Entropy of nonextreme charged rotating black holes in string theory

Mirjam Cvetič*
School of Natural Science, Institute for Advanced Study, Olden Lane, Princeton, New Jersey 08540

Donam Youm†
Physics Department, Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
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We give the explicit expression for four-dimensional rotating charged black hole solutions of $N=4$ (or $N=8$) superstring vacua, parametrized by the ADM mass, four charges [two electric and two magnetic charges, each arising from a different U(1) gauge factor], and the angular momentum (as well as the asymptotic values of four toroidal moduli of a two-torus and the dilaton-axion field). The explicit form of the thermodynamic entropy is parametrized in a suggestive way as a sum of the product of the ''left-moving'' and the ''right-moving'' terms, which may have an interpretation in terms of the microscopic degrees of freedom of the corresponding D-brane configuration. We also give an analogous parametrization of the thermodynamic entropy for the recently obtained five-dimensional rotating charged black holes parametrized by the ADM mass, three U(1) charges, and two rotational parameters (as well as the asymptotic values of one toroidal modulus and the dilaton). [S0556-2821(96)02716-6]

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I. INTRODUCTION

String theory has reached an exciting stage, where it has now become possible to address the long-standing problems of (quantum) gravity, i.e., the microscopic origin of the black hole entropy and possibly the issues of the black hole information loss for certain classical black hole solutions of effective string theory, whose charges can be identified with the Ramond-Ramond (RR) charges of type II string theory. In this case the black hole configurations can be identified with particular D-brane configurations, whose microscopic degrees can be calculated by applying the ''D-brane technology.''[1] In particular, the microscopic entropy of certain five-dimensional Bogomol'ni-Prasad-Sommerfield-(BPS) saturated static [2–4] and rotating [5] as well as certain four-dimensional BPS-saturated static [6,7] black holes of $N=4$ (or $N=8$) superstring vacua has been calculated in that manner.[2] In addition, the microscopic entropy for certain infinitesimal deviations from the BPS-saturated limit for static [3,11] and rotating [12] five-dimensional black holes has also been provided. A reliable microscopic calculation in terms of the D-brane configurations is possible only in the coupling regime where the classical black hole description is not valid and vice versa [2]. However, the topological arguments [barring (unlikely) phase transitions] for the microscopic calculation and the protection from quantum corrections (because of the large enough supersymmetry) for classical results of the BPS-saturated states allow one to extrapolate [2] the two results in the regime of each other's validity and the agreement between them has been obtained.

Incidentally, the ''D-brane technology'' allows one to calculate the microscopic entropy of certain types of black holes whose explicit (general) classical configurations have been constructed only *a posteriori*. In particular, five-dimensional (generating) solutions parametrized by the Arnowitt-Deser-Misner (ADM) mass, *three* U(1) charges [13], and *two* rotational parameters [14] have been obtained only most recently. Their BPS-saturated limit [10], as well as specific BPS-saturated [2,5] and nonextreme [11,12] solutions with special charge assignments, have also been provided only recently. We should, however, point out that in the case of special charge assignments [along with the subsequent rescaling of the asymptotic value(s) of the scalar field(s)], the global space-time properties of the solution remain the same as in the case of taking all the three charges different.

The situation for classical solutions of four-dimensional black holes of $N=4$ string vacua is somewhat better. General nonextreme static dyonic charged black hole solutions [15,16] were given and their BPS-saturated limit was understood [9], prior to the realization that these solutions have an interpretation as D-brane configurations. In particular, a dyonic BPS-saturated solution with four different charges [17] turns out to have a suitable parametrization in terms of the corresponding D-brane configuration, whose microscopic entropy has just been calculated [6,7].

As for general nonextreme solutions, the classical solutions are believed to suffer from quantum corrections and the microscopic calculation of the entropy need not match the classical result. However, in Ref. [13] the classical entropy of the nonextreme, five-dimensional static black hole solutions with three charges has been written in a suggestive manner, i.e., as a product of three terms, where each of them may have an interpretation in terms of the square root of

*On sabbatic leave from the University of Pennsylvania. Electronic address: cvetic@sns.ias.edu

†On leave from the University of Pennsylvania. Electronic addresses: youm@pupgg.princeton.edu; youm@sns.ias.edu

[1]For a review on D-brane physics, see Ref. [1].

[2]An earlier complementary approach to calculate the microscopic entropy of four-dimensional, BPS-saturated black holes was initiated in Ref. [8] and further elaborated in Refs. [9,10].

numbers of D-brane and anti-D-brane configuration(s). This expression coincides with the microscopic calculations of the entropy in special limits, but may hold [13] in general. For four-dimensional, four-charge solutions the classical entropy is also known [18], and when written in an analogous form it may also have an interpretation in terms of the number of the D-brane configurations, as announced [19] at the end of Ref. [13].

The purpose of this paper is to obtain the explicit form of the thermodynamic black hole entropy for general nonextreme *rotating* charged black hole solutions (of $N=4$ or $N=8$ string vacua) in five as well as four dimensions. For that purpose we present the explicit form of the four-dimensional, nonextreme rotating charged black hole solution, parametrized by the ADM mass, four different U(1) charges, and one rotational parameter. We present the explicit form of the thermodynamic entropy, which is written in a suggestive way as a sum of the ''left-moving'' and the ''right-moving'' terms, that may have an interpretation in terms of the degrees of freedom of the D-brane configuration. We shall also present an analogous expression for the classical entropy of recently constructed nonextreme five-dimensional solution [14] parametrized by the ADM mass, three U(1) charges, and two rotational parameters.

