Nonperturbative amplification of inhomogeneities in a self-reproducing universe

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We investigate the distribution of energy density in a stationary self-reproducing inflationary universe. We show that the main fraction of volume of the Universe in a state with a given density ρ at any given moment of proper time t is concentrated near the centers of deep exponentially wide spherically symmetric wells in the density distribution. Since this statement is very surprising and counterintuitive, we perform our investigation by three different analytical methods to verify our conclusions, and then confirm our analytical results by computer simulations. If one assumes that we are typical observers living in the Universe at a given moment of time, then our results may imply that we should live near the center of a deep and exponentially large void, which we will call an infloid. The validity of this particular interpretation of our results is not quite clear since it depends on the as-yet unsolved problem of measure in quantum cosmology. Therefore, at the moment we would prefer to consider our results simply as a demonstration of nontrivial properties of the hypersurface of a given time in the fractal self-reproducing universe, without making any far-reaching conclusions concerning the structure of our own part of the Universe. Still we believe that our results may be of some importance since they demonstrate that nonperturbative effects in quantum cosmology, at least in principle, may have significant observational consequences, including an apparent violation of the Copernican principle. [S0556-2821(96)03816-7]

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I. INTRODUCTION

According to the Copernican principle, the only special thing about the Earth is that we are living here. We are not at the center of the Universe, as people thought before. This point of view is reflected also in the so-called cosmological principle, which asserts that our place in the Universe is by no means special and that the space around us has to be homogeneous and isotropic after smoothing over small lumps of matter. This principle lies in the foundation of contemporary cosmology [1] since it has not only definite philosophical appeal but also an apparent observational confirmation by a host of data on the large scale structure of the Universe. However, theoretical interpretation of this principle is usually based on the big bang picture of the Universe and its evolution, inherently related to a simple geometry of the Friedmann-Robertson-Walker-type. The only theoretical justification of the homogeneity and isotropy of the Universe which is known to us at present is based on inflationary cosmology. But this theory simultaneously with explaining why our Universe locally looks so homogeneous predicts that on an extremely large scale the Universe must be extremely inhomogeneous [2]. Thus, after providing certain support to the cosmological principle, inflationary theory eventually removes it as having only limited validity. But until very recently we did not suspect that inflation may invalidate the Copernican principle as well, since there is nothing about inflation which would require us to live in the center of the Universe.

The situation became less obvious when we studied the global structure of an inflationary universe in the chaotic inflation scenario, and found that according to a very wide class of inflationary theories, the main fraction of volume of the Universe in a state with a given density ρ at any given moment of time t (during or after inflation) should be concentrated near the centers of deep exponentially wide spherically symmetric wells in the density distribution [3]. This result is based on investigation of nonperturbative effects in the theory of a fractal self-reproducing universe[1] in the chaotic inflation scenario [4].

Observational implications of this result depend on its interpretation. If we assume that we live in a part which is typical—and by ''typical'' we mean those parts of the Universe which have the greatest volume with other parameters (time and density) being equal—then our result implies that we should live near the center of one of the wells in the density distribution. There should be many such wells in the Universe, but each of them should be exponentially wide. In what follows we will call these wells ''infloids.'' An observer living near the center of an infloid will see himself ''in the center of the world,'' which would obviously contradict the Copernican principle.

One should clearly distinguish between the validity of our

[1]Self-reproduction of the Universe is possible in the new inflationary theory as well [5], but as we will see, in this theory the effect which we are going to discuss is negligibly small.

result and the validity of its interpretation suggested above. Even though the effect by itself is rather surprising we think that it is correct. We verified its validity by three independent analytical methods, as well as by computer simulations. Meanwhile, the validity of its interpretation is much less clear. The main problem is related to the ambiguity in the choice of measure in quantum cosmology [6]. There are infinitely many domains with similar properties in a self-reproducing inflationary universe. When we are trying to compare their volumes, we are comparing infinities. The results of this comparison depend on the choice of the regularization procedure. The prescription that we should compare volumes at a given time t in synchronous coordinates is intuitively appealing, but there exist other prescriptions which lead to different conclusions [3,6–8]. Until the interpretation problem is resolved, we will be unable to say for sure whether inflationary cosmology actually predicts that we should live in a center of a spherically symmetric well. Still this possibility is so interesting that it deserves a detailed investigation even at our present, admittedly rather incomplete level of understanding of quantum cosmology. This is the main purpose of our paper.

In Sec. II we will give a short review of the theory of a self-reproducing universe in the chaotic inflation scenario and discuss which type of phenomena should be called typical in such a universe. Then we will describe two approaches to the problem of estimating the typical magnitude of the quantum fluctuations under the volume-weighted measure. The first, developed in Sec. III, is based on counting the balance of probability factors. The second is based on the investigation of the probability distribution $P_P(\phi,t)$. This distribution describes the portion of the physical volume of the Universe which contains the field ϕ at the time t. According to [6], this distribution rapidly approaches a stationary regime, where the portion of the physical volume of the Universe containing the field ϕ becomes independent of time. Investigation of this distribution in Sec. IV will allow us to derive the results obtained in Sec. III in a different way. In Sec. V we will develop a path integral approach to the investigation of $P_P(\phi,t)$. The new method provides another way to confirm our results. However, this method is interesting by itself. It gives us a new powerful tool for investigation of the global structure of the self-reproducing universe, which may be useful independently of the existence of the effect discussed in this paper. In Sec. VI we will describe computer simulations which we used to verify our analytical results. Only then, after we make sure that our rather counterintuitive results are actually correct, will we describe their possible interpretation and their observational consequences. In Sec. VII we will describe the structure of infloids, their evolution after the end of inflation, and their observational manifestations. In Sec. VIII we will discuss our results, ambiguities of their interpretation, and formulate our conclusions. In the Appendix we present a generalization of our results for different time parametrizations.

II. SELF-REPRODUCING UNIVERSE

Let us consider the simplest model of chaotic inflation based on the theory of a scalar field ϕ minimally coupled to gravity, with the effective potential $V(\phi)$. If the classical field ϕ (the inflaton field) is sufficiently homogeneous in some domain of the Universe, then its behavior inside this domain is governed by the equations

$$\ddot{\phi}+3H\dot{\phi}=-V'(\phi), \tag{1}$$

$$H^2+\frac{k}{a^2}=\frac{8\pi}{3M_P^2}\left[\tfrac{1}{2}\dot{\phi}^2+V(\phi)\right]. \tag{2}$$

Here $H=\dot{a}/a, a(t)$ is the scale factor of the Universe, and $k=+1,-1$, or 0 for a closed, open, or flat universe, respectively. M_P is the Planck mass, which we will put equal to 1 in the rest of the paper.

Investigation of these equations has shown that for many potentials $V(\phi)$ [e.g., in all power-law $V(\phi)\sim\phi^n$ and exponential $V(\phi)\sim e^{\alpha\phi}$ potentials] there exists an intermediate asymptotic regime of slow rolling of the field ϕ and quasiexponential expansion (inflation) of the Universe [2]. At this stage, which is called inflation, one can neglect the term $\ddot{\phi}$ in Eq. (2), as well as the terms k/a^2 and $(4\pi/3)\dot{\phi}^2$ in Eq. (2). Therefore during inflation

$$H=\sqrt{\frac{8\pi V}{3}}, \quad \dot{\phi}=-\frac{V'(\phi)}{3H}. \tag{3}$$

In the theories $V(\phi)\sim\phi^n$ inflation ends at $\phi=\phi_e$, where $\phi_e\sim 10^{-1}n$.

Inflation stretches all initial inhomogeneities. Therefore, if the evolution of a universe were governed solely by classical equations of motion, we would end up with an extremely smooth universe with no primordial fluctuations to initiate the growth of galaxies. Fortunately, new density perturbations are generated during inflation due to quantum effects. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially in the expanding Universe. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more perturbations of the classical field with wavelengths greater than H^{-1}. The average amplitude of such perturbations generated during a time interval H^{-1} (in which the Universe expands by a factor of e) is given by

$$|\delta\phi(x)|=\frac{H}{2\pi}. \tag{4}$$

The phases of each wave are random. It is important also that quantum fluctuations occur independently in all domains of the inflationary universe of a size greater than the radius of the event horizon H^{-1}. Therefore, the sum of all waves at any given region of a size $O(H^{-1})$ fluctuates and experiences Brownian jumps in all directions in the space of fields.

The standard way of description of the stochastic behavior of the inflaton field during the slow rolling stage is to coarse grain it over separate domains of radius H^{-1} (we will call these domains ''h regions'' [9,10], to indicate that each of them has a radius coinciding with the radius of the event horizon H^{-1}) and consider the effective equation of motion of the long-wavelength field [11,4]:

$$\frac{d}{dt}\phi = -\frac{V'(\phi)}{3H(\phi)} + \frac{H^{3/2}(\phi)}{2\pi}\xi(t), \tag{5}$$

Here $\xi(t)$ is the effective white noise generated by quantum fluctuations.

