

Metric perturbations from quantum tunneling in open inflation

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We study the effect that quantum fluctuations produced during the nucleation of a single-bubble open inflationary universe have on the amplitude of temperature anisotropies in the microwave background. We compute the instanton action for the quantum tunneling between the false and true vacua in open inflation models and show that the amplitude of quantum fluctuations of the bubble wall is very sensitive to the gravitational effects of the true vacuum. We study the spectrum of quantum fluctuations of the bubble wall and confirm that there is only an inhomogeneous ($k^2 = -3$) discrete mode associated with transverse traceless fluctuations of the bubble wall. This supercurvature mode could, in principle, distort the anisotropy of the microwave background. We compute the amplitude of the gauge-invariant metric perturbations induced by the bubble wall fluctuations on a comoving hypersurface, and calculate the induced amplitude of temperature fluctuations in the microwave background, for arbitrary values of Ω_0 . We find that in the limit $\Omega_0 \approx 1$, the quadrupole dominates the angular power spectrum, such as in the usual Grishchuk-Zel'dovich effect. The resulting bounds on the amplitude of quantum fluctuations of the bubble wall from the absence of such an effect in the observed microwave background anisotropies are quite strong. We also study the contribution from a discrete long wavelength supercurvature mode ($k^2 \approx 2m^2/3H^2$) that appears in the spectrum of open de Sitter vacuum fluctuations. We constrain the parameters of the models of open inflation so that these modes do not distort the observed temperature anisotropy. [S0556-2821(96)00116-6]

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I. INTRODUCTION

Until recently, one of the most robust predictions of inflation was the extreme flatness of our local patch of the Universe. However, in the last few months there has been a lot of excitement about the possibility of producing an open universe from inflation [1–3]. An open universe could resolve the age crisis caused by the observations of a relatively large Hubble constant, $H_0 = 69 \pm 8$ km/s Mpc, which corresponds (for a flat Universe without a cosmological constant) to a very small age of the universe, $t_0 = 9.5 \pm 1.1$ Gyr [4], in conflict with the ages of globular clusters, 15.8 ± 2.1 Gyr [5]. An alternative solution could be the introduction of a non-zero cosmological constant which could accommodate both a flat and an old universe with a large expansion rate, but there still remains the question of why the cosmological constant is so small. But perhaps the true excitement comes from the fact that open inflation provides a new way of solving the classical problems of the hot big bang cosmology, the homogeneity and flatness problems. In standard inflation the two are intimately related and it is not possible to relax one (flatness) without affecting the other (homogeneity) [6]. Open inflation solves the homogeneity problem by inflating the Universe in a false vacuum and then nucleating a very symmetric bubble within which our Universe expanded to “almost” flatness.

The first models of open inflation [1] considered a single field trapped in a metastable state that later tunneled to the true vacuum with a nonzero energy density. The field then rolled down a very flat potential, inflating the required amount of e folds to produce an open universe. At the end of inflation the Universe reheated to give the well-known, hot big bang cosmology. These models had the unpleasant feature of strongly contrived potentials, since in order to tunnel

without producing too large inhomogeneities a large mass in the false vacuum is needed, while a very small mass for the inflaton field is required to give the observed amplitude of density perturbations in the cosmic microwave background (CMB). Linde and Mezhlumian [3] suggested a simple way out by including two fields, one with a large mass, responsible for tunneling, and the other with a very small mass, responsible for inflation in the true vacuum.

According to this picture we live inside a bubble that nucleated from de Sitter space by quantum tunneling with an extremely small probability. This ensures, first of all, that there will be no other nucleating events, at least in our past light cone, and, therefore, the initial state is pure de Sitter vacuum. Second, that the nucleated bubble is extremely spherically symmetric. Although the homogeneity problem is thus solved at the classical level, there might still be large quantum fluctuations during the process of quantum tunneling.

There are, in principle, two sources of metric perturbations in open inflation, the vacuum fluctuations of the inflaton field that are stretched to cosmological scales by the expansion, and quantum fluctuations of the bubble wall produced during bubble nucleation. The first have been extensively studied in recent papers [1,2,7–9]; the second have been addressed by Linde and Mezhlumian [3], and more recently by Hamazaki *et al.* [8], for an empty bubble. We study the fluctuations of the wall when the bubble is not empty. The calculations will be done in the thin wall approximation, which is valid for most potentials with a deep false vacuum minimum and a large potential barrier between the two vacua. Most results follow the Coleman–De Luccia formalism [10], valid when the tunneling occurs from de Sitter to Minkowski space-time. However, the new ingredient in open inflation is precisely the nonzero energy density of the true

vacuum which could still drive inflation to almost flatness. The instanton action associated with the more general quantum tunneling process from de Sitter to de Sitter was computed long ago by Parke [11]. We will use his results to calculate the tunneling action of open inflation. We compute the average amplitude of quantum fluctuations of the bubble wall from variations of the instanton action. Following the covariant formalism of Garriga and Vilenkin [12], we study the spectrum of inhomogeneous scalar modes associated with quantum fluctuations of the bubble wall, and find that there is only a discrete mode, with $k^2 = -3$.¹ This mode could, in principle, contribute very strongly to the anisotropy of the CMB. We study its contribution to the CMB in a gauge-invariant way and present the results in the Appendix, both analytically for $\Omega_0 \approx 1$ and numerically for $\Omega_0 < 1$. We analyze the constraints on the open inflation models from the absence of such an effect in the anisotropies of the microwave background, as observed by COBE [13]. It turns out that there are important constraints on the models, but not enough to rule them out.

Another important issue is whether large quantum fluctuations of the inflaton field before tunneling could propagate inside the bubble and distort the CMB. This is a relevant question in the case that the Universe in the false vacuum is actually in a process of self-reproduction, and thus extremely inhomogeneous [14]. In that case, very large scale metric perturbations could affect the amplitude of the lowest multipoles of the temperature anisotropies in the background radiation. This is the so-called Grishchuk-Zel'dovich effect [15]. We have recently evaluated this effect in the open universe case [16] and found strong constraints on the amplitude of very long wavelength perturbations contributing to the lowest CMB multipoles. In the case of open inflation models where the mass of the scalar field in the false vacuum is smaller than the Hubble parameter, there is a discrete supercurvature vacuum mode [2], with $k^2 \approx 2m^2/3H^2 < 1$, that could, in principle, distort the CMB, as discussed by Yamamoto *et al.* in Ref. [7]. We derive bounds on the parameters of open inflation models from the absence of such an effect in the microwave background anisotropies.