The paper is organized in the following way. In Sec. II the explicit form of the four-dimensional rotating charged solution and the thermodynamic entropy with a discussion of different limits is given. In Sec. III we write down the physical parameters and present an analogous form for the entropy of the five-dimensional rotating solution. In Sec. IV we comment on a potential interpretation of the microscopic entropy.

II. FOUR-DIMENSIONAL ROTATING SOLUTION

We shall present an explicit form of the (generating) solution for the four-charge rotating black hole solution of four-dimensional $N=4$ (or $N=8$) superstring vacua. We choose to parametrize the generating solution in terms of the massless fields of the heterotic string compactified on a six-torus [or Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector of the type IIA string compactified on T^6]. This solution has an equivalent parametrization (because of the string-string duality) in terms of the NS-NS fields of type IIA compactified on $K3\times T^2$ or T-dualized type IIB string. Because of the T duality (or U duality) of the type IIA string, the solutions parametrized in terms of the NS-NS charges have a map onto Ramond-Ramond (RR) charges and thus an interpretation in terms of D-brane configurations.

A. Effective action of heterotic string on tori

For the sake of completeness, we briefly summarize the results of the effective action of toroidally compactified heterotic string in D dimensions, following Refs. [20,21].

The compactification of the extra $(10-D)$ spatial coordinates on a $(10-D)$ torus can be achieved by choosing the following Abelian Kaluza-Klein ansatz for the ten-dimensional metric

$$\hat{G}_{MN}=\begin{pmatrix} e^{a\varphi}g_{\mu\nu}+G_{mn}A_\mu^{(1)m}A_\nu^{(1)n} & A_\mu^{(1)m}G_{mn} \\ A_\nu^{(1)n}G_{mn} & G_{mn} \end{pmatrix}, \quad (1)$$

where $A_\mu^{(1)m}$ $(\mu=0,1,\ldots,D-1;\ m=1,\ldots,10-D)$ are D-dimensional Kaluza-Klein U(1) gauge fields, $\varphi\equiv\hat{\Phi}-\frac{1}{2}\ln\det G_{mn}$ is the D-dimensional dilaton field, and $a\equiv 2/(D-2)$. Then, the affective action is specified by the following massless bosonic fields: the (Einstein-frame) graviton $g_{\mu\nu}$, the dilaton e^{φ}, $(36-2D)$ U(1) gauge fields $\mathcal{A}_\mu^i\equiv(A_\mu^{(1)m},A_{\mu m}^{(2)},A_\mu^{(3)I})$ defined as $A_{\mu m}^{(2)}\equiv\hat{B}_{\mu m}+\hat{B}_{mn}A_\mu^{(1)n}+\frac{1}{2}\hat{A}_m^I A_\mu^{(3)I}$, $A_\mu^{(3)I}\equiv\hat{A}_\mu^I-\hat{A}_m^I A_\mu^{(1)m}$, and the following symmetric $O(10-D,26-D)$ matrix of the scalar fields (moduli):

$$M=\begin{pmatrix} G^{-1} & -G^{-1}C & -G^{-1}a^T \\ -C^TG^{-1} & G+C^TG^{-1}C+a^Ta & C^TG^{-1}a^T+a^T \\ -aG^{-1} & aG^{-1}C+a & I+aG^{-1}a^T \end{pmatrix}, \quad (2)$$

where $G\equiv[\hat{G}_{mn}]$, $C\equiv[\frac{1}{2}\hat{A}_m^{(I)}\hat{A}_n^{(I)}+\hat{B}_{mn}]$, and $a\equiv[\hat{A}_m^I]$ are defined in terms of the internal parts of ten-dimensional fields. Then, the effective D-dimensional effective action takes the form

$$\begin{aligned}\mathcal{L}=\frac{1}{16\pi G_D}\sqrt{-g}\Big[&\mathcal{R}_g-\frac{1}{(D-2)}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \\ &+\frac{1}{8}g^{\mu\nu}\mathrm{Tr}(\partial_\mu ML\partial_\nu ML) \\ &-\frac{1}{12}e^{-2a\varphi}g^{\mu\mu'}g^{\nu\nu'}g^{\rho\rho'}H_{\mu\nu\rho}H_{\mu'\nu'\rho'} \\ &-\frac{1}{4}e^{-a\varphi}g^{\mu\mu'}g^{\nu\nu'}\mathcal{F}^i{}_{\mu\nu}(LML)_{ij}\mathcal{F}^j{}_{\mu'\nu'}\Big], \end{aligned} \quad (3)$$

where $g\equiv\det g_{\mu\nu}$, $\mathcal{R}_g$ is the Ricci scalar of $g_{\mu\nu}$, and $\mathcal{F}_{\mu\nu}^i=\partial_\mu\mathcal{A}_\nu^i-\partial_\nu\mathcal{A}_\mu^i$ are the $\mathrm{U}(1)^{36-2D}$ gauge field strengths.