Let us find the critical value ϕ_* such that for $V(\phi) < V(\phi_*)$ the classical slow roll dominates the evolution of the inflaton, while for $V(\phi) > V(\phi_*)$ the quantum fluctuations are more important. Within the characteristic time interval $\Delta t = H^{-1}$ for values of inflaton near the critical value ϕ_* the classical decrease $\Delta\phi = \dot{\phi}\Delta t$ of the inflaton, defined through Eqs. (3), is of the same magnitude as the typical quantum fluctuation generated during the same period, given by Eq. (4). After some algebra we get from Eqs. (3) and (4) the relation defining ϕ_* implicitly:

$$\frac{3H^3(\phi_*)}{2\pi V'(\phi_*)} = H(\phi_*)\frac{4\,V(\phi_*)}{V'(\phi_*)} \sim 1. \tag{6}$$

Let us consider for definiteness the theory $V(\phi) = \lambda\phi^4/4$. In this case Eq. (6) yields $\phi_* \sim \lambda^{-1/6}$. One can easily see that if $\phi < \phi_*$, then the decrease of the field ϕ due to its classical motion $\Delta\phi = 1/2\pi\phi$ is much greater than the average amplitude of the quantum fluctuations $\delta\phi = \sqrt{\lambda/6\pi}\phi^2$ generated during the same characteristic time interval H^{-1}. But for $\phi > \phi_*$, $\delta\phi(x)$ will exceed $\Delta\phi$; i.e., the Brownian motion of the field ϕ will become more rapid than its classical motion. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during this time is H^{-1}, the whole Hubble domain after the time H^{-1} becomes effectively divided into e^3 h regions, each containing almost homogeneous (but different from each other) field $\phi - \Delta\phi + \delta\phi$.

In almost half of these domains (i.e., in $e^3/2 \sim 10$ h regions) the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases. During the next time interval $\Delta t = H^{-1}$ the field grows again in the half of the new h regions. Thus, the total number of h regions containing growing field ϕ becomes equal to $(e^3/2)^2 = e^{2(3-\ln 2)}$. This means that until the fluctuations of field ϕ grow sufficiently large, the total physical volume occupied by permanently growing field ϕ (i.e., the total number of h regions containing the growing field ϕ) increases with time like $\exp[(3-\ln 2)Ht]$. This leads to the self-reproduction of inflationary domains with $\phi > \phi_*$ in the chaotic inflation scenario [4].

Note that the greater is the value of the effective potential, the greater is the rate of exponential expansion of the Universe. As a result, the main growth of the total volume of the Universe occurs due to exponential expansion of the domains with the greatest possible values of the Hubble constant $H = H_{\max}$ [4,6]. In some models there is no upper bound to the value of H [12,13]. However, in the simplest versions of chaotic inflation based on Einstein's theory of gravity there are several reasons to expect that there exists an upper bound for the rate of inflation [6,14,15].

In what follows we will assume that there is an upper bound $H_{\max}$ on the value of the Hubble constant during inflation. For definiteness we will assume that $H_{\max} = \sqrt{8\pi/3}$, which corresponds to the Planck boundary $V(\phi_P) = 1$. This is a rather natural assumption for chaotic inflation. However, one should note that in some models $H_{\max}$ may be much smaller. In particular, in the new inflation scenario $H_{\max} = \sqrt{8\pi V(0)/3}$ is many orders of magnitude smaller than 1.

The independence of the subsequent evolution of the h region on its previous history, the dominance of the domains where the inflaton field energy grows rather than decreases in the volume-weighted measure, and the upper bound for the energies at which the inflation can proceed are the three main features inherent to many models of inflation. When all these features are present the evolution of the inflationary universe as a whole approaches a regime which we called *global stationarity* in [6]. This stage is characterized by the stability of the distribution of regions with various local values of energy density and other parameters, while the number of such regions grows exponentially with a constant coefficient, proportional to the maximal possible rate of inflation $\lambda_1 = d_{\mathrm{fr}} H_{\max}$. Here d_{fr} is a model-dependent fractal dimension of the classical space [9,6], which is very close to 3 for small coupling constants of the inflaton field.

The new picture of the Universe is extremely unusual, and it may force us to reconsider our definition of what is typical and what is not. In particular, the standard theory of the large scale structure of the Universe is based on the assumption that a typical behavior of the scalar field at the last stages of inflation is described by Eqs. (3) and (4). This is indeed the case if one studies a single branch of an inflationary universe beginning at $\phi \ll \phi_*$. However, if one investigates the global structure of the universes at all ϕ and tries to find the typical behavior of *all* inflationary domains *with a volume-weighted measure*, the result may appear to be somewhat different.

III. STATIONARY INFLATION AND NONPERTURBATIVE EFFECTS

Suppose that we have one inflationary domain of initial size H^{-1}, containing a scalar field $\phi > \phi_*$. Let us wait 15×10^9 years (in synchronous time t in each part of this domain) and see what are the typical properties of those parts of our original domain which at the present moment have some particular value of density, e.g., $\rho = 10^{-29}$ g cm^{-3}. The answer to this question proves to be rather unexpected.

This domain exponentially expands, and becomes divided into many new domains of size H^{-1}, which evolve independently of each other. In many new domains the scalar field decreases because of classical rolling and quantum fluctuations. The rate of expansion of these domains rapidly decreases, and they give a relatively small contribution to the total volume of those parts of the Universe which will have density 10^{-29} g cm^{-3} 15×10^9 years later. Meanwhile, those domains where quantum jumps occur in the direction of growth of the field ϕ gradually push this field towards the upper bound where inflation can possibly exist, which is presumably close to the Planck boundary $V(\phi_P)\sim 1$. Such domains for a long time stay near the Planck boundary, and exponentially grow with Planckian speed. Thus, the longer they stay near the Planck boundary, the greater the contribution to the volume of the Universe they give.

However, the domains of interest for us eventually should roll down and evolve into the regions with density 10^{-29} g cm^{-3}. Thus, these domains cannot stay near the Planck boundary for an indefinitely long time, producing a new volume with Planckian speed. However, they will do their best if they stay there as long as possible, in order to roll down at the latest possible moment. In fact they will do even better if they stay near the Planck boundary even longer, to save time for additional rapid inflation, and then rush down with a speed exceeding the speed of classical rolling. This may happen if quantum fluctuations coherently add up to large quantum jumps towards small ϕ. This process is dual to the process of perpetual climbing up, which leads to the self-reproduction of an inflationary universe.

Of course, the probability of large quantum jumps down is exponentially suppressed. However, by staying longer near the Planck boundary inflationary domains get an additional exponentially large contribution to their volume. These two exponential factors compete with each other to give us an optimal trajectory by which the scalar field rushes down in those domains which eventually give the leading contribution to the volume of the Universe. From what we are saying it should be clear that the quantum jumps of the scalar field along such optimal trajectories should have a greater amplitude than their regular value $H/2\pi$, and they should preferably occur downwards. As a result, the energy density along these optimal trajectories will be smaller than the energy density of their lazy neighbors which prefer to slide down without too much jumping. This creates wells in the distribution of the energy density, which we called infloids [3].

Suppose that the extra time interval spent at the highest energies is $\tilde{\Delta}t$. Then we win the volume by a factor of $\exp(d_{\rm fr}H_{\rm max}\tilde{\Delta}t)$. However, to compensate for the lost time the inflaton field ϕ has to jump at least once (let us say, when it reaches the value ϕ) with amplitude $\tilde{\delta}\phi=n(\phi)H(\phi)/2\pi$ such that it covers in one jump the distance which would otherwise require time $\tilde{\Delta}t$ to slowly roll down:

$$\tilde{\Delta}t(\phi)=\frac{\tilde{\delta}\phi}{\dot{\phi}}=\frac{n(\phi)H(\phi)/2\pi}{\dot{\phi}}=n(\phi)\frac{4V(\phi)}{V'(\phi)}, \qquad (7)$$

where we introduced the factor $n(\phi)$ by which the jump is amplified, i.e., by which it is greater than the standard jump $H(\phi)/2\pi$. The probability of such a jump is suppressed by the factor $\exp[-\frac{1}{2}n^2(\phi)]$. The leading contribution to the volume of the Universe occurs due to the jumps which maximize the volume-weighted probability:

$$P\sim\exp[d_{\rm fr}H_{\rm max}\tilde{\Delta}t(\phi)-\tfrac{1}{2}n^2(\phi)]$$
$$=\exp\left(d_{\rm fr}H_{\rm max}n(\phi)\frac{4V(\phi)}{V'(\phi)}-\tfrac{1}{2}n^2(\phi)\right). \qquad (8)$$

Maximizing with respect to $n(\phi)$ gives the amplification factor as a function of the location of the jump on the inflaton trajectory:

$$n(\phi)=4d_{\rm fr}H_{\rm max}\frac{V(\phi)}{V'(\phi)}. \qquad (9)$$

In fact, we have found [3] that the typical trajectories which give the leading contribution to the volume of the Universe consist entirely of such subsequent jumps. In what follows we will give an alternative derivation of this result. Meanwhile, comparing with Eq. (6) one immediately sees that $n(\phi)\gg 1$ for $\phi<\phi_*$, since $d_{\rm fr}\sim 3$ and $H_{\rm max}\gg H(\phi)$ for such values of the inflaton field in chaotic inflation. Therefore, our treatment of these quantum fluctuations as large and rear quantum jumps is self-consistent.

To avoid misunderstandings one should note that a more accurate definition of the amplification coefficient would be $n(\phi)+1$. Indeed, in the absence of nonperturbative effects we would have $n(\phi)=0$ since perturbative jumps occur in both directions with equal probability. The coefficient $n(\phi)$ relates an additional amplitude of jumps *down* to the regular perturbative amplitude of the jumps in both directions. This subtlety will not be important for us here since we are interested in the case $n\gg 1$.

