Q^2 evolution of the spin-dependent asymmetry A_1^p at large and intermediate x

A. V. Kotikov*
Laboratoire de Physique Theorique Ecole Normale Supérieure de Lyon and Laboratoire d'Annecy-le-Vieux de Physique des Particules, Laboratoire d'Annecy-le-Vieux de Physique des Particules, Boîte Postale 110, F-74941, Annecy-le-Vieux Cedex, France

D. V. Peshekhonov†
Particle Physics Laboratory, Joint Institute for Nuclear Research, Dubna, Russia
(Received 17 July 1995; revised manuscript received 1 April 1996)

We consider the Q^2 evolution of asymmetry $A_1(x,Q^2)$ in the kinematical range $0.05 \leqslant x \leqslant 1.0$. To estimate possible effects on the spin-dependent structure function $g_1(x,Q^2)$, we apply our results to the E143 and SMC proton data. [S0556-2821(96)03215-8]

PACS number(s): 13.60.Hb, 11.55.Hx, 13.88.+e

I. INTRODUCTION

The spin-dependent structure function (SF) of the nucleon g_1 is extracted from the virtual photon-nucleon asymmetry $A_1(x,Q^2)$ according to the equation

$$g_1(x,Q^2)=A_1(x,Q^2)F_1(x,Q^2)\left(1+\frac{4M^2x^2}{Q^2}\right),$$

where $F_1(x,Q^2)$ is the spin average SF and M is proton mass.

The most stringent theoretical predictions on the spin-dependent structure functions refer to their first moments Γ_1. Experimentally data on A_1 are collected at different values of Q^2 for different values of x but the correct calculation of the Γ_1 value requires the same value of Q^2 at all x. The accuracy of previous experiments [1–5] allows one to analyze data under the assumption the asymmetry $A_1(x,Q^2)$ is Q^2 independent [3–7]. However, in view of more precise forthcoming data it is important to know the Q^2 dependence of the asymmetry following from QCD.

The assumption $A_1(x,Q^2)$ is Q^2 independent means the SF $g_1(x,Q^2)$ and $F_1(x,Q^2)$ have the same Q^2 dependence. This is right asymptotically for $x\rightarrow 1$ where the Q^2 evolution of the SF is determined by the nonsinglet (NS) anomalous dimension[1] (AD) $\gamma_{\rm NS}(\alpha,n)=\alpha\gamma_{\rm NS}^{(0)}(n)+\alpha^2\gamma_{\rm NS}^{(1)}(n)+O(\alpha^3)$ and by the first coefficients of the NS coefficient functions $B_j(\alpha,n)=1+\alpha b_j(n)+O(\alpha^2)$ $(j=F,g)$; see [8,9]. In the leading order (LO) of the perturbative theory the NS AD and the coefficient functions are the same for polarized and nonpolarized SF [10]. In the next-to-leading order (NLO) they are close, too. They are same within an accuracy $O(1/n^2)$ [10].

The behavior of the SF F_1 at large x can be presented in the LO approximation[2] [8] as

$$F_1(x,Q^2)\underset{x\rightarrow 1}{\sim}A_F[\alpha(Q^2)]^{-d_0}\frac{(1-x)^{\nu_F[\alpha(Q^2)]}}{\Gamma(1+\nu_F[\alpha(Q^2)])},$$

where

$$d_0=\frac{16}{3\beta_0}\left(\frac{3}{4}-C\right),\quad \nu_F[\alpha]=\nu_F-\frac{16}{3\beta_0}\ln\,\alpha(Q^2),$$

C is the Eulerian constant, and β_0 is the first coefficient of the β function in the α expansion.

The behavior of g_1 at large x can be presented in the same way by replacing the values of A_F and ν_F to A_g and ν_g. Then SF F_1 and g_1 have the same Q^2 behavior at $x\rightarrow 1$ if ν_F and ν_g are close.

The values of A_j and ν_j $(j=F,g)$ cannot be defined in the framework of perturbative theory, unfortunately. However, from the quark counting rule, $\nu_F[Q^2]$ and $\nu_g[Q^2]$ are close to 3 that is in agreement with the experimental data [2–5] (see reviews [11]). As a consequence the asymmetry $A_1=1$ at $x\rightarrow 1$ will be used in our analysis further.

In general, at moderate and small x, SF $F_1(x,Q^2)$ and $g_1(x,Q^2)$ have different and more complicated Q^2 dependences.

In this article we present the results on Q^2 evolution of the asymmetry $A_1(x,Q^2)$ in kinematical range $x \geqslant 0.05$, applied to the E143 [5] and SMC [3] proton data.

At large x $(x \geqslant 0.3)$ our analysis is done in the NS case of QCD evolution (due to the absence of the gluon terms at this kinematical range). This allows us to work in the NLO approximation (the nonsinglet polarized AD are known in this order of the theory). At $x<0.3$ we add the gluon distribution and work in the LO approximation because NLO singlet AD are unknown yet.[3]

*On leave of absence from Particle Physics Laboratory, JINR, Dubna, Russia. Electronic address: kotikov@lapphp0.in2p3.fr; kotikov@sunse.jinr.dubna.su

†Electronic address: dimitri@na47sun05.cern.ch; peshehon@sunse.jinr.dubna.su

[1] In contrast with the standard case, we use below $\alpha(Q^2)=\alpha_s(Q^2)/(4\pi)$.

[2] The NLO corrections may be found in [9].

[3] While completing this study, the NLO polarized AD have been calculated (see [12]).

II. LARGE x REGION

The evolution equation for the asymmetry $A_1(x,Q^2)$ can be determined from the evolution equations for SF $F_1(x,Q^2)$ and $g_1(x,Q^2)$:

$$\frac{dF_1(x,Q^2)}{d\ \ln(Q^2)}=-\frac{1}{2}\int_x^1\frac{dy}{y}\,\tilde{\gamma}_F\left(\frac{x}{y}\right)F_1(y,Q^2), \tag{1a}$$

$$\frac{dg_1(x,Q^2)}{d\ \ln(Q^2)}=-\frac{1}{2}\int_x^1\frac{dy}{y}\,\tilde{\gamma}_g\left(\frac{x}{y}\right)g_1(y,Q^2), \tag{1b}$$

$$\frac{dA_1(x,Q^2)}{d\ \ln(Q^2)}=-\frac{1}{2}\left\{\frac{1}{F_1(x,Q^2)}\int_x^1\frac{dy}{y}\,\tilde{\gamma}_g\left(\frac{x}{y}\right)g_1(y,Q^2)-\frac{g_1(x,Q^2)}{F_1^2(x,Q^2)}\int_x^1\frac{dy}{y}\,\tilde{\gamma}_F\left(\frac{x}{y}\right)F_1(y,Q^2)\right\}, \tag{1}$$

where the splitting functions $\tilde{\gamma}_j(x)=\alpha\gamma_{\rm NS}^{(0)}(x)+\alpha^2(\gamma_j^{(1)}(x)+2\beta_0 b_j(x))+O(\alpha^3)$ $(j=F,g)$ are the Mellin transforms of the corresponding[4] AD $\gamma_{\rm NS}^{(0)}(n)$, $\gamma_j^{(i)}(n)$, and Wilson coefficients $b_j(n)$.

Let us suppose that $\tilde{\gamma}_F(x)\approx\tilde{\gamma}_g(x)\equiv\gamma(x)$ (see [13]) and express the function $g_1(x,Q^2)$ in terms of $A_1(x,Q^2)$ and $F_1(x,Q^2)$:

$$\frac{dA_1(x,Q^2)}{d\ \ln(Q^2)}=\frac{-1}{2F_1(x,Q^2)}\int_x^1\frac{dy}{y}\,\gamma\left(\frac{x}{y}\right)[A_1(y,Q^2)-A_1(x,Q^2)]F_1(y,Q^2). \tag{2}$$

The evolution equations (1a), and (1b) are written for the SF but not for the parton distributions; that is why they include Wilson coefficient functions $b_j(n)$ on the right-hand side (RHS). These coefficients differ (see [10,9]) in polarized and nonpolarized cases:

$$b_g(n)-b_F(n)=\frac{8}{3}\,\frac{1}{n(n+1)},$$

$$\text{or}\ \ b_g(x)-b_F(x)=\frac{8}{3}\,x(1-x)$$

which leads to the correction in Eq. (2)$\sim 4/3\alpha^2\beta_0(1-x)^2$ which is small and we will neglect it in the analysis.

