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Thermodynamics of decaying vacuum cosmologies

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The thermodynamic behavior of decaying vacuum cosmologies is investigated within a manifestly covariant formulation. Such a process corresponds to a continuous, irreversible energy flow from the vacuum component to the created matter constituents. It is shown that if the specific entropy per particle remains constant during the process, the equilibrium relations are preserved. In particular, if the vacuum decays into photons, the energy density ρ and average number density of photons n scale with the temperature as $\rho \sim T^4$ and $n \sim T^3$. The temperature law is determined and a generalized Planckian-type form of the spectrum, which is preserved in the course of the evolution, is also proposed. Some consequences of these results for decaying vacuum FRW-type cosmologies as well as for models with ''adiabatic'' photon creation are discussed. [S0556-2821(96)00316-5]

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I. INTRODUCTION

The cosmological applications of vacuum decay have been extensively investigated in the literature, mainly in connection with inflationary universe scenarios [1–4]. More recently, motivated by the so-called ''cosmological constant problem'' as well as by the ''age problem'' of the standard Friedmann-Robertson-Walker (FRW) model (for reviews of such problems see Refs. [5–7]), many authors have also proposed phenomenological models with a slowly decaying vacuum energy density [8–20]. Roughly speaking, the basic difference between these two kinds of models comes from the fact that, in the former, the vacuum decays completely in a very short period in the very early Universe (phase transition), whereas in the latter, it decays continuously (slowly) in the course of the cosmic evolution. In the second class of models, the attempts to invent a mechanism rendering the cosmological constant almost exactly or exactly vanishing are replaced by the opposite and somewhat more natural idea that the vacuum energy density is a dynamic variable. It is assumed that the effective Λ term behaves like a fluid interacting with the other matter fields of the Universe (as in a multifluid model). As a consequence, the vacuum energy density is not constant since the energy-momentum tensor of the mixture must be conserved in the course of the expansion. In such Λ-variable models, the slow decay of the vacuum energy density may also provide the source term for matter and radiation, thereby suggesting a natural solution for the aforementioned puzzles. First, the explanation accounting for the present smallness of the effective cosmological constant may be deceptively simple: the cosmological constant is very small today because the Universe is too old [9]. Second, although small in comparison with the usual microphysics scales, the ''remnant'' cosmological constant may provide a good fit to the age of the Universe [15,18] (see also Ref. [16] for other kinematical tests).

In what follows, although the results presented here may be interesting for the first class of models, we are more interested in the macroscopic approach for continuous vacuum decay (variable Λ) models. To the best of our knowledge, the temperature evolution for the created matter at the expense of the vacuum component has not been computed from first principles. In particular, for the case of radiation, the lack of a well-defined temperature law as well as the related spectrum implies that the constraints coming from the measurements of the cosmic microwave background radiation (CMBR) cannot be studied without additional hypotheses. As we know, the isotropy of the CMBR and the Planckian form of its spectrum may be a crucial test for this kind of cosmologies. For instance, when the distortions of the Planck spectrum are discussed in the model proposed by Freese *et al.* [11], it is explicitly assumed that the vacuum does not decay into photons fully equilibrated to a Planck spectrum since in this case there are no distortions at all. On the other hand, in their nucleosynthesis analysis, the created photons are supposed to be quickly (and continuously) thermalized, with the total radiation energy density always satisfying the equilibrium relation $\rho_r \sim T^4$ during the radiation phase. Indeed, they had to make this assumption in order to be able to determine how the radiation number density, the temperature, and other physical quantities change with time [22]. This kind of assumption was further extensively adopted (see, for instance, Refs. [13,15,21]).

In this article, we focus our attention on the thermodynamic aspects of decaying vacuum models. As we shall see, if the vacuum is regarded as a second fluid component transferring energy continuously to the material component, the second law of thermodynamics constrains the whole process in such a way that the temperature law may be easily determined. In particular, we will establish under which conditions the equilibrium relations are preserved. These constraints lead us to introduce the idea of an ''adiabatic'' vacuum decay which, due to its simplicity, seems to be the most relevant process from a physical point of view. The related spectral distribution is derived and some conse-

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quences of this approach to FRW decaying vacuum models are also discussed.

