Heavy fermion screening effects and gauge invariance

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We show that the naively expected large virtual heavy fermion effects in low energy processes may be screened if the process under consideration contains external gauge bosons constrained by gauge invariance. We illustrate this by a typical example of the process $\gamma\gamma\rightarrow b\bar{b}$. Phenomenological implications are also briefly indicated. [S0556-2821(96)03417-0]

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Studying the effects of heavy particles in radiative corrections is of special importance for exploring new physics if the accelerator energy is not sufficient to directly produce them. With respect to these effects, there are two kinds of theories. In renormalizable theories with coupling constants independent of the heavy particle masses such as quantum electrodynamics (QED), these effects are not significant since the decoupling theorem [1] shows that the heavy particles completely decouple from the low energy physics in the heavy mass limit. In nondecoupling theories to which the decoupling theorem does not apply, these effects can be significant and are thus important for studying new physics. A typical example of the nondecoupling theory is the standard model (SM) of the electroweak theory, in which heavy particles may affect low energy physics in two separate ways. First, the heavy top quark is a necessary ingredient in chiral anomaly cancellation, and integrating it out will induce an effective Wess-Zumino-Witten term at low energies [2], which is constant in the heavy top limit. Second, particles in the SM acquire mass from the fixed vacuum expectation value, so that the heavy masses are proportional to the corresponding coupling constants, and thus the conditions for the validity of the decoupling theorem are not satisfied. This kind of nondecoupling can make certain observables depend on positive powers of the heavy particle masses which will blow up in the heavy mass limit. A well-known example is the one-loop heavy top correction to the parameter ρ reflecting the W, Z boson mass relation, which behaves as $G_F m_t^2$ [3] and originates from the custodial SU(2) symmetry [4] breaking by the large mass splitting between the top and bottom quarks. In the Higgs sector, however, a similar correction from a heavy Higgs boson is absent due to Veltman's screening theorem [5]. The naively expected leading terms of $O(G_F m_H^2)$ at one loop [5] and $O(G_F^2 m_H^4)$ at two loop [6] are canceled in the W, Z mass relation, and the survivals are the next-to-leading terms of $O(G_F M_W^2 \ln m_H^2)$ and $O(G_F^2 M_W^2 m_H^2)$, respectively. This phenomenon has been attributed in [7] to the vestige of the global custodial symmetry, and generalized to all orders in perturbation theory.

In this paper, we shall point out that screening effects may also appear in the heavy fermion sector if the low energy process under consideration contains external gauge bosons which are constrained by gauge invariance. The effects of heavy top quark on low energy processes involving one pair of light fermions have been examined by Lin *et al.* [8] with a nonlinear realization of custodial SU(2) symmetry. Here, instead of listing all effective operators to a given order, we shall make a simple and direct argument on the screening effects of heavy fermions so that heavy fermion effects may be less important than naively expected. Our discussion is based on a simple analysis of gauge invariance and dimension counting. Although we take the process $\gamma\gamma\rightarrow b\bar{b}$ as an example to illustrate the screening effect of the heavy top, which is of interest by itself in photon collider physics [9], the whole analysis applies to the general cases involving heavy fermions. We shall also briefly discuss the processes $H\rightarrow\gamma\gamma$, $b\rightarrow s\gamma$ and indicate the phenomenological implications.

At tree level, $\gamma\gamma\rightarrow b\bar{b}$ is a pure QED process. In the following, we first focus on its one-loop correction arising from a virtual heavy top and then generalize it to higher loops. As a theoretical study, we are only interested in the leading-m_t term corresponding to the heavy top limit. Whether this is a good approximation is an issue of phenomenology which is not the main purpose of this paper. In this limit we may set the bottom mass to zero, $m_b=0$. We work in the R_ξ gauge. The leading term is contributed by the exchange of the unphysical Goldstone boson $\phi^{\pm}$ (and at higher loops by the exchange of the unphysical Goldstone boson ϕ^0 and physical Higgs boson H as well). The nonleading terms which are of the same order as those from the ordinary electroweak corrections are ignored here. Note that the nonleading terms are $\xi_{W(Z)}$ dependent, and this dependence is canceled only when corrections from W, Z bosons are included. With this consideration, the relevant interaction Lagrangian at one-loop level is