II. QUANTUM TUNNELING

In this section we review the calculation of Parke [11] on the instanton action for the quantum tunneling between a false and a true vacuum in de Sitter space. We assume the potential has a large barrier between the two minima, so that the thin wall approximation remains valid, and a large mass in the false vacuum. One of the dangers of quantum tunneling, for a small mass of the tunneling field, is the existence of the Hawking-Moss instanton [17]. In this case, the field jumps to the top of the barrier between the two vacua and very slowly ‘‘rolls down’’ the potential. The problem then is that there are large quantum fluctuations which are not inflated away, and these large perturbations would unacceptably distort the observed anisotropy of the CMB. For that reason alone it is assumed that the mass of the tunneling field

should be much larger than the rate of expansion at the false vacuum.

The Euclidean tunneling action for a single scalar field can be written as

$$S_E = \int d^4x \sqrt{-g} \left[-\frac{1}{2\kappa^2} R + \frac{1}{2} (\partial\phi)^2 + U(\phi) \right], \quad (1)$$

where $\kappa^2 \equiv 8\pi G$ and the Euclidean O(4)-invariant metric is $ds_E^2 = d\tau^2 + a^2(\tau) d\Omega_3^2$. The curvature scalar is given by $R = -6a^{-2}(aa'' + a'^2 - 1)$, where a prime denotes derivative with respect to Euclidean time. Integrating by parts and using the Euclidean equations of motion, $a'^2 - 1 = a^2 \kappa^2 [\phi'^2/2 - U(\phi)]/3$, we find

$$\begin{aligned} S_E &= 2\pi^2 \int d\tau \left[a^3 \left(\frac{1}{2} \phi'^2 + U(\phi) \right) + \frac{3}{\kappa^2} (a^2 a'' + a a'^2 - a) \right] \\ &= -\frac{12\pi^2}{\kappa^2} \int d\tau a (1 - a^2 H^2), \end{aligned} \quad (2)$$

where $H^2 \equiv \kappa^2 U/3$. The instanton (or bounce) action which determines the probability of tunneling is given by $B = S_E(\phi) - S_E(\phi_F)$. We define $U_F \equiv U(\phi_F)$ and $U_T \equiv U(\phi_T)$ as the false and true vacuum energies, respectively, which characterize the end points of the quantum tunneling. Taking into account the contributions to the instanton action coming from both the wall and the interior of the bubble, Parke found the following expression for the bounce action [11]:

$$\begin{aligned} B(a) &= 2\pi^2 a^3 S_1 + \frac{4\pi^2}{\kappa^2} \left[\frac{1}{H_T^2} [(1 - a^2 H_T^2)^{3/2} - 1] \right. \\ &\quad \left. - \frac{1}{H_F^2} [(1 - a^2 H_F^2)^{3/2} - 1] \right], \end{aligned} \quad (3)$$

where

$$S_1 = \int_{\phi_F}^{\phi_T} d\phi \{2[U(\phi) - U_F]\}^{1/2}, \quad (4)$$

and $U_F - U_T \equiv \epsilon$. In general, we choose $\epsilon \ll U_T$ but this is not essential. For the thin wall approximation to be valid we require that the width of the bubble wall Δa be much smaller than its radius of curvature:

$$\frac{\Delta a}{a} \approx \frac{H_T(\phi_T - \phi_F)}{[2(U_0 - U_F)]^{1/2}} \ll 1, \quad (5)$$

where U_0 is the value of the potential at the maximum. The only requirement is that the barrier between ϕ_F and ϕ_T be sufficiently high, i.e., $U_0 \gg U_T$.

It is now possible to compute the radius of curvature of the bubble wall for which the action (3) is an extremum:

$$\begin{aligned} B'(a) &= \frac{12\pi^2 a}{\kappa^2} \left[\frac{\kappa^2}{2} S_1 a - (1 - a^2 H_T^2)^{1/2} + (1 - a^2 H_F^2)^{1/2} \right] \\ &= 0. \end{aligned} \quad (6)$$

¹ $k^2 = 1$ corresponds to the curvature scale.

An exact solution [11] can be written in terms of dimensionless parameters α and β :

$$a^2 H_T^2 = \frac{\alpha^2}{\alpha^2 + (1 + \alpha^2 \beta)^2}, \quad (7)$$

$$\alpha = a_0 H_T \equiv \frac{3S_1}{\epsilon} H_T, \quad \beta = \frac{\epsilon}{4U_T}. \quad (8)$$

The parameter α characterizes the strength of the gravitational interaction in the true vacuum. The extremal solution (7) is valid both in the limit $\alpha^2 \ll 1$, for which we recover the usual tunneling result, $a = a_0$, from de Sitter to Minkowski ($H_T = 0$); and in the limit $\alpha^2 \beta \gg 1$, which gives $a = 4/(\kappa^2 S_1)$. In both cases the radius of curvature satisfies $a \ll H_T^{-1}$. On the other hand, the largest radius of curvature occurs for $\alpha^2 \beta = 1$, that is $a = H_F^{-1}$.

The extremal action corresponds to the O(3,1) symmetric bubble. We are interested in deviations from perfect isotropy and homogeneity, i.e., on the quantum fluctuations generated during bubble nucleation. Linde and Mezhlumian [3] evaluated a typical quantum deviation of the radius of curvature of the bubble by computing the first quantum correction to the tunneling action, $S = S_0 + \hbar \Delta S$, where $\Delta S = B''(a)(\delta a)^2/2$, and the second derivative of the bounce action (3) at the extremum is exactly given by

$$\begin{aligned} B''(a) &= \frac{12\pi^2}{\kappa^2} [(1 - a^2 H_T^2)^{-1/2} - (1 - a^2 H_F^2)^{-1/2}] \\ &= -18\pi^2 \frac{S_1^2}{\epsilon} \frac{[\alpha^2 + (1 + \alpha^2 \beta)^2]^{1/2}}{(1 + \alpha^2 \beta)(1 - \alpha^2 \beta)} \\ &\simeq -18\pi^2 \frac{S_1^2}{\epsilon} (1 - \alpha^2 \beta)^{-1}. \end{aligned} \quad (9)$$

In order to evaluate a typical deviation of the curvature of the bubble, we can estimate $\Delta S \sim 1$, see Ref. [3], and, thus,

$$\delta a \simeq \frac{\sqrt{\epsilon}}{3\pi S_1} |1 - \alpha^2 \beta|^{1/2}. \quad (10)$$

In the limit $\alpha^2 \beta \ll 1$, we recover the results of Refs. [12,3]. On the other hand, the last factor could strongly affect the overall curvature of the nucleated bubble, when gravitational effects are important.

