-SB-95-22, hep-ph/9507238 (unpublished); Y. Nir, Phys. Lett. B **354**, 107 (1995).
- [38] M. Cvetič and P. Langacker, Phys. Rev. D **46**, 2759 (1992).
- [39] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986); Phys. Rev. D **42**, 1285 (1990); P. Langacker, R. D. Peccei, and T. Yanagida, Mod. Phys. Lett. A **1**, 541 (1986).
- [40] W. Buchmüller and T. Yanagida, Phys. Lett. B **302**, 240 (1993).