## Dipole moments of the $\tau$ lepton as a sensitive probe for physics beyond the standard model

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(Received 20 September 1995)

*CP*-violating dipole moments of leptons vanish at least to three loop order and are estimated to be  $(m_l/\text{MeV}) \times 1.6 \times 10^{-40} \ e$  cm in the standard model (SM), where  $m_l$  is the mass of the lepton. However, they can receive potentially large contributions in some beyond the SM scenarios and this makes them very sensitive probes of new physics. In this article we show that a nonuniversal interaction, involving leptoquarks to the quark-lepton pair of the third generation through helicity-unsuppressed couplings of the order of ordinary gauge couplings, can generate electric and weak dipole moments of the order of  $10^{-19} \ e$  cm for the  $\tau$  lepton. This is greater than pure supersymmetric (SUSY) and left-right (*LR*) contributions by almost three orders of magnitude. It is also greater than the mirror fermionic contribution by an order of magnitude. The measurements of  $d_{\tau}^z$  and  $d_{\tau}^{\gamma}$  at CERN LEP, SLC, and TCF are expected to reach sensitivities of  $10^{-18} \ e$  cm and  $10^{-19} \ e$  cm, respectively, in the near future. The observation of a nonvanishing dipole moment of  $\tau$  at these facilities would, therefore, strongly favor superstring-inspired light leptoquark-mediated interactions, over pure SUSY or *LR* interactions and perhaps also mirror-generated mixings without some sort of quark-lepton unification as its origin. [S0556-2821(96)05113-2]

PACS number(s): 14.60.Fg, 11.30.Er, 13.10.+q, 13.40.Em

#### I. INTRODUCTION

*CP* violation has so far been observed only in the decay of neutral kaons [1]. In the standard model (SM) CP violation arises from the complex Yukawa couplings which generate a nonvanishing phase in the quark mixing [Cabibbo-Kobayashi-Maskawa (CKM)] matrix [2]. On the other hand, in leptonic reactions, CP violation only comes through higher order corrections involving quark mixing. For the process considered here, the production of  $\tau^+ \tau^-$  in  $e^+ e^-$  collisions, the SM prediction for *CP*-violating effects is so small that it will not be measurable in any experiment currently proposed. To give a numerical estimate, CP-violating electric dipole moments (EDM's) of leptons vanish at least to three-loop order in the SM and are estimated to be of the order of  $1.6(m_1/\text{MeV}) \times 10^{-40} \text{ e cm}$ , where  $m_1$  is the mass of the lepton [3]. Observation of CPviolation in leptonic systems at current or near future experimental facilities would, therefore, signal beyond the SM interactions.