II. THERMODYNAMICS AND VACUUM DECAY

Let us consider a self-gravitating fluid satisfying the Einstein field equations (EFE's) with a variable Λ term:

$$G^{\alpha\beta}=\chi T_m^{\alpha\beta}+\Lambda g^{\alpha\beta}, \tag{1}$$

where $\chi=8\pi G$ and the Λ term is the vacuum energy-momentum tensor (EMT), corresponding to an energy density $\rho_v=\Lambda/8\pi G$ and pressure $p_v=-\rho_v$ (in our units $c=1$). $T_m^{\alpha\beta}$ is the EMT of the material component which is defined by

$$T_m^{\alpha\beta}=(\rho+p)u^\alpha u^\beta-pg^{\alpha\beta}, \tag{2}$$

where ρ is the fluid energy density and p is the pressure.

Since we are assuming a continuous energy transfer from the vacuum to the material component, the effective cosmological constant is a time-dependent parameter. In this way, the energy conservation law $(u_\alpha T^{\alpha\beta}{}_{;\beta}=0)$, which is contained in the EFE's, assumes the form

$$\dot{\rho}+(\rho+p)\theta=-\frac{\dot{\Lambda}}{8\pi G}, \tag{3}$$

where the overdot denotes covariant derivative along the world lines (for instance, $\dot{\rho}:=u^\alpha\rho_{;\alpha}$) and $\theta=u^\alpha_{;\alpha}$ is the scalar of expansion. For a FRW geometry, for instance, $\dot{\rho}$ is the time-comoving derivative and $\theta=3H$, where H is the Hubble parameter.

As we know, in order to have a complete fluid description, besides its EMT, it is necessary to define the particle current N^α and the entropy current S^α in terms of the fluid variables. The current N^α is given by

$$N^\alpha=nu^\alpha, \tag{4}$$

where n is the particle number density of the fluid component. Since material constituents are continuously generated by the decaying vacuum, the above four-vector satisfies a balance equation $N^\alpha_{;\alpha}=\psi$ or, equivalently,

$$\dot{n}+n\theta=\psi, \tag{5}$$

where ψ is the particle source ($\psi>0$) or sink ($\psi<0$) term. For decaying vacuum models ψ is positive, and must be related in a very definite way with the variation rate of Λ. Since the EMT of both components are isotropic, without loss of generality, we may define the entropy current in the form below

$$S^\alpha=n\sigma u^\alpha, \tag{6}$$

where σ is the specific entropy (per particle). If the Λ term is constant, the above entropy current is conserved. This is a consequence of the fact that in our approach we are neglecting the usual dissipative processes arising in relativistic simple fluids as well as a possible irreversible matter creation process at the expense of the gravitational field (see Refs. [23–26]). Accordingly, the existence of a nonequilibrium decay process means that

$$S^\alpha_{;\alpha}\geq 0, \tag{7}$$

as required by the second law of thermodynamics. Some words are necessary to clarify the meaning of the above condition. In principle, one might argue that the second law should be applied for the system as a whole, that is, including the vacuum component. However, assuming that the chemical potential of the vacuum fluid is identically zero, it follows from the vacuum equation of state that $\sigma_v=0$ and so also its entropy current is zero [see Eq. (10) below]. In other words, the vacuum plays the role of a condensate carrying no entropy, as happens in the two-fluid description usually employed in superfluid dynamics [27].

Before discussing the temperature evolution law, we need to obtain an expression relating $\dot{\Lambda}$ and ψ. As usual for nonequilibrium processes, such an expression must be defined in such a way that the entropy source is non-negative. To do that, we first remark that the equilibrium variables are related by Gibbs' law:

$$nTd\sigma=d\rho-\frac{\rho+p}{n}\,dn, \tag{8}$$

where T is the temperature. Hence, taking the time-comoving derivative of the above expression and using Eqs. (3) and (5), it is readily obtained that

$$S^\alpha_{;\alpha}:=n\dot{\sigma}+\sigma\psi=-\frac{\dot{\Lambda}}{8\pi GT}-\frac{\mu\psi}{T}, \tag{9}$$

where μ denotes the chemical potential of the created matter, which is defined by the usual Euler's relation:

$$\mu=\frac{\rho+p}{n}-T\sigma. \tag{10}$$

It should be noted that when $\psi=0$, we expect a vanishing time variation of Λ and so also of the entropy production. We recall that the vacuum decaying is the unique source of irreversibility (particle creation) considered in the present treatment. Such a condition can be expressed by the phenomenological ansatz

$$\frac{\dot{\Lambda}}{8\pi G}=-\beta\psi, \tag{11}$$

where β is a positive-definite parameter in order to guarantee that for $\psi>0$, we shall have $\dot{\Lambda}<0$. With this choice, Eq. (9) can be rewritten as

$$S^\alpha_{;\alpha}:=n\dot{\sigma}+\sigma\psi=\frac{\psi}{T}\,(\beta-\mu). \tag{12}$$