It is interesting that the coefficient of the amplification $n(\phi)$ can be directly related to the ratio of amplitudes of conventional scalar and tensor perturbations generated at the same scale at which the jump occurs. The amplitudes of these perturbations can be written as

$$A_S^{\rm pert}(\phi)=\left(\frac{\delta\rho}{\rho}\right)_S=c_S\frac{H^2(\phi)}{2\pi\dot{\phi}},$$
$$A_T^{\rm pert}(\phi)=\left(\frac{\delta\rho}{\rho}\right)_T=c_T\frac{H(\phi)}{M_P}. \qquad (10)$$

Here c_S and c_T are some coefficients of the order of unity. Using these expressions we can rewrite Eq. (9) for $d_{\rm fr}\sim 3$ in the form

$$n(\phi)=\frac{3c_T}{c_S}\frac{H_{\rm max}}{M_P}\frac{A_S^{\rm pert}(\phi)}{A_T^{\rm pert}(\phi)}. \qquad (11)$$

In the same way as the conventional amplitude of jumps $H/2\pi$ is related to the well-known perturbations of the background energy density, the ''nonperturbatively amplified'' jumps which we have just described are related to the ''nonperturbative'' contribution to deviations of the background energy density from its average value. A possible interpretation of this result is that at the length scale associated with the value of the field ϕ there is an additional nonperturbative contribution to the *monopole* amplitude:

$$A_S^{\rm nonpert}(\phi)=\left(\frac{3c_T}{c_S}\frac{H_{\rm max}A_S^{\rm pert}(\phi)}{M_P\,A_T^{\rm pert}(\phi)}\right)A_S^{\rm pert}. \qquad (12)$$

We will discuss the structure of infloids and their possible observational consequences in Sec. VII. Here we only note that Eq. (11) gives a simple tool for understanding of the possible significance of the effect under consideration. Indeed, in the simplest chaotic inflation models, such as the theory $(\lambda/n)\phi^n$, one has $H_{\rm max}\sim M_P$ and $A_S^{\rm pert}(\phi) \gg A_T^{\rm pert}(\phi)$; thus, one has $n(\phi)\gg 1$. On the other hand, in the versions of chaotic inflation scenario where inflation occurs near a local maximum of the effective potential (as in the new inflation models) $H_{\rm max}$ is many orders of magnitude smaller than M_P, and therefore the nonperturbative effects discussed above are negligibly small. Thus, investigation of nonperturbative effects can give us a rather unexpected possibility to distinguish between various classes of inflationary models. We will return to this issue at the end of the paper.

IV. NONPERTURBATIVE EFFECTS AND BRANCHING DIFFUSION

One of the best ways to examine nonperturbative effects is to investigate the probability distribution $P_P(\phi,t)$ to find a domain of a given physical volume in a state with a given field ϕ at some moment of time t. The distribution $P_P(\phi,t)$ obeys the following branching diffusion equation [10,16,6]:

$$\frac{\partial P_P}{\partial t}=\frac{\partial}{\partial\phi}\left[\frac{H^{3/2}(\phi)}{2\sqrt{2}\pi}\frac{\partial}{\partial\phi}\left(\frac{H^{3/2}(\phi)}{2\sqrt{2}\pi}P_P\right)+\frac{V'(\phi)}{3H(\phi)}P_P\right] + 3H(\phi)P_P. \tag{13}$$

This equation is valid only during inflation, which typically occurs within some limited interval of values of the field ϕ: $\phi_{\rm min}<\phi<\phi_{\rm max}$. In the simplest versions of the chaotic inflation model $\phi_{\rm min}\equiv\phi_e\sim 1$, where ϕ_e is the boundary at which inflation ends. Meanwhile, as we argued in the previous section, $\phi_{\rm max}$ is close to the Planck boundary ϕ_P, where $V(\phi_P)=1$. To find solutions of this equation one must specify the boundary conditions. The behavior of the solutions typically is not very sensitive to the boundary conditions at ϕ_e; it is sufficient to assume that the diffusion coefficient [and, correspondingly, the double derivative term in the right-hand side (RHS) of Eq. (13)] vanishes for $\phi<\phi_e$ [6]. The conditions near the Planck boundary play a more important role. In this paper we will assume that there can be no inflation at $V(\phi)>1$, which corresponds to the boundary condition $P_P(\phi,t)|_{\phi>\phi_P}=0$. At the end of the paper we will discuss possible modifications of our results if $\phi_{\rm max}$ differs from ϕ_P.

One may try to obtain solutions of Eq. (13) in the form of the eigenfunction series

$$P_P(\phi,t)=\sum_{s=1}^{\infty} e^{\lambda_s t}\pi_s(\phi) \underset{t\to\infty}{\sim} e^{\lambda_1 t}\pi_1(\phi), \tag{14}$$

where, in the limit of large time t, only the term with the largest eigenvalue λ_1 survives. The function $\pi_1(\phi)$ in the limit $t\to\infty$ has the meaning of a normalized *time-independent* probability distribution (so-called invariant probability density of the branching diffusion) to find a given field ϕ in a unit physical volume, whereas the factor $e^{\lambda_1 t}$ shows the overall growth of the volume of all parts of the Universe, which does not depend on ϕ in the limit $t\to\infty$. This ‘‘ground state’’ eigenfunction satisfies the equation

$$\frac{\partial}{\partial\phi}\left[\frac{H^{3/2}(\phi)}{2\sqrt{2}\pi}\frac{\partial}{\partial\phi}\left(\frac{H^{3/2}(\phi)}{2\sqrt{2}\pi}\pi_1(\phi)\right)+\frac{V'(\phi)}{3H(\phi)}\pi_1(\phi)\right] + 3H(\phi)\pi_1(\phi)=\lambda_1\pi_1(\phi). \tag{15}$$

In the limit when we can neglect the diffusion (second derivative) term it is easy to solve this equation:

$$\pi_1(\phi)=C(\phi_0)\frac{3H(\phi)}{V'(\phi)}\exp \times\left(-\int_{\phi}^{\phi_0}\left[\lambda_1\frac{3H(\zeta)}{V'(\zeta)}-\frac{9H^2(\zeta)}{V'(\zeta)}\right]d\zeta\right), \tag{16}$$

where we chose some starting point ϕ_0 and the corresponding normalization constant $C(\phi_0)$ which should match this approximate solution to the exact one at this point. As before, let us introduce the fractal dimension of classical space-time through $\lambda_1=d_{\rm fr}H_{\rm max}$ (see [9,6] for a detailed discussion of the fractal structure of a self-reproducing universe). Let us also introduce the critical value of inflaton $\phi_{\rm fr}$ at which the no-diffusion approximation for Eq. (15) breaks. Then, since $H_{\rm max}\gg H(\phi)$ for chaotic inflation, we can rewrite Eq. (16) as

$$\pi_1(\phi)=C(\phi_{\rm fr})\frac{3H(\phi)}{V'(\phi)}\exp\left(-\int_{\phi}^{\phi_{\rm fr}}d_{\rm fr}H_{\rm max}\frac{3H(\zeta)}{V'(\zeta)}d\zeta\right). \tag{17}$$

Substituting Eq. (17) into Eq. (15) we get the defining relation for the value of inflaton field $\phi_{\rm fr}$ at which the no-diffusion approximation breaks:

$$\lambda_1\frac{9H^5(\phi_{\rm fr})}{4\pi^2[V'(\phi_{\rm fr})]^2}\sim 1. \tag{18}$$

We can rewrite (the square root of) this relation in a form which makes the comparison with the definition of the other critical value ϕ_* more apparent:

$$\sqrt{\frac{d_{\rm fr}H_{\rm max}}{H(\phi_{\rm fr})}}\frac{3H^3(\phi_{\rm fr})}{2\pi V'(\phi_{\rm fr})}\sim 1. \tag{19}$$

Comparing Eq. (19) with Eq. (6), one finds that for all chaotic inflation models $\phi_{\rm fr}<\phi_*$ [one can assume self-consistently that $H_{\rm max}\gg H(\phi_{\rm fr})$ in such models]. The value of ϕ_* in Eq. (6) comes from comparing the slow-roll rate in a given h region with the typical amplitude of quantum fluctuations while considering only the h regions generated locally from the region which we picked. On the other hand, the value $\phi_{\rm fr}$ comes from comparing the slow-roll rate to the typical amplitude of fluctuations considering all h regions in the whole Universe which happen to have the same value of the inflaton field inside. The fact that the second constraint is more stringent is yet another indication of the considerably larger magnitude of the quantum fluctuations when we take into account the whole stationary Universe.

In the particular case of the simplest theory with $V(\phi)=\lambda\phi^4/4$, we have $H=\sqrt{2\pi\lambda/3}\phi^2$, $\phi_{\rm fr}\sim\lambda^{-1/8}\ll\phi_*$, and the dependence of the solution (17) on ϕ is [6]

$$\pi_1(\phi)\sim\phi^{\sqrt{(6\pi/\lambda)}d_{\rm fr}H_{\rm max}}. \tag{20}$$

This is an extremely strong dependence. For example, for the realistic value of the coupling constant $\lambda\sim10^{-13}$ chosen to fit the observable large scale structure of the Universe one has $d_{\rm fr}\approx3$. One may assume for definiteness that $H_{\rm max}=\sqrt{8\pi/3}$, corresponding to inflation with $V(\phi)=1$ (Planck density). Then one has an extremely sharp dependence $\pi_1\sim\phi^{1.2\times10^8}$. All surprising results we are going to obtain are rooted in this effect. One of the consequences is the distribution of energy density ρ. For example, during inflation $\rho\approx\lambda\phi^4/4$. Equation (20) implies that the distribution of density ρ is

$$P_P(\rho)\sim\rho^{3\times10^7}. \tag{21}$$

Thus at each moment of time t the Universe consists of an indefinitely large number of domains containing matter with all possible values of density, the total volume of all domains with density 2ρ being approximately 10^{10^7} times greater than the total volume of all domains with density ρ.