The D-dimensional effective action (3) is invariant under the $O(10-D,26-D)$ transformations (T duality) [20,21]:

$$\begin{aligned} &M\to\Omega M\Omega^T,\quad \mathcal{A}_\mu^i\to\Omega_{ij}\mathcal{A}_\mu^j,\quad g_{\mu\nu}\to g_{\mu\nu}, \\ &\varphi\to\varphi,\quad B_{\mu\nu}\to B_{\mu\nu}, \end{aligned} \quad (4)$$

where Ω is an $O(10-D,26-D)$-invariant matrix, i.e., with the property

$$\Omega^TL\Omega=L,\quad L=\begin{pmatrix} 0 & I_{10-D} & 0 \\ I_{10-D} & 0 & 0 \\ 0 & 0 & I_{26-D} \end{pmatrix}, \quad (5)$$

where I_n denotes the $n\times n$ identity matrix.

In particular, for $D=4$ the field strength of the one-form field is self-dual and the corresponding equations of motion and Bianchi identities are invariant under the SL(2,R) transformations (S duality) [21]:

$$\begin{aligned} &S\to\frac{aS+b}{cS+d},\quad M\to M,\quad g_{\mu\nu}\to g_{\mu\nu}, \\ &\mathcal{F}^i{}_{\mu\nu}\to(c\Psi+d)\mathcal{F}^i{}_{\mu\nu}+ce^{-2\varphi}(ML)_{ij}\tilde{\mathcal{F}}^j{}_{\mu\nu}, \end{aligned} \quad (6)$$

where $\tilde{\mathcal{F}}^{i\mu\nu}=(1/2\sqrt{-g})\,\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^{i}_{\rho\sigma}$ and $a,b,c,d\in R$ satisfy $ad-bc=1$. Here, $S\equiv\Psi+ie^{-\varphi}$ is a complex scalar field.[3]

At the quantum level, the parameters of both T- and S-duality transformations become integer valued, corresponding to the exact symmetry of the perturbative string theory and the conjectured nonperturbative symmetry of string theory, respectively.

B. Explicit solution

In order to obtain the explicit form of rotating charged solution, we employ the solution-generating technique, by performing symmetry transformations on a known solution. In particular, we perform four SO(1,1)$\subset$ O(8,24) boosts [15] on the four-dimensional Kerr solution, specified by the ADM mass m and the rotational parameter l (angular momentum per unit mass).[4] Here, O(8,24) is a symmetry of the effective three-dimensional action for stationary solutions of toroidally compactified heterotic string [22]. The four boosts $\delta_{e1},\delta_{e2},\ \delta_{p1}$, and δ_{p2} induce two electric $Q_2^{(1),(2)}$ and two magnetic charges $P_1^{(1),(2)}$ of U(1) gauge fields $A_{\mu 2}^{(1),(2)}$ and $A_{\mu 1}^{(1),(2)}$, respectively. The solution obtained in this manner is specified by the ADM mass, *four* U(1) charges, and one rotational parameter.[5] In addition, one can subsequently rescale the asymptotic values of the dilaton-axion field and the four toroidal moduli of two torus, i.e., the scalar fields that vary with spatial direction.

Thus, the starting point is the four-dimensional Kerr solution

$$\begin{aligned} ds^2=&-\frac{r^2+l^2\cos^2\theta-2mr}{r^2+l^2\cos^2\theta}dt^2+\frac{r^2+l^2\cos^2\theta}{r^2+l^2-2mr}dr^2\\ &+(r^2+l^2\cos^2\theta)d\theta^2+\frac{\sin^2\theta}{r^2+l^2\cos^2\theta}\\ &\times[(r^2+l^2)(r^2+l^2\cos^2\theta)+2ml^2r\sin^2\theta]d\phi^2\\ &-\frac{4mlr\sin^2\theta}{r^2+l^2\cos^2\theta}dtd\phi, \end{aligned} \tag{7}$$

where m is its ADM mass and l is the rotational parameter. The explicit sequence of the four boost transformations as well as technical details of relating the fields of the effective three-dimensional action and the four-dimensional fields are detailed in Ref. [15] (see also Ref. [14]).

The final expression of the nonextreme dyonic rotating black hole solution in terms of the (nontrivial) four-dimensional bosonic fields is of the form[6]

$$g_{11}=\frac{(r+2m\sinh^2\delta_{p2})(r+2m\sinh^2\delta_{e2})+l^2\cos^2\theta}{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{e2})+l^2\cos^2\theta},$$

$$g_{12}=\frac{2ml\cos\theta(\sinh\delta_{p1}\cosh\delta_{p2}\sinh\delta_{qe1}\cosh\delta_{e2}-\cosh\delta_{p1}\sinh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2})}{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{e2})+l^2\cos^2\theta},$$

$$g_{22}=\frac{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{e1})+l^2\cos^2\theta}{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{e2})+l^2\cos^2\theta},$$

$$B_{12}=-\frac{2ml\cos\theta(\sinh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2}-\cosh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\cosh\delta_{e2})}{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{e2})+l^2\cos^2\theta},$$

$$e^{\varphi}=\frac{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{p2})+l^2\cos^2\theta}{\Delta^{1/2}},$$

[3]Ψ is the axion which is equivalent to the two-form field $B_{\mu\nu}$ through the duality transformation $H^{\mu\nu\rho}=-(e^{2\varphi}/\sqrt{-g})\varepsilon^{\mu\nu\rho\sigma}\partial_\sigma\Psi$.