Hence, the entropy production rate will be non-negative in accordance with Eq. (7) only when the phenomenological coefficient β satisfies either $\beta\geq\mu$ if $\psi>0$, or $\beta\leq\mu$ if $\psi<0$. As first remarked by Salim and Waga [18], in the case of photons ($\mu=0$), we see from Eqs. (7) and (9) that only a cosmological constant decreasing with time is thermodynamically allowed. As a self-consistency check, we notice that the same result is derived from our phenomenological ansatz (11), together with Eq. (12) and the second law of thermodynamics. Keeping these considerations in mind, we discuss next the temperature law for a continuously decaying vacuum.

III. TEMPERATURE-EVOLUTION LAW

The time dependence of the temperature may be easily established from Eqs. (3), (5), and (8), by adopting T and n as basic thermodynamic variables. Inserting $\rho(T,n)$ into Eq. (3) and using Eq. (5), it follows that

$$\left(\frac{\partial\rho}{\partial T}\right)_n \dot{T}=\left[n\left(\frac{\partial\rho}{\partial n}\right)_T-\rho-p\right]\theta-\left(\frac{\partial\rho}{\partial n}\right)_T\psi-\frac{\dot{\Lambda}}{8\pi G}. \quad (13)$$

Now, since $d\sigma$ is an exact differential, Gibbs' law (8) yields the well-known thermodynamic identity

$$T\left(\frac{\partial p}{\partial T}\right)_n=\rho+p-n\left(\frac{\partial\rho}{\partial n}\right)_T, \quad (14)$$

and inserting Eq. (14) into Eq. (13), we obtain

$$\frac{\dot{T}}{T}=-\left(\frac{\partial p}{\partial\rho}\right)_n\theta-\frac{(\partial\rho/\partial n)_T}{T(\partial\rho/\partial T)_n}\psi-\frac{\dot{\Lambda}}{8\pi GT(\partial\rho/\partial T)_n}. \quad (15)$$

The first term on the right-hand side (RHS) of Eq. (15) is the usual equilibrium contribution. In this case, we see that for an expanding fluid, $\theta>0$ leads to $\dot{T}<0$ as it should be. The remaining terms display the out of equilibrium contributions due to the particle creation rate ψ and its source $\dot{\Lambda}$, which are related by Eq. (11). Note that Eq. (15) is a pure consequence of the relativistic nonequilibrium first order thermodynamics, e.g., the EFE's do not play any special role in its derivation. In addition, since many different processes may take place simultaneously, other contributions, such as bulk viscosity, gravitational matter creation, heat flow, diffusion, and all possible cross effects, should be taken into account. In any case, the method applied here may be easily extended by including the corresponding terms into the basic thermodynamic quantities. In what follows, I will discuss in detail an interesting particular case, which, due to its simplicity and possible physical applications (see Sec. V), deserves a special attention.

IV. ''ADIABATIC'' CASE

Let us first discuss under which conditions the equilibrium relations for the particle number and energy density are preserved in the presence of a decaying vacuum. Inserting the value of θ obtained from Eq. (5) and the value of Λ as given by the phenomenological law (11) into Eq. (15), it follows that

$$\frac{\dot{T}}{T}=\left(\frac{\partial p}{\partial\rho}\right)_n\frac{\dot{n}}{n}-\frac{1}{nT(\partial\rho/\partial T)_n}\left[T\left(\frac{\partial p}{\partial T}\right)_n+n\left(\frac{\partial\rho}{\partial n}\right)_T-n\beta\right]\psi. \quad (16)$$

The first term on the RHS of the above equation still resembles a typical equilibrium term; however, there are also out of equilibrium contributions encoded in it. To be more precise, suppose that the second term on RHS of Eq. (16) is absent and that the fluid satisfies the usual ''γ-law'' equation of state

$$p=(\gamma-1)\rho, \quad (17)$$

where the ''adiabatic index'' γ lies in the interval [0,2]. In this case, a straightforward integration of Eq. (16) furnishes $n^{(1-\gamma)}T=$const, and, for $\gamma\neq 1$,

$$n=\text{const}\times T^{1/(\gamma-1)}, \quad (18)$$

which has the same form for $n(T)$ as perfect adiabatic simple fluid. However, the number density of particles no longer satisfies the usual conservation law [see Eq. (5)]. It thus follows that the equilibrium relations will be preserved only if the second term on the RHS of Eq. (16) is identically zero. In this case, using Eq. (14) we see that the phenomenological parameter β assumes a remarkably simple form