Let us consider now all inflationary domains which contain a given field ϕ at a given moment of time t. One may ask the question, what was the value of this field in those domains at the moment $t-H^{-1}$? In order to answer this question one should add to ϕ the value of its classical drift $\dot\phi H^{-1}$ and the amplitude of quantum jumps $\Delta\phi$. The typical jump is given by $\delta\phi=\pm H/2\pi$. At the last stages of inflation this quantity is by many orders of magnitude smaller than $\dot\phi H^{-1}$. But in which sense are jumps $\pm H/2\pi$ typical? If we consider any particular initial value of the field ϕ, then the typical jump from this point is indeed given by $\pm H/2\pi$ under the conventional comoving measure. However, if we are considering all domains with a given ϕ and trying to find all those domains from which the field ϕ could originate back in time, the answer may be quite different. Indeed, the total volume of all domains with a given field ϕ at any moment of time t depends on ϕ extremely strongly: The dependence is exponential in the general case (17) or a power law with a huge power, like in the case of $\lambda\phi^4/4$ theory (20). This means that the total volume of all domains which could jump towards the given field ϕ from the value $\phi+\Delta\phi$ will be enhanced by a large additional factor $P_P(\phi+\Delta\phi)/P_P(\phi)$. On the other hand, the probability of large jumps $\Delta\phi$ is suppressed by the Gaussian factor $\exp[-2\pi^2(\Delta\phi)^2/H^2]$. Thus, under the established stationary probability distribution the probability of the inflaton field in a given domain to have experienced a quantum jump $\Delta\phi$ is given by

$$P(\Delta\phi)\sim\exp\left(d_{\rm fr}H_{\rm max}\frac{3H(\phi)}{V'(\phi)}\Delta\phi-\frac{2\pi^2(\Delta\phi)^2}{H^2(\phi)}\right). \tag{22}$$

One can easily verify that this distribution has a sharp maximum at

$$\Delta\phi_{\rm np}=d_{\rm fr}H_{\rm max}\frac{3H^3(\phi)}{4\pi^2V'(\phi)}=n(\phi)\frac{H(\phi)}{2\pi}, \tag{23}$$

and the width of this maximum is of the order $H/2\pi$. In other words, most of the domains of a given field ϕ are formed due to nonperturbative (hence the subscript ''np'') jumps which are greater than the ''typical'' ones by a factor $n(\phi)$ which coincides with our previous result (9). For future reference, we will write here this result in an equivalent form

$$n(\phi)=4\lambda_1\frac{V(\phi)}{V'(\phi)}. \tag{24}$$

The limit of applicability of this expression is below the energy level $V(\phi_{\rm fr})$ [see Eqs. (18) and (19) for the definition of the critical value $\phi_{\rm fr}$].

In particular, for the theory $\lambda\phi^4/4$ we have

$$n(\phi)=\lambda_1\phi. \tag{25}$$

For $H_{\rm max}=\sqrt{8\pi/3}$, $\lambda\ll1$, and $\phi\sim4.5$, which corresponds to today's horizon scale, this gives the amplification coefficient

$$n(\phi)=2\sqrt{6\pi}\phi\sim40. \tag{26}$$

V. VOLUME-WEIGHTED SLOW-ROLLING APPROXIMATION

We learned in the previous section that quantum fluctuations in volume-weighted measure have pretty large expectation value, which makes the jumps to go preferentially downwards (unlike in the comoving measure where there is no preferred direction of the fluctuations and therefore they have a zero expectation value). Indeed, such was the very meaning of our derivation of large jumps that they had to occur in the direction of the usual slow roll in order to make up the extra time spent by inflaton at higher energies. Therefore, we can conclude that the slow-rolling speed itself gets a correction corresponding to the rate at which such large jumps occur and their size. Since each such jump of size $n(\phi)H(\phi)/2\pi$ occurs during the time interval $H^{-1}(\phi)$, we can estimate the additional speed gained by the inflaton as $n(\phi)H^2(\phi)/2\pi$, thus bringing the overall slow-roll speed to the volume-weighted value [we substituted Eq. (9) for the value of $n(\phi)$, the amplification factor]:

$$\dot\phi=-\frac{V'(\phi)}{3H(\phi)}-d_{\rm fr}H_{\rm max}\frac{16V^2(\phi)}{3V'(\phi)}. \tag{27}$$

Here the minus sign in front of the correction term is due to the preferred direction of the jumps, bringing the slow-roll speed to a higher absolute value.

The limits of applicability of this expression are the same as for Eq. (23), i.e., below the energy density corresponding to the critical value $\phi_{\rm fr}$ of the inflaton field, defined by Eqs. (18) and (19). However, those limits simply tell where the approximate expression (27) is valid, while the effect of speeding up the slow roll of the inflaton is valid in a much wider range.

Let us derive a more general version of this result and, correspondingly, a more general expression for amplified quantum jumps (9) and (23) which will be valid for almost whole range of variation of the inflaton field. The volume-weighted probability distribution can be defined as the path integral over all realizations of noise taken with Gaussian weight modified by the volume factor [10,6]:

$$P_P(\phi,t)=\int \mathcal{D}\xi \exp\left\{\int^t\left(-\frac{1}{2}\xi^2(s)+3H(\phi_\xi(s))\right)ds\right\} \times \delta(\phi_\xi(t)-\phi). \tag{28}$$

Here $\phi_\xi(s)$ is the solution of Eq. (5) with a particular realization of the noise. The Gaussian path integral over the noise can be converted into the path integral over the histories of inflaton evolution [17] if we express the noise through concurrent value of inflaton $\phi(t)$ using the equation of motion (5):

$$\xi(t)=\frac{2\pi}{H^{3/2}(\phi)}\dot{\phi}+\frac{2\pi V'(\phi)}{3H^{5/2}(\phi)}. \tag{29}$$

It is convenient to make the variable transformation

$$z=\int_\phi \frac{2\pi}{H^{3/2}(\phi')}d\phi'. \tag{30}$$

In terms of this variable the definition of the white noise is rewritten in compact form

$$\xi(t)=-\dot{z}+W(z), \tag{31}$$

where we introduced the ''superpotential''[2] [we used the relation (30) to reexpress it in terms of the derivative with respect to z]

$$W(z)=\frac{2\pi}{3H^{5/2}(\phi)}\frac{dV(\phi)}{d\phi}=\frac{d}{dz}\left(\frac{3}{16V(z)}\right). \tag{32}$$

The path integral defining the volume-weighted measure in terms of $z(t)$ becomes, after substituting Eqs. (31) into Eqs. (28),

$$P_P(z,t)=\int \mathcal{D}z(s)J[z]\exp\left\{-\int^t\left(\frac{1}{2}[\dot{z}(s)-W(z(s))]^2 -3H(z(s))\right)ds\right\}\delta(z(t)-z). \tag{33}$$

The Jacobian $J[z]$ of the transformation from ξ to ϕ and then to z is preexponential [17] and unimportant for our current investigation. We will neglect it in what follows.

Let us find the trajectory $z(t)$ [which we will translate later into trajectory $\phi(t)$] which contributes most to the path integral (33). Such a saddle point trajectory will correspond to the typical history of the evolution of inflaton under a volume-weighted measure. The exponent in the path integral (33) looks like a Euclidean version of the Lagrangian action, which corresponds to an interpretation of the diffusion equation (13) as a Euclidean Schrödinger equation for a point particle. We can rewrite this action in Hamiltonian form using the conventional relation

$$\int^t \mathcal{L}dt=\int^{z(t)}p\,dz-\int^t\mathcal{H}dt, \tag{34}$$

where the canonical momentum is

$$p=\frac{\partial\mathcal{L}}{\partial\dot{z}}=\dot{z}-W(z). \tag{35}$$

Since the action does not contain an explicit time dependence, the Hamiltonian is conserved:

$$\mathcal{H}=\tfrac{1}{2}p^2+pW(z)+3H(z)=\lambda_1. \tag{36}$$

The reason why the conserved Hamiltonian is equal to the highest eigenvalue is that in the end we should get the time dependence of a type $\exp(\lambda_1 t)$ as warranted by the stationary solution (14). Meanwhile, the $\int p\,dz$ term of the action should give us the correct (semiclassical) field dependence of the probability density $P_P(z(\phi),t)$ (see below).

Solving the Hamiltonian constraint (36) with respect to p (we have to choose the positive solution of the equation for rolling down), and using Eq. (35), we obtain the equation for the typical volume-weighted trajectory:

$$\dot{z}=\sqrt{W^2(z)+2\lambda_1-6H(z)}. \tag{37}$$

This equation translates back in terms of the inflaton field variable into a volume-weighted slow-roll equation:

$$\dot{\phi}=-\sqrt{\left(\frac{V'(\phi)}{3H(\phi)}\right)^2+[d_{\rm fr}H_{\rm max}-3H(\phi)]\frac{H^3(\phi)}{2\pi^2}}. \tag{38}$$

For most of the inflaton range of variation (except very close to the Planck boundary) we can ignore the $3H$ term with respect to the $d_{\rm fr}H_{\rm max}$ term. The relative importance of the two remaining terms under the square root is governed by the critical value $\phi_{\rm fr}$ — below this level the first term is more important, while above it the second one dominates. Not surprisingly, below $\phi_{\rm fr}$ this equation coincides with Eq. (27). However, its validity limits are much wider, allowing us to use it beyond $\phi_{\rm fr}$, to which the applicability of Eq. (27) was limited.