Let us suppose that we have the measured values of the symmetry in Ix bins at the range $x\in[0.3;1.0]$, these x bins are small enough so that $A_1(x,Q^2)$ has no x dependence into the bins, i.e.,

$$A_1(x,Q^2)=\sum_{i=1}^{I+1}A_{1_i}(Q^2)\Theta(x_i-x)\Theta(x-x_{i-1}), \tag{3}$$

where function $\Theta(z)$ is defined as

$$\Theta(z)=\begin{cases}1, & z>0,\\ 0, & z<0,\end{cases}$$

and the value of the asymmetry in the last bin is known and fixed:

$$A_1(x_{I+1},Q^2)\equiv A_1(1,Q^2)=1. \tag{4}$$

Equation (2) and conditions (3,4) allow us to write the relation for each x bin to recalculate the measured values of the asymmetry $A_1(x_i,Q_i^2)$ at fixed Q^2 (see Appendix A).

To simplify obtained formulas (A1)–(A4) we evaluate $A_1(x,Q^2)$ at the upper edge of the x bins and introduce the new variable:

$$R_i(Q^2)=[A_{1_i}(Q^2)-A_{1_{I+1}}]F_1(x_i,Q^2). \tag{5}$$

The evolution procedure starts at the last bin $]x_I,1.]$.

$x\in]x_I,1.0]$:

$$R_{I+1}=0\leftrightarrow A_{1_{I+1}}=\text{const}. \tag{6}$$

This result confirms the fact that the asymmetry is Q^2 independent at $x=1$.

$x\in]x_{I-1},x_I]$:

$$R_I(Q^2)=R_I(Q_I^2),\quad \text{i.e.,}\ \ R_I(Q^2)=R_I=\text{const}. \tag{7}$$

$x\in]x_{I-2},x_{I-1}]$.

Following Eqs. (A3,7) we have

$$R_{I-1}(Q^2)=R_{I-1}(Q_{I-1}^2)+R_I L_{I-1,I}(t) \tag{8}$$

where

$$L_{I-1,I}(t)=\int_{t_{I-1}}^{t}d\xi\,T_{I-1,I}(\xi)$$

$$\text{and}\ \ T_{I-1,I}(\xi)=\frac{1}{F_1(x_I,\xi)}\,\frac{d}{d\xi}\,[F_1(x_{I-1},\xi)-F_1(x_I,\xi)]$$

with $t_i=\ln(Q_i^2/\Lambda^2)$ and $t=\ln(Q^2/\Lambda^2)$.

$x\in[x_{I-3},x_{I-2}]$.

From Eq. (A4) at $i=I-2$ and Eqs. (7) and (8) we obtain

$$\begin{aligned}R_{I-2}(Q^2)-R_{I-2}(Q_{I-2}^2)&=\int_{t_{I-2}}^{t}d\xi\sum_{k=I-1}^{I}R_k(\xi)T_{k-1,k}(\xi)\\&=\int_{t_{I-2}}^{t}d\xi\left\{\left[R_{I-1}(Q_{I-2}^2)+R_I\int_{t_{I-2}}^{t}d\xi\,T_{I-1,I}(\xi)\right]\times T_{I-2,I-1}(\xi)+R_I T_{I-1,I}(\xi)\right\}\\&=R_{I-1}(Q_{I-2}^2)L_{I-2,I-1}(t)+R_1 L_{I-2,I}(t),\end{aligned} \tag{9}$$

where

---

[4] The AD $\gamma_j^{(1)}(n)$ $(j=F,g)$ coincide with the ''+'' and ''−'' components of NS AD $\gamma_{\rm NS}^{(1)\pm}(n)$ (see [10]), i.e., $\gamma_F^{(1)}(n)\equiv\gamma_{\rm NS}^{(1)+}(n)$ and $\gamma_g^{(1)}(n)\equiv\gamma_{\rm NS}^{(1)-}(n)$. The difference $\gamma_g^{(1)}(n)-\gamma_F^{(1)}(n)=128/(3n^6)+O(n^{-7})$ (see [13]) can be neglected.

$$L_{I-2,I}(t)=\int_{t_{I-2}}^{t} d\xi T_{I-2,I-1}(\xi)\int_{t_{I-2}}^{\xi} d\eta T_{I-1,I}(\eta) + \int_{t_{I-2}}^{t} d\xi T_{I-1,I}(\xi) \equiv \int_{t_{I-2}}^{t} d\xi \widetilde{T}_{I-2,I-1}(\xi)\int_{t_{I-2}}^{\xi} d\eta T_{I-1,I(\eta)}$$

and $\widetilde{T}_{I-2,I-1}(\xi)=T_{I-2,I-1}+\delta(t-\xi)$

Let us consider the *generic* bin now.

$x\in[x_{i-1},x_i]$.

By analogy with Eqs. (8,9) the evolution equation of $R_i(Q^2)$ can be written as

$$R_i(Q^2)=R_i(Q_i^2)+\sum_{k=i+1}^{I} R_k(Q_i^2)L_{i,k}(Q^2), \tag{10}$$

where

$$L_{i,k}(Q^2)=\int_{t_i}^{t} d\xi \widetilde{T}_{i,i+1}(\xi)\int_{t_i}^{\xi} d\eta \widetilde{T}_{i+l,i+2}(\eta)\cdots \times\int_{t_i}^{\delta} d\tau \widetilde{T}_{I-2,I-1}(\tau)\int_{t_i}^{\tau} T_{I-1,I}(\zeta)d\zeta.$$

We have equations which describe Q^2 evolution of the asymmetry in the range of $x\geqslant 0.3$. The final equation [Eq. (10)] shows an advantage of our method as soon as it does not contain the Mellin convolution. The Q^2 dependence is expressed as a multiple integral of some known functions and can be calculated directly.

III. AN EXPANSION TO THE INTERMEDIATE x

Let us consider the case when the gluon term is not negligible. The evolution equations (1a,1b) on the SF $F_1(x,Q^2)$, $g_1(x,Q^2)$ and the asymmetry $A_1(x,Q^2)$ (1) have to be modified then:

$$\frac{dF_1(x,Q^2)}{d\,\ln(Q^2)}=-\frac{\alpha}{2}\int_x^1\frac{dy}{y}\left\{\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)F_1(y,Q^2) + \gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2)\right\}, \tag{11a}$$

$$\frac{dg_1(x,Q^2)}{d\,\ln(Q^2)}=-\frac{\alpha}{2}\int_x^1\frac{dy}{y}\left\{\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)g_1(y,Q^2) + \overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)\right\}, \tag{11b}$$

$$\frac{dA_1(x,Q^2)}{d\,\ln(Q^2)}=-\frac{\alpha}{2}\left\{\frac{1}{F_1(x,Q^2)}\int_x^1\frac{dy}{y}\left[\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)g_1(y,Q^2) + \overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)\right] - \frac{g_1(x,Q^2)}{F_1^2(x,Q^2)} \times\int_x^1\frac{dy}{y}\left[\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)F_1(y,Q^2) + \gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2)\right],\tag{11}$$

where $G(x,Q^2)=g_++g_-$ and $\Delta G=g_+-g_-$ are gluon distributions in the polarized and nonpolarized cases, respectively. Functions $\gamma_{\phi\phi}^{(0)}(x)$, $\gamma_{\phi g}^{(0)}(x)$, and $\overline{\gamma_{\phi g}^{(0)}}(x)$ are the Mellin transforms of the LO singlet unpolarized and polarized AD $\gamma_{\phi\phi}^{(0)}(n)$, $\gamma_{\phi g}^{(0)}(n)$, and $\overline{\gamma_{\phi g}^{(0)}}(n)$, respectively.

