Hyperon weak radiative decays in chiral perturbation theory

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We investigate the leading-order amplitudes for weak radiative decays of hyperons in chiral perturbation theory. We consistently include contributions from the next-to-leading order weak-interaction Lagrangian. It is shown that due to these terms Hara's theorem is violated. The data for the decay rates of the charged hyperons can be accounted for. However, at this order in the chiral expansion, the four decay rates of the neutral hyperons satisfy relations which are in disagreement with the data. The asymmetry parameters for all the decays cannot be accounted for without higher-order terms. We briefly comment on the effect of the 27-plet part of the weak interaction. [S0556-2821(96)01317-3]

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I. INTRODUCTION

Weak radiative decays of hyperons, $B_i \rightarrow B_f + \gamma$, have received attention for a long time, both experimentally [1–6] and theoretically [7–20]. There have been several theoretical approaches to this problem. One of the two major approaches uses hadronic degrees of freedom [9–12] while the alternative is solely based on the quark picture of hyperons [13–15]. While the data are known to be consistent with the lower bounds on the amplitudes derived from unitarity constraints [9,11], none of the theoretical models has managed to give a satisfactory account on details of the data, in particular the rates for the four neutral decay modes.

One of the issues that has been emphasized in the literature is the apparent violation of Hara's theorem [7,8], which states that the parity-violating amplitudes for $\Sigma^+ \rightarrow p + \gamma$ and $\Xi^- \rightarrow \Sigma^- + \gamma$ vanish in the limit of SU(3) symmetry. It predicts, in contradiction with experiments, that the asymmetry parameter for $\Sigma^+ \rightarrow p + \gamma$ should be quite small. (See Ref. [21] for a review of the relevant arguments.)

The hadronic models did not have a great deal of success in explaining the details of the data. All the models of this type (except those which include vector mesons) preserve Hara's theorem in their formulations. A general analysis which include SU(3) breaking [13] actually predicts a small, positive asymmetry for the Σ^+ decay while the data show that it is negative and relatively large. Models that assume vector-meson dominance [16] can introduce effects that violate Hara's theorem due to mixing of the vector meson with the photon. In models using quarks, it was pointed out [14] that the diagrams in which a W boson is exchanged between two constituent quarks can give rise to violation of Hara's theorem. In addition, models which include vector-meson dominance are in better agreement with the data, though the situation is still not satisfactory. The experimentally observed negative asymmetry parameter for Σ^+ decay is best accounted for using QCD sum rules [17]. Other approaches can be found in Refs. [18–20]. A detailed overview on both experimental and theoretical aspects of weak radiative decays of hyperons is given in Refs. [4,21,22].

Chiral perturbation theory (ChPT) [23,24] has been shown to be a useful way of describing low-energy hadronic processes, especially those that involve only mesons. It is an effective field theory in terms of hadronic degrees of freedom based on the symmetry properties of QCD. For application to processes involving baryons it is most consistently formulated in the heavy-baryon formulation [25], in which the SU(3)-invariant baryon mass $\dot{m}$ is removed by a field transformation (see also Ref. [26], where a similar transformation is carried out). In this approach an amplitude for a given process is expanded in external pion four-momenta q, baryon *residual* four-momenta k, and the quark mass m_s. We will neglect the up- and down-quark masses. We will collectively write down q, k, and m_s as E. (As we will discuss later, we will adopt the convention that k and m_s are of the same order in the chiral expansion.) The perturbation theory is reliable only when E is smaller than the chiral symmetry-breaking scale Λ_χ. In the heavy-baryon formulation there is an additional expansion in $1/\dot{m}$. However, all these terms can be effectively absorbed in counterterms of the theory [27].

Weak radiative decays of hyperons have been studied before in the context of ChPT by Jenkins, Luke, Manohar and Savage [20] and Neufeld [19]. Jenkins *et al.* and Neufeld calculated the amplitude up to the one-loop level. These loop diagrams give contributions to the amplitudes which are at least of order $O(E^2)$ in the chiral expansion. However, tree-level direct emission diagrams from the next-to-leading order weak Lagrangian [27], which give contribution of order $O(E)$ to the amplitudes, were not considered. The reasons such terms might be neglected consistently is the fact that they are not needed for the renormalization. However, in general, since ChPT should be based on a most general Lagrangian, [24] they also should be included. We will see that as a consequence of not taking these terms into account, the analysis for weak radiative decay of both Jenkins *et al.* and Neufeld should satisfy Hara's theorem.