We can write down a good approximation for the field-dependent normalized probability density $\pi_1(\phi)$, omitting the less important preexponential terms:

[2] This name is due to the fact that $W(z)$ plays the role of a superpotential in a supersymmetry- (SUSY-)Schrödinger-like version of the Fokker-Planck equation.

$$\pi_1(\phi)=\exp\left\{-\int^{z(\phi)}p\,dz\right\}$$
$$=\exp\left\{-\int_\phi d\zeta\left(\sqrt{\left(\frac{3V'(\zeta)}{16V^2(\zeta)}\right)^2+[d_{\rm fr}H_{\rm max}-3H(\phi)]\frac{8\pi^2}{H^3(\phi)}}-\frac{3V'(\zeta)}{16V^2(\zeta)}\right)\right\}. \tag{39}$$

Of course, below $\phi_{\rm fr}$ this expression also coincides with its counterpart (17) derived previously. This result has remarkable properties which will be studied further in [18].

Using the volume-weighted slow-roll equation (38) we can derive a general expression for the amplified quantum jump size. It is given by the change in ϕ within time interval H^{-1} calculated according to Eq. (38), less the regular expression for the change of field due to the slow roll in a comoving measure:

$$\Delta\phi_{\rm np}=\sqrt{\left(\frac{V'(\phi)}{3H^2(\phi)}\right)^2+[d_{\rm fr}H_{\rm max}-3H(\phi)]\frac{H(\phi)}{2\pi^2}}-\frac{V'(\phi)}{3H^2(\phi)}. \tag{40}$$

This gives the following expression for the amplification factor (the ration of $\Delta\phi_{\rm np}$ and the conventional amplitude $H/2\pi$):

$$n(\phi)=\sqrt{\left(\frac{2\pi V'(\phi)}{3H^3(\phi)}\right)^2+\frac{2d_{\rm fr}H_{\rm max}-6H(\phi)}{H(\phi)}}-\frac{2\pi V'(\phi)}{3H^3(\phi)}. \tag{41}$$

The consistency conditions for our results (38)–(41) arise from several assumptions which we made in their derivation and whose validity should be maintained. The first one is that the slow rolling approximation be valid; i.e., $\ddot{\phi}\ll 3H(\phi)\dot{\phi}$. The second is that the amplification factor be greater than 1. The third condition is that the saddle-point approximation used to derive these results be valid, which means that $p\gg 1$. And the final, fourth condition is the implicit assumption that large quantum jumps which occur in a single h region do not make the gradient energy inside that region greater than the potential energy of the inflaton field (which, of course, would immediately invalidate the inflationary approximation). One can easily check that all four conditions lead to the same, very relaxed restrictions — the energy density of the inflaton field $V(\phi)$ must be lower than the Planck density (or, more precisely, lower than the energy density corresponding to the maximal rate of expansion $H_{\rm max}$). Thus, we can use the results obtained above in most of the variation range for the inflaton field in chaotic inflation.

One can easily check that for $\phi<\phi_{\rm fr}$, $H(\phi)\ll H_{\rm max}$, Eq. (41) yields

$$n(\phi)=4d_{\rm fr}H_{\rm max}\frac{V(\phi)}{V'(\phi)}. \tag{42}$$

This expression coincides with the expression for the amplification coefficient which we obtained earlier by two other methods; see Eqs. (9) and (23).

VI. NUMERICAL SIMULATIONS

A. Basic idea of computer simulations

Even though we verified our results by several different methods, they are still very unusual and counterintuitive. Therefore we performed a computer simulation of stochastic processes in an inflationary universe, which allows one to obtain an additional verification of our results and to calculate the amplification factor $n(\phi)$ numerically. We have used two different methods of computer simulations. The first one is more direct and easy to understand. Its basic idea can be explained as follows.

We have studied a set of domains of initial size H^{-1} filled with a large homogeneous field ϕ. We considered large initial values of ϕ, which leads to the self-reproduction of inflationary domains. From the point of view of stochastic processes which we study, each domain can be modeled by a single point with the field ϕ in it. Our purpose was to study the typical amplitude of quantum jumps of the scalar field ϕ in those domains which reached some value $\phi_0=O(1)$ close to the end of inflation. Then we calculate the amplification factor $n(\phi_0)$ for various ϕ_0.

Each step of our calculations corresponds to a time change $\Delta t=uH_0^{-1}$. Here $H_0\equiv H(\phi_0)$, and u is some number, $u<1$. The results do not depend on u if it is small enough. The evolution of the field ϕ in each domain consists of several independent parts. First of all, the field evolves according to classical equations of motion during inflation, which means that it decreases by $uV'/3HH_0$ during each time interval uH_0^{-1}. Second, it makes quantum jumps by $\delta\phi=(H/2\pi)(\sqrt{uH/H_0})r_i$. Here r_i is a set of normal random numbers, which are different for each inflationary domain. To account for the growth of the physical volume of each domain we used the following procedure. We followed each domain until its radius had grown 2 times, and after that we considered it as eight independent domains. In accordance with our condition $P_P(\phi,t)|_{\phi>\phi_P}=0$, we removed all domains where the field ϕ jumped to the super-Planckian densities $V(\phi)>1$. Therefore our method removes the overall growth factor $e^{\lambda_1 t}$ in the expression $P_P\sim e^{\lambda_1 t}\pi_1(\phi)$ and directly gives the time-independent function $\pi_1(\phi)$ which we are looking for. Indeed we have checked that after a sufficiently large time t the distribution of domains followed by the computer with a good accuracy approached the stationary distribution $\pi_1(\phi)$ which we have obtained in [6] by a completely different method; see Fig. 1. We used it as a consistency check for our calculations. In what follows we will not distinguish between P_P and the time-independent factor $\pi_1(\phi)$.

We kept in the computer memory information about all jumps of each domain during the last time interval H_0^{-1} be-

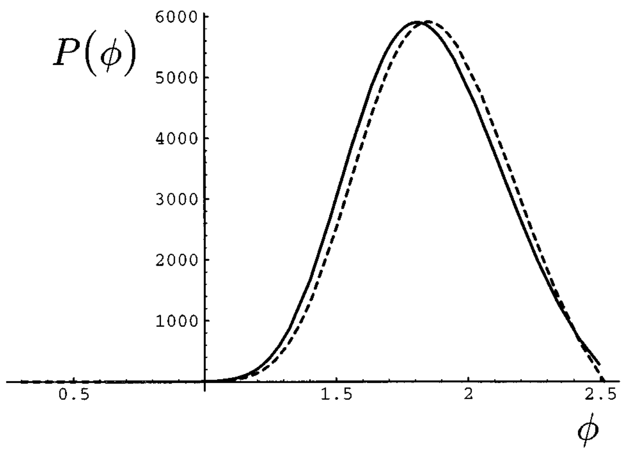

FIG. 1. Probability distribution $P(\phi)$ for $V=\lambda\phi^4/4$, $\lambda=0.1$. The dashed line is the numerical solution to a differential equation describing $P(\phi)$. The solid curve is obtained using computer simulations described in this paper. A small deviation between the solid curve and the dashed line is due to the finite size of each step and the finite grid size.

fore the field ϕ inside this domain becomes smaller than ϕ_0. This made it possible to evaluate an average sum of all jumps of those domains in which the scalar field became smaller than ϕ_0 within the last time interval H_0^{-1}. Naively, one could expect this value to be smaller than $H_0/2\pi$, since the average amplitude of the jumps is $H_0/2\pi$, but they occur both in the positive and negative directions. However, our simulations confirmed our analytical result $\Delta\phi=\lambda_1\phi H_0/2\pi$. In other words, we have found that most of the domains which reach the hypersurface $\phi=\phi_0$ within a time interval $\Delta t=H_0^{-1}$ do it by rolling accompanied by persistent jumps down, which have a combined amplitude $\lambda_1\phi_0$ times greater than $H_0/2\pi$.

B. Details of the method

Even though this method of calculations may seem quite straightforward (it is the so-called event-tracing Monte Carlo method), in reality it must be somewhat modified. The main problem is obvious if one recalls our expression for the probability distribution $P_P\sim e^{\lambda_1 t}\pi_1(\phi)$ at small ϕ: $P_P\sim e^{\lambda_1 t}\phi^{(\sqrt{6\pi/\lambda})\lambda_1}$. As we already mentioned (omitting the time-dependent factor), this yields $P_P\sim\phi^{10^8}$ for the realistic value $\lambda\sim 10^{-13}$. It is extremely difficult to work with distributions which are so sharp.

Therefore in our computer simulations we have studied models with $\lambda\sim 0.1$, which makes computations possible. On the other hand, when one increases the value of λ an additional problem arises. Our simple expression $P_P\sim\phi^{(\sqrt{6\pi/\lambda})\lambda_1}$ has been obtained in the limit of very small λ, which is not perfectly accurate for $\lambda\sim 0.1$. Therefore we will represent P_P in a more general form $P(\phi)=\phi^{g(\phi)}$, where $g(\phi)$ approaches a constant value $(\sqrt{6\pi/\lambda})\lambda_1$ for $\phi\ll\lambda^{-1/8}$. One should also take into account that the classical decrease $c(\phi)=V'(\phi)/3HH_0$ of the field ϕ during the time H_0^{-1} and the standard deviation $s(\phi)=(H/2\pi)\sqrt{H/H_0}$ (the average amplitude of quantum fluctuations during the time H_0^{-1}) are not constant throughout the region where the effect takes place. In such a situation an expression for the amplification coefficient $n(\phi)$ will be slightly different from our simple expression $n=\lambda_1\phi$. Therefore we should first derive here a more accurate expression for $n(\phi)$, and then compare it with the results of our simulations.