As in the previous section function $g_1(x,Q^2)$ is expressed in terms of $A_1(x,Q^2)$ and $F_1(x,Q^2)$:

$$\frac{dA_1(x,Q^2)}{d\,\ln(Q^2)}=\frac{-\alpha}{2F_1(x,Q^2)}\int_{x_1}^1\frac{dy}{y}\left[\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)[A_1(y,Q^2) - A_1(x,Q^2)]F_1(y,Q^2)+\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2) - \gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)A_1(x,Q^2)G(y,Q^2)\right]. \tag{12}$$

We still use conditions (3,4) for the extended x region. Equations (12,3,4) allow us to write ‘‘I’’ equations to recalculate the measured values of the asymmetry $A_1(x_i,Q_i^2)$ at fixed Q^2 (see Appendix B). To simplify the picture and by analogy with the previous section we evaluate the asymmetry $A_1(x,Q^2)$ at the upper edge of each x bin and work in a term of the variable $R_i(Q^2)$ defined below.

Here we start the evolution procedure at bin $]x_{I-1},x_I]$.

$x\in]x_{I-1},x_I]$:

$$R_I(Q^2)=R_I(Q_I^2)-K_{I,I}(Q^2), \tag{13}$$

where $K_{I,I}(Q^2)=\int_{t_I}^{t} d\xi Q_I(\xi)$ and

$$Q_I(\xi)=\frac{\alpha(\xi)}{2}\int_{x_1}^1\frac{dy}{y}\left[\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,\xi) - A_{1_{I+1}}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi)\right].$$

$x\in]x_{I-2},x_{I-1}]$.

According to Eq. (B2) we have

$$R_{I-1}(Q^2)=R_{I-1}(Q_{I-1}^2)+R_I(Q_{I-1}^2)L_{I-1,I}(t)-K_{I-1,I}(t) -K_{I-1,I-1}(t), \tag{14}$$

where

$$L_{I-1,I}(t)=\int_{t_{I-1}}^{t} d\xi P_{I-1,I}(\xi)$$

and

$$P_{I-1,I}(\xi)=T_{I-1,I}(\xi)+\frac{1}{F_1(x_{I-1},\xi)}\frac{\alpha(\xi)}{2}\int_{x_{I-1}}^{x_1}\frac{dy}{y}\times\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi);$$

$$K_{I-1,I}(t)=\int_{t_{I-1}}^{t}d\xi P_{I-1,I}(\xi)\int_{t_{I-1}}^{\xi}d\eta Q_I(\eta)$$

$$\in[x_{I-3},x_{I-2}].$$

From Eq. (B2) at $i=I-2$ and Eqs. (7,13) we obtain

$$R_{I-2}(Q^2)-R_{I-2}(Q_{I-2}^2)=R_{I-1}(Q_{I-2}^2)L_{I-2,I-1}(t)+R_I(Q_{I-2}^2)L_{I-2,I}(t)-\sum_{k=I-2}^{I}K_{I-2,k}(t), \qquad (15)$$

where

$$L_{I-2,I}(t)=\int_{t_{I-2}}^{t}d\xi\widetilde{P}_{I-2,I-1}(\xi)\int_{t_{I-2}}^{\xi}d\eta P_{I-1,I}(\eta)$$

and

$$K_{I-2,I}(t)=\int_{t_{I-2}}^{t}d\xi\widetilde{P}_{I-2,I-1}(\xi)\int_{t_{I-2}}^{\xi}d\eta P_{I-1,I}(\eta)\times\int_{t_{I-1}}^{\eta}d\zeta Q_I(\zeta)$$

with $\widetilde{P}_{I-2,I-1}(\xi)=P_{I-2,I-1}+\delta(t-\xi)$.
Consider now the *generic* bin:
$x\in[x_{i-1},x_i]$.
From Eq. (B3) by analogy with Eqs. (8,9,13,14) the evolution equation of $R_i(Q^2)$ can be written in the form

$$R_i(Q^2)=\sum_{k=i}^{I}[R_k(Q_i^2)L_{i,k}(Q^2)-K_{i,k}(Q^2)], \qquad (16)$$

where

$$L_{i,i}(t)=1 \quad \text{and} \quad K_{i,i}(t)=\int_{t_i}^{t}d\xi Q_i(\xi),$$

$$L_{i,k}(Q^2)=\int_{t_i}^{t}d\xi\widetilde{P}_{i,i+1}(\xi)\int_{t_i}^{\xi}d\eta\widetilde{P}_{i+1,i+2}(\eta)\cdots\times\int_{t_i}^{\delta}d\tau\widetilde{P}_{I-2,I-1}(\tau)\int_{t_i}^{\tau}P_{I-1,I}(\zeta)d\zeta,$$

$$K_{i,k}(Q^2)=\int_{t_i}^{t}d\xi\widetilde{P}_{i,i+1}(\xi)\int_{t_i}^{\xi}d\eta\widetilde{P}_{i+1,i+2}(\eta)\cdots\times\int_{t_i}^{\delta}d\tau\widetilde{P}_{I-2,I-1}(\tau)\int_{t_i}^{\tau}P_{I-1,I(\zeta)}d\zeta\int_{t_i}^{\zeta}d\delta Q_i(\delta),$$

$$P_{i,i+1}(\xi)=T_{i,i+1}(\xi)+\frac{1}{F_1(x_{i+1},\xi)}\frac{\alpha(\xi)}{2}\times\int_{x_i}^{x_{i+1}}\frac{dy}{y}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi),$$

$$Q_i(\xi)=\frac{\alpha(\xi)}{2}\int_{x_i}^{1}\frac{dy}{y}\left[\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,\xi)-A_{1_{I+1}}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi)\right].$$

The polarized and unpolarized gluon terms $G(\chi,Q^2)$ and $\Delta G(x,Q^2)$ we describe in the form

$$G(x,Q^2)=g(x,Q^2)\Theta(x_K-x)$$

and

$$\Delta G(x,Q^2)=\Delta g(x,Q^2)\Theta(x_K-x),$$

where $x_K\in[x_{\min},1.0]$. It allows us to represent the functions $P_{i,i+1}$ and $Q_i(\xi)$ as

$$P_{i,i+1}(\xi)=T_{i,i+1}(\xi)+\frac{\Theta(x_K-x_{i+1})}{F_1(x_{i+1},\xi)}\frac{\alpha(\xi)}{2}\times\int_{x_i}^{x_{i+1}}\frac{dy}{y}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi),$$

$$Q_i(\xi)=\frac{\alpha(\xi)}{2}\Theta(x_K-x_{i+1})\int_{x_i}^{x_K}\frac{dy}{y}\left[\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,\xi)-A_{1_{I+1}}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,\xi)\right].$$

So, Eq. (16) describes the Q^2 dependence of the asymmetry in the range $x<0.3$. By analogy with the previous section the Q^2 evolution is expressed as a multiple integral of the known functions and can be calculated directly.

IV. RESULTS

To apply our calculations we took the E143 [5] and SMC [3] proton data as an input. The spin independent SF $F_1(x,Q^2)$ was defined as

$$F_1(x,Q^2)=\frac{F_2(x,Q^2)}{2x[1+R(x,Q^2)]} \qquad (17)$$

and we used parametrizations of F_2 and R by the New Muon Collaboration (NMC) [14] and SLAC global fit [15], respectively.[5] Duke-Owens parametrization [16] was used for gluon distribution $g(x,Q^2)$. The polarized gluon distribu-

[5]The terms responsible for high-twist effects were removed from parametrizations. The consideration of the high-twist effects is above our analysis. However, we hope that their importance is strongly reduced when we consider the SF ratio but not SF itself.