$$\beta=\frac{\rho+p}{n}. \quad (19)$$

The next step is to show that the value of β deduced above also guarantees the equilibrium relation for the energy density. In fact, by combining Eqs. (3), (11), (17), and (19), we readily obtain

$$\dot{\rho}+\gamma\rho\theta=\gamma\rho\frac{\psi}{n}. \quad (20)$$

Therefore, comparing Eq. (20) with Eq. (5), it follows that

$$\frac{\dot{\rho}}{\rho}=\gamma\frac{\dot{n}}{n}, \quad (21)$$

the solution of which is $\rho=\text{const}\times n^{\gamma}$, or, using Eq. (18),

$$\rho=\eta T^{\gamma/(\gamma-1)}, \quad (22)$$

where η is a γ-dependent integration constant. The above expression is the equilibrium energy density-temperature relation for a γ fluid. In particular, for a photon fluid (γ=4/3), one obtains from Eqs. (18) and (22), respectively, $n\sim T^3$ and $\rho\sim T^4$, just the well-known relations valid for blackbody radiation. Therefore, the condition expressed by Eq. (19) preserves the usual equilibrium relations and so it should have a rather simple physical interpretation.

In order to clarify the physical meaning of relation (19) we return to the entropy production expression given by Eq. (12). Inserting the value of β given above into Eq. (12), it is easy to see that the variation rate of the specific entropy may be written as

$$\dot{\sigma}=\frac{\psi}{nT}\left(\frac{\rho+p}{n}-\mu-T\sigma\right), \quad (23)$$

and from Euler's relation (10) we see that $\dot{\sigma}=0$. Therefore, the equilibrium relations are preserved only if the specific entropy per particle of the created particles is constant. In other words, when the specific entropy remains constant, or equivalently, β is given by Eq. (19), no finite-thermalization time is required since the particles originated from the decaying vacuum are created in equilibrium with the already existing ones. Naturally, the process as a whole is out of equilibrium. In fact, since $\sigma = s/n$, the condition $\dot{\sigma}=0$ leads, with the help of Eq. (5), to a balance equation for the entropy density [see also the discussion below Eq. (29)]

$$\dot{s} + s\theta = \frac{s\psi}{n}, \tag{24}$$

which for $\psi=0$ reduces to the usual ''continuity'' equation for an adiabatic flow.

V. ''ADIABATIC'' DECAYING VACUUM AND FRW-TYPE COSMOLOGIES

Let us now consider the FRW line element

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2\right), \tag{25}$$

where R is the scale factor and $k=0,\pm 1$ is the curvature parameter.

In such a background the Einstein field equations (EFE's) for the nonvacuum component plus a cosmological Λ term are

$$8\pi G\rho + \Lambda = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{26}$$

$$8\pi G p - \Lambda = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}, \tag{27}$$

where ρ and p, as usual, are assumed to obey the γ-law equation of state (17).

As we have seen, in the ''adiabatic'' case, the temperature T satisfies Eq. (18) and the energy density is given by Eq. (22) regardless of the microscopic details of the vacuum decay. In addition, for a FRW geometry, the comoving volume scales as $V \sim R^3$ and, up to a constant factor, $N = nR^3$. Thus, using Eq. (18) we may write the temperature evolution law

$$N^{1-\gamma} T R^{3(\gamma-1)} = \text{const.} \tag{28}$$

As expected, if N is conserved (no vacuum decay), the usual equilibrium law is recovered. It should be noticed that the above temperature law has a rather general character. It can be applied regardless of the specific creation mechanism operating in the FRW geometry. As a matter of fact, it depends only on the validity of the ''adiabaticity'' condition (19), thereby implying that the equilibrium relation (18) is preserved. For instance, the above law is the same as the one deduced in Ref. [25] for an ''adiabatic'' particle creation at the expense of the gravitational field. In particular for photon creation ($\gamma=4/3$), Eq. (28) reduces to

$$N^{-1/3} T R = \text{const}, \tag{29}$$

instead of the usual TR=const of the standard FRW model.

In the homogeneous case, $\sigma = S/N$, where S and N are the total entropy and number of particles, and since we are considering the ''adiabatic'' case, it follows that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N}. \tag{30}$$

Hence, the burst of entropy is closely related with the created matter due to the decaying vacuum.