In this paper we consistently calculate the leading-order amplitude of weak radiative decays of hyperons in ChPT. At this order, no loop contributions need to be considered. However, one does need to take into account the higher-order terms in the weak chiral Lagrangian. We will show that this

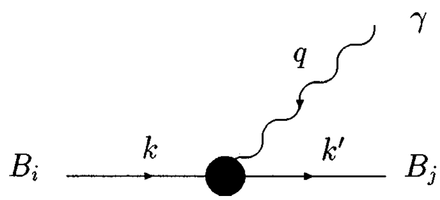

FIG. 1. Kinematics for weak radiative hyperon decays. B_i and B_f denote the initial and final hyperon, respectively. The baryon momenta k and k' are *residual* momenta, defined in the Appendix.

gives rise to violation of Hara's theorem. As a consequence the decay rates for the charged decays $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$ can be accounted for consistently. We will show that, in leading order, ChPT predicts the ratios of the decay amplitudes of all the neutral channels as functions of the baryon masses only. We will compare these predictions with the data. Furthermore, the asymmetry parameters still vanish in this leading-order calculation. We shall explain why this is not necessarily inconsistent with the data in the expansion scheme of ChPT. We will also discuss the contribution of the 27-plet component of the weak Lagrangian to the amplitude.

This paper is organized as follows. We will start in the next section by discussing the general formalism of hyperon weak radiative decay. In Sec. III we will calculate the amplitude in leading-order ChPT, including contributions from the weak- and strong-interaction Lagrangian with higher-order terms. In Sec. IV we will discuss briefly the contribution from the 27-plet. Next, in Sec. V, we compare our results with the data and with previous calculations in ChPT. Finally, Sec. VI contains a summary and our conclusions.

II. GENERAL FORMALISM

In this section we will consider some general features of weak radiative hyperon decay and summarize the data. To deal with baryons we will work in the heavy-baryon formalism [25] briefly outlined in the appendix.

As shown in the appendix, in the heavy-baryon formalism and in the gauge

$$v \cdot \epsilon = 0\,,$$

the general amplitude for the hyperon weak radiative decay process

$$B_i(m_i) \to B_f(m_f) + \gamma \tag{1}$$

is given by

$$\epsilon_\mu(q)\mathcal{M}^\mu = 2\epsilon_\mu(q)\overline{U_v}(k')(q_\nu[S_v^\mu,S_v^\nu]A + \Delta m S_v^\mu B)U_v(k), \tag{2}$$

where U_v and $\overline{U_v}$ are the heavy-baryon spinors of the initial and final baryons, respectively, and ϵ_μ is the photon polarization vector. The momenta are defined in Fig. 1. The ''form factors'' A and B in Eq. (2) correspond to the parity-conserving and parity-violating part of the amplitude, respectively. The factor $\Delta m \equiv m_i - m_f$ multiplying B appears by convention: it is introduced in order to reproduce the parity-violating form factor in the conventional relativistic formalism [see Eq. (A6) of the appendix]. This factor plays a crucial role in the discussion of Hara's theorem.

Hara's theorem concerns the parity-violating amplitude in the limit of U-spin symmetry. (U-spin transformations interchange an s and d quark.) Assuming U-spin symmetry, Hara's theorem can be easily obtained from Eq. (2). If we have U-spin symmetry, the mass difference between p and Σ vanishes:

$$m_{\Sigma^+} - m_p = 0\,. \tag{3}$$

If we also assume that the parity-violating form factor B has *no* pole in $m_{\Sigma^+} - m_p$, we find from Eq. (2) that the parity-violating amplitude for $\Sigma^+ \to p+\gamma$ vanishes. However, as we will see in the following, the assumption that B is nonsingular may not be correct in the framework of ChPT. Such a possibility is consistent with Low's theorem [28] and was already pointed out in quite a general context in Refs. [29,30] on which we will comment later.

There are two possible independent observables in this process. Using Eq. (2) and the photon-polarization sum in the gauge $v \cdot \epsilon = 0$,

$$\sum_\lambda \epsilon_\lambda^\mu(k)\epsilon_\lambda^\nu(k) = -g^{\mu\nu} - \frac{k^\mu k^\nu}{(v\cdot k)^2} + \frac{k^\mu v^\nu + k^\nu v^\mu}{v\cdot k}, \tag{4}$$

we find for the decay rate the usual expression

$$\Gamma = \frac{\omega^3}{\pi}(|A|^2 + |B|^2), \tag{5}$$

where ω is the photon energy in the lab frame given by

$$\omega = \frac{m_i^2 - m_f^2}{2m_i}. \tag{6}$$

As required, the decay rates are regular in the chiral limit even if the form factors are singular, since the aforementioned potential singular behavior of A and B is compensated by the phase-space factor ω. The second observable, related to the angular dependence, is the asymmetry parameter given by

$$\alpha = \frac{2\,\mathrm{Re}(AB^*)}{|A|^2 + |B|^2}. \tag{7}$$

The present data on the decay rates and asymmetry parameters is summarized in Table I.

TABLE I. Present status of decay rates and asymmetry parameters. The numbers are the combined weighted means from Ref. [21]. Both the decay rate and the asymmetry parameter for $\Sigma^0 \to \Lambda + \gamma$ have not been measured.