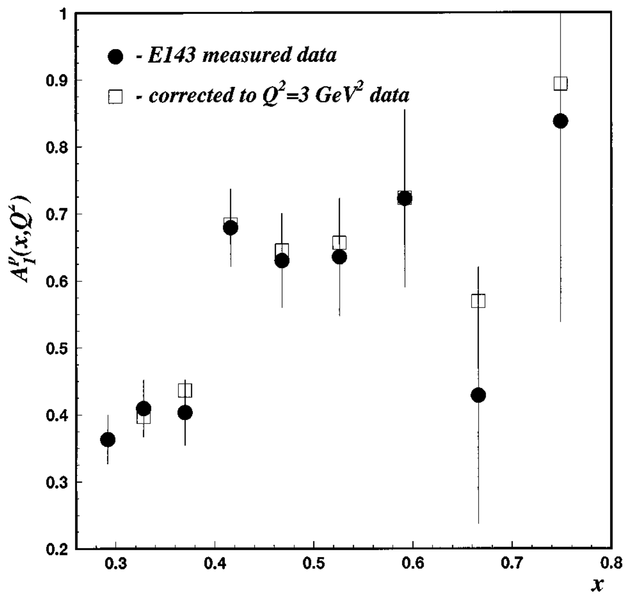

FIG. 1. E143 measured virtual photon-nucleon asymmetry A_1^p [9] (shown as closed points) in comparison with evolved to $Q^2{=}3$ GeV^2 (open squares).

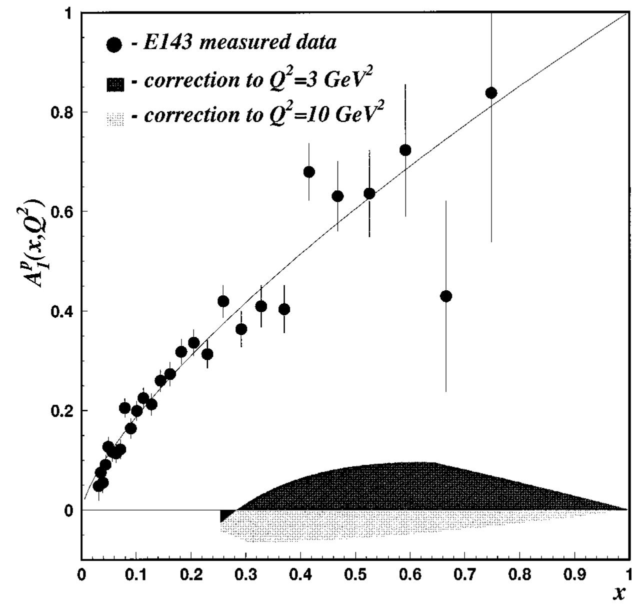

FIG. 2. E143 measured data on the asymmetry A_1 as a function of x and their parametrization. Variations of the asymmetry caused by the Q^2 evolution procedure shown as dark and light areas for $Q^2{=}3$ GeV^2 and $Q^2{=}10$ GeV^2, respectively.

tion is less known (see discussions in [17]) and we apply the suggestion by Brodsky (see [18]) and parametrize it in the form

$$\Delta g(x,Q^2)\equiv\frac{1-(1-x)^2}{1+(1-x)^2}\,g(x,Q^2).$$

E143 [Spin Muon Collaboration (SMC)] has measured the proton asymmetry in the kinematical range $1.3<Q^2<10(70)$ GeV^2 and $0.029(0.003)\geqslant x\geqslant 0.8(0.7)$. The average Q^2 of the E143 and SMC proton data is $\langle Q^2\rangle=3$ GeV^2 and $\langle Q^2\rangle=10$ GeV^2, respectively.

First we evolved E143 data (using the original experimental binning) which have been measured at the range $x>0.3$ to $Q^2=3$ GeV^2 (see Fig. 1).

It is seen from the figure the corrections on A_1 coming due to evolution procedure are small in an accuracy of an existing experimental data but not negligible.

The main advantage of our method (representation of the evolution equation as the sum of the multiple integrals) gives its main technical restriction: the number and the multiplication of the integrals.

The number of E143 and SMC measured x bins is too high to apply our method directly on the experimental data. To decrease the complexity of the analysis we parametrize both E143 and SMC measurements as a functions of x (see Fig. 2 and Fig. 3) and create a new, more convenient (for us) binning. Using these smooth curves as an input we got corrections on the asymmetries coming due to evolution to $Q^2=3$ GeV^2 and 10 GeV^2. The values of the correction are shown in Figs. 2, 3 as the grey areas (dark for $\langle Q^2\rangle=3$ GeV^2 and light for $\langle Q^2\rangle=10$ GeV^2 in Fig. 2 and dark for $\langle Q^2\rangle=10$ GeV^2 in Fig. 2).

The next figures show the function $xg_1^p(x,Q^2)$ calculated (E143 data were used) at $Q^2=3$ GeV^2 and 10 GeV^2 with an original and Q^2 corrected values of the asymmetry (see Figs. 4, 5). The functions g_1 were defined as

$$g_1^m(x,Q^2)=A_1^{\rm meas}(x)F_1(x,Q^2)\left(1+\frac{4M^2x^2}{Q^2}\right),$$

$$g_1^c(x,Q^2)=A_1^{\rm corr}(x)F_1(x,Q^2)\left(1+\frac{4M^2x^2}{Q^2}\right),$$

and $A_1^{\rm meas,corr}$ correspond to original and Q^2-evolved values of the asymmetry, respectively.

Figure 6 shows the same calculation at $Q^2=10$ GeV^2 produced for SMC data. The main uncertainty of our method came from the condition (3)—the asymmetry A_1 has no x dependence into the bin. Figure 7 shows the x dependence of this source of the uncertainty. As it is seen from the Figs. 3–6 we do not evolve data below $x=0.04$ where the uncertainy of the method is greater than 30%.

Finally, we estimate the influence of the evolution procedure to the first moment value Γ_1^p. To do this, we calculate the values of the spin-dependent structure function $g_1^p(x,\langle Q^2\rangle)$, using the corrected values of the asymmetry $A_1(x,\langle Q^2\rangle)$ and parametrizations of $F_2(x,\langle Q^2\rangle)$ and $R(x,\langle Q^2\rangle)$. For SMC data [3] obtained results are shown in Fig. 6 for $\langle Q^2\rangle\equiv 10$ GeV^2. To calculate the integral at $x\leqslant 0.05$ we suggest that the measured asymmetry is Q^2 independent at the x range. To get the integral for experimentally unmeasured small x region ($x\leqslant 0.003$) we use the estimation by SMC [3]. The first moment values calculated in these suggestions are

$$\Gamma_1^p(10\ \ {\rm GeV}^2)=\int_0^1 g_1(x,10\ \ {\rm GeV}^2)dx=0.125\pm 0.002,$$

which differs from the SMC published result to $\Delta\Gamma=-0.006$ ± 0.002. Presented uncertainty (± 0.002) is caused by the method.

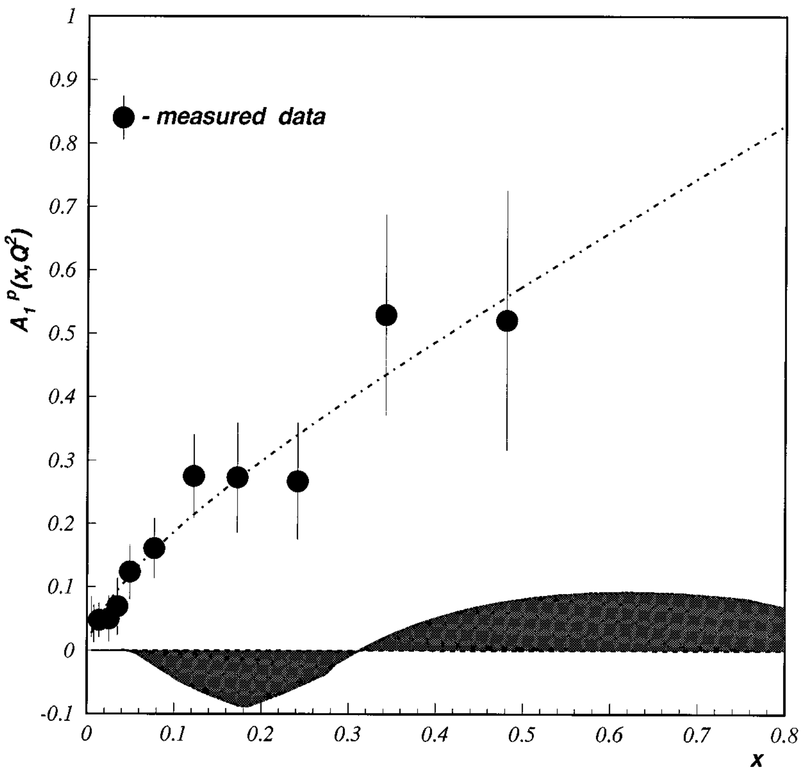

FIG. 3. SMC measured data on the asymmetry A_1 as a function of x and their parametrization. Variations of the asymmetry caused by the Q^2 evolution procedure shown as a dark area (Q^2=10 GeV^2).