We are actually interested in the spectrum of inhomogeneous quantum fluctuations of the bubble wall, which would appear from the inclusion of gradient terms in the bounce action. These inhomogeneous scalar modes were studied by Garriga and Vilenkin [12] for an empty bubble, using a covariant formalism in an embedding de Sitter space. In the context of open inflation models, the bubble is not empty and the radius of curvature of the bubble at the moment of nucleation is smaller than the de Sitter horizon scale H_F^{-1} . The geometry of the three-dimensional bubble is characterized by its extrinsic and intrinsic curvatures. The unperturbed bubble world sheet has an induced metric $g_{ab} = \partial_a x^\mu \partial_b x_\mu$, where the subindices $\{a, b\}$ label coordinates on the bubble wall, and $\{\mu, \nu\}$ label space-time coordinates. The extrinsic curvature of the bubble wall is $K_{ab} = -(\dot{a}/a)g_{ab}$. However, as the

bubble expands, $\dot{a}/a = H \coth(Ht) \rightarrow H$, the curvature scale of the bubble approaches the horizon scale H^{-1} and remains fixed. Here, H stands for the Hubble constant of the embedding de Sitter space.

We are interested in perturbations in the space-time coordinates of the bubble wall. Since only motion transverse to the bubble wall is physically observable (the rest can be eliminated by a coordinate transformation), we will only consider linear perturbations of the type $\delta x^\mu = \varphi n^\mu$, where φ is a scalar that characterizes the fluctuations normal to the surface ($n^\mu = Hx^\mu$). The metric perturbations become $\delta g_{ab} = -2\varphi K_{ab} + \partial_a \varphi \partial_b \varphi + \varphi^2 K_a^c K_{cb} \simeq -2\varphi K_{ab}$, to first order. The equation of motion for the scalar fluctuation φ can be obtained from the variation of the extrinsic curvature scalar ($K = -3H = \text{const}$), $\delta K = \nabla^2 \varphi + K_{ab} K^{ab} \varphi - K_{ab} \partial^a \varphi \partial^b \varphi - \varphi^2 K_{ab} K^{ac} K_c^b \simeq \nabla^2 \varphi + K_{ab} K^{ab} \varphi = 0$, to first order,²

$$\delta K = \nabla^2 \varphi + 3H^2 \varphi = \frac{k^2 + 3}{a^2} \varphi = 0. \quad (11)$$

The bubble wall fluctuations thus correspond to an inhomogeneous scalar mode characterized by $k^2 = -3$, with the peculiar property that the associated curvature perturbation is transverse traceless [12,8],

$$\delta \bar{R}_{ab}^{(3)} = -H \delta \bar{K}_{ab} = -H(\nabla_a \nabla_b \varphi + H^2 g_{ab} \varphi), \quad (12)$$

while the Ricci scalar remains unperturbed,

$$\delta R^{(3)} = -H \frac{k^2 + 3}{a^2} \varphi = 0. \quad (13)$$

In principle, there could have been other inhomogeneous modes at bubble nucleation, but the fact that the bubble wall asymptotically acquires a fixed curvature determines that only the inhomogeneous scalar mode with $k^2 = -3$ survives on the surface of the bubble.

III. METRIC PERTURBATIONS

We now study the effect that quantum fluctuations of the bubble wall produce on the microwave background. In order to do this, we have to relate the metric perturbations in the (2+1)-dimensional bubble wall, at a fixed radial coordinate, with metric perturbations on a three-dimensional comoving equal-time hypersurface inside the bubble. For that purpose, we recall the open de Sitter coordinates of Ref. [1]: Region I contains the interior of the bubble and is parametrized (in units of H^{-1}) by $ds^2 = -d\zeta^2 + \sinh^2 \zeta (d\xi^2 + \sinh^2 \xi d\Omega^2)$ with coordinates (ζ, ξ) , while region II is outside the bubble and is described by the metric $ds^2 = d\sigma^2 + \sin^2 \sigma (-d\tau^2 + \cosh^2 \tau d\Omega^2)$ with coordinates (τ, σ) . In these coordinates the bubble wall is a timelike hypersurface at a fixed coordinate σ in region II, which can be analytically continued into a spacelike hypersurface at a fixed comoving time $\zeta = i\sigma$

²Note that the full expression $\delta K = 0$ still corresponds to a mode with $k^2 = -3$.

inside the bubble. Thus, perturbations in the bubble wall hypersurface propagate inside as metric perturbations in a comoving equal-time hypersurface. The (2+1)-dimensional $k^2 = -3$ mode of Ref. [12] corresponds analytically to the three-dimensional open universe discrete mode with $k^2 = -3$ discussed by Hamazaki *et al.* [8].

We want to evaluate, in linear perturbation theory, the primordial metric perturbations associated with these quantum fluctuations. The most general scalar metric perturbations can be written as [18–20]

$$ds^2 = a^2(\eta) [-(1+2A)d\eta^2 + 2B_{|i}dx^i d\eta + \{(1+2\mathcal{R})\gamma_{ij} + 2E_{|ij}\}dx^i dx^j], \quad (14)$$

where $\{i, j\}$ label the three-dimensional open space coordinates with metric γ_{ij} . The four linear scalar perturbations are not independent. Under a gauge transformation $\tilde{\eta} = \eta + \xi^0(\eta, x^k)$, $\tilde{x}^i = x^i + \gamma^{ij}\xi_{|j}(\eta, x^k)$, the metric perturbations transform as

$$\tilde{A} = A - \xi^{0'} - \frac{a'}{a}\xi^0, \quad \tilde{\mathcal{R}} = \mathcal{R} - \frac{a'}{a}\xi^0, \quad (15)$$

$$\tilde{B} = B + \xi^0 - \xi', \quad \tilde{E} = E - \xi, \quad (16)$$

where a prime denotes derivative with respect to conformal time η . There are, however, only two independent gauge-invariant gravitational potentials [20],

$$\Phi = A + \frac{1}{a}[a(B-E)']', \quad (17)$$

$$\Psi = \mathcal{R} + \frac{a'}{a}(B-E)', \quad (18)$$

which are further related through the perturbed Einstein equations

$$\Phi + \Psi = 0, \quad (19)$$

$$4\frac{k^2+3}{a^2}\Psi = 2\delta\rho. \quad (20)$$

Here, $\delta\rho$ is the gauge-invariant density perturbation [18]. Note that for the $k^2 = -3$ mode of bubble wall fluctuations, the amplitude of density perturbations is identically zero. This is a very special mode, as was first pointed out by Lifshitz and Khalatnikov [21]. We will study in detail its effect on metric perturbations.