$B_i \to B_f + \gamma$	Γ [10^{-18} GeV]	α	Ref.
$\Lambda \to n + \gamma$	4.07 ± 0.35	–	[1]
$\Xi^0 \to \Lambda + \gamma$	2.4 ± 0.36	0.43 ± 0.44	[2]
$\Xi^0 \to \Sigma^0 + \gamma$	8.1 ± 1.0	0.20 ± 0.32	[3]
$\Sigma^+ \to p + \gamma$	10.1 ± 0.5	-0.76 ± 0.08	[4,5]
$\Xi^- \to \Sigma^- + \gamma$	0.51 ± 0.092	1.0 ± 1.3	[6]

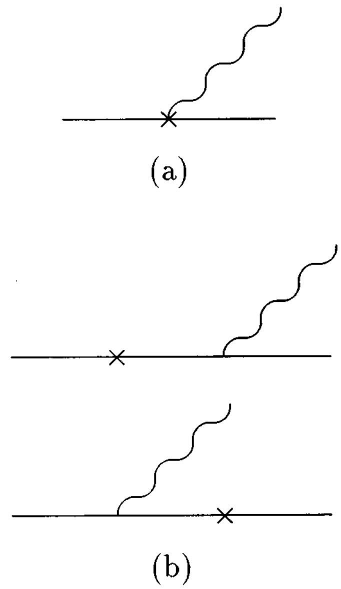

FIG. 2. Feynman diagrams for weak radiative hyperon decay in leading-order chiral perturbation theory. The cross-sign denotes the weak interaction.

III. LEADING-ORDER AMPLITUDE

We will now turn to the calculation of the hyperon weak radiative decays in leading-order ChPT in the heavy-baryon formalism. The necessary weak ChPT Lagrangian, up to terms of order E, has been given in Ref. [27]. We shall consider only the CP-even part of the Lagrangian. The diagrams contributing to the leading-order amplitude are tree diagrams given in Fig. 2. There are two kinds of diagrams: the direct emission diagrams Fig. 2(a), and the baryon pole diagrams Fig. 2(b). Loop diagrams can be omitted in our calculation, since they give rise to contributions of higher order.

Since the full Lagrangian, including the Lagrangian in the weak-interaction sector, has been given elsewhere, we give here only the terms directly relevant to the hyperon weak radiative decay in leading order. The baryons are represented by the usual SU(3) matrix

$$H=\begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda+\frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda-\frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \quad (8)$$

and, since pions do not enter this tree-level description, we can take for the other fields the expansions

$$D^\mu=\partial^\mu-ieQA^\mu, \quad \Delta^\mu=0, \quad \sigma=\chi,$$
$$\rho=0, \quad \lambda=\lambda_6, \quad \lambda'=\lambda_7, \quad (9)$$

where Q is the quark charge matrix

$$Q=\tfrac{1}{3}\mathrm{diag}(2,-1,-1), \quad (10)$$

$\lambda_{6,7}$ are the Gell-Mann matrices (giving rise to $|\Delta S|=1$ transitions),

$$\lambda_6=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad (11)$$

and χ the SU(3)-breaking mass matrix

$$m_s\mathrm{diag}(0,0,1). \quad (12)$$

In leading order, the decuplet does not play a role and we can restrict ourselves to the terms

$$\mathcal{L}_s^{(1,0)}=i\mathrm{Tr}[\bar{H}[v\cdot D,H]], \quad (13)$$

$$\mathcal{L}_s^{(0,1)}=A_1\mathrm{Tr}[\bar{H}\{\sigma,H\}]+A_2\mathrm{Tr}[\bar{H}[\sigma,H]]+A_3\mathrm{Tr}[\bar{H}H]\times\mathrm{Tr}[\sigma], \quad (14)$$

$$\mathcal{L}_s^{(2,0)}=B_1\mathrm{Tr}[\bar{H}[D^\mu,[D_\mu,H]]]+B_3\mathrm{Tr}[\bar{H}[S_v^\mu,S_v^\nu]\times\{[D_\mu,D_\nu],H\}]+B_4\mathrm{Tr}[\bar{H}[S_v^\mu,S_v^\nu][[D_\mu,D_\nu],H]], \quad (15)$$

$$\mathcal{L}_w^{(0,0)}=h_D\mathrm{Tr}[\bar{H}\{\lambda,H\}]+h_F\mathrm{Tr}[\bar{H}[\lambda,H]], \quad (16)$$

$$\begin{aligned}\mathcal{L}_w^{(1,0)}=&\,ia_5\mathrm{Tr}[\bar{H}S_v^\mu\{\lambda,[D_\mu,H]\}+\bar{H}S_v^\mu[D_\mu,\{\lambda,H\}]]\\&+ia_6\mathrm{Tr}[\bar{H}S_v^\mu[\lambda,[D_\mu,H]]+\bar{H}S_v^\mu[D_\mu,[\lambda,H]]]\\&+a_7\mathrm{Tr}[\bar{H}S_v^\mu\{[D_\mu,\lambda'],H\}]\\&+a_8\mathrm{Tr}[\bar{H}S_v^\mu[[D_\mu,\lambda'],H]]. \quad (17)\end{aligned}$$

In our notation the superscripts (i,j) denote a term of order $k^i m_s^j$ in the chiral expansion, and the subscripts s and w identify the strong and weak interaction, respectively. As mentioned in Sec. I, the terms in $\mathcal{L}_w^{(1,0)}$ [Eq. (17)], were not taken into account in previous ChPT calculations.