For E143 data the Q^2 evolution illustrated by Figs. 4, 5 leads to the changes $\Delta\Gamma=0.003\pm0.001$ and $\Delta\Gamma=-0.002\pm0.001$ for the first moments $\Gamma_1^p(3\ \mathrm{GeV}^2)$ and $\Gamma_1^p(10\ \mathrm{GeV}^2)$, respectively.

V. CONCLUSION

Based on these results we can conclude that the Q^2 dependence of the asymmetry $A_1(x,Q^2)$ [and hence SF

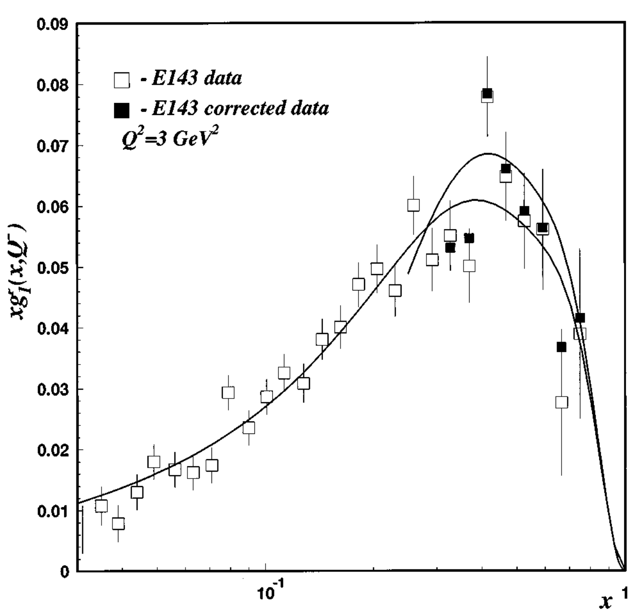

FIG. 4. E143 data. Structure function $xg_1^p(x)$ calculated at Q^2=3 GeV^2 in the assumption A_1 has no Q^2 dependence (shown as open points) and with the asymmetry evolved by our method (closed points). Curves correspond to $xg_1^p(x)$ (at Q^2=3 GeV^2) calculated with initial and Q^2-evolved parametrizations of the asymmetry.

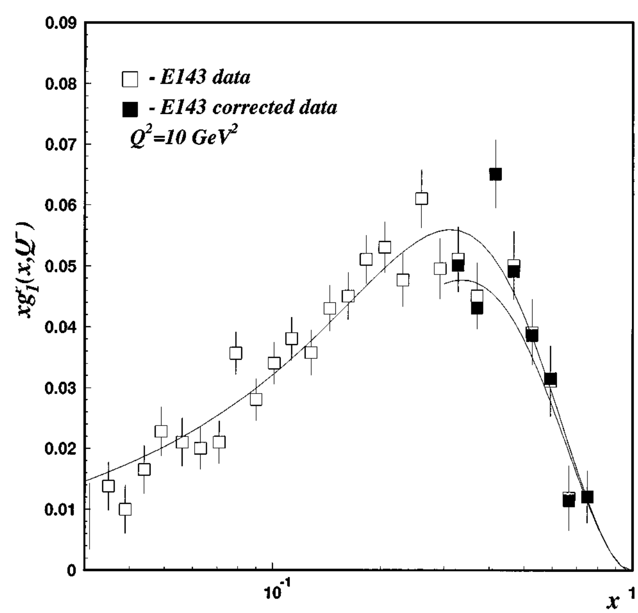

FIG. 5. E143 data. Structure function $xg_1^p(x)$ calculated at Q^2=10 GeV^2 in the assumption A_1 has no Q^2 dependence (shown as open points) and with the asymmetry evolved by our method (closed points). Curves correspond to $xg_1^p(x)$ (at Q^2=10 GeV^2) calculated with initial and Q^2-evolved parametrizations of the asymmetry.

$g_1(x,Q^2)$ itself] is not negligible, essentially at large x values. It is important to add the Q^2 evolution procedure to data analysis in view of the forthcoming more precise measurement and to compare experimental data measured in different $x-Q^2$ kinematical ranges.

Notice that the corrections caused by the Q^2 evolution of the $A_1(x,Q^2)$ are opposite in sign at large and intermediate x. Thus the value of Γ_1^p does not change a lot when the Q^2 evolution of the $A_1(x,Q^2)$ is incorporated.

We have compared our results with other predictions [19–21] and have found the results to be in qualitative agreement in the discussed kinematical range.

Here we would like to note that the addition of NLO corrections at the intermediate x values does not change the results of the present analysis too much. In our approach these extra contributions are connected basically with gluon distribution, because NLO corrections to quark distributions are taken into account automatically by using NLO representation of $dF_1/d\ln Q^2$ [see the RHS of Eqs. (A1,B1b)]. The nonpolarized gluon distribution does not effect to $g_1(Q^2)$ [it contributes only to $A_1(x,Q^2)$] and hence to $\Gamma_1(Q^2)$. The effect of polarized gluon distribution is small (see, for example, [20]) in the considered x range and will be essential only at smaller x values (see [21–23]). The small x range has become the subject of intensive study (see [17] for a recent review of the situation in the spin-dependent case), but it is beyond the scope of the present analysis.

Note added. After completion of this study we became aware of Ref. [24], where the Q^2 dependence of asymmetry $A_1(x,Q^2)$ was studied. Our results are in qualitative agreement with the SLAC analysis.

ACKNOWLEDGMENTS

This work was partially supported by the Russian Fund for the Fundamental Investigations (95-02-04314-a).

APPENDIX A

In the following we briefly resume how evolution equation (10) was obtained. We consider each x bin separately starting with the last one and evolve the asymmetry value $A_{1_{\mathrm{bin}_k}}(x_{\mathrm{bin}_k},Q^2_{\mathrm{bin}_k})$ based on Eqs. (2,3,4).

$x\in[x_I,1.0]$:

$$\frac{dA_{1_{I+1}}(Q^2)}{d\ln(Q^2)}=\frac{-1}{2F_1(x,Q^2)}\int_{x_I}^1\frac{dy}{y}\,\gamma\left(\frac{x}{y}\right)[A_{1_{I+1}}(Q^2)-A_{1_{I+1}}(Q^2)]F_1(y,Q^2)\equiv 0,$$

that derives to $A_{1_{I+1}}=\mathrm{const}$, i.e., $A_1(x,Q^2)$ [in agreement with [20,21] and Eq. (4)] is not evolving at $x\to 1$.

$x\in]x_{I-1},x_I]$:

$$\frac{dA_{1_I}(Q^2)}{d\ln(Q^2)}=\frac{-1}{2F_1(x,Q^2)}\int_{x_I}^1\frac{dy}{y}\,\gamma\left(\frac{x}{y}\right)\times[A_{1_{I+1}}-A_{1_I}(Q^2)]F_1(y,Q^2)$$
$$=[\text{and using Eq. (1a)}]$$
$$=[A_{1_{I+1}}-A_{1_I}(Q^2)]\,\frac{dF_1(x_I,Q^2)}{d\ln Q^2}\,\frac{1}{F_1(x,Q^2)}. \tag{A1}$$

Note that Eq. (A1) has no Mellin convolution. This very important result allows us to evaluate the asymmetry value without AD.