The scalar metric perturbations can be separated into $A(\eta, x^i) = A(\eta)Q(x^i)$,³ where $Q(x^i)$ are the scalar harmonics of a spatially open universe, solutions of the Helmholtz equation [22],

$$L^2 Q(\xi, \Omega) = -k^2 Q(\xi, \Omega), \quad (21)$$

where $L^2 = \sinh^{-2}\xi\partial_\xi(\sinh^2\xi\partial_\xi) + \sinh^{-2}\xi L_\Omega^2$ is the open universe Laplacian. These solutions have the general form

$Q_{klm}(\xi, \Omega) = \Pi_{kl}(\xi)Y_{lm}(\theta, \phi)$, see Appendix. The scalar harmonics can be used to construct a traceless tensor

$$Q_{ij} = \frac{1}{k^2}Q_{|ij} + \frac{1}{3}\gamma_{ij}Q, \quad (22)$$

satisfying $Q_i{}^i = 0$, see Eq. (21), as well as

$$\nabla^i Q_{ij} = -\frac{2}{3}\frac{k^2+3}{k^2}\nabla_j Q. \quad (23)$$

We can now investigate the contribution of the discrete $k^2 = -3$ mode to the primordial perturbations in an open universe. In principle, it is possible to analyze the amplitude of metric perturbations on any hypersurface [18]. However, we believe it is most convenient to study them on comoving hypersurfaces, where they have a clear physical meaning as curvature perturbations. Furthermore, the perturbed bubble wall hypersurface has the property of being also a uniform-expansion ($\delta K = 0$) hypersurface, see Eq. (11). It turns out that, for $k^2 = -3$ (and only for this mode), the same gauge transformation that takes you to a comoving hypersurface, also takes you to a uniform-Hubble-constant hypersurface [18],

$$\delta K = -\frac{3}{a}\left[\mathcal{R}' - \frac{a'}{a}A + \frac{k^2}{3}(B-E)'\right]Q \equiv 0, \quad (24)$$

$$\delta\bar{K}_{ij} = -\frac{k^2}{a}(B-E)Q_{ij} \equiv \frac{3}{a}\left(\mathcal{R}' - \frac{a'}{a}A\right)Q_{ij}, \quad (25)$$

with intrinsic curvature separated into its trace and traceless parts:

$$\delta R^{(3)} = 4\frac{k^2+3}{a^2}\mathcal{R}Q \equiv 0, \quad (26)$$

$$\delta\bar{R}_{ij}^{(3)} = -\frac{k^2}{a^2}\mathcal{R}Q_{ij}. \quad (27)$$

Furthermore, for $k^2 = -3$, the dynamic equations can be written as [18]

$$A = -\frac{w}{1+w}\eta, \quad (28)$$

$$\left[a^2\left(\mathcal{R}' - \frac{a'}{a}A\right)\right]' = \frac{k^2}{3}(\mathcal{R}+A)a^2, \quad (29)$$

where $w = p/\rho$, and $\eta = (\delta p - \delta\rho dp/d\rho)Q$ is the gauge-invariant nonadiabatic part of matter perturbations, which vanishes for the single-field (adiabatic) perturbations of open inflation. In that case, the remaining scalar perturbation satisfies the equation

$$\mathcal{R}'' + 2\frac{a'}{a}\mathcal{R}' + \mathcal{R} = 0. \quad (30)$$

During inflation, $a(\eta) = a_0/\sinh\eta$, there is an exact solution to this equation given by

³From now on A, B , etc. stand for the η -dependent functions.

$$\mathcal{R} = C \frac{a'}{a^2} = C \frac{\dot{a}}{a}, \quad (31)$$

where $C = \delta a$ is the amplitude of quantum fluctuations of the bubble wall, see Eq. (10). It is easy to see that during inflation \mathcal{R} becomes constant:

$$\mathcal{R}_0 = H \delta a. \quad (32)$$

This curvature perturbation is a very peculiar one. Let us evaluate the gauge-invariant potentials Φ and Ψ in Eq. (17). Substituting the solution (31), together with $\mathcal{R}' = -C/a$, we find

$$\Phi = \frac{1}{a} (a\mathcal{R}')' = 0, \quad (33)$$

$$\Psi = \mathcal{R} + \frac{a'}{a} \mathcal{R}' = 0. \quad (34)$$

Therefore, this mode has vanishing gauge-invariant potentials, as well as vanishing gauge-invariant density perturbations. One might be tempted to dismiss this mode altogether; however, it is possible to see that the traceless parts of the extrinsic and intrinsic curvatures do not vanish:

$$\delta \bar{K}_{ij} = \frac{3}{a} \mathcal{R}' Q_{ij} = -\frac{3}{a^2} C Q_{ij}, \quad (35)$$

$$\delta \bar{R}_{ij}^{(3)} = \frac{3}{a^2} \mathcal{R} Q_{ij} = \frac{3}{a^2} C \frac{a'}{a} Q_{ij}. \quad (36)$$

Therefore, this $k^2 = -3$ mode can be understood as a transverse traceless perturbation, see Eqs. (22) and (23). This was pointed out in Refs. [8,23]. During inflation, $a'/a^2 = \dot{a}/a \rightarrow H = \text{const}$, and thus $\delta \bar{R}_{ij}^{(3)} = -H \delta \bar{K}_{ij}$, which coincides with the bubble wall fluctuations' relation (12), as expected.

Spatial curvature, $a^{-2} = H^2(1 - \Omega)$, will vanish during the last stages of inflation and will be negligible ($\Omega \approx 1$) during radiation. It is then easy to show with Eq. (30) that \mathcal{R} remains approximately constant. However, as we enter the matter era, $a(\eta) = a_0(\cosh \eta - 1)$, spatial curvature will become important, in order to produce an open universe ($\Omega < 1$). As a consequence, \mathcal{R} is no longer a constant and evolves with Eq. (30). There is an exact solution to this equation during the matter era,

$$\mathcal{R} = \mathcal{R}_0 G(\eta) \equiv \mathcal{R}_0 3 \frac{\eta \sinh \eta - 2(\cosh \eta - 1)}{(\cosh \eta - 1)^2}, \quad (37)$$

satisfying $G(0) = 1$. We are interested, however, in the gauge-invariant gravitational potential (17),

$$\Phi = -\left(\mathcal{R} + \frac{a'}{a} \mathcal{R}' \right) = -\frac{3}{5} \mathcal{R}_0 F(\eta), \quad (38)$$

where

$$F(\eta) \equiv 5 \frac{\sinh^2 \eta - 3 \eta \sinh \eta + 4(\cosh \eta - 1)}{(\cosh \eta - 1)^3}, \quad (39)$$

satisfying $F(0) = 1$.

In the next section we will evaluate the amplitude of temperature anisotropies from all possible modes present in open inflation. These include the $k^2 = -3$ mode associated with bubble wall fluctuations, as well as the discrete supercurvature mode with $k^2 = 2m^2/3H^2$ present in the spectrum of open de Sitter vacuum fluctuations. There is also the continuum of subcurvature modes associated with quantum fluctuations of the inflaton field during the second stage of inflation.