The parameters A_1,A_2, and A_3 in the strong Lagrangian $\mathcal{L}_s^{(0,1)}$ determine the four SU(2)-invariant masses of the octet baryons up to first order in m_s. Therefore, they provide one prediction, which is the Gell-Mann–Okubo mass relation, and fit the physical baryon masses within about 5%. We will choose these parameters such that the baryon masses are fitted best. Except for these mass terms in the strong Lagrangian $\mathcal{L}_s^{(0,1)}$ all the terms in the Lagrangian obey SU(3) symmetry. Therefore, all SU(3)-breaking effects in the amplitudes in our formulation are due to the intermediate baryon propagator in the pole diagrams in Fig. 2(b). Since we have chosen $m_u=m_d=0$, and the small mass effects due to the electromagnetic interaction can be ignored here, the baryon masses obey isospin symmetry and we will use the obvious notation m_N, m_Λ, m_Σ, and m_Ξ to represent the average mass of the corresponding isospin multiplets. Note that in calculating the decay rates the phase space gives rise to additional sources of SU(3)-breaking mass differences. However, in that case, we shall use the observed masses.

The terms with $B_1, \ldots, B_3$ in the strong-interaction Lagrangian $\mathcal{L}_s^{(2,0)}$ enter the amplitude for weak radiative decays through the baryon electromagnetic vertex in the pole diagrams of Fig. 2(b), while h_D and h_F in the weak-interaction Lagrangian $\mathcal{L}_w^{(0,0)}$ enter through the weak baryon mixing in these same diagrams.

Finally, the terms containing the parameters $a_5, \ldots, a_8$ in the weak Lagrangian $\mathcal{L}_w^{(1,0)}$ give rise to the direct emission diagrams shown in Fig. 2(a). However, since $[\lambda_7, Q]=0$, the terms with a_7 and a_8 do not contribute to hyperon weak radiative decays in leading order, and can be ignored in the following.

The diagrams in Fig. 2 lead to the following results for the parity-conserving form factors A in leading order ChPT:

$$A_{\Lambda\to n\gamma}=-\frac{eB_3}{3\sqrt{6}}\left(\frac{3h_F+h_D}{m_\Lambda-m_N}-3\frac{h_F-h_D}{m_\Sigma-m_N}\right), \tag{18a}$$

$$A_{\Sigma^+\to p\gamma}=0\,, \tag{18b}$$

$$A_{\Sigma^0\to n\gamma}=\sqrt{3}A_{\Lambda\to n\gamma}, \tag{18c}$$

$$A_{\Xi^0\to\Lambda\gamma}=-\frac{eB_3}{3\sqrt{6}}\left(\frac{3h_F-h_D}{m_\Xi-m_\Lambda}-3\frac{h_F+h_D}{m_\Xi-m_\Sigma}\right)$$
$$=-\frac{m_\Lambda-m_N}{m_\Xi-m_\Lambda}\,\frac{m_\Sigma-m_N}{m_\Xi-m_\Sigma}A_{\Lambda\to n\gamma}, \tag{18d}$$

$$A_{\Xi^0\to\Sigma^0\gamma}=\sqrt{3}A_{\Xi^0\to\Lambda\gamma}$$
$$=-\sqrt{3}\frac{m_\Lambda-m_N}{m_\Xi-m_\Lambda}\,\frac{m_\Sigma-m_N}{m_\Xi-m_\Sigma}A_{\Lambda\to n\gamma}, \tag{18e}$$

and

$$A_{\Xi^-\to\Sigma^-\gamma}=0\,. \tag{18f}$$

The final result in Eq. (18d) follows from the Gell-Mann–Okubo mass relation which is satisfied to the order we consider, as discussed earlier. The above expression for the parity-conserving amplitude were obtained before, on general ground, in Ref. [8] based on (1) pole-diagram dominance, (2) U-spin symmetry, which is preserved by charge operator Q, and (3) SU(3)-breaking through only the Gell-Mann–Okubo mass relations. All these ingredients are consistent with our current formulation of ChPT and therefore it is not surprising they are reproduced here. In particular, the weak next-to-leading order Lagrangian, which was never considered in the literature before, happens not to contribute to the parity-conserving amplitude. The symmetry relations derived by Li and Liu [29] deviate from those in Eqs. (18d) and (18e) because they did not take a diagrammatic approach and the explicit pole structure of the diagrams is absent in their calculation.