From a theoretical standpoint the description of CP violation in the framework of the SM does not offer any explanation of its origin. Many extensions of the SM have been proposed which try to offer deeper insight into the mechanism of *CP* violation [4]. Some of them predict *CP*-violating effects in interactions where there is no significant contribution from the SM. In addition, the predicted magnitude of these effects differs from one extension of the SM to the other, so that their experimental search could not only detect some beyond SM physics, but also shed some light on its nature. The most sensitive and classic tests in this field are the searches for EDM's of the neutron, electron, muon, and  $\tau$ . No nonvanishing EDM has been found so far, and upper limits have been set at  $d_n^{\gamma} < 1.1 \times 10^{-25} \ e \ \text{cm}$ ,  $d_{\mu}^{\gamma} < 1.9 \times 10^{-26} \ e \ \text{cm}$ ,  $d_{\mu}^{\gamma} < 1.1 \times 10^{-18} \ e \ \text{cm}$ , and  $d_{\tau}^{\gamma} < 5 \times 10^{-17} \ e \ cm$  [5]. A new search in this field is the search for a weak dipole (WDM) of  $\tau$  ( $d_{\tau}^{z}$ ) at the z peak. This quantity is best constrained from the CERN  $e^+e^-$  collider LEP data to have its real part  $(\text{Red}_{\tau}^z) \leq 6.7 \times 10^{-18}$ e cm and an imaginary part  $(\text{Im}d_{\tau}^{z}) \leq 4.5 \times 10^{-17}$  e cm at 95% C.L. [6]. These upper limits were obtained by measuring the expectation values of certain optimal variables which constitute the dominant CP-violating part of the matrix element for  $\tau^+ \tau^-$  production. The method of optimal variables is different from the idea originally proposed by Bernreuther et al., which consisted of measuring CP-odd tensor correlations [7] among the charged final state particles in the reactions  $e^+e^- \rightarrow \tau^- + \tau^- \rightarrow X^+ \overline{\nu}_{\tau} X^- \nu_{\tau}$ . One such tensor correlation was looked for at LEP and 95% C.L. limits of  $(\text{Red}_{\tau}^{z} < 7.0 \times 10^{-17})$ e cm from OPAL and Re  $d_{\pi}^{z} < 3.7 \times 10^{-17}$  e cm from ALEPH were obtained from a sample of 650 000 z's [8]. Recently, it has been shown [9] that certain CP-odd vector correlations in the reaction  $e^+e^- \rightarrow \tau^+ \tau^- \rightarrow X^+ \overline{\nu}_{\tau} X^- \nu_{\tau}$  are enhanced significantly when the  $e^-$  and  $e^+$  beams are longitudinally polarized. This makes them sensitive to the real and imaginary parts of the WDFF at the SLAC linear collider (SLC). In the presence of a substantial polarization of  $e^+$  and  $e^-$  beams, the same correlations also become sensitive to the real and imaginary parts of the EDM when the  $\tau^+ \tau^-$  production is no longer dominated by Z exchange, but instead by photon exchange as in a  $\tau$ -charm factory (TCF). For a polarization  $P_e = \pm 0.75$  and  $10^6$  Z's at SLC, these vector correlations could probe  $\operatorname{Re} d_{\tau}^{z}$  and  $\operatorname{Im} d_{\tau}^{z}$  with sensitivities of  $3 \times 10^{-17}$ and  $1.2 \times 10^{-16}$  e cm, which are comparable with the limits obtained from tensor correlations and with unpolarized beams. On the other hand, at TCF with 42% average polarization of each beam and a total yield of  $2 \times 10^7 \tau^+ \tau^-$  pairs it would be possible to attain sensitivities of  $10^{-19} e$  cm for  $\operatorname{Red}_{\tau}^{\gamma}$  and  $5 \times 10^{-16}$  for  $e \operatorname{cm}$  for  $\operatorname{Im} d_{\tau}^{\gamma}$  with vector correlations. This is an order of magnitude better than the sensitivity achievable with unpolarized beams.

Theoretically, it would be interesting to consider some extension of the SM that predicts real parts of the weak di-

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pole form factor (WDFF) and EDM at a level that is close to the precision range achievable at present or in near future experimental facilities. Since  $\tau$  belongs to the third or most massive generation among the fermion families, one possibility to generate measurable dipole moments of  $\tau$  would be to make use of the nonuniversal interaction that gives rise to large  $m_t$  in loop-induced corrections. In this article we shall therefore consider dipole moments of the  $\tau$  lepton due to leptoquarks (LQ's) that couple  $\tau$  to t through helicityunsuppressed couplings. Being flavor diagonal, the couplings are not subject to flavor-changing suppression either and can be as large as electromagnetic (em) coupling [10]. We find that such light  $(m_{LQ} \approx 100 \text{ GeV})$  scalar leptoquarks that couple to both  $\tau_L$  and  $\tau_R$  with couplings of magnitude  $|g_L| \approx |g_R| \approx e$  can give rise to  $d_{\tau}^z$  and  $d_{\tau}^{\gamma}$  of the order of  $10^{-19}$  e cm for  $m_t \approx 175$  GeV. We also find that the pure supersymmetric (SUSY) or LR contributions to  $d_{\tau}^{\gamma}$  in models without some sort of quark lepton unification are of the order of  $10^{-22} e$  cm. The models considered in this article are all invariant under the combined CPT transformation, and therefore  $\text{Im}(d_{\tau}^{\gamma})$  and  $\text{Im}(d_{\tau}^{z})$  turn out to be zero. Since the predicted values for  $d_{\tau}^{\gamma}$  and  $d_{\tau}^{z}$  due to LQ's lie close to the precision range for measuring these dipole moments at LEP, SLC, and TCF, their observation in the near future would favor such leptoquark scenario over SUSY or LR scenarios without some kind of quark-lepton unification as their origin. This in turn would imply some superstring-inspired grand unified model such as E(6), which can contain such light LO's without violating baryon number and lepton number conservation. On the other hand, a negative result would not favor any particular extension of the SM. Nevertheless, the hierarchy of the predicted values of the dipole moments in different scenarios and the proximity of some of them to the current precision range warrant a vigorous and continued search for  $\tau$  dipole moments to unravel the nature of beyond SM physics.

