COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Scattering of very light charged particles

J. C. Taylor
DAMTP, Cambridge University, Silver Street, Cambridge, United Kingdom
(Received 23 October 1995)

I advance arguments against the view that the Lee-Nauenberg-Kinoshita theorem is relevant in practice to the scattering of charged particles as their mass tends to zero. [S0556-2821(96)01216-7]

PACS number(s): 11.55.Bq, 11.10.Jj, 11.15.Bt

Scattering cross sections for reactions involving, in the initial state, a charged particle of mass m contain (in perturbation theory) $\ln(m)$ terms, which become large as $m\to 0$. These logarithms come from virtual photons which are nearly parallel to the charged particle. In 1964, following earlier work of Kinoshita [1], Lee, and Nauenberg (KLN) [1] proved that, by summing over an appropriate set of initial states, the $\ln(m)$ terms could be removed. Since then it seems not to have been generally agreed whether the KLN theorem (as regards initial states) is relevant to physically realistic situations. (For further references, see [2].)

In 1992, Contopanagos and Einhorn (CE) [2] published a paper which, amongst many other things, seemed to claim with some certainty that KLN initial states sum do represent physical reality. They went further and studied qualitatively the parameters necessary to characterize the KLN initial states in a typical physical, realistic situation. This has the great virtue of focusing the argument in a concrete way.

I remain unconvinced of the physical relevance of the KLN theorem for initial states, and, lest the very thorough CE paper should be thought to close the argument, I write this work to emphasize the questions which, to my mind, remain. I do not claim any complete understanding, but I hope this work may at any rate provoke further discussion.

In QED, soft divergences are well understood in the Bloch-Nordsiek theory, and also coherent state theory is applicable. Further, the application of the KLN theorem to final-state collinear divergences is uncontroversial. So I concentrate on initial-state collinear divergences in QED, and just ignore soft divergences. (Alternatively, one could use as a model ϕ^3 theory in six dimensions, which has collinear but not soft divergences.)

CE does make a very definite claim. In the second paragraph of the paper they state ''we shall show that the requisite initial-state sum does inevitably occur in massless theories.'' The authors are aware of the surprising nature of this claim. In Sec. IV they say: ''The equality displayed in Eq. (4.1) requires a specific relative weighting among degenerate initial states, viz., the same phase space normalizations that apply to final states. While this relation is an indisputable mathematical fact, it carries the paradoxical implication that initial-state degeneracy is to be associated with a certain relative weight between, say, an incoming single electron of definite energy and an electron of much lower energy accompanied by a hard but nearly collinear photon. This conflicts with the intuitive notion of an electron beam as well as the idea that one may prepare arbitrary linear combinations of states in Hilbert space. A complete resolution of this paradox requires a more careful analysis of the measurement process. While we have not carried out such a study, we believe it would show that . . . .'' The general tone of the CE paper, however, is one of great confidence in physical relevance of the initial-state sums.

Something very strong is being claimed: that whatever the physical situation, collider or fixed target, etc., there will be some KLN initial-state (with some choice of parameters) which corresponds to it.

I begin by briefly summarizing CE's formulation, which has the advantage of being rather concrete. In the spirit of CE, I will usually consider an ''electron'' of very small mass m, rather than a truly massless one. This avoids mathematical questions about Hilbert spaces.

CE begin by observing that, in the $m=0$ limit, the usual Mǿller operators

$$\Omega^{(\pm)}_{H,H_0}=\lim_{t\to\pm\infty} e^{iHt}e^{-iH_0t} \tag{1}$$

do not exist, and so neither does the usual S matrix

$$S=[\Omega^{(-)}_{H,H_0}]^{\dagger}\Omega^{(+)}_{H,H_0}. \tag{2}$$

They therefore construct an ''asymptotic Hamiltonian'' $H_A(\Delta)$, which consists of H_0 and parts of the interaction $(H-H_0)$ involving degenerate initial and final states. For example, $H_A(\Delta)$ might contain those parts of $(H-H_0)$ describing the interaction of an electron and a photon with an opening angle less than Δ. (Actually, CE allow for different Δ's for the initial and final states, but for simplicity I will disregard this complication.)

CE then define $\Omega^{(\pm)}_{H,H_A}$ and $\Omega^{(\pm)}_{H_A,H_0}$ analogously to Eq. (1), and define

$$S_A=[\Omega^{(-)}_{H,H_A}]^\dagger\Omega^{(+)}_{H,H_A}=\Omega^{(-)}_{H_A,H_0}S[\Omega^{(+)}_{H_A,H_0}]^\dagger. \tag{3}$$

CE argue that this is a good operator. There are then two questions: what matrix elements of S_A to take, and how to fix Δ. I will discuss these questions in turn.

The obvious matrix elements of S_A to take are those between eigenstates of H_A. But we do not know what these eigenstates are. CE elect to take matrix elements of S_A between eigenstates of H_0 (Fock states). They assert (i) that such matrix elements are insensitive to m, and (ii) that the value of such a matrix element is to some extent independent of the particular Fock states chosen. For example, it makes no difference to replace a one-electron state by a superposition of states containing in addition any number of nearly collinear photons as long as their opening angles are less than Δ.

These two properties, if true, guarantee that Fock state matrix elements of S_A provide a natural and unambiguous prescription for getting results which are insensitive to m. These properties are therefore important for the plausibility of the program.