For the parity-violating amplitude B we find

$$B_{\Lambda\to n\gamma}=0\,, \tag{19a}$$

$$B_{\Sigma^+\to p\gamma}=\frac{e(a_5-a_6)}{m_\Sigma-m_N}, \tag{19b}$$

$$B_{\Sigma^0\to n\gamma}=0\,, \tag{19c}$$

$$B_{\Xi^0\to\Lambda\gamma}=0\,, \tag{19d}$$

$$B_{\Xi^0\to\Sigma^0\gamma}=0\,, \tag{19e}$$

and

$$B_{\Xi^-\to\Sigma^-\gamma}=-\frac{e(a_5+a_6)}{m_\Xi-m_\Sigma}. \tag{19f}$$

Note that the mass differences Δm in the denominators in [Eq. (19)] arise because of the Δm in the parity-violating part of the general amplitude Eq. (2). From the point of view of ChPT, a_i are the fundamental parameters that should be treated as constants, i.e., they can not compensate for the Δm in the denominators in Eq. (19). As a result, the B form factors become singular in the SU(3)-invariant limit, contrary to the usual implicit assumption in the derivation of Hara's theorem.

Let us now take a closer look at our results. The pole diagrams only contribute to the parity-conserving form factor A, in accordance with the Lee-Swift theorem [33]. Since the pole contributions to the charged decay modes $\Sigma^+\to p+\gamma$ and $\Xi^-\to\Sigma^-+\gamma$ cancel, we find a nonzero parity-conserving form factor only for the neutral decays $\Lambda\to n+\gamma$, $\Sigma^0\to n+\gamma$, $\Xi^0\to\Lambda+\gamma$, and $\Xi^0\to\Sigma^0+\gamma$.

The terms in weak Lagrangian in Eq. (17) all contain $[Q,H]$. Since $[Q,H]=0$ for neutral baryons, the direct emission diagrams do not contribute to any of the neutral decays. For the same reason also the parameters B_1 and B_4 do not give contributions to the neutral decays in the pole diagrams.

Since for all the decays either A or B vanishes, we immediately conclude that the asymmetry parameters, defined by Eq. (7), still vanish in this (leading) order. However, we can show, by qualitative arguments, that this does not need to imply a contradiction between ChPT and the measured asymmetry parameter for $\Sigma^+\to p+\gamma$ in Table I. Assuming, in the spirit of ChPT, that the form factors for charged decay modes can be expanded as

$$A=a_1\lambda, \quad B=b_0+\lambda b_1, \tag{20}$$

with $\lambda\approx 0.3(\approx m_s/\Lambda_\chi)$, and that a_1, b_0, and b_1 are of about equal magnitude. It leads, using Eq. (7), to an asymmetry parameter of about 0.6 in magnitude, which is roughly in agreement with the data for $\Sigma^+\to p+\gamma$.

In leading order, we find, from Eq. (18),

$$\mathcal{M}^\mu_{\Sigma^0\to n+\gamma}=\sqrt{3}\mathcal{M}^\mu_{\Lambda\to n+\gamma}, \tag{21}$$

$$\mathcal{M}^\mu_{\Xi^0\to\Lambda+\gamma}=-\frac{1}{\sqrt{3}}\,\frac{m_\Lambda-m_N}{m_\Xi-m_\Lambda}\,\frac{m_\Sigma-m_N}{m_\Xi-m_\Sigma}\mathcal{M}^\mu_{\Sigma^0\to n+\gamma}, \tag{22}$$

$$\mathcal{M}^\mu_{\Xi^0\to\Sigma^0+\gamma}=\sqrt{3}\mathcal{M}^\mu_{\Xi^0\to\Lambda+\gamma}. \tag{23}$$

Therefore, all ratios of the neutral decay amplitudes depend only on the baryon masses and not on any constant from

$\mathcal{L}_s^{(2,0)}$ $\mathcal{L}_w^{(0,0)}$ or $\mathcal{L}_w^{(1,0)}$ Their magnitudes, on the other hand, also depend on a particular combination of the three parameters B_3, h_D, h_F.

IV. INCLUSION OF THE 27-PLET

The Lagrangian [Eq. (17)] corresponds to the part of the weak interaction that transforms as $(8_L,1_R)$ under $SU(3)_L \times SU(3)_R$. Since the weak interaction consists of a product of two left-handed flavor-SU(3) currents, the effective chiral Lagrangian also has a $(27_L,1_R)$ component. This 27-plet part of the weak Lagrangian can be included in the same way as the octet part [27]. Closer inspection shows that inclusion of the 27-plet corresponds simply to replacing h_D and h_F in Eqs. (18) and (19) by

$$h_D \rightarrow h_D + 2h_{27} \tag{24}$$

in the charged channels, and by

$$h_D \rightarrow h_D - 3h_{27} \tag{25}$$

in the neutral channels, where h_{27} is the coupling constant from the leading-order weak-interaction Lagrangian that transforms as a 27-plet. From the $\Delta I = 1/2$ rule of the weak nonleptonic decays, h_{27} is expected to be a small parameter compared to h_D and h_F. While it enters the charged and neutral channels differently, the inclusion of the 27-plet into the analysis does not alter our results, both for the decay rates and for the asymmetry parameters.