It is worth mentioning that, in comparison with the standard model, the macroscopic formulation discussed here has only one additional free parameter, namely, the variation rate of the Λ term, or equivalently from Eq. (11), the particle creation rate ψ. In fact, in the ''adiabatic'' case, the phenomenological β parameter is completely determined by condition (19). In principle, either $\dot{\Lambda}$ or ψ must be computed from a more fundamental model for the decaying vacuum. Of course, at the level of a definite equation of motion, this is equivalent to assuming *a priori* a functional expression for $\Lambda(t)$ itself, as has been usually done in the literature (see Refs. [10–20]). Note also that any ''adiabatic'' decaying vacuum FRW-type cosmology has its temperature law determined by Eq. (29). However, how T scales with R depends, naturally, on the specific decay rate of Λ, since it will determine from Eqs. (11) and (5) the specific $N(R)$ function. Such a function will also define through Eq. (30) the amount of entropy produced. In this way, both the cooling rate and entropy generation in decaying vacuum cosmologies are highly model-dependent functions, as should be expected.

Another interesting question is closely related to the spectrum of the CMBR. As we know, in the standard model, both the equilibrium relations and the Planckian form of the spectrum are preserved in the course of the expansion. The latter result follows naturally from the fact that $\nu \sim R^{-1}$ (kinematical condition for FRW geometry) and $T \sim R^{-1}$. Our results show that when ''adiabatic'' photon creation takes place, the equilibrium relations are preserved, while T necessarily follows a more general temperature law given by Eq. (29). In this case, one may be tempted to conclude that all models with photon creation fail the crucial test provided by the present isotropy and spectral distribution of the CMBR (see, for instance, Steigman [29] and reference quoted therein). However, such a conclusion is not so neat as it appears at first sight. For instance, suppose that in the ''adiabatic'' case, the spectrum assumes the form (a derivation is outlined in the appendix)

$$\rho_T(\nu) = \left(\frac{N(t)}{N_0}\right)^{4/3} \frac{8\pi h}{c^3} \frac{\nu^3}{\exp\left[\left(\frac{N(t)}{N_0}\right)^{1/3} \frac{h\nu}{kT}\right] - 1}, \tag{31}$$

where $N(t)$ is the comoving time-dependent number of photons and N_0 is the constant value of N evaluated at some fixed epoch, say, the present time. When there is no creation, $N(t) = N_0$, and the usual Planckian form is recovered. Since the frequency scales as $\nu \sim R^{-1}$, as a consequence of the temperature law (29), the exponential factor in Eq. (31) is clearly preserved in the course of the expansion. In addition, it is readily seen that the equilibrium relations are recovered using such a spectrum. In fact, from Eq. (21) it follows that $n \sim \rho^{3/4}$, and by introducing a new variable $x = (N/N_0)^{1/3}(h\nu/kT)$, it is easy to see that

$$\rho(T) = \int_0^\infty \rho_T(\nu)\,d\nu = aT^4, \tag{32}$$

where a is the usual radiation density constant. In this way, the spectrum given by Eq. (31) seems to be the most natural generalization of Planck's radiation formula in the presence of ''adiabatic'' photon production. Since it cannot, on experimental grounds, be distinguished from the usual blackbody spectrum with no matter creation, models with ''adiabatic'' photon creation may be compatible with the present isotropy and spectral distribution of the microwave background. This conclusion is extremely general and may be applied even for Dirac-type cosmologies (the case for G-variable cosmologies will be discussed in detail elsewhere). It is worth mentioning that Eq. (31) is quite different from the form originally proposed by Canuto and Narlikar [30] to circumvent the criticism of Steigman [29] (see also the paper of Narlikar and Rana [31]). The main difference comes from the fact that the temperature evolution law (29) has now been incorporated into the exponential factor of the above spectrum. Although the usual Planckian spectrum cannot be distinguished at present from Eq. (31), this does not mean that the same happens for high redshifts. For instance, one may check that the wavelength λ_m for which $\rho_T(\lambda)$ assumes its maximum value now satisfies the displacement law

$$\lambda_m T = 0.289\left(\frac{N(t)}{N_0}\right)^{1/3} \text{ cm K}, \tag{33}$$

which reduces to the usual Wien's law for $N = N_0$. Hence, since in the past, $N(t) < N_0$, for a given redshift z, the typical energy of photons whose spectral distribution is given by Eq. (31), will be smaller than that described by the usual Planckian spectrum. More precisely, since the scale factor as a function of the redshift is given by $R = R_0(1+z)^{-1}$, we see from Eq. (29) that