$$\mathcal{L}_1=\frac{g_2 m_t}{\sqrt{2}M_W}(\phi^+\overline{t_R}b_L+\phi^-\overline{b_L}t_R)+eA_\mu(Q_t\overline{t}\gamma^\mu t+Q_b\overline{b}\gamma^\mu b)$$
$$+ieA_\mu(\phi^-\partial^\mu\phi^+-\phi^+\partial^\mu\phi^-)+e^2A_\mu A^\mu\phi^+\phi^-, \qquad (1)$$

where Q_t and Q_b are the electric charges of the top and bottom quarks, respectively. Beyond one loop, the terms

$$\mathcal{L}_2=-\frac{g_2 m_t}{2M_W}H\overline{t}t+i\frac{g_2 m_t}{2M_W}\phi^0\overline{t}\gamma_5 t \qquad (2)$$

should also be added. Here we have ignored the small quark mixing. Note that only the left-handed component of the b-quark couples to the top quark, so that taking $m_b=0$ is safe and will not produce collinear or mass singularity because the collinear configuration is forbidden by the conservation of angular momentum.

Now we analyze the Lorentz structure of the one-loop amplitude for the process $\gamma(k_1,\epsilon_\mu^{(1)})\gamma(k_2,\epsilon_\nu^{(2)})\to b(p_1)\overline{b}(p_2)$ from the following physical requirements: (1) on-shell conditions, $k_i^2=p_i^2=0$, $\not{p}_2 v=0=\overline{u}\not{p}_1$; (2) terms proportional to $k_{1\mu}$ or $k_{2\nu}$ being automatically canceled and thus dropped from the beginning; (3) left-handedness of the b. It is then straightforward to write down the complete set of independent structures for the amplitude:

$$i\mathcal{A}_{\mu\nu}^{\text{one loop}}=\frac{ie^2}{(4\pi)^2}\frac{G_F m_t^2}{2\sqrt{2}}\overline{u_L}[Q_t^2A_{\mu\nu}^{(t)}+Q_tQ_bA_{\mu\nu}^{(tb)}+Q_b^2A_{\mu\nu}^{(b)}]v_L,$$
$$A_{\mu\nu}^{(i)}=(\not{k}_1-\not{k}_2)[g_{\mu\nu}h_1^{(i)}+p_{1\mu}p_{1\nu}h_2^{(i)}+k_{2\mu}k_{1\nu}h_3^{(i)}+k_{1\nu}p_{1\mu}h_4^{(i)}+k_{2\mu}p_{1\nu}h_5^{(i)}]+\gamma_\mu(k_{1\nu}h_6^{(i)}+p_{1\nu}h_7^{(i)})+\gamma_\nu(k_{2\mu}h_8^{(i)}+p_{1\mu}h_9^{(i)})+i\epsilon_{\rho\alpha\mu\nu}\gamma^\rho\gamma_5(k_1-k_2)^\alpha h_{10}^{(i)}, \qquad (3)$$

where the form factors $h_a^{(i)}$ are functions of the Mandelstam variables s,t,u and are related to each other by crossing symmetry. From the naive dimension counting and the fact that the leading terms are independent of $\xi_W M_W^2$ and that $\mathcal{A}^{\text{one loop}}$ should be finite as the energy $\sqrt{s}\to 0$, it is tempting to conclude that $h_1^{(i)}$ and $h_{6-10}^{(i)}$ would behave as m_t^{-2} in the heavy top limit, and would thus contribute a leading term of $O(G_F m_t^0)$ in $\mathcal{A}^{\text{one loop}}$. However, *this naively expected behavior actually does not appear due to an additional constraint from* $\mathrm{U}(1)_{em}$ *gauge invariance*. To put it simply, gauge invariance dictates the lowest dimension that a gauge invariant structure should carry so that the above analysis breaks down.[1] The use of gauge invariant structures or bases was considered by Bardeen and Tung in the 1960s [11]. In the present case, the amplitude can be expanded in a complete set of gauge-invariant structures:

$$i\mathcal{A}_{\mu\nu}^{\text{one loop}}=\frac{ie^2}{(4\pi)^2}\frac{G_F m_t^2}{2\sqrt{2}}\Sigma_{a=1}^5[Q_t^2f_a^{(t)}+Q_tQ_bf_a^{(tb)}+Q_b^2f_a^{(b)}]\overline{u_L}O_{\mu\nu}^a v_L,$$
$$O_{\mu\nu}^1=(\not{k}_1-\not{k}_2)(k_{2\mu}k_{1\nu}-g_{\mu\nu}k_1\cdot k_2),$$
$$O_{\mu\nu}^2=(\not{k}_1-\not{k}_2)(p_{1\mu}p_{1\nu}k_1\cdot k_2-k_{2\mu}p_{1\nu}k_1\cdot p_1-k_{1\nu}p_{1\mu}k_2\cdot p_1+g_{\mu\nu}k_1\cdot p_1k_2\cdot p_1),$$
$$O_{\mu\nu}^3=2\gamma_\nu(-p_{1\mu}k_1\cdot k_2+k_{2\mu}k_1\cdot p_1)+(\not{k}_1-\not{k}_2)(g_{\mu\nu}k_1\cdot p_1-p_{1\mu}k_{1\nu}), \qquad (4)$$
$$O_{\mu\nu}^4=2\gamma_\mu(-p_{1\nu}k_1\cdot k_2+k_{1\nu}k_2\cdot p_1)-(\not{k}_1-\not{k}_2)(g_{\mu\nu}k_2\cdot p_1-p_{1\nu}k_{2\mu}),$$
$$O_{\mu\nu}^5=i\epsilon_{\rho\alpha\mu\nu}\gamma^\rho\gamma_5(k_1-k_2)^\alpha k_1\cdot k_2+(\not{k}_1-\not{k}_2)(p_{1\nu}k_{2\mu}-p_{1\mu}k_{1\nu})+k_{2\mu}\gamma_\nu(2k_2\cdot p_1-k_1\cdot k_2)+k_{1\nu}\gamma_\mu(2k_1\cdot p_1-k_1\cdot k_2).$$

Note that $O_{\mu\nu}^{3,4,5}$ are gauge invariant only in the on-shell sense. $O_{\mu\nu}^{1,2}$ are crossing odd, $O_{\mu\nu}^5$ is crossing even, and $O_{\mu\nu}^{3,4}$ are crossing exchanged, so are their form factors $f_a^{(i)}$. One may use alternative sets of structures, but a nice feature of the above one is that each structure is uniquely characterized by its first term. Again, by dimension counting and the finiteness of $\mathcal{A}^{\text{one loop}}$ as $\sqrt{s}\to 0$, we deduce that, in the heavy top limit, $f_2\sim m_t^{-6}$, $f_{a\neq 2}\sim m_t^{-4}$ up to logarithms of the form $[1+\text{const}\times\ln(\xi_W M_W^2/m_t^2)]$ which take into account the infrared singularity of box diagrams in the Landau gauge $\xi_W=0$. Indeed, *there are no leading terms, and* $\mathcal{A}^{\text{one loop}}$ *is then dominated by the next-to-leading terms of* $O(G_F m_t^{-2}[1+\text{const}\times\ln(\xi_W M_W^2/m_t^2)])$.

At first sight it seems that the top quark decouples from $\gamma\gamma\to b\overline{b}$ in its large mass limit. This is certainly not the case. The heavy top effects are only screened with leading terms cancelled in observables. To see this we go to higher loops. The above analysis in terms of form factors applies to the L-loop case after only a slight modification of the factor $G_F m_t^2/16\pi^2$ in Eqs. (3) and (4), i.e., $G_F m_t^2/16\pi^2\to(G_F m_t^2/16\pi^2)^L$. So for the L-loop correction,

$$\mathcal{A}^{L\text{ loop}}=O(G_F^L m_t^{2(L-2)}) \quad \text{up to logarithms.} \qquad (5)$$

This is totally different from the decoupling of heavy fermions in QED but is quite similar to the *screening* phenomenon in the Higgs sector.

Two comments are in order.

(1) As pointed out above, the next-to-leading term is generally $\xi_{W,Z}$ dependent. This gives us a lesson that whenever the naively expected leading term is absent in some observables, we should be careful in simplifying the computation by ignoring the internal weak-gauge-boson contributions. Especially, when there are infrared singularities associated with unphysical Goldstone bosons in the Landau gauge, we must

[1] The importance of U(1) gauge invariance for calculating amplitudes involving unstable bosons was emphasized recently by Argyres *et al.* [10].

include the contributions from internal W,Z bosons to obtain a physical result even just to keep the first nonvanishing term in the heavy top limit.