IV. TEMPERATURE ANISOTROPIES

In this section we study the constraints that temperature anisotropies in the CMB impose on the models of open inflation. For any model, the value of Ω_0 today depends very critically upon the number of e folds of inflation from the tunneling event to the end of inflation. For $N_e = 65$ we find Ω_0 very close to 1. In fact, since

$$|1 - \Omega_0| \sim \exp(-2N_e) \times 10^{56}, \quad (40)$$

a few e folds of inflation less will produce a wide open universe. In most models, the second stage of inflation within the bubble occurs in the usual way with a very flat scalar potential, where 65 e folds correspond to a value of the inflaton field $\sigma \approx 3 M_p$. In that case, tunneling to a value of the inflaton just below $3 M_p$ would produce an open universe [3]. In the single-field models of open inflation proposed in Refs. [1,2], the tunneling field is also the inflaton field. As we discussed in Sec. II, in order that the field does not tunnel to the top of the potential, and thus produce large amplitude metric perturbations, we need a mass at the false vacuum which is larger than the rate of expansion, $M_F \gg H_F$. This condition is enough to suppress the amplitude of metric perturbations before tunneling. However, in the two-field models of Ref. [3] the tunneling field ϕ could be very heavy while the inflaton field σ has a small mass, both in the false and the true vacua. In this case, Sasaki *et al.* have shown that there exists a discrete supercurvature mode in the spectrum of vacuum fluctuations in the open de Sitter space [2]. This mode could, in principle, affect the lowest multipoles of the temperature anisotropy of the CMB, in what is known as the Grishchuk-Zel'dovich effect, studied in Ref. [16] for an open universe.

The dominant effect on large scales is known as the Sachs-Wolfe effect. Due to this effect, metric perturbations on the surface of last scattering are responsible for temperature fluctuations in the CMB [24],

$$\frac{\delta T}{T} = \frac{1}{3} \Phi(0) + 2 \int_0^{\eta_0} \Phi'(\eta_0 - r) dr, \quad (41)$$

where $\eta_0 = \text{arccosh}(2/\Omega_0 - 1)$ is the distance to the last scattering surface. For $\Omega_0 < 2/(1 + \cosh 1) \approx 0.786$, the surface of last scattering is located beyond the curvature scale.

We can expand the observed temperature anisotropies in terms of spherical harmonics,

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi), \quad (42)$$

and evaluate the angular power spectrum $C_l \equiv \langle |a_{lm}|^2 \rangle$, defined as the ensemble average of the l th multipole of the CMB temperature anisotropy. Observations, in fact, suggest that

$$l(l+1)C_l \lesssim \frac{24\pi}{5} \frac{Q_{\text{rms}}^2}{T_0^2} \approx 8 \times 10^{-10}, \quad (43)$$

for the lowest multipoles of the CMB anisotropies, where $Q_{\text{rms}} \approx 20 \mu\text{K}$ [25].

The contribution of different metric perturbation modes to the multipole components of the angular power spectrum can be separated into a discrete part, which includes the bubble wall fluctuations, labeled by $\nu=2$; the supercurvature mode $k_{\text{VL}}^2 \approx 2m_F^2/3H_F^2 < 1$ of open de Sitter vacuum [2], plus a continuum of subcurvature modes from fluctuations of the inflaton field in the last stage of inflation:

$$\begin{aligned} l(l+1)C_l = & 2\pi^2 l(l+1) \langle \mathcal{R}_0^2 \rangle_{\text{wall}} I_{\nu l}^2 \\ & + 2\pi^2 l(l+1) N_l^2 B_l^2 k_{\text{VL}}^2 \langle \mathcal{R}^2 \rangle_{\text{VL}} \\ & + 2\pi^2 l(l+1) \int_1^\infty \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k) I_{kl}^2. \end{aligned} \quad (44)$$

Here, $\mathcal{P}_{\mathcal{R}}(k)$ is the spectrum of primordial curvature perturbations. For the $\nu=2$ mode associated with the bubble wall, $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$ is the average square amplitude of the curvature perturbation (32). The ‘‘window function’’ I_{kl}^2 indicates how a given scale contributes to the C_l 's. We have evaluated $I_{\nu l}^2$ for the $\nu=2$ mode in the Appendix. The window function $N_l^2 B_l^2$ associated with the very long wavelength supercurvature mode k_{VL}^2 was computed in Ref. [16].

We analyze now the different sources of metric perturbations in models of open inflation and the constraints that observations of the microwave background impose on the models.

A. The subcurvature modes

Let us consider the temperature anisotropies produced by the continuum spectrum of subcurvature modes; see also Ref. [7]. The amplitude of scalar metric perturbations produced by quantum fluctuations of the inflaton field σ during the second phase of inflation is approximately given by [9]

$$\mathcal{R} = -H \frac{\delta\sigma}{\dot{\sigma}}, \quad (45)$$

which gives a nearly scale-invariant spectrum,

$$\langle \mathcal{R}^2 \rangle^{1/2} = \frac{4V}{\sigma V'} \frac{H\sigma}{M_p^2} \approx \frac{6H}{M_p}. \quad (46)$$

The observed temperature anisotropies of the microwave background at large scales [13] are consistent with such a spectrum of metric perturbations,

$$l(l+1)C_l = \frac{2\pi}{25} \langle \mathcal{R}^2 \rangle \approx \text{const.} \quad (47)$$

Assuming that the observed anisotropies (43) are due solely to quantum fluctuations of the inflaton requires $H/M_p \approx 10^{-5}$. This is a very general constraint. In the case of a massive inflaton, this bound requires its mass in the true vacuum to be

$$\frac{m_T}{M_p} \approx 2 \times 10^{-6}. \quad (48)$$

All models of open inflation should satisfy this constraint. Let us now discuss the more interesting discrete modes.

B. The $k^2 = -3$ mode

This mode is associated with quantum fluctuations of the bubble wall at tunneling that survive as scalar perturbations after the bubble has acquired a fixed comoving curvature. In order to evaluate its amplitude, let us parametrize the tunneling potential of Sec. II by

$$U(\phi) = U_F + \frac{\lambda}{4} \phi^2 (\phi - \phi_0)^2 - \epsilon \left(\frac{\phi}{\phi_0} \right)^4. \quad (49)$$

The two minima occur at $\phi_F = 0$ and $\phi_T \approx \phi_0 \equiv M\sqrt{2/\lambda}$, and the maximum of the potential is at $U_0 = \lambda \phi_0^4/64 = M^4/16\lambda$. Following Ref. [3], we will define $\epsilon \equiv \mu U_0$, with $\mu \ll 1$ for the thin wall approximation to be valid.