[4]Within toroidally compactified heterotic string the approach to obtain charged solutions from the neutral one was spelled out in Ref. [23]. This method was used to obtain, e.g., general rotating, electrically charged solutions in four dimensions [23], higher dimensional general electrically charged static solutions [24], and rotating solutions (with one rotational parameter) [25], as well as the general four-dimensional static dyonic solutions [17,16]. Related techniques were recently used to obtain a class of five-dimensional charged (rotating) solutions [11,12,17].

[5]A subset of T- and S-duality transformations, i.e., [SO(6)$\times$SO(22)]/[SO(4)$\times$SO(20)]$\subset O(6,22)$ and U(1)$\subset$SL(2,R) transformations, respectively, which do not affect the four-dimensional space-time, provides 51 additional charge parameters, which allow for a general solution specified by 28 electric and 28 magnetic charges subject to one constraint. Thus, the generating solution for the most general charged rotating solution should be specified by *five* charge parameters. In part, because of the technical difficulties we postpone the analysis in this case.

[6]The four-dimensional Newton's constant is taken to be $G_N^{D=4}=\frac{1}{8}$ and we follow the convention of, e.g., Ref. [26], for the definitions of the ADM mass, charges, dipole moments, and angular momenta.

$$ds_E^2=\Delta^{1/2}\Big[-\frac{r^2-2mr+l^2\cos^2\theta}{\Delta}dt^2+\frac{dr^2}{r^2-2mr+l^2}+d\theta^2$$
$$+\frac{\sin^2\theta}{\Delta}\{(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{p2})(r+2m\sinh^2\delta_{e1})(r+2m\sinh^2\delta_{e2})$$
$$+l^2(1+\cos^2\theta)r^2+W+2ml^2r\sin^2\theta\}d\phi^2-\frac{4ml}{\Delta}\{(\cosh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\cosh\delta_{e2}$$
$$-\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2})r+2m\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2}\}\sin^2\theta dt d\phi\Big], \tag{8}$$

where

$$\Delta\equiv(r+2m\sinh^2\delta_{p1})(r+2m\sinh^2\delta_{p2})(r+2m\sinh^2\delta_{e1})(r+2m\sinh^2\delta_{e2})+(2l^2r^2+W)\cos^2\theta,$$
$$W\equiv 2ml^2(\sinh^2\delta_{p1}+\sinh^2\delta_{p2}+\sinh^2\delta_{e1}+\sinh^2\delta_{e2})r+4m^2l^2(2\cosh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\cosh\delta_{e2}$$
$$\times\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\sinh\delta_{e2}-2\sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e1}\sinh^2\delta_{e2}-\sinh^2\delta_{p2}\sinh^2\delta_{e1}\sinh^2\delta_{e2}$$
$$-\sinh^2\delta_{p1}\sinh^2\delta_{e1}\sinh^2\delta_{e2}-\sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e2}-\sinh^2\delta_{p1}\sinh^2\delta_{p2}\sinh^2\delta_{e1})+l^4\cos^2\theta. \tag{9}$$

The axion field a also varies with spatial coordinates, but its expression turns out to be cumbersome.

The ADM mass, U(1) charges $Q_2^{(1),(2)}, P_1^{(1),(2)}$, and the angular momentum J, can be expressed in terms of m, l and four boosts in the following way:

$$M=4m(\cosh^2\delta_{e1}+\cosh^2\delta_{e2}+\cosh^2\delta_{p1}+\cosh^2\delta_{p2})-8m,$$
$$Q_2^{(1)}=4m\cosh\delta_{e1}\sinh\delta_{e1},\quad Q_2^{(2)}=4m\cosh\delta_{e2}\sinh\delta_{e2},$$
$$P_1^{(1)}=4m\cosh\delta_{p1}\sinh\delta_{p1},\quad P_1^{(2)}=4m\cosh\delta_{p2}\sinh\delta_{p2},$$
$$J=8lm(\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_{p1}\cosh\delta_{p2}-\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_{p1}\sinh\delta_{p2}). \tag{10}$$

The electric dipole moments $D_1^{(1,2)}$ and the magnetic dipole moments $\mu_2^{(1,2)}$ of the above solution can also be obtained by considering the asymptotic behavior of the solutions near spatial infinity:

$$D_1^{(1)}=-4lm(\sinh\delta_{e1}\sinh\delta_{e2}\cosh\delta_{p1}\sinh\delta_{p2}-\cosh\delta_{e1}\cosh\delta_{e2}\sinh\delta_{p1}\cosh\delta_{p2}),$$
$$D_1^{(2)}=-4lm(\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_{p1}\cosh\delta_{p2}-\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_{p1}\sinh\delta_{p2}),$$
$$\mu_2^{(1)}=4lm(\sinh\delta_{p1}\sinh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2}-\cosh\delta_{p1}\cosh\delta_{p2}\sinh\delta_{e1}\cosh\delta_{e2}),$$
$$\mu_2^{(2)}=4lm(\sinh\delta_{p1}\sinh\delta_{p2}\sinh\delta_{e1}\cosh\delta_{e2}-\cosh\delta_{p1}\cosh\delta_{p2}\cosh\delta_{e1}\sinh\delta_{e2}). \tag{11}$$

The solution corresponds effectively to the six-dimensional target-space background with the four toroidal moduli of two-torus (T^2) and the dilaton-axion field varying with the spatial directions.