$$T = T_0(1+z)\left(\frac{N(t)}{N_0}\right)^{1/3}, \tag{34}$$

where T_0 is the present-day value of T. This relation has some interesting physical consequences. First, we observe that universes with ''adiabatic'' photon creation are, for any value of $z > 0$, cooler than the standard model. Such a prediction may be experimentally verified, for instance, observing atomic or molecular transitions in absorbing clouds at high redshifts. In this way, it provides a crucial test for models endowed with ''adiabatic'' photon production, which is accessible with the present day technology. In this connection, Songaila *et al.* [32] recently reported the detection of the first fine structure of neutral carbon atoms in the $z = 1.776$ absorption-line system. Assuming that no other significant sources of excitation are present, the relative population of the level yielded a temperature of $T = 7.4 \pm 0.8$ K while using the standard relation it should be 7.58 K. Although, in accordance with this prediction, it is too early to interpret such measurements as a new successful test of the standard model. As remarked by Mayer [33], it is very difficult to pin down the amount of local excitation in the observed clouds using independent observations. In this way, this result must strictly be considered as an upper bound for the temperature of the Universe in the above-mentioned redshift. In principle, improved observational techniques as well as some reasonable estimates of the possible sources of excitations, may lead to a smaller value of the temperature, in conflict with the standard prediction. In this case, as happens with the cosmological constant and age problems, a decaying vacuum cosmology may become an interesting possibility to fit the data. In the future, temperature-redshift measurements of sufficient accuracy may constrain the free parameters of any specific decaying Λ model, or more generally, any kind of cosmology endowed with ''adiabatic'' creation of photons. Note also that Eq. (34) gives us a simple qualitative explanation of why models with decaying vacuum may solve the cosmological age problem which plagues the class of FRW models. In fact, since for a given redshift z the Universe is cooler than the standard model, more time is required to attain a fixed temperature scale in the early Universe. Some quantitative examples are given in Refs. [13–17].

As is well known, there is no derivation specifying either how fast the vacuum decays or how it couples with matter and/or radiation. As discussed earlier, this is equivalent to determining the function $N(t)$. In the present case, regardless of the form of such a function, it seems that the radiation must be produced through an induced decay mechanism, because the energy is always injected ''adiabatically,'' that is, fully equilibrated with the generalized Planck spectrum. Such a possibility was noted but not discussed by Freese *et al.* [11]. On the other hand, if one assumes that the vacuum couples only with the radiation, this means that baryons (and antibaryons) are not produced in the course of the evolution. Therefore, such as in the standard FRW model, the number density of the nonrelativistic component is conserved, that is, n_b scales with R^{-3}, and from Eq. (34) we may write

$$\sigma_{rb} = \sigma_{r0}\,\frac{N(t)}{N_0}, \tag{35}$$

where $\sigma_{rb} = 4aT_r^3/3n_b$ is the radiation-specific entropy (per baryon) and σ_{r0} its present value. The above expression means that the photon-baryon ratio increases as the Universe expands, however, at a rate which is strongly model dependent. Since the function $N(t)$ has not been determined from first principles, the constraint from nucleosynthesis code cannot be seen as a definitive answer, at least while we do not know how to select the best phenomenological description for the decaying vacuum. In this way, it is not surprising that models based on different decay rates and/or initial conditions, lead to somewhat opposite conclusions about such constraints (compare, for instance, Refs. [11, 14]).

VI. CONCLUSION

The thermodynamic behavior of variable Λ models has been investigated in the framework of a first-order relativistic theory for irreversible processes. The main results derived here may be summarized in the following statements:

(1) Decaying vacuum models with photon creation may be compatible with the constraints of the cosmic background radiation only when the creation occurs under ''adiabatic'' conditions, e.g., when the equilibrium relations are preserved. This is equivalent to, say, that the entropy per photon remains constant during the creation process (see Sec. IV).

(2) If the vacuum decays ''adiabatically'' in particles obeying the equation $p=(\gamma-1)\rho$, the temperature law is given by $N^{1-\gamma}T^{3}V^{\gamma-1}=$const, where $N(t)$ is the instantaneous number of particles and V the comoving volume. In the case of radiation (γ=4/3), it reduces to $N^{-1}T^{3}V=$const, and for a FRW geometry, $V\sim R^{3}$, we have $N^{-1/3}TR=$const [see Eq. (29)].