It is now possible to evaluate the instanton contribution from the bubble wall [Eq. (4)],

$$S_1 = \frac{M^3}{3\lambda} \left(1 + \frac{11}{32} \frac{\epsilon}{U_0} \right) \approx \frac{M^3}{3\lambda}, \quad (50)$$

while α and β are, see Eq. (7),

$$\alpha = \frac{16}{\mu} \frac{H}{M}, \quad \beta = \frac{\pi}{24} \frac{\mu}{\lambda} \frac{M^4}{M_p^2 H^2}, \quad (51)$$

$$\alpha^2 \beta = \frac{32\pi}{3\mu\lambda} \frac{M^2}{M_p^2}. \quad (52)$$

The average amplitude of the metric perturbation produced by quantum fluctuations in the radius of curvature of the bubble at nucleation, $\mathcal{R}_0 = H\delta a$, can then be written as

$$\begin{aligned} \langle \mathcal{R}_0^2 \rangle_{\text{wall}}^{1/2} & \approx \frac{\sqrt{\mu\lambda}}{4\pi} \frac{H}{M} |1 - \alpha^2 \beta|^{1/2} \\ & = \sqrt{\frac{2}{3}} \frac{H}{\pi M_p} \left| \frac{1 - \alpha^2 \beta}{\alpha^2 \beta} \right|^{1/2}. \end{aligned} \quad (53)$$

Note that we recover the result of Ref. [3] in the limit $\alpha^2 \beta \ll 1$. The constraints on $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$ from the CMB temperature fluctuations are discussed in the Appendix, see Eqs. (A12) and (A13Q). Using Eq. (53), we find

$$\left| \frac{1 - \alpha^2 \beta}{\alpha^2 \beta} \right| \lesssim 18\pi^2, \quad 0.1 \lesssim \Omega_0 \lesssim 0.4, \quad (54)$$

$$\left| \frac{1 - \alpha^2 \beta}{\alpha^2 \beta} \right| \lesssim \frac{54\pi^2}{(1 - \Omega_0)^2}, \quad 0.4 \lesssim \Omega_0 \lesssim 1. \quad (55)$$

We can now bound the parameters of the model (7):

$$\mu\lambda \leq 192\pi^3 \frac{M^2}{M_P^2}, \quad 0.1 \leq \Omega_0 \leq 0.4, \quad (56)$$

$$\mu\lambda \leq \frac{576\pi^3}{(1-\Omega_0)^2} \frac{M^2}{M_P^2}, \quad 0.4 \leq \Omega_0 \leq 1. \quad (57)$$

Let us give some values to the parameters. Suppose that $M \sim 10^{-3}M_P$, see Ref. [3], and take, e.g., $\Omega_0 \approx 0.3$. Then

$$\mu\lambda < 6 \times 10^{-3}. \quad (58)$$

This is a relatively weak bound on λ for small values of μ . On the other hand, for $\Omega_0 = 0.8$, the bound becomes $\mu\lambda < 0.4$, much weaker and thus much easier to satisfy. Furthermore, as Ω_0 tends to 1, the bound disappears altogether.

In summary, the bound on the parameters of open inflation models from quantum fluctuations of the bubble wall depends significantly on the value of Ω_0 . In most cases, the bound is not very strong and it is possible to find models satisfying the constraints.

C. The discrete vacuum mode

For open inflation models in which one of the scalar fields has a mass in the false vacuum much smaller than the Hubble rate of expansion, there exists a discrete supercurvature mode in the spectrum of open de Sitter vacuum fluctuations [2],

$$k_{\text{VL}}^2 = 1 - \left[\left(\frac{9}{4} - \frac{m_F^2}{H_F^2} \right)^{1/2} - \frac{1}{2} \right]^2 \approx \frac{2}{3} \frac{m_F^2}{H_F^2}, \quad (59)$$

that propagates inside the bubble. This is the case of one of the two-field models of Ref. [3], where the inflaton in the false vacuum is light. The metric perturbation for this mode is

$$\langle \mathcal{R}^2 \rangle_{\text{VL}} \approx \left(\frac{4V_F(\sigma)}{V_F'(\sigma)} \frac{H_F}{M_P} \right)^2 \approx 36 \frac{H_F^2}{M_P^2}, \quad (60)$$

which could, in principle, affect the anisotropy of the microwave background, due to a large value of H_F .

The Grishchuk-Zel'dovich effect gives the contribution to the microwave background anisotropy from a very large scale metric perturbation. In an open universe, one can constrain the amplitude of a discrete, very long wavelength supercurvature mode $0 < k_{\text{VL}}^2 < 1$ to satisfy⁴ [16]

$$k_{\text{VL}}^2 \langle \mathcal{R}^2 \rangle_{\text{VL}} \leq 4 \times 10^{-8}, \quad (61)$$

in the range $0.25 \leq \Omega_0 \leq 0.8$, in order not to affect the observed low multipole anisotropies of the microwave background, see Eq. (43). The constraint is somewhat weaker, $k_{\text{VL}}^2 \langle \mathcal{R}^2 \rangle_{\text{VL}} < 2 \times 10^{-9} (1 - \Omega_0)^{-2}$, for $0.8 \leq \Omega_0 < 1$, see Ref. [16].

Using the bound (61), we find a relatively weak bound on the allowed mass of the inflaton field in the false vacuum,

$$\frac{m_F}{M_P} \leq 4 \times 10^{-5}, \quad (62)$$

which allows a mass somewhat larger than that in the true vacuum (48).

Another possibility is to choose the inflaton field to be massless in the false vacuum and thus the discrete mode (59) would correspond to a homogeneous ($k^2 = 0$) mode, which does not affect the CMB, see Ref. [16]. This is the case of hybrid open inflation [3]. One could also have a very massive inflaton field in the false vacuum, as in the ‘‘supernatural’’ open inflation model of Ref. [3], in which case there is no discrete supercurvature vacuum mode.

In summary, either the scalar fields are very massive in the false vacuum, such that no supercurvature modes are present in the spectrum of metric perturbations, or they are very light so that their associated supercurvature mode does not distort the observed anisotropy of the microwave background.