In the above expressions the asymptotic values of four moduli of T^2 are taken to have canonical values $g_{11\infty}=g_{22\infty}=1$, $B_{12\infty}=g_{12\infty}=0$, but can be rescaled (along with the physical charges) by an arbitrary O(2,2) constant matrix [cf., Eq. (4)]. Also, the canonical choice of the asymptotic value of the dilaton-axion field, $\varphi_\infty=\Psi_\infty=0$, can be rescaled (along with the physical charges) by an arbitrary SL(2,R) constant matrix [cf. Eq. (6)]. Note, none of these rescaling transformations changes the four-dimensional space-time, only the physical interpretation of the charge parameters in Eq. (10) changes. We will primarily stick to the representation with the canonical choices of the asymptotic values of the scalar fields.[7]

[7]On the other hand, a judicial choice of different asymptotic values of scalar fields is useful (see, e.g., Refs. [11,12]) for calculating reliably the microscopic entropy for the infinitesimal deviations from the BPS-saturated limits.

The solution (8) has the inner r_- and the outer r_+ horizons at

$$r_\pm = m \pm \sqrt{m^2 - l^2}, \tag{12}$$

provided $m \geqslant |l|$. In this case the solution has the global space-time of the Kerr-Newman black hole, i.e., the rotating charged black hole of the Maxwell-Einstein gravity, with the ring singularity at $r = \min\{Q_2^{(1)}, Q_2^{(2)}, P_1^{(1)}, P_1^{(2)}\}$ and $\theta = \pi/2$. When $\delta_{e1} = \delta_{e2} = \delta_{p1} = \delta_{p2}$, i.e., $Q_2^{(1)} = Q_2^{(2)} = P_1^{(1)} = P_1^{(2)}$, all the toroidal moduli and the axion-dilaton field are constant, and thus the solution *is* the Kerr-Newman solution.[8]

The extreme solution, i.e., the case when the inner and the outer horizons coincide, is reached when $m \to |l|^+$. In this case, the global space-time is that of extreme Kerr-Newman solution.

The BPS-saturated limit, i.e., when the configuration saturates the Bogomol'ni bound for the ADM mass, is reached when $m \to 0$, while the charges $Q_2^{(1),(2)}$ and $P_1^{(1),(2)}$, and the angular momentum J are kept finite. For the charges to be constant, the boosts $\delta_{e1,e2,p1,p2} \to \infty$, while keeping $m e^{2\delta_{e1,e2,p1,p2}}$ constant. In order for J to remain *nonzero*, the rotational parameter l has to remain *nonzero*. Thus, the BPS saturated charged solution with the nonzero angular momentum J has a naked singularity,[9] since the constraint $m \geqslant |l|$ is not satisfied. Thus, the existence of the naked singularities in the case of rotating BPS-saturated solution persists even in the case of four nonzero charges.[10]

Thus, the only regular BPS-saturated solution is in this case the static solution (with zero angular momentum $J = 0$, i.e., $l \to 0$). Its global space-time is that of extreme Reissner-Nordström black holes.

C. Entropy of the four-dimensional rotating solution

The thermodynamic (Bekenstein-Hawking) entropy is of the form $S = (1/4G_N)A$, where A is the surface area, $A = \int d\theta d\phi \sqrt{g_{\theta\theta} g_{\phi\phi}}|_{r=r_+}$, determined at the outer horizon $r_+ = m + \sqrt{m^2 - l^2}$. In the case of Eq. (8) we were able to cast the thermodynamic entropy in the form

$$\begin{aligned} S &= 16\pi \left[m^2 \left(\prod_{i=1}^4 \cosh\delta_i + \prod_{i=1}^4 \sinh\delta_i \right) \right. \\ &\quad \left. + m\sqrt{m^2 - l^2} \left(\prod_{i=1}^4 \cosh\delta_i - \prod_{i=1}^4 \sinh\delta_i \right) \right] \\ &= 16\pi \left[m^2 \left(\prod_{i=1}^4 \cosh\delta_i + \prod_{i=1}^4 \sinh\delta_i \right) \right. \\ &\quad \left. + \left\{ m^4 \left(\prod_{i=1}^4 \cosh\delta_i - \prod_{i=1}^4 \sinh\delta_i \right)^2 - J^2 \right\}^{1/2} \right], \end{aligned} \tag{13}$$

where m, l are again the ADM mass and the angular momentum per unit mass of the Kerr solution (7) and $\delta_{1,2,3,4} \equiv \delta_{e1,e2,p1,p2}$ are the four boosts specifying the four charges (10). In the second line the entropy is cast in terms of the angular momentum J of the charged solution.

Even though one expects the thermodynamic entropy to be cast in a form which is a square root of an expression (which depends on charges and angular momentum), it can be written in a form (13) which is a *sum* of two terms, i.e., the sum of the ''left-moving'' and the ''right-moving'' contributions. Each term is *symmetric* in terms of the four boost parameters (and, thus, in terms of the four charges). On the other hand, Eq. (13) bears an asymmetry with respect to the angular momentum, i.e., only the *right-moving* term contains the angular momentum, which acts to reduce the right-moving contribution to the entropy.

When the angular momentum is zero ($J = 0$, i.e., $l = 0$), the entropy is of the form [18][11]

$$S = 32\pi m^2 \prod_{i=1}^4 \cosh\delta_i, \tag{14}$$

which is a generalization of the static five-dimensional result with three charges [13] to the static four-dimensional result with four charges.