Consider a point for which $\phi=\phi_0$ at some time t, at which the stationary regime is already established. At the earlier time $t-H_0^{-1}$ this point was approximately at $\phi=\phi_0+c(\phi_0)+x$, where $x=\delta\phi$ is the sum of all quantum jumps experienced by the field ϕ at this point during the last time interval H_0^{-1}. Consider the probability $P(\delta\phi)$ that the field ϕ jumped to ϕ_0 from the point $\phi_0+c(\phi_0)+x$. This probability distribution is equal to the distribution $P_P(\phi)$ times the probability of undergoing a quantum fluctuation of length $\delta\phi$:

$$P(x)\propto P_P[\phi_0+c(\phi_0)+x]\exp\left(-\frac{x^2}{2s^2}\right). \tag{43}$$

The position of the maximum of the distribution $P(x)$ is given by

$$\frac{P_P'[\phi_0+c(\phi_0)+x]}{P_P[\phi_0+c(\phi_0)+x]}=\frac{x}{s^2}. \tag{44}$$

To solve this equation for x we need to know $P_P(\phi)$. As earlier we assume $P_P(\phi)=\phi^{g(\phi)}$ where $g(\phi)$ varies slowly with ϕ. If $g'\phi\ln\phi\ll\phi^{-1}\ln g$ (which happens to be a good approximation for $\phi\sim 1$), Eq. (44) can be easily solved, and the expression for $n(\phi_0)$ looks as follows:

$$\begin{aligned} n(\phi_0)&=\frac{x}{s(\phi_0)}\\ &\approx\frac{1}{2s(\phi_0)}\{\sqrt{[\phi_0+c(\phi_0)]^2+4g(\phi_0)s^2(\phi_0)}\\ &\quad-[\phi_0+c(\phi_0)]\}. \end{aligned} \tag{45}$$

One can obtain a slightly more accurate expression by taking into account the dependence of g, c, and s on ϕ. Note that in the situation which we are going to investigate $\phi_0\gg c(\phi_0)\gg x\gg s(\phi_0)$. In the limit when g, s, and c can be considered constant, and $\phi_0\gg s,c$ this equation leads to our earlier expression $n(\phi_0)=\lambda_1\phi_0$.

In order to use Eq. (45) we also need to know $g(\phi)$ for our problem. We approximate $g(\phi)$ by a second order polynomial in ϕ and substitute $P(\phi)=\phi^{a_0+a_1\phi+a_2\phi^2}$ into the differential equation for $P(\phi)$. Local analysis around $\phi=\phi_0$ shows $P(\phi)\approx\phi^{56-23\phi-4\phi^2}$. This approximation is accurate for $\phi\sim 1$.

C. Numerical calculation of $n(\phi)$

Even for not very small λ the distribution P_P remains extremely sharp. We have made our simulations with $\lambda=0.1$, in which case $P_P\sim\phi^{60}$. This means that if we want to follow the evolution of a single domain with $\phi=0.5$, then we should simultaneously keep track of $2^{60}-10^{18}$ domains with $\phi=1$. Therefore the simple event-tracing Monte Carlo approach which we described above can be quite adequate for the investigation of P_P near its maximum, but not for the study of P_P far away from the maximum of the distribution.

A more advanced approach is to represent the distribution by evenly spaced points with weight proportional to the distribution. In other words, we rewrite the probability distribution as a finite sum of nearly δ-functional distributions:

$$P_P(\phi,t)\approx\sum_{i=1}^{N} p_i(\phi,t), \tag{46}$$

where

$$p_i(\phi,t)=\begin{cases} P_P(\phi_i) & \text{for } \phi_i\leqslant\phi<\phi_{i+1}, \\ 0 & \text{otherwise.}\end{cases} \tag{47}$$

At each step of the simulation we investigate the evolution of the distributions p_i during the time $\Delta t=uH_0^{-1}$. The following equation takes into account classical decrease of the field ϕ, quantum fluctuations, and inflation:

$$p_i(\phi,t+\Delta t)\propto p_i(\phi,t)\frac{1}{s(\phi_i)}\exp\left(-\frac{(\phi-\phi_i+c)^2}{2\,s^2(\phi_i)}\right)\times\exp\left(\frac{3uH(\phi)}{H(\phi_0)}\right). \tag{48}$$

We find $P_P(\phi,t+\Delta t)$ by computing the sum of $p_i(\phi,t+\Delta t)$. Then we normalize the distribution $P_P(\phi,t+\Delta t)$ and again subdivide into a new set of p_i, in accordance with Eq. (46). We repeat this process until the resulting distribution P_P approaches a stationary regime.

The most tricky part of the algorithm is to find the amplification factor $n(\phi_0)$. To do that, we associate another distribution $x_i(\phi,t)$ with each ϕ_i. Here $x_i(\phi,t)$ is the sum of quantum fluctuations during the last time interval H^{-1}, along all trajectories which ended up in the interval $\phi_i\leqslant\phi<\phi_{i+1}$ at the time t. We combine all $x_i(\phi,t)$ into a single distribution $X(\phi,t)$, and evolve it in the same way as $P_P(\phi,t)$, dividing it into the nearly δ-functional distributions $x_i(\phi,t)$ at every iteration. This is possible because $x_i(\phi,t)$ is approximately Gaussian and its standard deviation is small compared with its mean. When the $P_P(\phi,t)$ converges, $x_0(\phi_0,t)$ approximates x, from which $n(\phi_0)$ can be calculated.

Decreasing step size u increases accuracy of $P_P(\phi)$ until some point, after which the accuracy starts to decrease. This decrease is explained by the fact that evolved p_i's are too sharp and therefore are represented inaccurately. To avoid this, having fixed N, we must keep u high enough, so that the smallest quantum fluctuation s is wider than the grid spacing. The grid spacing is proportional to $1/N$ and s is proportional to $\sqrt{u}$, and so the minimal $\sqrt{u}$ is proportional to $1/N$. The execution time until convergence is proportional to $N^2/\sqrt{u}$, or for the minimal u it is proportional to N^3. In practice the largest N for which the algorithm converges in a reasonable amount of time is of the order of 10^3.

D. Results of numerical calculations

The first step is to verify the numerical algorithm by comparing the probability distribution $P(\phi)$ it computes with a solution obtained by solving equation (15) obtained in [6].

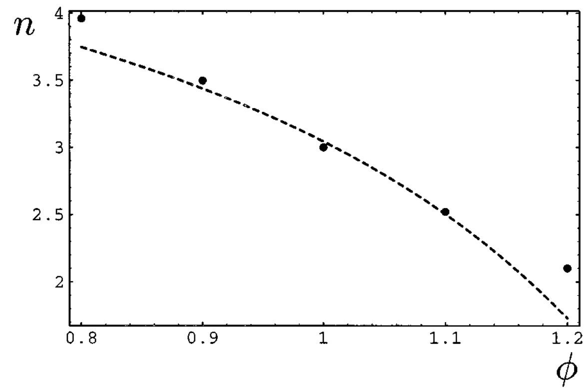

FIG. 2. Comparison between the analytical expression for $n(\phi)$ (dashed line) and the values for $n(\phi)$ obtained by computer simulations. While the analytical expression is not absolutely precise due to various assumptions (such as constancy of g, c, and s during the time H^{-1}), it does give approximately correct values for n.

Figure 1 shows that $P(\phi)$ is very close to the correct probability distribution. The deviation between the two decreases with step size.

The second step is to verify out formula for n. Figure 2 shows that numerically computed values of n for different ϕ_0 are close to the ones predicted by the analytical result. The deviation is explained by the approximations made in the analytical solution (constancy of g, c, and s during time H^{-1}). We have found, also, that the typical deviation of the amplitude of jumps from their average value $nH/2\pi$ is of the order of $H/2\pi$, as suggested by Eq. (22). This will be important for our subsequent considerations.

VII. SPATIAL STRUCTURE OF INFLOIDS

As one can see from Eq. (38), the value of the field $\phi(t)$ corresponding to the typical volume-weighted trajectories moves down more rapidly that one would expect from the classical slow-roll equation $\dot{\phi}=-V'(\phi)/3H(\phi)$. This is exactly the reason why such nonperturbatively enhanced trajectories, being surrounded by usual classical neighbors, should correspond to the minima in the distribution of density. To analyze the spatial structure of the Universe near the points corresponding to the optimal volume-weighted trajectory (38) one should remember that in terms of the ordinary comoving measure P_c the probability of large fluctuations is suppressed by the factor $\exp[-n^2(\phi)/2]$. It is well known that exponentially suppressed perturbations typically give rise to spherically symmetric bubbles [19]. Let us show first of all that the main part of the volume of the Universe in a state with a given ϕ (or with a given density ρ) corresponds to the centers of these bubbles, which we called infloids.

Consider again the collection of all parts of the Universe with a given ϕ (or a given density) at a given time t. We have found that most of the jumps producing this field ϕ during the previous time interval H^{-1} occurred from domains containing the field ϕ in a narrow interval of values near $\phi-\dot{\phi}/H+n(\phi)H/2\pi$. The width of this interval was found to be of the order of $H/2\pi$, which is much smaller than the typical depth of our bubble, $\Delta\phi\sim n(\phi)H/2\pi$, since

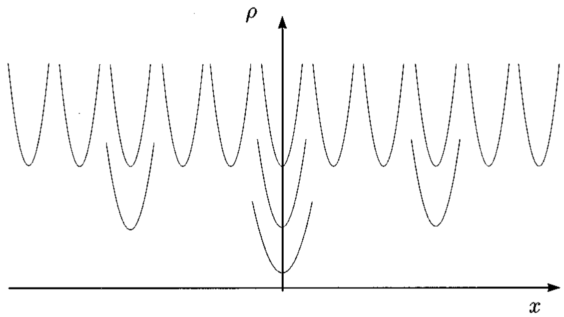

FIG. 3. A schematic illustration which shows the number of infloids with a given density and distribution of matter near their centers.

we have $n(\phi)\gg 1$ for all chaotic inflation models. Now suppose that the domain containing the field ϕ appears not at the center of the bubble, but at its wall. This would mean that the field near the center of the bubble is even smaller than ϕ. Such a configuration could be created by a jump from $\phi-\dot{\phi}/H+n(\phi)H/2\pi$ only if the amplitude of the jump is greater than $n(\phi)H/2\pi$. However, we have found that the main contribution to the volume of domains with a given ϕ is produced by jumps of an amplitude $[n(\phi)\pm 1]H/2\pi$, the greater deviation from the typical amplitude $n(\phi)H/2\pi$ being exponentially suppressed. This means that the scalar field ϕ can differ from its value at the center of the bubble by no more than the usual amplitude of scalar field perturbations $H/2\pi$, which is smaller than the depth of the bubble by a factor of $n^{-1}(\phi)$. Thus, the main fraction of the volume of the Universe with a given ϕ (or with a given density of matter) can be only slightly outside the center. This may lead to a small contribution to the anisotropy of the microwave background radiation.