V. CONCLUSIONS

In this paper we have computed the amplitude of metric perturbations produced by quantum fluctuations of the bubble wall at the moment of tunneling. These could, in principle, be a source of temperature fluctuations in the microwave background, in the context of the present models of single-bubble open inflation. By taking into account the corrections due to gravitational effects at the tunneling event, we have found that a nonzero energy density in the true vacuum could strongly modify the amplitude of fluctuations in the bubble wall. These fluctuations can be understood as discrete, long wavelength modes associated with perturbations in the curvature of the bubble. In the open de Sitter coordinates, the bubble wall is a timelike hypersurface at a fixed radial coordinate which asymptotically determines a spacelike hypersurface at a fixed comoving time inside the bubble. Small fluctuations in the curvature of the bubble propagate inside as perturbations in the time it takes to end inflation [26], and thus generate metric perturbations on comoving hypersurfaces. However, quantum fluctuations of the bubble wall generate only a discrete, inhomogeneous ($k^2 = -3$) mode [12,8]. It is possible to calculate the effect that this transverse traceless scalar metric perturbation produces on the microwave background. We have computed this effect for arbitrary values of Ω_0 , and described the results in the Appendix. See also Ref. [23] for the limit $\Omega_0 \approx 1$. In Sec. IV we constrain the parameters of open inflation models to avoid distortions in the observed temperature anisotropies. The resulting bounds on the amplitude of bubble wall quantum fluctuations are quite severe, although not enough to rule out these models.

Furthermore, in the single-field models of Refs. [1,2], the inflaton potential is fine tuned so that the field has a very large mass in the false vacuum. This ensures not only that the tunneling occurs in the thin wall approximation and not along the Hawking-Moss instanton, but also that there are no supercurvature modes that propagate inside the bubble [8].

⁴Note that in Ref. [16] there is a factor of $2\pi^2$ missing in the amplitude of the angular power spectrum, which makes the bounds on the curvature perturbation stronger.

However, in the two-field models of Ref. [3], the tunneling field has a very large mass in the false vacuum, but the inflaton field does not (except in the ‘‘supernatural inflation’’ model). This implies two things, first that there is a discrete, supercurvature vacuum mode [2] that propagates inside the bubble, and second that the amplitude of such a long wavelength perturbation could be rather large. These two features could, in principle, be enough to destroy the isotropy of the CMB to a level incompatible with observations, see Ref. [7]. Using the bounds on the amplitude and wavelength of such a perturbation from the open universe Grishchuk-Zel’dovich effect [16], we find an upper bound on the mass of the inflaton field at the false vacuum which is easily satisfied by all models.

As a consequence, the present models of open inflation seem to work well with very reasonable parameters, at least as reasonable as those of standard inflation. A different issue is whether these models will turn out to be the correct description of the origin of our patch of the Universe. As mentioned in the Introduction, another possible solution to the age crisis could be that the Universe is flat with a nonvanishing cosmological constant. Fortunately, cosmology has become a science and within a few years we will be able to tell, from the shape and amplitude of the spectrum of temperature fluctuations, whether our patch of the Universe is indeed open or flat [27]. A different possibility is that the present observations of the Hubble parameter turn out to be wrong and the actual value is well within the range allowed by a flat universe without a cosmological constant.

In any case, it is encouraging to see that the inflationary paradigm is able to accommodate an open universe, even if we never have to make use of it. Much more difficult would be to compute a probability distribution for the value of Ω_0 . Such an attempt was made in Refs. [3] and [28]. We believe the problem of probability measure in cosmology is not yet settled and we still have to learn how to pose the appropriate questions.

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APPENDIX: CMB TEMPERATURE FLUCTUATIONS FROM A $K^2 = -3$ MODE

The open universe scalar harmonics can be written as $Q_{klm}(\xi, \Omega) = \Pi_{kl}(\xi) Y_{lm}(\theta, \phi)$, with $Y_{lm}(\theta, \phi)$ the usual spherical harmonics and

$$\Pi_{kl}(\xi) = N_{\nu l} \frac{P_{\nu-1/2}^{-l-1/2}(\cosh\xi)}{\sqrt{\sinh\xi}}, \quad (\text{A1})$$

$$N_{\nu l} = [\Gamma(l+1+\nu)\Gamma(l+1-\nu)/2]^{1/2}, \quad (\text{A2})$$

in terms of the associated Legendre polynomials. Here, $\nu \equiv \sqrt{1-k^2}$ is real for supercurvature modes, $k^2 < 1$, and we have normalized the modes following Refs. [8,23]. The higher multipoles can be obtained from [29]:

$$P_{\nu-1/2}^{1/2}(\cosh\xi) = \sqrt{\frac{2}{\pi \sinh\xi}} \cosh\nu\xi, \quad (\text{A3})$$

$$P_{\nu-1/2}^{-1/2}(\cosh\xi) = \sqrt{\frac{2}{\pi \sinh\xi}} \frac{\sinh\nu\xi}{\nu}, \quad (\text{A4})$$

with the recurrence relation

$$(\nu^2 - l^2)P_{\nu-1/2}^{-l-1/2}(\cosh\xi) = P_{\nu-1/2}^{3/2-l}(\cosh\xi) - (2l-1)\coth\xi P_{\nu-1/2}^{1/2-l}(\cosh\xi). \quad (\text{A5})$$

The scalar harmonics for the first two multipoles can be written as

$$Q_{200}(\xi, \Omega) = N_0 \cosh\xi Y_{00}(\theta, \phi), \quad (\text{A6})$$

$$Q_{21m}(\xi, \Omega) = N_1 \sinh\xi Y_{1m}(\theta, \phi), \quad (\text{A7})$$

where the norms N_0 and N_1 diverge, see Eq. (A1). This is not a problem since the only contribution of these modes to curvature perturbations comes through the transverse traceless tensor Q_{ij} , see Eq. (35), and is easy to check that, for $k^2 = -3$, it vanishes identically for the first two multipoles, $l=0$ and $l=1$. However, it does not vanish for the higher multipoles and thus the transverse traceless curvature perturbation (35) gets contributions from all the $l \geq 2$ multipoles [23]. Let us calculate the quadrupole with Eq. (A5). It is

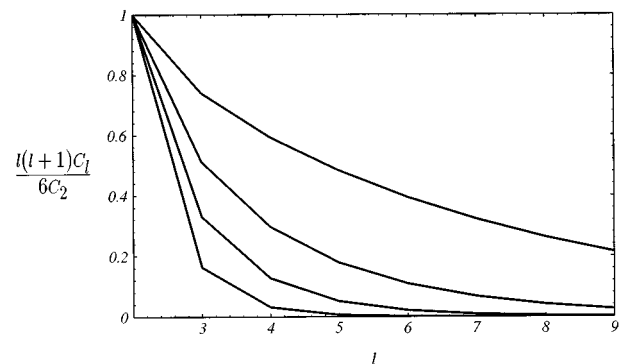


FIG. 1. The shape of the radiation angular power spectra $l(l+1)C_l$ induced by the $k^2 = -3$ mode of bubble wall fluctuations, normalized to the quadrupole, for the first nine multipoles. The curves correspond to $\Omega_0 = 0.2, 0.4, 0.6, 0.8$, from top to bottom. It is clear that none of them is compatible with a flat spectrum. Note that we recover the usual Grishchuk-Zel’dovich effect of a dominating quadrupole in the limit $\Omega_0 \simeq 1$.