The contents of this article are divided into the following sections. In Sec. II we present the effective Lagrangian, describing the couplings of scalar and vector LQ's to quarklepton pairs, which will be used in this article to calculate  $d_{\tau}^{\gamma}$  and  $d_{\tau}^{z}$ . Here we also present the expressions for *CP*-conserving magnetic moments  $(\delta \mu_{\tau}^{\gamma}, \delta \mu_{\tau}^{z})$ and *CP*-violating dipole moments  $(d_{\tau}^{\gamma}, d_{\tau}^{z})$  due to an  $S_{1}$  type of LQ that incidentally gives rise to the most dominant contributions. In Sec. III we present the estimates of the magnitudes and relative phase of left-handed (LH) and righthanded (RH) leptoquark couplings that will be used in this article to calculate the dipole and magnetic moments. We also show the consistency of these estimates with several pieces of experimental data. In Sec. IV we estimate the dipole moments and magnetic moments of  $\tau$  due to LQ's. Here we also present the estimates of pure SUSY and LR contributions to  $d_{\tau}^{\gamma}$  and compare the contributions of different scenarios. In Sec. V we show that for the parameter values assumed in this article the estimates of  $d_{\tau}^{\gamma}$  and  $\delta \mu_{\tau}^{\gamma}$  are consistent with the current experimental limits on  $B(\tau \rightarrow \mu \gamma)$ and  $\delta a_{\tau}$  (anomalous magnetic moment of tau). Finally, in Sec. VI we present the conclusions of our study.

# II. LEPTOQUARK-INDUCED DIPOLE MOMENTS OF THE $\tau$

The effective Lagrangian with the most general dimensionless  $SU(3)_c \times SU(2)_l \times U(1)_y$  invariant couplings of scalar and vector leptoquarks that can give rise to dipole moments of charged leptons can be written as [10]

$$L_{\text{eff}} = (g_{1L} \overline{q}_{R}^{c} \iota \tau_{2} l_{L} + g_{1R} \overline{t}_{L}^{c} e_{R}) S_{1} + (g_{2L} \overline{d}_{L}^{c} \gamma^{\mu} l_{L} + g_{2R} \overline{q}_{R}^{c} \gamma_{\mu} e_{R}) V_{2\mu}^{+} + (h_{2L} \overline{u}_{R} l_{L} + h_{2R} \overline{q}_{L} \iota \tau_{2} e_{R}) R_{2}^{+} + (h_{1L} \overline{q}_{L} \gamma^{\mu} l_{L} + h_{1R} \overline{d}_{R} \gamma^{\mu} e_{R}) U_{1\mu} + \text{H.c.}.$$
(1)

Here  $q_L$ ,  $l_L$  are LH quark and lepton doublets, and  $e_R$ ,  $d_R$ ,  $u_R$  are RH charged leptons, down and up quarks, respectively.  $\psi^c$  is a charge-conjugated fermion field. The indices of the LQ's give the dimension of their SU(2) representation. Color, weak isospin, and generation indices have been suppressed. The subscripts *L* and *R* of the coupling constants stand for lepton chirality.