We should emphasize that all our results are based on the investigation of the global structure of the Universe rather than of the structure of each particular bubble. This is why we assert that our effect is *nonperturbative*. If one neglects that the Universe is a fractal and looks only at one particular bubble (i.e., at the one in which we live now), then one can find that inside each bubble there is a plenty of space far away from its center. Therefore one could conclude that there is nothing special about the centers of the bubbles. However, when determining the fraction of domains near the centers we were comparing the volumes of *all* regions of *equal* density at equal time. Meanwhile, the density $\rho_{\rm wall}$ of matter on the walls of a bubble is greater than the density $\rho_{\rm center}$ in its center. As we have emphasized in the discussion after Eq. (20), the total volume of *all* domains of density $\rho_{\rm wall}$ is greater than the total volume of all domains of density $\rho_{\rm center}$ by the factor $(\rho_{\rm wall}/\rho_{\rm center})^{3\times 10^7}$. Thus, it is correct that the volume of space outside the center of the bubble is not smaller than the space near the center. However, going outside the center brings us to the region of a different density, $\rho_{\rm wall}>\rho_{\rm center}$. Our results imply that one can find much more space with $\rho=\rho_{\rm wall}$ not at the walls of our bubble, but near the centers of *other* bubbles.

This situation can be very schematically illustrated by Fig. 3. We do not make an attempt to show the spatial distribution of infloids. Rather we show the density distribution near the center of each of them. All these regions basically are very similar, but at any particular moment of time t there are much more regions with large density since they appeared from the regions which inflated at the nearly Planckian density for a longer time. With time the whole set of curves should go lower, to smaller ρ. However, at each moment of time there will be domains with all possible values of ρ, so that the distribution of all curves does not change in time (stationarity). If one looks at the whole picture without discriminating between states with different values of the density, it may seem that there is much more space outside of the centers of the bubbles. However, at any given moment of time t the main fraction of volume of the Universe in a state *with a given density* ρ is concentrated near the centers of spherically symmetric bubbles. One may look, for example, at the density corresponding to the centers of the third row of curves. At this density one may live either near the center of any of the 11 infloids or at the walls of only 3 of them. The fraction of the volume near the centers would be much greater if we try to show the realistic distribution $P_P(\rho)\sim\rho^{3\times 10^7}$ of the number of domains with a given density in the theory $\lambda\phi^4/4$.

The nonperturbative jumps down should occur on all scales independently. One may visualize the whole process as follows. At each given moment most of the volume of the Universe where the field ϕ takes some particular value appears close to the centers of infloids created by the nonperturbative jumps by $n(\phi)H/2\pi$. The new jumps occur each time H^{-1} independently of the previous history of the regions with a given ϕ. Therefore the leading contribution to the volume will be given by those rare centers of infloids where the field ϕ jumps down by $n(\phi)H/2\pi$ again and again. That is why the typical volume-weighted trajectories permanently go down with a speed exceeding the speed of classical rolling by $n(\phi)H^2/2\pi$; see Eqs. (38) and (41).

One may visualize the resulting distribution of the scalar field in the following way. At some scale r the deviation of the field ϕ from homogeneity can be approximately represented as a well of a radius r with the depth $n(\phi)H/2\pi$. Near the bottom of this well there is another well of a smaller radius $e^{-1}r$ and approximately of the same depth $n(\phi)H/2\pi$. Near the center of this well there is another well of a radius $e^{-2}r$, etc. In particular, in the theory $\lambda\phi^4/4$ the depth of each well will be $3H_{\rm max}H\phi/2\pi$. Of course, this is just a discrete model. The shape of the smooth distribution of the scalar field is determined by the equation

$$\frac{d\phi}{d\ln rH}=\frac{3H_{\rm max}H\phi}{2\pi}=\sqrt{\frac{3\lambda}{2\pi}}H_{\rm max}\phi^3, \tag{49}$$

which gives

$$\phi^2(r)\approx\frac{\phi^2(0)}{1-H_{\rm max}\phi^2(0)\sqrt{(6\lambda/\pi)}\ln rH}\quad \text{for } r>H^{-1}. \tag{50}$$

Note that $\phi(r)\approx\phi(0)$ for $r<H^{-1}$ (there are no perturbations of the classical field on this scale).

This distribution is slightly altered by the usual small perturbations of the scalar field. At a distance much greater than their wavelength from the center of the well these perturbations have the usual magnitude $H/2\pi$. Thus, our results do not lead to considerable modifications of the usual density perturbations which lead to galaxy formation. However, the presence of the deep well (50) can significantly change the local geometry of the Universe.

In the inflationary scenario with $V(\phi)=(\lambda/4)\phi^4$ fluctuations which presently have a scale comparable with the horizon radius $r_h\sim10^{28}$ cm have been formed at $\phi\sim5$ (in the units $M_P=1$). As we have mentioned already $3H_{\rm max}\approx2\sqrt{6\pi}\sim8.68$ for our choice of boundary conditions, [6] and the typical nonperturbative jump down on the scale of the present horizon should be $3H_{\rm max}\phi\sim40$ times greater than the standard jump; see Eq. (26). In the theory $(\lambda/4)\phi^4$ the standard jumps lead to density perturbations of the amplitude

$$\frac{\delta\rho}{\rho}\sim\frac{2\sqrt{6\lambda\pi}}{5}\phi^3\sim5\times10^{-5}$$

(in the normalization of [2]). Thus, according to our analysis, the nonperturbative decrease of density on each length scale different from the previous one by the factor e should be about

$$\frac{\delta\rho}{\rho}\sim H_{\rm max}\frac{6\sqrt{6\lambda\pi}}{5}\phi^4\sim2\times10^{-3}.$$

This allows one to evaluate the shape of the resulting well in the density distribution as a function of the distance from its center. One can write the following equation for the scale dependence of density:

$$\frac{1}{\rho}\frac{d\rho}{d\ln(r/r_0)}=-H_{\rm max}\frac{6\sqrt{6\lambda\pi}}{5}\phi^4, \tag{51}$$

where r is the distance from the center of the well. Note that

$$\phi=\frac{1}{\sqrt{\pi}}\left(\ln\frac{r}{r_0}\right)^{1/2}$$

in the theory $(\lambda/4)\phi^4$ [2]. Here r_0 corresponds to the smallest scale at which inflationary perturbations have been produced. This scale is model dependent, but typically at present it is about 1 cm. This yields

$$\frac{\Delta\rho}{\rho_c}\equiv\frac{\rho(r)-\rho(r_0)}{\rho(r_0)}=\frac{2H_{\rm max}\sqrt{6\lambda}}{5\pi\sqrt{3\pi}}\ln^3\frac{r}{r_0}. \tag{52}$$

This gives the typical deviation of the density on the scale of the horizon [where $\ln(r_h/r_0)\sim60$] from the density at the center:

$$\frac{\Delta\rho}{\rho_c}\sim750\frac{\delta\rho}{\rho}\sim4\times10^{-2}.$$

It is very tempting to interpret this effect in such a way that the Universe around us becomes locally open, with $1-\Omega\sim10^{-1}$. Indeed, our effect is very similar to the one discussed in [20,21], where it was shown that the Universe becomes open if it is contained in the interior of a bubble created by the O(4) symmetric tunneling. Our nonperturbative jumps look very similar to tunneling with the bubble formation. However, unlike in the case considered in [20,21], our bubbles appear on all length scales.

The results discussed above refer to the density distribution at the moment when the corresponding wavelengths were entering horizon. At the later stages gravitational instability should lead to growth of the corresponding density perturbations. Indeed, we know that density perturbations on the galaxy scale have grown more than 10^4 times in the linear growth regime until they reached the amplitude $\delta\rho/\rho\sim1$, and then continued growing even further. The same can be expected in our case, but even in a more dramatic way since our ''density perturbations'' on all scales are much greater than the usual density perturbations which are responsible for galaxy formation. This would make the center of the well very deep; its density should be many orders of magnitude smaller than the density of the Universe on the scale of horizon. This is not what we see around.

This problem can be easily resolved. Indeed, our effect (but not the amplitude of the usual density perturbations) is proportional to $H_{\rm max}$, which is the maximal value of the Hubble constant compatible with inflation. If, for example, the maximal energy scale in quantum gravity or in string theory is given not by 10^{19} GeV, but by 10^{18} GeV, then the parameter $H_{\rm max}$ will decrease by a factor 10^{-2}. Therefore the nonperturbative effects can be strongly suppressed in the models which we studied in this paper. As we already mentioned, nonperturbative effects in new inflation are even much smaller, since there $H_{\rm max}$ is always many orders of magnitude smaller than 1. Inflationary Brans-Dicke cosmology in cases when the probability distribution P_P is stationary also leads to negligibly small nonperturbative effects [13]. Thus it is easy to make our effect very small without disturbing the standard predictions of inflationary cosmology. However, it is quite possible that we will not have any difficulties even with very large $n(\phi)$ if we interpret our results more carefully.