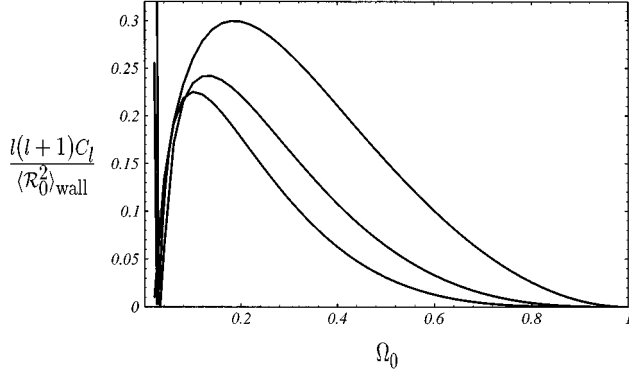


FIG. 2. The amplitude of the first three contributing multipoles, $l=2,3,4$ (from top to bottom), of the angular power spectrum in units of $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$, as a function of Ω_0 . It is clear that for a large range of Ω_0 , the quadrupole dominates the spectrum.

clear from this equation that the mode $k^2 = -3$, or $\nu=2$, is very special since the left-hand side (LHS) seems to vanish for $l=2$. However, the right-hand side (RHS) also vanishes and thus we should find the quadrupole in the limit $\nu \rightarrow 2$,

$$\Pi_{22}(\xi) = \sqrt{\frac{6 \sinh 4\xi - 8 \sinh 2\xi + 12\xi}{\pi}} \frac{48 \sinh^3 \xi}{48 \sinh^3 \xi}. \quad (\text{A8})$$

The rest of the multipoles can now be obtained from this expression together with the recurrence relation (A5).

We are interested in the multipole components of temperature fluctuations induced by this discrete mode. The best way to analyze its effect on the microwave background is to study this scalar metric perturbation in the comoving, uniform-Hubble-constant hypersurface gauge, in terms of the gauge-invariant potential Φ , as we did in Sec. III.

Let us evaluate the multipole components of the temperature anisotropies associated with the bubble wall fluctuations. A formalism that includes supercurvature modes ($k^2 < 1$) was developed in Ref. [9], where it was found that they contribute to the CMB temperature anisotropies as realizations of a homogeneous random field. Furthermore, in Ref. [8] it was shown that the $k^2 = -3$ mode also corresponds to a homogeneous random field, once we subtract the nonphysical monopole and dipole contributions.

In Sec. IV, we gave the expression of the angular power spectrum of the observed temperature fluctuations coming from the various metric perturbations, see Eq. (44). We will concentrate here on the contribution of the discrete $\nu=2$ mode associated with the bubble wall quantum fluctuations. $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$ is the average square amplitude of the metric perturbation (32). The ‘‘window function’’ $I_{\nu l}^2$ indicates how this mode contributes to the C_l 's,

$$\nu I_{\nu l} = \frac{1}{5} \Pi_{\nu l}(\eta_0) + \frac{6}{5} \int_0^{\eta_0} \Pi_{\nu l}(r) F'(\eta_0 - r) dr, \quad (\text{A9})$$

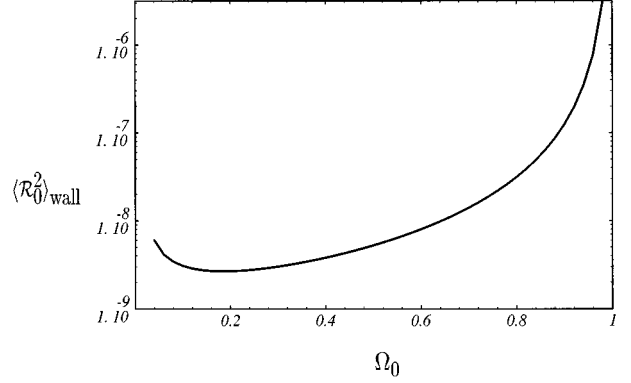


FIG. 3. Limits on $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$, based on current observational limits on $l(l+1)C_l < 8 \times 10^{-10}$. The allowed values of $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$ are those below the curve. In a large range of Ω_0 , the quadrupole provides the strongest constraint.

where $F(\eta)$ is given by Eq. (39).

It is possible to compute analytically its contribution to the first multipoles of the angular power spectrum, for values of Ω_0 close to one:

$$C_2 = \frac{\pi}{3} \left(\frac{8}{25} \right)^2 \langle \mathcal{R}_0^2 \rangle_{\text{wall}} (1 - \Omega_0)^2, \quad (\text{A10})$$

$$C_3 = \frac{\pi}{15} \left(\frac{16}{35} \right)^2 \langle \mathcal{R}_0^2 \rangle_{\text{wall}} (1 - \Omega_0)^3. \quad (\text{A11})$$

In general, we find $C_l \sim \langle \mathcal{R}_0^2 \rangle_{\text{wall}} (1 - \Omega_0)^l$, and thus the quadrupole dominates over the rest of the multipoles, such as in the case of open universe Grishchuk-Zel'dovich effect [16].

For $\Omega_0 < 1$, we have to evaluate numerically the window functions for the different multipoles, and compute their contributions to the CMB. In Fig. 1 we show the shape of the angular power spectrum, normalized to the quadrupole, as a function of multipole number l , for various values of Ω_0 . It is clear that none of them is compatible with a flat spectrum. Note that we recover the usual Grishchuk-Zel'dovich effect of a dominating quadrupole in the limit $\Omega_0 \approx 1$. In Fig. 2 we show the amplitude of the first contributing multipoles of the angular power spectrum in units of $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$, as a function of Ω_0 . It is clear that for a large range of Ω_0 , the quadrupole dominates the spectrum.

In Fig. 3 we show the limits on the curvature perturbation $\langle \mathcal{R}_0^2 \rangle_{\text{wall}}$ as a function of Ω_0 , from the observational limits on $l(l+1)C_l < 8 \times 10^{-10}$. We can parametrize this bound as

$$\langle \mathcal{R}_0^2 \rangle_{\text{wall}} \leq 3 \times 10^{-9}, \quad 0.1 \leq \Omega_0 \leq 0.4, \quad (\text{A12})$$

$$\langle \mathcal{R}_0^2 \rangle_{\text{wall}} \leq \frac{10^{-9}}{(1 - \Omega_0)^2}, \quad 0.4 \leq \Omega_0 \leq 1. \quad (\text{A13})$$

This is our main result. We will use it to constrain the models of open inflation in Sec. IV.

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