(3)The usual Planckian spectrum has been generalized to include ''adiabatic'' photon creation [see Eq. (31) and the appendix]. Such a form is uniquely determined by the radiation temperature law. In this way, when the ''adiabatic'' condition has been implemented, it can be applied regardless of the specific creation mechanism operating in the spacetime. For instance, it holds for models endowed with ''adiabatic'' creation at the expense of the gravitational field as discussed in Refs. [23–26]. The new spectrum is preserved in the course of the evolution and is clearly consistent with the present isotropy of the CMBR. Instead of the matter creation process, the observed slight anisotropy (such as in the standard model) must be associated with the usual physical processes taking place during the matter-dominated phase.

(4) The measurement of the Universe temperature at high redshifts is a crucial test for models endowed with ''adiabatic'' creation, in particular, for decaying vacuum cosmologies. For a given redshift z, the temperature is smaller than that one predicted by the standard model [see Eq. (34)].

Finally, we remark that the results presented here may be generalized by allowing additional contributions for the EMT (different creation mechanisms). However, if the ''adiabatic'' condition is imposed, neither the temperature law nor the form of the spectrum will be modified. Specific models will be studied in a forthcoming communication.

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APPENDIX: ''ADIABATIC'' BLACKBODY SPECTRUM

In this appendix, a formula for blackbody radiation when the photon creation process takes place ''adiabatically,'' is derived. As we have seen, in this case the temperature law is given by [see our Eq. (29)]

$$N^{-1/3}TR=\text{const.} \tag{A1}$$

Since the wavelength λ scales with R, the above equation means that if one compresses or expands a hollow cavity containing blackbody radiation in such a way that photons are ''adiabatically'' introduced in it, we may write for each wave component

$$N^{-1/3}\lambda T=\text{const.} \tag{A2}$$

The above quantity plays the role of a generalized ''adiabatic'' invariant in the sense of Ehrenfest [34]. When N is constant, the usual adiabatic invariant for expanding blackbody radiation is recovered.

Let T_1 be the temperature in the instant $t=t_1$, and focus our attention on the band $\Delta\lambda_1$ centered on the wavelength λ_1 whose energy density is $\rho_{T_1}(\lambda_1)\Delta\lambda_1$. In a subsequent time $t=t_2$, when the temperature T_1 changed to T_2, due to an ''adiabatic'' expansion, the energy of the band changed to $\rho_{T_2}(\lambda_2)\Delta\lambda_2$ and according to Eq. (A2), $\Delta\lambda_1$ and $\Delta\lambda_2$ are related by

$$\frac{\Delta\lambda_2}{\Delta\lambda_1}=\left(\frac{N(t_2)}{N(t_1)}\right)^{1/3}\frac{T_1}{T_2}. \tag{A3}$$

As shown earlier in Sec. IV, the thermodynamic equilibrium relations are preserved and since distinct bands do not interact, it follows that

$$\frac{\rho_{T_2}(\lambda_2)\Delta\lambda_2}{\rho_{T_1}(\lambda_1)\Delta\lambda_1}=\left(\frac{T_2}{T_1}\right)^{4}. \tag{A4}$$

By combining the above result with Eq. (A3) and using again Eq. (A2), we obtain for an arbitrary component

$$\rho_T(\lambda)\lambda^5=\text{const}\times N^{4/3}. \tag{A5}$$

In the Planckian case, $N=N_0=$const, the above expression reduces to $\rho_T(\lambda)\lambda^5=$const, as it should be. Without loss of generality, taking into account (A2), the above result may be rewritten as (we have normalized N by its value N_0 without photon creation)

$$\rho_T(\lambda)=\left(\frac{N}{N_0}\right)^{4/3}\lambda^{-5}\phi^*\left[\left(\frac{N}{N_0}\right)^{-1/3}\lambda T\right], \tag{A6}$$

where ϕ^* is an arbitrary function of its argument. In terms of frequency, since $\rho_T(\nu)d\nu=\rho_T(\lambda)|d\nu/d\lambda|d\lambda$, it follows that

$$\rho_T(\nu)=\left(\frac{N}{N_0}\right)^{4/3}\nu^3\phi\left[\left(\frac{N}{N_0}\right)^{-1/3}\frac{T}{\nu}\right], \tag{A7}$$

where ϕ is proportional to ϕ^*. The above equation is a generalized form of the well-known Wien's law [35]. In order to recover the usual Planckian distribution, the arbitrary function must be

$$\phi = \frac{8\pi h}{c^3} \frac{1}{\exp\left[\left(\frac{N(t)}{N_0}\right)^{1/3} \frac{h\nu}{kT}\right] - 1},$$

with Eq. (A7) taking the form assumed in Eq. (31): namely,

$$\rho_T(\nu) = \left(\frac{N(t)}{N_0}\right)^{4/3} \frac{8\pi h}{c^3} \frac{\nu^3}{\exp\left[\left(\frac{N(t)}{N_0}\right)^{1/3} \frac{h\nu}{kT}\right] - 1}. \quad \text{(A8)}$$

Note that no reference has been made to the specific source of photons. The above-sketched derivation depends only on the temperature law as given by Eq. (A2), or equivalently, that creation occurs preserving the equilibrium relations (''adiabatic'' creation).