VIII. INTERPRETATION AND POSSIBLE IMPROVEMENTS OF THE PROBABILITY MEASURE

An implicit hypothesis behind our interpretation is that we are typical, and therefore we live and make observations in those parts of the Universe where most other people do. One may argue that the total number of observers which can live in domains with given properties (e.g., in domains with a given density) should be proportional to the total volume of these domains at a given time. However, our existence is determined not only by the local density of the Universe but by the possibility for life to evolve for about 5×10^9 years on a planet of our type in a vicinity of a star of the type of the Sun. If, for example, we have density 10^{-29} g cm^{-3} in a small vicinity of the center of the infloid, and density 10^{-27} g cm^{-3} on the horizon scale, then the age of our part of the Universe (or, to be more accurate, the time after the end of inflation) will be determined not by the density near the center of the infloid, by the large scale density 10^{-27} g cm^{-3}, and it will be only about 1×10^9 years.

Moreover, any structures such as galaxies or clusters cannot be formed near the centers of the infloids since the density there is very small. Indeed, on each particular scale the jump down completely overwhelms the amplitude of the usual density perturbations. The bubble cannot contain any galaxies at the distance from the center comparable with the galaxy scale, it cannot contain any clusters at the distance comparable with the size of a cluster, etc. In other words, the center would be devoid of any structures necessary for the existence of our life.

Thus, the naive idea that the number of observers is proportional to volume does not work at the distances from the centers which are smaller than the present size of the horizon. Even though at any given moment of time most of the volume of the Universe at the density 10^{-29} g cm^{-3} is concentrated near the centers of infloids, the corresponding parts of the Universe are too young and do not have any structures necessary for our existence. Volume alone does not mean much. We live on the surface of the Earth even though the volume of empty space around us is incomparably greater.

One may argue that the disparity between the age of the local part of the Universe and its density appears only if one considers perturbations on a scale smaller than the horizon. Therefore it still may be true that we should live in the centers of huge bubbles, which have a shape (52) for $r > H_0^{-1}$, where H_0^{-1} is the size of the present horizon. If the cutoff occurs at $r \gg H_0^{-1}$, this may not lead to any observable consequences at all. However, if the cutoff occurs at $r \sim H_0^{-1}$, the resulting geometry may resemble an open Universe with a scale-dependent effective parameter $\Omega(r)$ [3]. Thus a more elaborate choice of the probability measure may lead to observable effects which will be interesting and less dramatic than the effects described in the previous section. In order to make any definite conclusions about the preferable parts of the Universe one should study probability distributions which include several other factors in addition to density. This should be a subject of a separate investigation. The main goal of our paper was to demonstrate that in a certain class of theories with a rather reasonable choice of probability measure the nonperturbative effects may be quite substantial. This result by itself was very surprising, and we believe that it deserves further investigation despite all uncertainties involved.

An additional ambiguity in the interpretation of our results appears due to the dependence of the distribution P_P on the choice of time parametrization. Indeed, there are many different ways to define ''time'' in general relativity. If, for example, one measures time not by a clock but by rulers and determines time by the degree of a local expansion of the Universe, then in this ''time'' the rate of expansion of the Universe does not depend on its density. As a result, our effect is absent in this time parametrization [3]. The reason why the results depend on the time parametrization is deeply related to the properties of a self-reproducing universe. The total volume of all parts of such a universe diverges in the large time limit. Therefore when we are trying to find which parts of the Universe have a greater volume we are comparing infinities. There are some methods to regularize these infinities in a way that would make the final results only mildly dependent on the choice of time parametrization [7,8]. However, there are many such methods, and the final results are exponentially sensitive to the choice of the method [8]. In this paper we used the standard time parametrization which is most closely related to our own nature (time measured by number of oscillations rather than by the distance to the nearby galaxies). But maybe we should use another time parametrization (see the Appendix) or even integrate over all possible time parametrizations. Right now we still do not know what is the right way to go. We do not even know if it is right that we are typical and that we should live in domains of the greatest volume; see the discussion of this problem in [6,8,15].

Therefore at present we would prefer to consider our results simply as a demonstration of the nontrivial properties of the hypersurface of a given time in the fractal self-reproducing universe, without making any far-reaching conclusions concerning the structure of our own part of the Universe. However, we must admit that we are amazed by the fact that the main fraction of the volume of the inflationary universe in a state with a given density ρ at any given moment of proper time t should be concentrated near the centers of deep spherically symmetric wells. We confirmed this result by four different methods, and we believe that it is correct. Until the interpretation problem is resolved, it will remain unclear whether our result is just a mathematical curiosity or can be considered as a real prediction of properties of our own part of the Universe. At present we can neither prove nor disprove the last possibility, and this by itself is a very unexpected conclusion. A few years ago we would say that the possibility that we live in a local ''center of the world'' definitely contradicts basic principles of cosmology. Now we can only say that it is an open question to be studied both theoretically and experimentally. If somebody asks whether we should live in the center of the world, we will be unable to give a definite answer. But if observations show us that the answer is yes, we will know why.

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APPENDIX

Let us consider a different time parametrization, related to the proper time by local path dependent transformation:

$$t \to \tau(t) = \int^t ds\, T(\phi_\xi(s)), \tag{A1}$$

where $T(\phi)$ is a positive function, and its argument in Eq. (A1) is a solution of Eq. (5) with a particular realization of the white noise. The stochastic Langevin equation in this parametrization looks like

$$\frac{d\phi}{d\tau} = -\frac{V'(\phi)}{3H(\phi)T(\phi)} + \frac{H^{3/2}(\phi)}{2\pi T^{1/2}(\phi)}\xi(\tau), \tag{A2}$$

The branching diffusion equation in an arbitrary time parametrization can be written as

$$\frac{\partial}{\partial\tau}P_P(\phi,\tau)=\frac{1}{2}\frac{\partial}{\partial\phi}\left[\frac{H^{3/2}(\phi)}{2\pi T^{1/2}(\phi)}\frac{\partial}{\partial\phi}\left(\frac{H^{3/2}(\phi)}{2\pi T^{1/2}(\phi)}P_P(\phi,\tau)\right)\right]+\frac{\partial}{\partial\phi}\left(\frac{V'(\phi)}{3H(\phi)T(\phi)}P_P(\phi,\tau)\right)+\frac{3H(\phi)}{T(\phi)}P_P(\phi,\tau). \tag{A3}$$

Its solution will generally be a stationary probability function with an overall constant expansion factor just like in Eq. (14). The value of the constant λ_1 will depend on the parametrization.

We can find the volume-weighted slow-roll trajectory of the inflaton field in an arbitrary parametrization very similarly to the approach used for proper time, but we have to keep in mind that it is no longer true that $\lambda_1=d_{fr}H_{max}$. The result is

$$\frac{d\phi}{d\tau}=-\sqrt{\left(\frac{V'(\phi)}{3H(\phi)T(\phi)}\right)^2+\left(\lambda_1-3\frac{H(\phi)}{T(\phi)}\right)\frac{H^3(\phi)}{2\pi^2T(\phi)}}. \tag{A4}$$

Since the conventional (i.e., calculated under the comoving probability) amplitude of the quantum jumps generated during the typical time interval $\Delta\tau\sim T(\phi)H^{-1}(\phi)$ in the given time parametrization is still given by the usual quantity $H/2\pi$ [see the Langevin equation (A2) above], then the definition for amplification factor becomes

$$n(\phi)=\sqrt{\left(\frac{2\pi V'(\phi)}{3H^3(\phi)}\right)^2+\left(\lambda_1-3\frac{H(\phi)}{T(\phi)}\right)\frac{2T(\phi)}{H(\phi)}}-\frac{2\pi V'(\phi)}{3H^3(\phi)}. \tag{A5}$$

In the particular case of the time parametrization $T=H$, which corresponds to the scale factor $a(t)$ playing the role of time τ, we get

$$\frac{d\phi}{d\tau}=-\sqrt{\left(\frac{V'(\phi)}{3H^2(\phi)}\right)^2+(\lambda_1-3)\frac{H^2(\phi)}{2\pi^2}} \tag{A6}$$

and

$$n(\phi)=\sqrt{\left(\frac{2\pi V'(\phi)}{3H^3(\phi)}\right)^2+2(\lambda_1-3)}-\frac{2\pi V'(\phi)}{3H^3(\phi)}. \tag{A7}$$

Since $\lambda_1<3$, in this time parametrization the volume-weighted slow roll (A6) is not faster but slightly slower than the conventional slow roll. As a result, most of the volume on the hypersurfaces of constant ''time'' τ will be concentrated near the spherically symmetric hills (rather than wells) in the energy density. However, the amplification factor is always very small.

The change of time parametrization (A1) corresponds to one of the possible ways to choose regularization procedure for evaluation of divergent probabilities in an eternally expanding universe [8]. Other types of regularization procedure were proposed in [7,8]. In particular, the regularization scheme suggested in [7] is essentially equivalent to choosing the $T=H$ parametrization which we discussed above [8]. One can easily verify that in the limit $\phi\ll\phi_{fr}$ our equations for the $T=H$ parametrization, Eqs. (A6) and (A7), yield the same results for the nonperturbative jumps as the ones obtained in [7]. As is argued in, [8] from the point of view of the interpretation of our results it is not obvious that this regularization has any advantages as compared to a more intuitive and straightforward approach used in the main part of this paper. However, each regularization scheme and each time parametrization gives an additional interesting information about the structure of an inflationary universe. Therefore we presented in this appendix an extension of our results for the more general class of time parametrizations (A5).

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