We shall assume that in the underlying extension of the SM there is some symmetry that prevents the LQ's from giving rise to baryon- and lepton-number-violating decays. Such a situation indeed occurs in a four-dimensional E(6)grand unified model derived from a ten-dimensional E(8) $\times E(8)$  heterotic superstring theory [11]. There, a discrete symmetry arising from the topological properties of the compact manifold causes the diquark couplings to vanish. The couplings and masses of such LQ's have to satisfy much weaker bounds. In fact, in the low energy superstring models we obtain relatively small masses for the  $S_1$  leptoquark  $(\approx 50-1000 \text{ GeV})$  [12]. At the CERN large electron-positron collider (LEP), the experiments have established a lower bound  $m_{LQ} \ge 45-73$  GeV for scalar leptoquarks [13]. On the other hand, the search for scalar leptoquark decaying into an electron-jet pair in  $p\overline{p}$  colliders has constrained their masses to be  $m_{LO} \ge 112$  GeV [14]. Finally, the experiments at the DESY ep collider HERA constrain their masses to be  $m_{\rm LO} \ge 92-184$  GeV depending on the leptoquark type and couplings. In this article we shall take the LQ couplings and masses to be bounded by low energy processes and by the recent LEP data, since we do not have a detailed knowledge of the compact manifold where we realized the compactification. Low energy experiments imply that if there is one or more LQ's for each quark-lepton generation, it is possible to have flavor-diagonal couplings as large as ordinary gauge couplings for LQ masses of order 100 GeV [10]. In addition, the strong helicity suppression on the product  $g_{1L}g_{1R}$  or  $h_{2L}h_{2R}$  from the flavor-conserving decay  $\pi^+ \rightarrow e^+ \nu_e$  (which implies chiral couplings for LQ's of the first generation) does not apply for the  $\tau$  which belongs to the third generation. Note that LQ's can give rise to *CP*-violating dipole moments only if they couple to charged leptons of both chiralities. For scalar LQ's, the dipole moments of  $\tau$  get a large contribution from the chirality-flipping top quark mass in the loop diagram. However, for vector LQ's, dipole moments of  $\tau$  get contribution from the bottom quark mass in the loop integral and are therefore much smaller. In addition, it is difficult to incorporate vector LQ's in a low energy effective theory below 1 TeV. On the other hand, the  $SU(2)_w$  singlet, charge-1/3 scalar leptoquark  $S_1$  occurs in the superstring-inspired E(6) grand unified model. Further, because of reasons mentioned in Sec. II, they can be relatively light  $(m_{S_1} \approx 100 \text{ GeV})$  without giving rise to proton decay. In this article we shall therefore consider dipole moments of  $\tau$  due to  $S_1$  only. The SU(2)<sub>w</sub> doublet, scalar leptoquark  $R_2$  gives rise to similar contributions to  $d_{\tau}^{\gamma}$  and  $d_{\tau}^{z}$ , but we shall not consider it here. We find that at one-loop order the exchange of  $S_1$  leads to the following effective Lagrangian describing the interaction of  $\gamma$  with the magnetic and dipole moments of  $\tau$ :

$$L_{\rm eff} = (\iota e/3) N_c m_l I(p,q) \overline{\tau} \sigma_{\mu\nu} [{\rm Re}(g_{1L}^* g_{1R}) + \iota \gamma_5 {\rm Im}(g_{1L}^* g_{1R})] \tau F^{\mu\nu}, \qquad (2)$$

where q and p are the four-momenta of the photon and the incoming  $\tau$ ,  $N_c$  is the number of colors, and

$$I(p,q) = \int \left[ d^4 l / (2\pi)^4 \right] \{ 1 / (l^2 - m_t^2) [(l+q)^2 - m_t^2] \\ \times [(l-p)^2 - m_{S_1}^2] \}.$$
(3)

Similarly, the effective Lagrangian describing the coupling of *Z* to the weak magnetic and dipole moments of  $\tau$  turns out to be

$$L_{\rm eff} = (\iota e/2c_w s_w) N_c m_t [(\frac{1}{2} - \frac{2}{3} s_w^2) I(p,q) + \frac{1}{2} A(p,q)] \overline{\tau} \sigma_{\mu\nu} [\operatorname{Re}(g_{1L}^* g_{1R}) + \iota \gamma_5 \operatorname{Im}(g_{1L}^* g_{1R})] \tau Z^{\mu\nu}, \qquad (4)$$

where  $c_w = \cos \theta_w s_w = \sin \theta_w$ , and

$$A(p,q)q^{\nu} + B(p,q)p^{\nu} = \int [d^{4}l/(2\pi)^{4}] \{ l^{\nu}/(l^{2} - m_{t}^{2}) \\ \times [(l+q)^{2} - m_{t}^{2}] [(l-p)^{2} - m_{s_{1}}^{2}] \}.$$
(5)

I(p,q), A(p,q), and B(p,q) are scalar functions of  $p^2$ ,  $q^2$ , and  $p \cdot q$ .