[1] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[2] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 220 (1982).
[4] S. W. Hawking and I. Moss, Phys. Lett. **110B**, 35 (1982).
[5] A. Zee, in *High Energy Physics*, Proceedings of the 20th Annual Orbis Scientiae, edited by B. Kursunoglu, S. L. Mintz, and A. Perlmutter (Plenum, New York, 1985).
[6] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[7] S. M. Carroll, W. H. Press, and E. L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992); **393**, 3 (1992).
[8] L. M. Krauss and M. S. Turner, Gen. Relativ. Gravit. **27**, 1137 (1995).
[9] P. J. E. Peebles, Astrophys. J. **284**, 439 (1984).
[10] M. Ozer and M. O. Taha, Phys. Lett. B **171**, 363 (1986); Nucl. Phys. **B287**, 776 (1987).
[11] K. Freese, F. C. Adams, J. A. Frieman, and E. Mottola, Nucl. Phys. **B287**, 797 (1987).
[12] W. Chen and Y. S. Wu, Phys. Rev. D **41**, 695 (1990).
[13] A.-M. M. Abdel-Rahman, Phys. Rev. D **45**, 3497 (1992).
[14] A.-M. M. Abdel-Rahman, Gen. Relativ. Gravit. **27**, 573 (1995).
[15] T. Nishioka and Y. Fujii, Phys. Rev. D **45**, 2140 (1992).
[16] J. C. Carvalho, J. A. S. Lima, and I. Waga, Phys. Rev. D **46**, 2404 (1992).
[17] I. Waga, Astrophys. J. **414**, 436 (1993).
[18] D. Pavon, Phys. Rev. D **43**, 375 (1991); J. M. Salim and I. Waga, Class. Quantum Grav. **10**, 1767 (1993).
[19] J. A. S. Lima and J. M. F. Maia, Phys. Rev. D **49**, 5597 (1994); J. A. S. Lima and M. Trodden, *ibid*. **53**, 4280 (1996).
[20] J. Lopez and D. V. Nanopoulos, Mod. Phys. Lett. A **11**, 1 (1996).
[21] J. Matygasek, Phys. Rev. D **51**, 4154 (1995).
[22] In order to maintain the Planckian distribution, Freese *et al.* [11] also assumed explicitly that the frequency of photons redshifts, as $\nu \sim R^{-(1-x)}$, where $x = \rho_v/(\rho + \rho_v)$. This assumption is in direct contradiction with the well-established kinematical result, $\nu \sim R^{-1}$, for FRW-type geometries. In particular, for decaying vacuum cosmologies, this means that the usual Planck form is inevitably destroyed. A new spectrum taking into account photon creation and preserving its form in the course of the evolution is discussed in Sec. V.
[23] I. Prigogine, J. Geheniau, E. Gunzig, and P. Nardone, Gen. Relativ. Gravit. **21**, 767 (1989).
[24] J. A. S. Lima, M. O. Calvão, and I. Waga, in *Frontier Physics, Essays in Honor of Jayme Tiomno* (World Scientific, Singapore, 1991).
[25] M. O. Calvão, J. A. S. Lima, and I. Waga, Phys. Lett. A **162**, 223 (1992).
[26] J. A. S. Lima and A. S. M. Germano, Phys. Lett. A **170**, 373 (1992).
[27] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1986).
[28] J. A. S. Lima, A. S. M. Germano, and L. R. Abramo, Phys. Rev. D **53**, 4287 (1996).
[29] G. Steigman, Astrophys. J. **221**, 407 (1978).
[30] V. Canuto and J. V. Narlikar, Astrophys. J. **236**, 6 (1980).
[31] J. V. Narlikar and N. C. Rana, Phys. Lett. A **77**, 219 (1980).
[32] A. Songaila *et al.*, Nature (London) **371**, 43 (1994).
[33] D. M. Mayer, Nature (London) **371**, 13 (1994).
[34] P. Ehrenfest, Philos. Mag. **33**, 500 (1917).
[35] M. Planck, *The Theory of Heat Radiation* (Dover, New York, 1991).