To be in accordance with condition (3) we have to fix the x position into the bin and evolve the asymmetry at this fixed x point. In agreement with our suggestion [see Eqs. (2,3,4)] that x bin is small enough, we can choose the most convenient x point for the analysis. It is $\langle x\rangle_{]x_{I-1},x_I]}=x_I$. Then

$$\ln|A_{1_I}(Q^2)-A_{I+1}|=-\ln F_1(x_I,Q^2)+\mathrm{const}$$

and Eq. (A1) is simplified as

$$A_{1_I}(Q^2)=A_{1_{I+1}}+(A_{1_I}(Q_I^2)-A_{1_{I+1}})\,\frac{F_1(x_I,Q_I^2)}{F_1(x_I,Q^2)}.$$

$x\in]x_{I-2},x_{I-1}]$:

$$F_1(x,Q^2)\,\frac{dA_{1_{I-1}}(Q^2)}{d\ln(Q^2)}=-\frac{1}{2}\left\{\int_{x_{I-1}}^{x_I}\frac{dy}{y}\,\gamma\left(\frac{x}{y}\right)[A_{1_I}(Q^2)-A_{1_{I-1}}(Q^2)]F_1(y,Q^2)+\int_{x_I}^1\frac{dy}{y}\,\gamma\left(\frac{x}{y}\right)[A_{1_{I+1}}-A_{1_{I-1}}(Q^2)]F_1(y,Q^2)\right\}$$
$$=[A_{1_I}(Q^2)-A_{1_{I-1}}(Q^2)]\left(\frac{dF_1(x_{I-1},Q^2)}{d\ln(Q^2)}-\frac{dF_1(x_1,Q^2)}{d\ln(Q^2)}\right)+[A_{1_{I+1}}-A_{1_{I-1}}(Q^2)]\,\frac{dF_1(x_I,Q^2)}{d\ln(Q^2)}$$
$$=[A_{1_{I+1}}-A_{1_{I-1}}(Q^2)]\,\frac{dF_1(x_{I-1},Q^2)}{d\ln(Q^2)}+[A_{1_{I+1}}-A_{1_I}(Q^2)]\,\frac{d}{d\ln Q^2}\,[F_1(x_I,Q^2)-F_1(x_{I-1},Q^2)].$$

The Mellin convolution disappears again. The solution at fixed $x=x_{I-1}$ is

$$[A_{1_{I-1}}(Q^2)-A_{1_{I+1}}]=\frac{F_1(x_{I-1},Q^2_{I-1})}{F_1(x_{I-1},Q^2)}\left\{[A_{1_{I-1}}(Q^2_{I-1})-A_{1_{I+1}}]-\int_{\ln Q^2_{I-1}}^{\ln Q^2}dt\,\frac{d[F_1(x_I,t)-F_1(x_{I-1},t)]/dt}{F_1(x_{I-1},Q^2_{I-1})}[A_{1_I}(t)-A_{1_{I+1}}]\right\}. \tag{A2}$$

$x\in]x_{i-1},x_i]$.
Looking through the bins step by step, we have found the form of the equation which describes the variation of $A_1(x,Q^2)$ as a function of Q^2 in any x bin:

$$\begin{aligned}F_1(x,Q^2)\frac{dA_{1_i}(x,Q^2)}{d\ln Q^2}&=-\frac{1}{2}\left\{\int_{x_i}^{x_{i+1}}\frac{dy}{y}\gamma\left(\frac{x}{y}\right)[A_{1_{i+1}}(Q^2)-A_{1_i}(Q^2)]F_1(y,Q^2)+\cdots+\int_{x_{I-1}}^{x_I}\frac{dy}{y}\gamma\left(\frac{x}{y}\right)[A_{1_I}(Q^2)\right.\\&\left.\quad-A_{1_i}(Q^2)]F_1(y,Q^2)+\int_{x_I}^{1}\frac{dy}{y}\gamma\left(\frac{x}{y}\right)[A_{1_{I+1}}-A_{1_i}(Q^2)]F_1(y,Q^2)\right\}\\&=[A_{1_{i+1}}(Q^2)-A_{1_i}(Q^2)]\frac{d}{d\ln Q^2}[F_1(x_i,Q^2)-F_1(x_{i+1},Q^2)]+\cdots+[A_{1_I}(Q^2)\\&\quad-A_{1_i}(Q^2)]\frac{d}{d\ln Q^2}[F_1(x_{I-1},Q^2)-F_1(x_I,Q^2)]+[A_{1_{I+1}}-A_{1_i}(Q^2)]\frac{d}{d\ln Q^2}F_1(x_I,Q^2)\\&=[A_{1_{I+1}}(Q^2)-A_{1_i}(Q^2)]\frac{d}{d\ln Q^2}F_1(x_i,Q^2)+\sum_{k=i+1}^{I}[A_{1_k}(t)-A_{1_{I+1}}]\frac{d}{d\ln Q^2}[F_1(x_{k-1},Q^2)\\&\quad-F_1(x_k,Q^2)]\}.\end{aligned} \tag{A3}$$

By analogy with the previous the solution of Eq. (A3) can be written at $x=x_i$ as

$$[A_{1_i}(Q^2)-A_{1_{I+1}}]=\frac{F_1(x_i,Q_i^2)}{F_1(x_i,Q^2)}\left\{[A_{1_i}(Q_i^2)-A_{1_{I+1}}]+\int_{\ln Q_i^2}^{\ln Q^2}dt\sum_{k=i+1}^{I-1}[A_{1_k}(t)-A_{1_{I+1}}]\frac{d(F_1(x_{k-1},t)-F_1(x_k,t))/dt}{F_1(x_i,Q_i^2)}\right\}. \tag{A4}$$

APPENDIX B

Here we briefly consider how evolution equation (16) containing gluons was obtained. By analogy with Appendix A we consider each x bin separately starting with the next to last one and evolve the asymmetry value $A_{1_{\text{bin}_k}}(x_{\text{bin}_k},Q^2_{\text{bin}_k})$ based on Eqs. (12,3,4).
$x\in]x_{I-1},x_I]$:

$$\begin{aligned}F_1(x,Q^2)\frac{dA_{1_I}(Q^2)}{d\ln(Q^2)}&=\frac{-\alpha}{2}\left[\int_{x_I}^{1}\frac{dy}{y}\gamma^{(0)}_{\phi\phi}\left(\frac{x}{y}\right)[A_{1_{I+1}}-A_{1_I}(Q^2)]F_1(y,Q^2)+\int_x^1\left(\frac{dy}{y}\overline{\gamma^{(0)}_{\phi g}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)-A_{1_I}(Q^2)G(y,Q^2)\right)\right]\\&=[\text{and using Eq. (11a)}]=[A_{1_{I+1}}-A_{1_I}(Q^2)]\frac{dF_1(x_I,Q^2)}{d\ln Q^2}-\frac{\alpha}{2}\left[\int_x^1\frac{dy}{y}\overline{\gamma^{(0)}_{\phi g}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)\right.\\&\quad\left.-A_{1_{I+1}}\int_{x_I}^1\frac{dy}{y}\gamma^{(0)}_{\phi g}\left(\frac{x}{y}\right)G(y,Q^2)-A_{1_I}(Q^2)\int_x^{x_I}\frac{dy}{y}\gamma^{(0)}_{\phi g}\left(\frac{x}{y}\right)G(y,Q^2)\right].\end{aligned} \tag{B1a}$$

Using our experience from Appendix A, below we evolve the asymmetry in point x_i for every $]x_{i-1},x_i]$ bin. We put $x=x_I$ in Eq. (B1a) and have

$$F_1(x,Q^2)\frac{dA_{1_I}(Q^2)}{d\ln(Q^2)}=[A_{1_{I+1}}-A_{1_I}(Q^2)]\frac{dF_1(x_I,Q^2)}{d\ln Q^2}-\frac{\alpha}{2}\int_{x_I}^1\frac{dy}{y}\left[\overline{\gamma^{(0)}_{\phi g}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)-A_{1_{I+1}}\gamma^{(0)}_{\phi g}\left(\frac{x}{y}\right)G(y,Q^2)\right]. \tag{B1b}$$