#### III. ESTIMATES OF $|g_{1L}^*g_{1R}|$ AND THE *CP*-VIOLATING PHASE $\delta$

Since the LQ's considered in this article do not lead to baryon- or lepton-number-violating decays, in general their couplings of either helicity (but not both) can be of the order of the em coupling if their masses are of the order of 100 GeV. The restriction to couplings of either helicity, but not both, arises from the helicity suppressed decay  $\pi^+ \rightarrow e^+ \nu_e$ and applies only to LQ couplings to a quark-lepton pair of the first generation [10]. In fact, one finds that  $|g_{1L}g_{1R}|^{1/2} < m_{S_1}/10$  TeV for LQ couplings to fermions of the first generation. However, this helicity constraint does not apply for the  $\tau$  since it is quite massive and both  $|g_{1L}|$ and  $|g_{1R}|$  can be simultaneously of the order of e. The recent LEP data on  $z \rightarrow \tau^+ \tau^-$  decay can be used to impose constraints on the masses and couplings for the third generation LO's, which couple to the top quark. It would be interesting to examine the implications of those constraints on the dipole moments of the  $\tau$  lepton. Mizukoshi *et al.* [16] evaluated the one-loop contribution due to LQ's to all LEP observables and made a global fit to extract 95% confidence level limits on the LQ masses and couplings. The limits obtained by them are most stringent for LQ's that couple to the top quark since their contributions are enhanced by powers of the top quark mass. Moreover, the limits are slightly better for LH couplings than for RH couplings for a given LQ. From the allowed region in the  $m_{LQ}-g_{1L}(g_{1R})$  plane for an  $S_1$  LQ, we find that for  $m_{LQ}\approx 100$  GeV the LEP limits are  $|g_{1L}| < 0.5$  and  $|g_{1R} < 0.5$ . The values of the LQ mass  $(m_{LQ}\approx 100 \text{ GeV})$  and couplings  $(|g_{1L}| \approx |g_{1R}| \approx e \approx 0.3)$  assumed by us in this article are therefore close to and consistent with the limits implied by the LEP data.

The *CP*-violating phase  $\delta$  (where we define  $g_{1L}^*g_{1R} = |g_{1L}^*g_{1R}|e^{\iota\delta}$ ) can be estimated, or rather an upper limit on it can be derived, from the experimental limit on  $d_e^{\gamma}$ . In order to do that we shall assume that the phase  $\delta$  for the third generation is of the same order as that of the first generation. Any hierarchy in the dipole moments of leptons of different generations will arise from the chirality-flipping mass in the loop diagram and from the constraint on flavor-changing LQ couplings. Under these assumptions we find that (using naive dimensional analysis)

$$d_e^{\gamma} \approx -\frac{2}{3} e N_c |g_{1L}^* g_{1R}| \sin \delta \frac{\xi}{16\pi^2} \frac{m_u}{m_{S_1}^2}.$$
 (6)

where  $\xi$  is a number of order 1 which arises in evaluating the loop integral. Note that for the couplings of  $S_1$  to the quarklepton pair of first generation  $|g_{1L}^*g_{1R}| \leq m_{S_1}/10$  TeV. From the experimental limit  $d_e^{\gamma} \leq 2 \times 10^{-26} e$  cm [5], it then follows that  $\sin \delta \leq 5/6.3 \xi \approx 1$ . This implies that  $\cos \delta \approx 1$  and, therefore, the LQ contribution to  $\delta \mu_{\tau}^{\gamma}$  turns out to be of the same order as  $d_{\tau}^{\gamma}$ . The fact that  $\delta \mu_{\tau}^{\gamma}$  is of the same order as  $d_{\tau}^{\gamma}$  makes the former very small and consistent with the experimental limit on  $|a_{\tau}^{\text{expt}} - a_{\tau}^{\text{SM}}|$ , where  $a_{\tau}$  is the anomalous magnetic moment of  $\tau$ .

# IV. ESTIMATES OF DIPOLE MOMENTS AND MAGNETIC MOMENTS OF $\tau$ IN DIFFERENT SCENARIOS

In order to evaluate  $d_{\tau}^{\gamma}$  and  $\delta \mu_{\tau}^{\gamma}$ , we have to find the value of I(p,q) for  $p^2 = m_{\tau}^2$  and  $-2p \cdot q = q^2 = 0$  corresponding to on-shell  $\tau$  and  $\gamma$ . On the other hand, to find  $d_{\tau}^z$  and  $\delta \mu_{\tau}^z$  we have to find the values of I(p,q) and A(p,q) for  $p^2 = m_{\tau}^2$  and  $-2p \cdot q = q^2 = M_z^2$  corresponding to on-shell  $\tau$  and Z. From the expressions for the effective Lagrangians gives in Sec. III, we find that

$$\delta\mu_{\tau}^{\gamma} = -(2\iota e/3)N_c m_t I(p,q) \operatorname{Re}(g_{1L}^*g_{1R}) \approx 1.1 \times 10^{-19} e \text{ cm},$$
(7a)

$$d_{\tau}^{\gamma} = -(2\iota e/3)N_c m_t I(p,q) \operatorname{Im}(g_{1L}^* g_{1R}) \approx 1.1 \times 10^{-19} e \operatorname{cm}$$
(7b)