V. DISCUSSION OF THE RESULTS

Before discussing the relation [Eq. (23)] we first investigate closely the different parameters in Eqs. (18) and (19) which contribute to the amplitude. Since we find distinct results for the neutral and charged channels, we will discuss them separately. We will also compare with previous calculations in ChPT.

A. Charged channels and Hara's theorem

Relevant for the two charged channels are the two parameters a_5 and a_6 from the next-to-leading order weak Lagrangian $\mathcal{L}_w^{(1,0)}$ Clearly, we can fit these two parameters to the two experimental decay rates as given in Table I. Due to the quadratic relation between the amplitude and the decay rate we find the following four combinations for the (dimensionless) parameters a_5 and a_6:

$$a_5 = 2.8\epsilon \times 10^{-8} \quad \text{and } a_6 = -1.6\epsilon \times 10^{-8}, \tag{26}$$

or

$$a_5 = -1.6\epsilon \times 10^{-8} \quad \text{and } a_6 = 2.8\epsilon \times 10^{-8}, \tag{27}$$

where $\epsilon = \pm 1$. With these parameters, the experimental data is reproduced. However, no prediction can be made yet. Nevertheless, our results for a_5 and a_6 are relevant since they contribute to other weak processes also, such as semileptonic decays of baryons.

More interestingly, due to the direct emission diagrams from $\mathcal{L}_w^{(1,0)}$ Hara's theorem is violated: Even in the U-spin symmetric limit, the parity-violating part of the decay amplitude is nonzero. Li and Liu [29] already argued, based on the low-energy theorem [28], that the form factor B can potentially be singular in m_s, and thus provided an explanation of the violation of Hara's theorem. In a later investigation, Gaillard [30] pointed out while such singular behavior may exist, it cannot arise from the pole diagrams similar to those in Fig. 2(b). Both observations are based on a context more general than ChPT and both are consistent with our conclusion. We have shown that, in the context of ChPT, the form factor B can indeed be singular due to the direct emission diagrams, while the potentially singular contributions from the pole diagrams cancel [30], to the order of our approximation.

TABLE II. Magnetic moments of the octet baryons and the transitional moment $\Sigma^0 \rightarrow \Lambda + \gamma$ in leading-order chiral perturbation theory. The first column contains the expressions in leading-order ChPT, the second column the experimental values, and the third column the fitted values, with $B_3 = -1.13\ \text{GeV}^{-1}$ and $B_4 = -0.82\ \text{GeV}^{-1}$. The average difference between the fitted and experimental moments is 19%. The constants B_3 and B_4 are from the next-to-leading order strong Lagrangian $\mathcal{L}_s^{(2,0)}$ [see Eq. (15)]. Note that only the constant B_3 plays a role in hyperon radiative decays.

B	$\mathcal{M}_{\text{th}}\ [e]$	$\mathcal{M}_{\text{expt}}\ [\mu_N]$	$\mathcal{M}_{\text{fitted}}\ [\mu_N]$
p	$-B_3/3 - B_4$	2.79	2.25
n	$2B_3/3$	-1.91	-1.41
Λ	$B_3/3$	-0.61	-0.71
Σ^+	$-B_3/3 - B_4$	2.42	2.25
Σ^-	$-B_3/3 + B_4$	-1.16	-0.84
Ξ^-	$-B_3/3 + B_4$	-0.68	-0.84
Ξ^0	$2B_3/3$	-1.25	-1.41
$\Sigma^0 \rightarrow \Lambda$	$B_3/\sqrt{3}$	± 1.6	–

B. Neutral channels

As can be seen in Eq. (18), the relevant parameters for the neutral channels are B_3, h_D, and h_F. The parameter B_3 is from the strong interaction Lagrangian [Eq. (15)], and determines (together with the constant B_4) the magnetic moments of the octet baryons. The first column of Table II shows the magnetic moments of the baryons in leading-order ChPT. Fitting B_3 to the experimental data, shown in the second column of Table II, gives

$$B_3 = -1.3\ \text{GeV}^{-1}. \tag{28}$$

The resulting fitted magnetic moments as given in the third column of Table II. Note that these results give rise to the Coleman and Glashow [31] relations between the baryon magnetic moments.

The parameters h_D and h_F can be obtained from hyperon nonleptonic decay. Considering s-wave nonleptonic decay data gives the best values, [32]

$$h_D = -0.58\mu, \quad h_F = 1.40\mu, \tag{29}$$

where $\mu = G_F m_\pi^2 \sqrt{2} f_\pi \approx 3 \times 10^{-8}$ GeV.

Using these values, together with that for B_3, we arrive at the decay rates for hyperon radiative decay as given in Table

TABLE III. Decay rates for the four neutral hyperon radiative decays, taking for the parameters h_D and h_F the values in Eq. (29) obtained from nonleptonic hyperon decays. The first column shows the observed rates from Ref. [22]. All decay rates are in units of 10^{-18} GeV.