In the (regular) BPS-saturated limit ($m \to 0$, $l \to 0$, while $m e^{2\delta_{e1,e2,p1,p2}}$ are kept constant), as well as the extreme limit $m \to |l|^+$, the ''right-moving'' term in Eq. (13) is zero; however, in terms of physical parameters (charges and angular momentum), the entropy is different in each case. In the BPS-saturated limit,

$$S = 32\pi m^2 \prod_{i=1}^4 \cosh\delta_i = 2\pi [P_1^{(1)} P_1^{(2)} Q_2^{(1)} Q_2^{(2)}]^{1/2}, \tag{15}$$

while, in the extreme limit,

$$\begin{aligned} S &= 16\pi m^2 \left(\prod_{i=1}^4 \cosh\delta_i + \prod_{i=1}^4 \sinh\delta_i \right) \\ &= 2\pi [J^2 + P_1^{(1)} P_1^{(2)} Q_2^{(1)} Q_2^{(2)}]^{1/2}. \end{aligned} \tag{16}$$

[8]The case with $\delta_{p1} = \delta_{p2} = 0$ is a generating solution of a general electrically charged rotating solution of Ref. [23]. The case with $\delta_{e1} = \delta_{p1}$, $\delta_{e2} = \delta_{p2}$, i.e., $Q_2^{(1)} = P_1^{(1)}$, $Q_2^{(2)} = P_1^{(2)}$, was recently constructed in Ref. [27].

[9]The conformal two-dimensional σ model, whose target space corresponds to dyonic rotating BPS-saturated solutions, also yields solutions with naked singularities [28].

[10]This fact was also known for the case of four-dimensional, electrically charged solutions [25], whose static BPS-saturated subset has the null horizon. On the other hand, in five dimensions, the charged solution with three nonzero charges has a regular BPS-saturated limit with the nonzero angular momentum [5,10,17]. In six dimensions, the electrically charged, BPS-saturated, rotating solution is also regular [25].

[11]In Ref. [18], the expression for the entropy has a typographical error, i.e., the square root is missing. Also, instead of in terms of boosts, the expression is given in terms of physical charges and the nonextremality parameter $\beta \equiv 2m$.

Note that in the case when the right-moving contribution is absent, both expressions (15) and (16) are independent of the asymptotic values of the moduli and the dilaton coupling and have a straightforward generalization to the manifestly S- and T-duality invariant form [9]. Namely, when expressed in terms of the conserved quantized electric and magnetic charge lattice vectors $\vec{\alpha},\vec{\beta}\in\Lambda_{6,22}$ (of toroidally compactified heterotic string), the surface area can be written as

$$S=2\pi[J^2+\{(\vec{\alpha}^T L\vec{\alpha})(\vec{\beta}^T L\vec{\beta})-(\vec{\alpha}^T L\vec{\beta})^2\}]^{1/2}. \quad (17)$$

III. ENTROPY OF THE FIVE-DIMENSIONAL ROTATING SOLUTION

In this section we write explicitly the entropy for five-dimensional rotating charged black holes specified by the ADM mass, three charges, and two rotational parameters. These solutions were obtained [14] by performing three SO(1,1)⊂O(8,24) boosts, on the five-dimensional (neutral) rotating solution parametrized by the mass m and two rotating parameters l_1 and l_2. The three boosts δ_{e1},δ_{e2}, and δ_e specify, respectively, the two electric charges $Q_1^{(1)},Q_1^{(2)}$ of the two U(1) gauge fields, i.e., the Kaluza-Klein $A_{\mu\,1}^{(1)}$ and the two-form $A_{\mu\,1}^{(2)}$ gauge fields associated with the first compactified direction, and the charge Q, i.e., the electric charge of the vector field, whose field strength is dual to the field strength $H_{\mu\nu\rho}$ of the five-dimensional two-form field $B_{\mu\nu}$.

For the sake of completeness, we quote the result [14] for a space-time metric of the solution

$$\begin{aligned}
ds_E^2=\overline{\Delta}^{1/3}\Bigg[&-\frac{(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta-2m)}{\overline{\Delta}}dt^2+\frac{r^2}{(r^2+l_1^2)(r^2+l_2^2)-2mr^2}dr^2+d\theta^2\\
&+\frac{4m\cos^2\theta\sin^2\theta}{\overline{\Delta}}[l_1l_2\{(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)-2m(\sinh^2\delta_{e1}\sinh^2\delta_{e2}+\sinh^2\delta_e\sinh^2\delta_{e1}+\sinh^2\delta_e\sinh^2\delta_{e2})\}\\
&+2m\{(l_1^2+l_2^2)\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e-2l_1l_2\sinh^2\delta_{e1}\sinh^2\delta_{e2}\sinh^2\delta_e\}]d\phi d\psi\\
&-\frac{4m\sin^2\theta}{\overline{\Delta}}[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e-l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e)\\
&+2ml_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e]d\phi dt-\frac{4m\cos^2\theta}{\overline{\Delta}}[(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&\times(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e-l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e)+2ml_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e]d\psi dt\\
&+\frac{\sin^2\theta}{\overline{\Delta}}[(r^2+2m\sinh^2\delta_e+l_1^2)(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)\\
&+2m\sin^2\theta\{(l_1^2\cosh^2\delta_m-l_2^2\sinh^2\delta_m)(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+4ml_1l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e\\
&-2m\sinh^2\delta_{e1}\sinh^2\delta_{e2}(l_1^2\cosh^2\delta_e+l_2^2\sinh^2\delta_e)-2ml_2^2\sinh^2\delta_e(\sinh^2\delta_{e1}+\sinh^2\delta_{e2})\}]d\phi^2+\frac{\cos^2\theta}{\overline{\Delta}}[(r^2+2m\sinh^2\delta_e\\
&+l_2^2)(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+2m\cos^2\theta\{(l_2^2\cosh^2\delta_e-l_1^2\sinh^2\delta_e)\\
&\times(r^2+l_1^2\cos^2\theta+l_2^2\sin^2\theta)+4ml_1l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e-2m\sinh^2\delta_{e1}\sinh^2\delta_{e2}(l_1^2\sinh^2\delta_e\\
&+l_2^2\cosh^2\delta_e)-2ml_1^2\sinh^2\delta_e(\sinh^2\delta_{e1}+\sinh^2\delta_{e2})\}]d\psi^2\Bigg], \qquad (18)
\end{aligned}$$