and

$$\delta\mu_{\tau}^{z} = -(\iota e/c_{w}s_{w})N_{c}m_{t}[(\frac{1}{2}-\frac{2}{3}s_{w}^{2})I(p,q)$$

$$+ \frac{1}{2} A(p,q) ] \operatorname{Re}(g_{1L}^* g_{1R}) \approx 2 \times 10^{-19} e \text{ cm}, \quad (8a)$$
$$d_{\tau}^{z} = -(\iota e/c_{w} s_{w}) N_{c} m_{t} [(\frac{1}{2} - \frac{2}{3} s_{w}^{2}) I(p,q)$$
$$+ \frac{1}{2} A(p,q) ] \operatorname{Im}(g_{1L}^* g_{1R}) \approx 2 \times 10^{-19} e \text{ cm}, \quad (8b)$$

for  $m_t \approx 175$  GeV,  $m_{S_1} \approx 100$  GeV,  $|g_{1L}^*g_{1R}| \approx e^2 \approx 0.1$ , and  $\cos \delta \approx \sin \delta \approx 1$ . The relevant loop integrals appearing in the above expressions have been evaluated numerically. An order of magnitude estimate of the magnetic moments and dipole moments can also be obtained by naive dimensional analysis.

We will now consider the electric dipole moments of  $\tau$  in SUSY and *LR*-symmetric models which do not incorporate some sort of quark-lepton unification and hence do not have LQ's. In SUSY models there can be inherently supersymmetric contributions that are large. For example, at the one-loop level a nonvanishing dipole moment can arise from a  $\tau$  going into a scalar  $\tau$  and a neutralino ( $\tilde{\gamma}, \tilde{Z}, \tilde{H}$ ). For the photino-mediated diagram we get [17,18]

$$\frac{d_{\tau}^{\gamma}}{e} = \frac{\alpha}{\pi} \frac{m_{\tau} \text{Im}(Am_{\widetilde{\gamma}})}{m^3} f(x)$$

where  $m_{\tilde{\gamma}}$  is the mass of the photino, *m* is the scale for low energy SUSY breaking,  $x = (m_{\tilde{\gamma}}/m)$ , *A* is a complex parameter of order unity, and f(x) is a Polchinski-Wise function [13]. The *CP*-violating phase  $\arg(Am_{\tilde{\gamma}})$  arises from the effective soft SUSY-breaking terms and can be bounded from  $d_e^{\gamma}$  [18]. Using the experimental bound  $d_e^{\gamma} \le 2 \times 10^{-26}$ *e* cm, we find  $\arg(Am_{\tilde{\gamma}}) < 10^{-2}$ . Hence  $d_{\tau}^{\gamma} \le 7.2 \times 10^{-23}$ *e* cm. Inherently, the supersymmetric contribution to  $d_{\tau}^{\gamma}$  is therefore less than the LQ contribution by almost three orders of magnitude.

For *LR*-symmetric models a sizable  $d_{\tau}^{\gamma}$  can arise if the  $\tau$  couples to a RH heavy neutrino. Chang, Pal, and Nieves [19] find that, for each  $W_i$  and  $\chi_A$  running in the loop ( $W_i$  and  $\chi_A$  are mass eigenstates for charged gauge bosons and neutrinos, respectively),

$$d_{\tau}^{iA} = -\frac{eg^2 m_A}{64\pi^2 M_i^2} U_{Li} U_{Ri} \text{Im}(P_{3A}Q_{3A}) \left(\frac{r_{Ai}^2 - 11r_{Ai} + 4}{(r_{Ai} - 1)^2} + \frac{6r_{Ai}^2 \ln r_{Ai}}{(r_{Ai} - 1)^3}\right),$$
(9)

where  $m_A = \text{mass}$  of  $\chi_A$ ,  $M_i = \text{mass}$  of  $W_i$ , and  $r_{Ai} = m_A^2/M_i^2$ . U and P,Q are unitary matrices that relate gauge eigenstates to the mass eigenstates of the charged bosons and neutrinos, respectively. For  $M_2 \ge M_1$  we get

$$d_{\tau}^{\gamma} \approx (1.0 \times 10^{-24} e \text{ cm}) \sin 2\zeta \sum_{A} \left(\frac{m_{A}}{1 \text{ MeV}}\right) \text{Im}(P_{3A}Q_{3A}) D_{A},$$
(10)

where  $D_A$  is the expression in large parentheses in Eq. (9) and  $\zeta$  is the  $W_L$ - $W_R$  mixing angle. From current algebra analysis of purely nonleptonic strange decays, one obtains  $\zeta < 0.004$ . Hence