$B_i \to B_f + \gamma$	$\Gamma_{\rm expt}$	$\Gamma_{\rm Eq.(29])}$
$\Lambda \to n + \gamma$	4.07	0.018
$\Sigma^0 \to n + \gamma$	-	0.16
$\Xi^0 \to \Lambda + \gamma$	2.4	0.087
$\Xi^0 \to \Sigma^0 + \gamma$	8.1	0.067

III. It shows a huge disagreement with the observed rates. The difference between the experimental and predicted decay rate is more than a factor of 200 for $\Lambda \to n + \gamma$, while the difference for the other channels is about two orders of magnitude.

This discrepancy, however, is highly dependent on the values used for h_D and h_F. Independent of any particular values chosen for the parameters from the Lagrangian, we can use Eq. (23) to predict three ratios between the four neutral decay rates. We find

$$\frac{\Gamma_{\Sigma^0 \to n+\gamma}}{\Gamma_{\Lambda \to n+\gamma}} = 8.1, \tag{30}$$

$$\frac{\Gamma_{\Xi^0 \to \Lambda+\gamma}}{\Gamma_{\Lambda \to n+\gamma}} = 4.9(0.59 \pm 0.14), \tag{31}$$

$$\frac{\Gamma_{\Xi^0 \to \Sigma^0+\gamma}}{\Gamma_{\Lambda \to n+\gamma}} = 3.7(1.99 \pm 0.42), \tag{32}$$

where the experimental values obtained from Table I are written in parentheses. While the first ratio cannot be obtained from experimental data, since $\Sigma^0 \to n + \gamma$ has not been measured, the predicted values for the other ratios are only within about a factor 8 and 2, respectively, in accordance with the observed ratios. In contrast with the case of the asymmetry parameters, one cannot hope that this disagreement may be resolved by higher-order effects, if ChPT is a consistent expansion scheme with the next-to-leading order corrections about 30% suppressed.

To compare our results with other recent work on hyperon weak radiative decays in the framework of ChPT [19,20], first note that Neufeld [19] did not adopt the heavy-baryon formulation and therefore his perturbative scheme may be doubtful [25]. In Ref. [20] the next-to-leading order weak Lagrangian $\mathcal{L}_w^{(1,0)}$ was not taken into account. Instead, Ref. [20] concentrated on analyzing the potentially dominant part of the loop corrections. These loop corrections are of higher order in the power counting than the contributions of the next-to-leading order weak Lagrangian which we analyzed. For the charged modes, we obtained in leading-order-vanishing-parity-conserving A form factors, while Ref. [20] obtained nonzero values using the differences of physical magnetic moments of the baryons as input. Note that in ChPT these differences between the magnetic moments follow from operators corresponding to $\mathcal{L}_s^{(2,1)}$. A more complete analysis of ChPT to this order needs to be done to justify the approach used in Ref. [20]. We have obtained nonzero parity-violating form factors B through $\mathcal{L}_w^{(1,0)}$. In Ref. [20] only nonzero results were obtained from the loop diagrams which are higher-order corrections to our results. For the neutral decay channels, we obtained nonzero A form factors through the pole diagrams and our results are consistent with that of Ref. [20] if the Coleman-Glashow values for magnetic moments are used in their equations. We obtained vanishing B form factors to the order of our approximation, while Ref. [20] obtained nonzero results through the loop corrections.

To put things in perspective, the higher-order effects are needed to explain some data such as the asymmetries. However, it is very important to have a complete lowest-order analysis before one proceeds to higher-order loop effects. Furthermore, not all the problems that ChPT encounters in the lowest order analysis such as ours are expected to disappear with the higher-order effects included. For ChPT to be feasible, higher-order effects should not be relevant to solve large leading-order discrepancies with data such as the ratios of decay rates of the neutral modes.

VI. SUMMARY AND CONCLUSIONS

We have studied the process of hyperon weak radiative decay in the framework of heavy-baryon chiral perturbation theory. In particular, we have put emphasis on the effect of including a complete next-to-leading order weak Lagrangian [27] in the description. We used it to obtain the leading-order decay amplitudes. In previous calculations [19,20], these leading contributions are missing.

In the leading order, all the ratios for decay amplitudes of the four *neutral channels*, $\Lambda \to n + \gamma$, $\Sigma^0 \to n + \gamma$, $\Xi^0 \to \Sigma^0 + \gamma$, and $\Xi^0 \to \Lambda + \gamma$, are simple functions of the baryon masses only. This follows from the observation that the direct-emission contributions vanish and only pole diagrams contribute. Comparing with experimental data these ratios are up to a factor of 8 off. It is interesting to note that taking the values of h_D and h_F from analysis of nonleptonic weak decays of hyperons leads to predictions which disagree with the observed decay rates for more than two orders of magnitude.

Clearly, these disagreements indicate something deficient about the applications of ChPT to the neutral decays. In the spirit of ChPT, the higher-order corrections should be smaller than the leading order, which is clearly not the case for the neutral channels. Therefore, this problem is not going to be resolved by the inclusion of higher-order effects such as the loop contributions calculated in Ref. [20]. It may be an indication that our current treatment of SU(3)-breaking effects is flawed. Or alternatively, it may indicate that higher resonances, such as the vector mesons, have to be included in the analysis of ChPT as hinted by the relative success of the vector-meson-dominance models [16].