where

$$\overline{\Delta}\equiv(r^2+2m\sinh^2\delta_{e1}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_{e2}+l_1^2\cos^2\theta+l_2^2\sin^2\theta)(r^2+2m\sinh^2\delta_e+l_1^2\cos^2\theta+l_2^2\sin^2\theta). \quad (19)$$

The ADM mass M, U(1) charges Q's and the angular momenta $J's$ are given in terms of the three boost parameters $\delta_{e1,e2,e}$ and the three parameters m,l_1,l_2 of the neutral rotating solution as[12]

[12]The five-dimensional Newton's constant is taken to be $G_N^{D=5}=\pi/4$.

$$M=2m(\cosh^2\delta_{e1}+\cosh^2\delta_{e2}+\cosh^2\delta_e)-3m,$$

$$Q_1^{(1)}=2m\cosh\delta_{e1}\sinh\delta_{e1},\quad Q_1^{(2)}=2m\cosh\delta_{e2}\sinh\delta_{e2},\quad Q=2m\cosh\delta_e\sinh\delta_e,$$

$$J_\phi=4m(l_1\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e-l_2\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e),$$

$$J_\psi=4m(l_2\cosh\delta_{e1}\cosh\delta_{e2}\cosh\delta_e-l_1\sinh\delta_{e1}\sinh\delta_{e2}\sinh\delta_e). \tag{20}$$

Note the solution is effectively the six-dimensional solution with the toroidal modulus g_{11} and the dilaton field φ varying with the spatial coordinates [17].

The solution has the inner r_- and the outer r_+ horizons at

$$r_\pm^2=m-\frac{1}{2}l_1^2-\frac{1}{2}l_2^2\pm\frac{1}{2}\sqrt{(l_1^2-l_2^2)^2+4m(m-l_1^2-l_2^2)}, \tag{21}$$

provided $2m\geqslant(|l_1|+|l_2|)^2$.

We write explicitly the classical entropy $S=(1/4G_N)A$, where A is the surface area $A=\int d\theta d\phi d\psi\sqrt{g_{\theta\theta}(g_{\phi\phi}g_{\psi\psi}-g_{\phi\psi}^2)}|_{r=r_+}$, determined at the outer horizon r_+. The entropy can be written in the form

$$\begin{aligned}S&=4\pi\left[m\{2m-(l_1-l_2)^2\}^{1/2}\left(\prod_{i=1}^3\cosh\delta_i+\prod_{i=1}^3\sinh\delta_i\right)+m\{2m-(l_1+l_2)^2\}^{1/2}\left(\prod_{i=1}^3\cosh\delta_i-\prod_{i=1}^3\sinh\delta_i\right)\right]\\&=4\pi\left[\left\{2m^3\left(\prod_{i=1}^3\cosh\delta_i+\prod_{i=1}^3\sinh\delta_i\right)^2-\frac{1}{16}(J_\phi-J_\psi)^2\right\}^{1/2}+\left\{2m^3\left(\prod_{i=1}^3\cosh\delta_i-\prod_{i=1}^3\sinh\delta_i\right)^2-\frac{1}{16}(J_\phi+J_\psi)^2\right\}^{1/2}\right],\end{aligned} \tag{22}$$

where $\delta_{1,2,3}\equiv\delta_{e1,e2,e}$ and m, $l_{1,2}$ are the ADM mass and the two rotational parameters of the five-dimensional rotating (neutral) solution, respectively. In the second line, the entropy is cast in terms of boosts (specifying the three charges) and the two angular momenta $J_{\psi,\phi}$ of the charged solution. Again, the classical entropy can be cast [Eq. (22)] as a *sum* of two terms, i.e., the sum of the ''left-moving'' and the ''right-moving'' contributions.

The form of the entropy, as a sum of the two terms, has been derived in the case of infinitesimal deviations from the BPS-saturated limit in Ref. [12], and its microscopic degrees of freedom were identified with the left- and the right-moving contributions of the D-brane world-volume Hilbert space with $J_{L,R}\equiv\frac{1}{2}(J_\phi\mp J_\psi)$ identified as the left- (or right-) moving charges of the $\mathrm{U}(1)_{L,R}$ $(N=4)$ superconformal (world-sheet) algebra.