$$d_{\tau}^{\gamma} \leq 10^{-26} \frac{\mathrm{Im}(\mu_D)_{\tau\tau}}{1 \mathrm{MeV}},$$

where  $\mu_D$  is the Dirac mass in the neutrino mass matrix. The upper bound on  $\text{Im}(\mu_D)_{\tau\tau}$  can be estimated from the LH neutrino mass  $m_{\nu_{\tau}} \approx (\mu_D)_{\tau\tau}^2 / \mu_N$ , where  $\mu_N$  is the mass of the RH neutrino. For  $\mu_N \approx 1$  TeV and  $m_{\nu_{\tau}}^{\text{expt}} \leq 35$  MeV, we get  $\text{Im}(\mu_D)_{\tau\tau} \leq 6$  GeV. Thus  $d_{\tau}^{\gamma} \leq 2.4 \times 10^{-22}$  e cm. Inherently, the *LR* contribution to  $d_{\tau}^{\gamma}$  is therefore also less than the LQ contribution by three orders of magnitude.

The EDM's of ordinary fermions can also receive large contributions if the theory contains mirror fermions. The EDM's in this case arise from mixings between ordinary and exotic mirror fermions. Very tight bounds have been placed on these mixings by Langacker and London [20] from a combined analysis of various experiments and by Bhattacharayya *et al.* [21] using the LEP data. Using these constraints on mixings and maximizing the *CP*-violating phase, one can derive limits on mirror fermionic contributions to various dipole moments. In particular, using the limits on the mixing angles from the LEP data on  $z \rightarrow \tau^+ \tau^-$  decay, Joshipura [22] has shown that a  $d_{\tau}^{\gamma}$  of the order of  $2.1 \times 10^{-20} \ e \ cm$  can be generated. Note first that the  $d_{\tau}^{\gamma}$ generated by mirror fermions is still an order of magnitude smaller than the leptoquark contribution derived by us in this article. Second, although mirror fermions can occur in a low energy effective theory which does not incorporate any quark-lepton unification, their existence becomes natural in the context of grand unified models based on large enough orthogonal groups.

### V. LEPTOQUARK CONTRIBUTION TO $B(\tau \rightarrow \mu \gamma)$ AND $\delta a_{\tau}$

The rare decay  $\tau \rightarrow \mu \gamma$  takes place through the transition magnetic and electric dipole moments between  $\tau$  and  $\mu$ . Leptoquark contributions to  $B(\tau \rightarrow \mu \gamma)$  and  $d_{\tau}^{\gamma}$  can therefore be related. It can be shown that

$$d_{\tau}^{\gamma} = [B(\tau \to \mu \gamma)]^{1/2} (Gm_{\tau}/2\sqrt{6\pi}e) \delta', \qquad (11)$$

where, for the  $S_1$  leptoquark with the top quark contribution dominating.

$$\delta' \approx \sqrt{2 \operatorname{Im}[(g_{1L}^*)_{33}(g_{1R})_{33}]/[|(g_{1L}^*)_{32}(g_{1R})_{33}|^2} + |(g_{1L}^*)_{33}(g_{1R})_{32}|^2]^{1/2}.$$
(12)

Here the subscripts *i*, *j* of the couplings denote the generations to which the quark and lepton belong. For  $|(g_{1L}^*)_{33}|\approx|(g_{1R})_{33}|\approx\frac{1}{3}$  and  $|(g_{1L}^*)_{32}|\approx|(g_{1R})_{32}|\approx10^{-3}$ (which is the present bound on flavor-changing LQ couplings) [10], we find that  $\delta' \approx (\sqrt{2}/3) 10^3 \sin \delta$ , where  $\delta$  is the *CP*-violating phase. From the experimental upper limit [5]  $B(\tau \rightarrow \mu \gamma) \leq 4 \times 10^{-6}$ , it then follows that  $d_{\tau}^{\gamma} \leq 7.2 \times 10^{-19}$ *e* cm. Hence the upper limit on  $d_{\tau}^{\gamma}$  derived from  $B(\tau \rightarrow \mu \gamma)$  and some plausible assumptions regarding the magnitude of flavor-changing couplings is consistent with the limit obtained from  $d_{e}^{\gamma}$ . Alternatively, for the values of the parameters assumed in this article, our LQ model predicts a value for  $B(\tau \rightarrow \mu \gamma)$  that is close to the present experimental bound.