The solution of Eq. (B1b) may be represented in the form

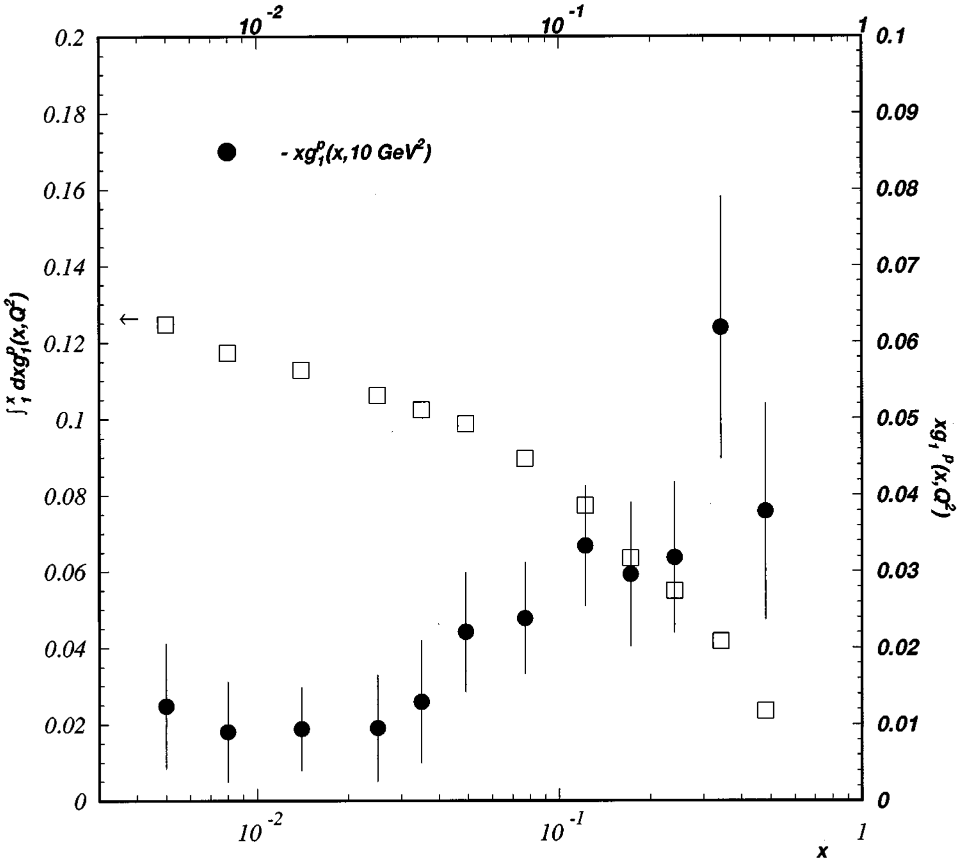

FIG. 6. SMC data. Structure function $xg_1^p(x)$ calculated at $Q^2=10$ GeV^2 with the asymmetry evolved by our method (shown as closed points). The value of $\int_1^x dxg_1(x)$ is shown as the open points.

$$A_{1_I}(Q^2)=A_{1_{I+1}}+(A_{1_I}(Q_I^2)-A_{1_{I+1}})\,\frac{F_1(x_I,Q_I^2)}{F_1(x_I,Q^2)}-\frac{1}{F_1(x_I,Q^2)}\int_{\ln Q_I^2/\Lambda^2}^{\ln Q^2/\Lambda^2}dt\,\frac{\alpha(t)}{2}\int_{x_I}^{1}\frac{dy}{y}\biggl[\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,t)$$
$$-A_{1_{I+1}}\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,t). \quad \text{(B1)}$$

$x\in\,]x_{I-2},x_{I-1}]$:

$$F_1(x,Q^2)\,\frac{dA_{1_{I-1}}(Q^2)}{d\ln(Q^2)}=-\frac{\alpha}{2}\Biggl\{\int_{x_{I-1}}^{x_I}\frac{dy}{y}\,\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)[A_{1_I}(Q^2)-A_{1_{I-1}}(Q^2)]F_1(y,Q^2)+\int_{x_I}^{1}\frac{dy}{y}\,\gamma_{\phi\phi}^{(0)}\left(\frac{x}{y}\right)[A_{1_{I+1}}$$
$$-A_{1_{I-1}}(Q^2)]F_1(y,Q^2)+\int_{x_{I-1}}^{1}\frac{dy}{y}\left(\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)-\int_{x_{I-1}}^{1}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)A_1(y,Q^2)G(y,Q^2)\right)\Biggr\}$$
$$=[A_{1_I}(Q^2)-A_{1_{I-1}}(Q^2)]\left(\frac{dF_1(x_{I-1},Q^2)}{d\ln(Q^2)}-\frac{dF_1(x_I,Q^2)}{d\ln(Q^2)}\right)+[A_{1_{I+1}}-A_{1_{I-1}}(Q^2)]\,\frac{dF_1(x_I,Q^2)}{d\ln(Q^2)}$$
$$-\frac{\alpha}{2}\Biggl[\int_{x_{I-1}}^{1}\frac{dy}{y}\,\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)-A_{1_{I+1}}\int_{x_I}^{1}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2)$$
$$-A_{1_I}(Q^2)\int_{x_{I-1}}^{x_I}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2)\Biggr]$$
$$=[A_{1_{I+1}}-A_{1_{I-1}}(Q^2)]\,\frac{dF_1(x_{I-1},Q^2)}{d\ln(Q^2)}$$
$$+[A_{1_{I+1}}-A_{1_I}(Q^2)]\,\frac{d}{d\ln Q^2}\,[F_1(x_I,Q^2)-F_1(x_{I-1},Q^2)]-\frac{\alpha}{2}\Biggl[\int_{x_{I-1}}^{1}\frac{dy}{y}\,\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,Q^2)$$
$$-A_{1_{I+1}}\int_{x_I}^{1}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2)-A_{1_I}(Q^2)\int_{x_{I-1}}^{x_I}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,Q^2).$$