Interestingly, even for a general nonextreme solution, the classical entropy (22) retains the form as a sum of two pieces, one containing the ''left-moving'' and another one the ''right-moving'' contributions, thus suggesting that even for a generic nonextreme case the expression may have a microscopic interpretation in terms of degrees arising from two (left-moving and right-moving) noninteracting D-brane world-volume sectors. Note also, that each term is *symmetric* under the permutation of the three boost parameters and thus under the permutation of the three charge assignments.

The (regular) BPS-saturated limit, i.e., the limit where the ADM mass saturates the Bogomol'ni bound, is reached [14] by taking $m\to0$, $l_{1,2}\to0$, and $\delta_{e1,e2,e}\to\infty$, while $Q_{1,2}=\frac{1}{2}me^{\delta_{e1,e2}}$, $Q=\frac{1}{2}me^{\delta}$, and $l_{1,2}/m^{1/2}$ are kept finite. In this case, the right-moving contribution disappears. Interestingly, in the extreme limit [the inner and outer horizons (21) coinciding], which corresponds to the choice $2m\to(l_1+l_2)^2$, the right-moving contributions again disappear; however, the actual value of the entropy in terms of the physical parameters is different from the BPS-saturated limit.

For zero angular momentum, the entropy formula again rearranges itself as a single term [13]:

$$S=8\sqrt{2}\pi m^{3/2}\prod_{i=1}^3\cosh\delta_i, \tag{23}$$

being fully symmetric under permutations of charges. In this case, the microscopic entropy can be calculated in certain limits, but it was pointed out [13] that its validity as a microscopic entropy may be true in general and that each (''dressed'') boost e^{δ_i} $[e^{-\delta_i}]$ may be interpreted as a square root of the number of the corresponding D-brane (anti-D-brane) configurations.

A more general expression for the entropy (22) has a suggestive form indicating that the relevant charge degrees of freedom should be identified with the left- and right-moving (D-brane world-volume) sectors, which appear in combinations $\Pi_{i=1}^3\cosh\delta_i\pm\Pi_{i=1}^3\sinh\delta_i$, respectively.

IV. COMMENTS ON THE *D*-BRANE INTERPRETATION

We obtained the explicit forms of the classical entropy for the four- and five-dimensional rotating charged black hole

solutions with four charges and one rotational parameter, and three charges and two rotational parameters, respectively. These solutions can be viewed as ''generating'' solutions for black holes of $N=4$ (or $N=8$) string vacua.

Even though we chose to parametrize the classical solutions in terms of fields of the toroidally compactified heterotic sting (or equivalently, in terms of the NS-NS sector fields of the toroidally compactified type IIA sting), these solutions map, because of string-string duality (or U duality), onto configurations with RR charges of type IIA string compactified on $K3\times T^2$ (or RR charges of type IIA string compactified on T^6). Thus, they have an interpretation in terms of the (intersecting) D-brane configuration.

Interestingly, even though nonextreme classical solutions may receive quantum corrections, for both four-dimensional and the five-dimensional solutions, the classical entropies (13) and (22) are given as sums of the ''left-moving'' and the ''right-moving'' contributions, which is suggestive of a microscopic interpretation in terms of two contributions arising from the (noninteracting) left-moving and right-moving sectors of the (intersecting) D-brane world-volume Hilbert spaces.

There is an interesting parallel between the structures of the entropy of the four-dimensional (13) and the five-dimensional (22) solutions. The effect of the fourth charge in four dimensions is an additional factor (in the products) associated with the fourth boost (i.e., the fourth charge). In either case, the expression is fully symmetric under permutations of boosts (charges). On the other hand, in going from five to four dimensions, the left-moving angular momentum disappears, while the right-moving angular momentum effectively remains nonzero, as if the limit $J_\psi \to J_\phi$ is taken. From the microscopic point of view, this suggests that in the case of rotating four-dimensional configurations, the D-brane world volume (world sheet) is specified by the $(N=2)$ superconformal algebra of the right-moving sector only, and, therefore, the states are identified under the $\mathrm{U}(1)_R$ superconformal currents only.

Since the structure of the classical entropy (13) [or (22)] suggests a full symmetry among the four (or the three) charges, it may be preferable to identify the (intersecting) D-brane configuration of four (or three) different types of D branes [29] whose world-volume excitations would account for the different charge degrees of freedom in a symmetric way.[13] In particular, the four-dimensional static generating solution of $N=4$ (or $N=8$) superstring vacua can be interpreted [29] (in terms of the type IIA string) as an intersecting D-brane configuration of $Q_2^{(1)}$ zero branes, and $Q_2^{(2)}$, $P_1^{(1)}$, and $P_1^{(2)}$ four branes wrapping around $K3$, $S_1^2\times T^2$, and $S_2^2\times T^2$ [or wrapping around (4567), (6789), and (4589) directions of T^6], respectively. Here, $S_{1,2}^2$ are the two cycles of $K3$. Calculations of the microscopic entropy for such intersecting D-brane configurations may lead to a symmetric treatment of the charge degrees of freedom, as well as to a possible understanding of the separate (noninteracting) contribution of the left-moving and right-moving (world-volume) degrees of freedom.

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[13]Note, however, that this is not the approach used in recent calculations of the microscopic entropy.

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