In our LQ model the contributions to  $\tau$  lepton's anomalous magnetic moments  $\delta \mu_{\tau}^{\gamma}$  and  $\delta \mu_{\tau}^{z}$  turn out to be of the same order as the *CP*-violating dipole moments  $d_{\tau}^{\gamma}$  and  $d_{\tau}^{z}$ . This makes  $\delta \mu_{\tau}^{\gamma}$  and  $\delta \mu_{\tau}^{z}$  quite small, since the scale of CP-violating dipole moments is naturally small. Nevertheless, it would be interesting to see if the estimate of  $\delta \mu_{\tau}^{\gamma}$  is with the experimental consistent bound on  $\delta a_{\tau} = |a_{\tau}^{\text{expt}} - a_{\tau}^{\text{SM}}|$ . The contribution of  $S_1$  leptoquark to  $\delta a_{\tau}$  is given by  $\delta a_{\tau} \approx 2.4 N_c \operatorname{Re}(g_{1L}^* g_{1R}) m_{\tau} m_t \times 10^{-7} \approx 1.9 \times 10^{-5}$  if we take  $\cos \delta \approx 1$ . The SM model contribution [5] is given by  $a_{\tau}^{\text{SM}} \approx 0.001$  177 3, whereas the experimental limit [5] is  $a_{\tau}^{\text{expt}} \approx 0.01$  at 95% C.L. The LQ contribution to  $\delta a_{\tau}$  is therefore well within the experimental bound of  $\delta a_{\tau} \leq 0.01.$ 

#### VI. CONCLUSIONS

In this article we have considered a particular species of light scalar LQ which occur in superstring-inspired E(6) grand unified model and whose contributions to the electric and weak dipole moments of  $\tau$  lepton are of the order of  $10^{-19} \ e$  cm. This should be compared with the precision range  $\delta \operatorname{Re}(d_{\tau}^{\gamma}) \leq 10^{-19} \ e$  cm and  $\delta \operatorname{Re}(d_{\tau}^z) \leq 10^{-18} \ e$  cm that can be achieved at current or proposed experimental facilities. The closeness of the predicted values to the precision range makes both this scenario and the experimental search of dipole moments of  $\tau$  interesting and worth pursuing. We have estimated the dipole moments for LQ couplings whose magnitudes are of the order of the em coupling and relative phase of O(1). For smaller values of these parameters, the

estimated dipole moments would be smaller. We have also shown that the inherently SUSY and LR contributions to the EDM of  $\tau$  are of the order of  $10^{-22} e$  cm, which is too small to be observed in any of the proposed experiments. The mirror fermionic contribution to  $d_{\tau}^{\gamma}$ , which is of order  $10^{-20}$ e cm, is also smaller than the projected precision range of the future experimental facilities by an order of magnitude. The observation of a nonvanishing dipole moment of  $\tau$  in the near future would therefore favor light LQ-mediated interaction over pure SUSY or LR interactions and perhaps also mirror-generated mixings (without some sort of quark lepton unification) as its origin. This in turn would imply some superstring-inspired grand unified model such as E(6). Dipole moments of  $\tau$  could therefore be used as a sensitive probe for unraveling the nature of physics beyond the SM. Thus a strong search program for dipole moments of  $\tau$  at LEP, SLC, and TCF is strongly warranted. Finally, we have shown that our estimates of  $d_{\tau}^{\gamma}$  and  $\delta \mu_{\tau}^{\gamma}$  are consistent with the experimental constraints on  $B(\tau \rightarrow \mu \gamma)$  and  $\delta a_{\tau}$ . An interesting feature of our estimates is that the CP-conserving magnetic moments turn out to be of the same order as the *CP*-violating dipole moments if we assume  $\sin \delta \approx \cos \delta \approx 1$ . Further, since the LQ interactions considered in this article are invariant under the *CPT* transformation, the imaginary parts of the dipole moments necessarily turn out to be zero. A *CPT*-odd observable  $\mathcal{O}$  can have a nonzero expectation value only in the presence of an absorptive part of the amplitude. Since the final state interactions, which could give rise to an absorptive part, are negligible in weak  $\tau$  decays, O must be proportional to  $\text{Im}d_{\tau}^{\gamma}$  or  $\text{Im}d_{\tau}^{z}$ . Measurement of some CPT-odd quantity can therefore be used to search for imaginary parts of dipole moments of  $\tau$  and verify the prediction of our LQ model.

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