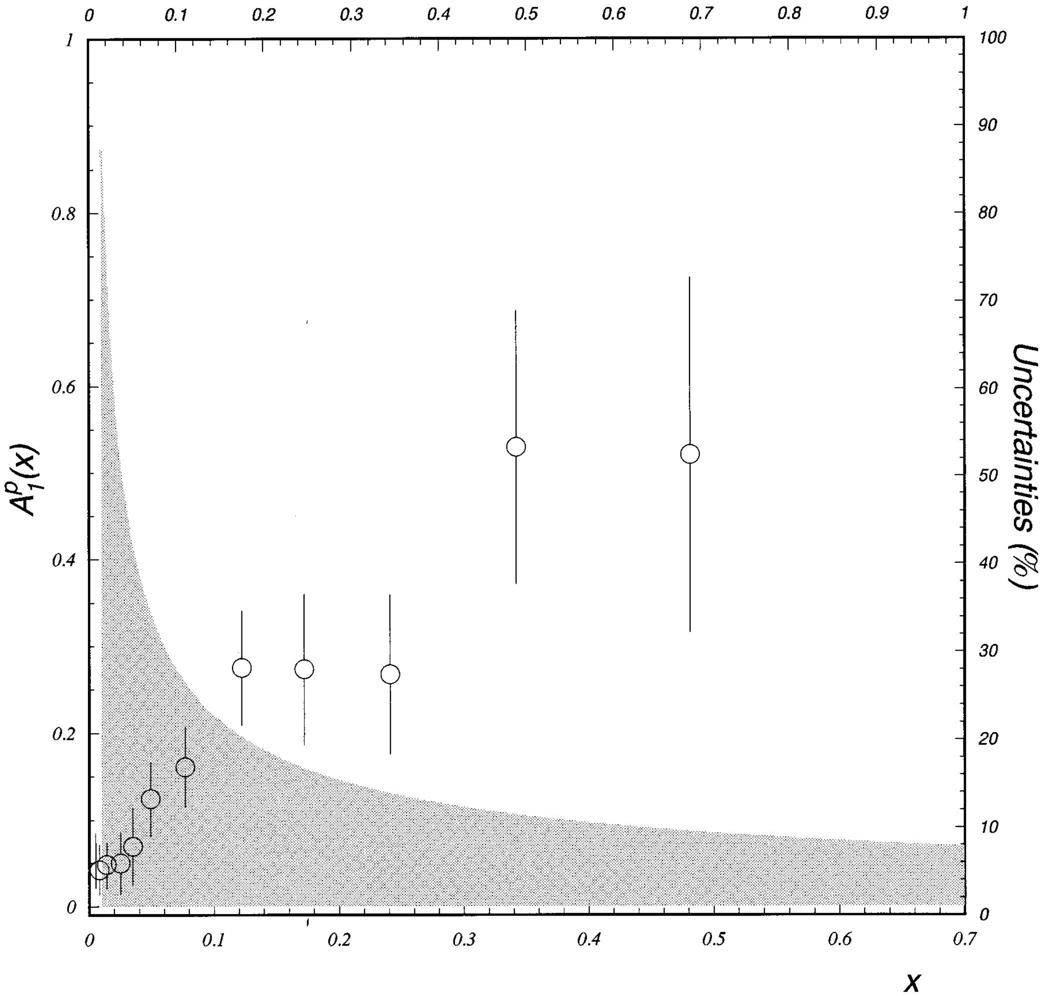

FIG. 7. The uncertainty of our method caused by the condition (3).

Analogously to the previous case we have the general solution in the form

$$[A_{1_{I-1}}(Q^2)-A_{1_{I+1}}]=\frac{F_1(x_{I-1},Q^2_{I-1})}{F_1(x_{I-1},Q^2)}\left\{[A_{1_{I-1}}(Q^2_{I-1})-A_{1_{I+1}}]-\int_{\ln Q^2_{I-1}}^{\ln Q^2}dt\,\frac{d[F_1(x_I,t)-F_1(x_{I-1},t)]/dt}{F_1(x_{I-1},Q^2_{I-1})}[A_{1_I}(t)-A_{1_{I+1}}]\right\}$$
$$-\frac{1}{F_1(x_{I-1},Q^2)}\int_{\ln Q^2_{I-1}/\Lambda^2}^{\ln Q^2/\Lambda^2}dt\,\frac{\alpha(t)}{2}\left[\int_{x_{I-1}}^{1}\frac{dy}{y}\,\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,t)\right.$$
$$\left.-\sum_{l=I}^{I+1}A_{1_l}(Q^2)\int_{x_l-1}^{x_l}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,t).\right. \tag{B2}$$

$x\in\,]x_{i-1},x_i]$.

By analogy with the priliminary steps and the analysis of Appendix A, we can easily obtain the solution for $A_i(Q^2)$:

$$[A_{1_i}(Q^2)-A_{1_{I+1}}]=\frac{F_1(x_i,Q_i^2)}{F_1(x_i,Q^2)}\left\{[A_{1_i}(Q_i^2)-A_{1_{I+1}}]+\int_{\ln Q_i^2}^{\ln Q^2}dt\sum_{k=i+1}^{I}[A_{1_k}(t)-A_{1_{I+1}}]\,\frac{d(F_1(x_{k-1},t)-F_1(x_k,t))/dt}{F_1(x_i,Q_i^2)}\right.$$
$$-\frac{1}{F_1(x_i,Q^2)}\int_{\ln Q_i^2/\Lambda^2}^{\ln Q^2/\Lambda^2}dt\,\frac{\alpha(t)}{2}\left[\int_{x_i}^{1}\frac{dy}{y}\,\overline{\gamma_{\phi g}^{(0)}}\left(\frac{x}{y}\right)\Delta G(y,t)\right.$$
$$\left.\left.-\sum_{k=i+1}^{I+1}A_{1_k}(Q^2)\int_{x_k-1}^{x_k}\frac{dy}{y}\,\gamma_{\phi g}^{(0)}\left(\frac{x}{y}\right)G(y,t)\right]\right\}. \tag{B3}$$

[1] EM Collaboration, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).
[2] SLAC-E80 Collaboration, M. J. Alguard *et al.*, Phys. Rev. Lett. **37**, 1261 (1976); **41**, 70 (1978); SLAC-E130 Collaboration, G. Baum *et al.*, *ibid.* **51**, 1135 (1983).
[3] SM Collaboration, B. Adeva *et al.*, Phys. Lett. B **302**, 533 (1993).
[4] SLAC-E142 Collaboration, P. L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993).
[5] SLAC-E143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995).
[6] J. Ellis and M. Karliner, Phys. Lett. B **313**, 131 (1993).
[7] F. E. Close, Report No. RAL-93-034, 1994 (unpublished); F. E. Close and R. G. Roberts, Phys. Lett. B **316**, 165 (1993).
[8] D. I. Gross, Phys. Rev. Lett. **32**, 1071 (1974); D. I. Gross and S. B. Treiman, *ibid.* **32**, 1145 (1974); F. Martin, Phys. Rev. D **19**, 1382 (1979); C. Lopez and F. J. Yndurain, Nucl. Phys. **B171**, 231 (1980).
[9] C. Lopez and F. J. Yndurain, Nucl. Phys. **B183**, 157 (1981); A. V. Kotikov, Report No. E2-88-422, 1988 (unpublished).
[10] J. Kodaira *et al.*, Phys. Rev. D **20**, 627 (1979); Nucl. Phys. **B159**, 99 (1979).
[11] M. Anselmino, A. Efremov, and E. Leader, Phys. Rep. **261**, 1 (1995); J. Ellis and M. Karliner, Report No. CERN-TH.279/95 and TAUP-2297-95, 1995 (unpublished); B. L. Ioffe, *The Nucleon Spin Structure*, The Proceeding of Quarks-94 (World Scientific, Singapore, 1995), p. 14.
[12] R. Merting and W. L. van Neervan, Z. Phys. C **70**, 637 (1996).
[13] D. A. Ross and C. T. Sachrajda, Nucl. Phys. **B149**, 497 (1979).
[14] NM Collaboration, P. Amaudruz *et al.*, Phys. Lett. B **295**, 159 (1992).
[15] SLAC Collaboration, L. W. Whitlow *et al.*, Phys. Lett. B **250**, 193 (1990).
[16] D. W. Duke and J. F. Owens Phys. Rev. D **30**, 49 (1984).
[17] S. Forte, Report No. CERN-Th/95-305 (hep-ph/9511345) (unpublished); R. D. Ball, Report No. Edinburg 95/558 (hep-ph/9511330) (unpublished); T. Morii, S. Tanaka, and T. Yamanishi, Report No. KOBE-FHD-95-05 (hep-ph/9508231) (unpublished); T. Gehrmann and W. J. Stirling, Report No. DTP/95/78 (hep-ph/9510243) (unpublished); Phys. Rev. D **53**, 6100 (1996).
[18] S. J. Brodsky, M. Burkardt, and I. Schmidt, Nucl. Phys. **441**, 197 (1995).
[19] A. V. Kotikov and D. V. Peshekhonov, Report No. ENSLAPP-A-499/94, 1994 (unpublished).
[20] G. Altarelli, P. Nason, and G. Ridolfi, Phys. Lett. B **320**, 152 (1994); T. Gehrmann and W. J. Stirling, Z. Phys. C **65**, 470 (1995); M. Gluck, E. Reya, and W. Vogelsang, Phys. Lett. B **359**, 210 (1995).
[21] M. Gluck *et al.*, Phys. Rev. D **53**, 4775 (1996).
[22] R. D. Ball, S. Forte, and G. Ridolfi, Reports No. CERN-Th/95-266, No. Edinburg 95/556, and No. GeF-TH-9/95 (hep-ph/9510449) (unpublished); T. Gehrmann and W. J. Stirling, Phys. Lett. B **365**, 347 (1996).
[23] R. D. Ball, S. Forte, and G. Ridolfi, Nucl. Phys. **B444**, 287 (1995); **B449**, 680(E) (1995).
[24] SLAC-E143 Collaboration, K. Abe *et al.*, Phys. Lett. B **364**, 61 (1995).