For the two *charged channels* $\Sigma^+ \to p + \gamma$ and $\Xi^- \to \Sigma^- + \gamma$ no similar relations as for the neutral channels can be extracted from ChPT to this order. Contrary to the case of the neutral channels, only the direct emission diagrams contribute. The contributions of all the pole diagrams add up to zero at this order. The two observed decay rates can be used to fix exactly the two parameters a_5 and a_6 from the next-to-leading order weak Lagrangian. Although we derive no prediction within the charged channels, the parameters extracted will be relevant for other observables such as weak semileptonic decays, making a future comparison feasible.

We have shown that, even in leading-order ChPT, Hara's theorem is violated by direct emission diagrams contributing to the parity-violating part of the amplitude. Going through the original assumptions in the proof of the theorem, our result indicates that the parity-violating form factors in the amplitude are singular in the limit of U-spin [or SU(3)] symmetry in the context of ChPT. This singular behavior leads to the failure of Hara's theorem.

The asymmetry parameters vanish in our leading-order calculation. However, we have shown that measured asymmetry parameter for $\Sigma^+ \rightarrow p+\gamma$ is of the right order of magnitude as the estimated higher-order correction. In other words, our results indicate that the asymmetry parameters are sensitive to loop effects and parameters in the higher-order Lagrangian.

In conclusion, our results indicate that there are some serious weaknesses associated with the current formulation of ChPT when it is applied to hyperons. We believe that a more careful formulation of SU(3)-breaking effects, together with the inclusion of vector mesons in the theory, may provide a better framework to understand hyperon radiative decays within ChPT. Both possibilities are under investigation.

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APPENDIX: GENERAL FORM OF WEAK RADIATIVE DECAY IN HEAVY-BARYON CHIRAL PERTURBATION THEORY

In this appendix we briefly review the heavy-baryon formulation of ChPT, and give the general amplitude of weak radiative decay in this formulation.

The nucleon mass in the chiral limit $\dot{m}$ is comparable with the chiral symmetry-breaking scale Λ_χ. To make a consistent chiral expansion possible, it can be removed by redefining the baryon field according to [25]

$$B_v = e^{i\dot{m} v\cdot x} B, \tag{A1}$$

where v^μ is the baryon four-velocity satisfying $v^2=1$. Next, one defines the projected fields

$$H = P_v^+ B_v, \quad h = P_v^- B_v, \tag{A2}$$

where P_v^+ and P_v^- are the projection operators

$$P_v^\pm = \frac{1 \pm \not{v}}{2}. \tag{A3}$$

The minus component field h is suppressed by $1/\dot{m}$ compared to H. It can be easily seen that, in momentum space, derivatives of H produce powers of

$$k^\mu = p^\mu - \dot{m} v^\mu, \tag{A4}$$

with p^μ the four-momentum of the baryon, which is (for processes at low energies) a small quantity. This *residual* baryon momentum k is the effective expansion parameter in this formulation of baryon ChPT. Effects of $1/\dot{m}$ can arise through h in higher order. However, these $1/\dot{m}$ corrections can be absorbed in the higher-order counterterms of the theory [27].

The general amplitude for the weak radiative decay

$$B_i(p) \rightarrow B_f(p') + \gamma \tag{A5}$$

is given by

$$\epsilon_\mu(q)\mathcal{M}^\mu = \epsilon_\mu(q)\overline{u(p')} i\sigma^{\mu\nu} q_\nu (A + B\gamma_5) u(p). \tag{A6}$$

Defining the operator $S_v^\mu \equiv (1/2) P_v^+ \gamma^\mu \gamma_5 P_v^+$ and using

$$P_v^+ \gamma_5 P_v^+ = 0, \quad P_v^+ \gamma^\mu P_v^+ = P_v^+ v^\mu, \tag{A7}$$

$$P_v^+ \sigma^{\mu\nu} P_v^+ = -2i[S_v^\mu, S_v^\nu], \tag{A8}$$

$$P_v^+ \sigma^{\mu\nu}\gamma_5 P_v^+ = -2i(v^\mu S_v^\nu - v^\nu S_v^\mu), \tag{A9}$$

we find that the general form of weak radiative decay in the heavy-baryon formalism is given by

$$\begin{aligned}\epsilon_\mu(q)\mathcal{M}^\mu = 2\epsilon_\mu(q)\overline{U_v}(k')\{q_\nu[S_v^\mu, S_v^\nu]A \\ + (S_v^\mu \Delta m_B - v^\mu S_v \cdot q)B\} U_v(k),\end{aligned} \tag{A10}$$

where Δm_B is the mass difference between the initial and final baryon, and U_v and $\overline{U_v}$ are plus components of u and $\overline{u}$ in Eq. (38). In the gauge $v\cdot\epsilon=0$, we then finally arrive at the form as used in Eq. (2).

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