(2) Consider the phenomenology at the photon colliders. Since the contributions from a virtual top quark are generally suppressed (or screened) in $\gamma\gamma$ processes not containing external tops, heavy top effects induced from physics beyond the SM should also be small. We have computed the one-loop radiative corrections to $\gamma\gamma \to b\bar{b}$ from the exchange of charged Higgs $H^{\pm}$ in the two Higgs doublet model. As an illustrating example we present here the form factor $f_1^{(t)}$ as

$$f_1^{(t)} = \cot^2\beta \int_0^1 dx \int_0^1 dy \int_0^1 dz (F_1 + F_2 + F_3),$$

$$F_1 = 2x^2(1-x)(1-y)z^4(-1+z-xyz)[\Lambda_1^{-2}(t) - \Lambda_1^{-2}(u)],$$

$$F_2 = 2x^2(1-x)(1-y)z^4(-1+z-xyz)[\Lambda_2^{-2}(u) - \Lambda_2^{-2}(t)], \tag{6}$$

$$F_3 = 2xyz^2(1-z)^2(1-y-xz+yz)[\Lambda_3^{-2}(t) - \Lambda_3^{-2}(u)],$$

where, for $\xi = t,u$,

$$\Lambda_1(\xi) = m_t^2 z + M_{H^{\pm}}^2(1-z) - sx(1-x)yz^2 - \xi x(1-y)z(1-z),$$

$$\Lambda_2(\xi) = m_t^2(1-z) + M_{H^{\pm}}^2 z - sx(1-x)yz^2 - \xi x(1-y)z(1-z), \tag{7}$$

$$\Lambda_3(\xi) = m_t^2 z + M_{H^{\pm}}^2(1-z) + s(1-x)yz(1-z) - \xi(x-y)z(1-z).$$

For $m_b = 4.5$ GeV (for tree contribution only), $m_t = 176$ GeV, $M_{H^{\pm}} = 400$ GeV, $\cot\beta = 5$, $\sqrt{s} = 100-400$ GeV, and using the spectrum function of back-scattered laser light [12], we find that the relative shift in the total cross section is less than 10^{-4}. The contribution from unphysical Goldstone boson $\phi^{\pm}$ is essentially the same as above, but with $\cot^2\beta$ removed and $M_{H\pm}^2$ replaced by the mass squared of $\phi^{\pm}$. It is clear that $f_1^{(t)} = O(m_t^{-4})$ up to logarithms in the heavy top limit, so that it will not contribute to the leading terms of $O(m_t^0)$ in $\mathcal{A}_{\mu\nu}^{\text{one loop}}$, as stated previously.

The above analysis applies to other processes as well. For example, since m_t is the largest scale in the decays $b \to s\gamma$ [13] and $H \to \gamma\gamma$ (or $gg \to H$ [14]) and the one-loop momentum integrals are seemingly linearly divergent, one would naively expect that the decay amplitudes behave as m_t^2. Actually this leading behavior is screened by the appearance of photons in final states. Because of the $U(1)_{\text{em}}$ gauge invariance, the effective Lagrangians are, respectively,

$$\mathcal{L}_{\text{eff}}^1 = Ae \frac{m_b}{v} \frac{m_t}{v} \overline{s_L} \sigma_{\mu\nu} b_R F^{\mu\nu},$$

$$\mathcal{L}_{\text{eff}}^2 = Be^2 \frac{m_t}{v} H F^{\mu\nu} F_{\mu\nu}, \tag{8}$$

where a factor of m_b has to appear in $\mathcal{L}_{\text{eff}}^1$ to flip the helicity since we have set $m_s = 0$. $A = a/m_t$, $B = b/m_t$, and a,b are finite pure numbers in the heavy top limit. Thus this only leads to a next-to-leading behavior which is constant in m_t. The m_t^2 dependence first appears at two loops [15], as argued above.

To summarize, we emphasize the importance of local gauge invariance in causing the screening of the heavy fermion effects in our discussion. In spontaneously broken gauge theories such as the SM, although the heavy top quark does not decouple as in QED, its effects may be *screened* in low energy processes involving photons. Intuitively, for processes containing external photons (or gluons), local gauge invariance makes the photons (gluons) carry higher powers of momenta than naively expected, so that the powers of the heavy fermion mass (as the heaviest mass scale) will be lowered as compared with the naive expectation. This kind of screening is different from Veltman's in the sense that the latter is due to the algebraic symmetry structure in the Higgs sector of the SM [7].

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