CE verify property (i) to lowest nontrivial order by explicit calculation. I have no reason to know whether it remains true to higher orders.

Now consider property (ii). Suppose the initial state was of the form

$$|i\rangle=a(\delta)|\mathbf{p}\rangle+\int d^3\mathbf{k}\,b(\delta,\mathbf{k})|\mathbf{p}-\mathbf{k},\mathbf{k}\rangle, \tag{4}$$

where the first term is a one-electron state and the second a superposition of electron-photon states, determined by the function b which is assumed to vanish when the angle between the electron and photon momenta, $\mathbf{p}$, $\mathbf{k}$, exceeds δ where $\delta<\Delta$. The coefficient a is necessary to normalize Eq. (4), and in general will depend logarithmically upon $|\mathbf{p}|/m$ and upon δ.

Consider now a matrix element of the form

$$\langle f|S_A(\Delta)|i\rangle, \tag{5}$$

where the final state $\langle f|$ has a similar form to Eq. (4). The claim in question allows us to ignore the contributions to Eq. (5) from the two-particle states in Eq. (4). This is the step I question.

First I observe that CE seems to neglect the normalization a in Eq. (4). But perhaps this is justified by some elaboration of the theory which is not explicitly stated in CE; so I will ignore this point.

Two reasons are given in CE to believe the claim. The first is an explicit calculation to order e^2 given in Appendix B of CE. I agree that the claim is justified to this order, but I shall argue that this order is special; and so verification to this order gives no evidence about higher orders.

The second justification is given in Appendix C of CE. It goes as follows. From Eq. (3), CE derives

$$S_A=T\exp\left[-i\int_{-\infty}^{\infty}dt\,\widehat{V}_I(t)\right], \tag{6}$$

where

$$\widehat{V}_I=e^{iH_At}V_Ie^{-iH_At}=e^{iH_At}(H-H_A)e^{-iH_At}. \tag{7}$$

Then the argument is simply that V_I, by construction, contains no vertices for the absorption of an electron-photon system with an opening angle less than Δ. But this argument seems to me to ignore the distinction between V_I and $\widehat{V}_I$. To check the claim to order e^2, one requires Eq. (6) to order e only; and to this order $V_I=\widehat{V}_I$. But, to higher orders, there are contributions from terms such as $[H_A,V_I]$ in Eq. (7), and then there are vertices for absorption of electron photon states with opening angles less than Δ.

Now I discuss CE's choice of Δ. For the example of an electron beam in a collider, they say that Δ may be identified (at least in order of magnitude) with r/L, where L is the distance from the final focus to the interaction point, and r is the dimension of the beam spot there. This is thus a purely geometrical region, not involving $\hbar$. But there is an argument which points to a completely different order of magnitude. If the electron propagates for a finite distance L it cannot, by the uncertainty principle, be exactly on shell. We may, in principle, consider the production and interaction of the electron as a single process, in which there is an intermediate electron propagator which is very near its pole. But in the presence of exactly massless photons the propagator probably does not have a pole at $E=(\mathbf{p}^2+m^2)^{1/2}$, but a branch point there. An electron propagating over a distance L probably samples a length of the branch cut of order $\hbar/L$. This would correspond to an opening angle $\Delta\simeq(EL/\hbar)^{-1/2}$.

In an accompanying paper [4], CE analyze ''evanescent'' processes, such as helicity flip of an electron emitting a collinear photon. Although the matrix element is proportional to m, the phase space has a factor $1/m^2$, and so the rate appears to be finite as $m\to 0$. CE claim that KLN initial states will cancel this effect when $m/E<\Delta$. My remarks above apply equally to the order of magnitude of Δ here.

Finally I briefly mention a non-Abelian case. Here there are soft divergences, uncancelled by the final-state Bloch-Nordsieck mechanism, when there are colored particles in the initial state even if these have mass [3]. For example, one could take bb reactions in a hypothetical unconfined world. To remove these soft divergences, the coherent initial states would have to include soft gluons moving in all directions: collinearity with the quarks is not relevant. It is difficult to see how the ''accelerator'' producing the quark beams could also produce coherent gluons converging on the annihilation point from *all* directions.

This example may not be so removed from physics. An extension of the QCD factorization theorem to higher twist would require a meaning to be given to quark cross sections. (The problem only appears at higher twist, because the uncancelled soft divergences are suppressed by a factor E^{-2}.)

I am grateful to J. C. Collins, D. R. T. Jones, and P. V. Landshoff for discussions on the subject of this work, and especially grateful to M. B. Einhorn for his patient explanations about [2].

[1] T. Kinoshita, J. Math. Phys. **3**, 650 (1962); T. D. Lee and M. Nauenberg, Phys. Rev. **133**, B1549 (1964).
[2] H. F. Contopanagos and M. B. Einhorn , Phys. Rev. D **45**, 1291 (1992).
[3] R. Doria, J. Frenkel, and J. C. Taylor, Nucl. Phys. **B168**, 93 (1980); A. Andrasi *et al.*, *ibid.* **B183**, 445 (1981); C. Di'Lieto *et al.*, *ibid.* **B183**, 223 (1981).
[4] H. F. Contopanagos and M. B. Einhorn, Phys. Rev. D **45**, 1322 (1992).