Isocurvature and adiabatic fluctuations of the axion in chaotic inflation models and large scale structure

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In chaotic inflation models, quantum fluctuations from axion fields lead to overproduction of domain walls and overly large isocurvature fluctuations, which is inconsistent with observations of cosmic microwave background anisotropies. These problems are solved by assuming a very flat potential for the Peccei-Quinn scalar field. As the simplest possibility, we consider a model where the Peccei-Quinn scalar is an inflaton itself, and show that the isocurvature fluctuations can be comparable with the adiabatic ones. We investigate the cosmological implications of the case when both adiabatic and isocurvature fluctuations exist, and find that the amplitude of the matter spectrum becomes smaller than that for the pure adiabatic case. This leads to a relatively high bias parameter ($b \simeq 2$) which is favored by the current observations. [S0556-2821(96)00816-8]

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I. INTRODUCTION

The axion [1–4] is the Nambu-Goldstone boson associated with the breaking of Peccei-Quinn symmetry, and was invented as a natural solution to the strong CP problem in QCD [5]. The Peccei-Quinn symmetry-breaking scale F_a is stringently constrained by laboratory experiments, astrophysics, and cosmology; the allowed range of F_a is between 10^{10} GeV and 10^{12} GeV [6] in standard cosmology. Axions are also cosmologically attractive since they can be cold dark matter (CDM) if F_a takes higher values in the allowed region.

The inflationary universe [7,8] was invented to solve problems in standard cosmology (flatness problem, horizon problem, monopole problem, etc.). In particular, the chaotic inflation model [9] is the simplest and most promising candidate that produces an inflationary universe. In chaotic inflation, some scalar field ϕ, which is called the inflaton, has a very flat potential $V(\phi)=\lambda\phi^4/4$ with $\lambda\sim 10^{-13}$. In the chaotic conditions of the early universe, the inflaton may have an expectation value much greater than the Planck mass and then slowly roll down to the true minimum of the potential. During the slow-rolling epoch, the universe expands exponentially.

When we consider the axion in a chaotic inflationary universe, we confront two serious problems associated with large quantum fluctuations generated during exponential expansion of the universe. One is the domain wall problem [10]. At the inflation epoch the axion field $a(x)$ is massless and its fluctuations are given by $\langle a\rangle = H/(2\pi)$, where H is the Hubble constant at that epoch. Since the phase of the Peccei-Quinn scalar θ_a is related to $a(x)$ by $\theta_a = a(x)/F_a$, the fluctuations of θ_a are given by

$$\delta\theta_a = \frac{H}{2\pi F_a}. \tag{1}$$

In chaotic inflation with the potential $\lambda\phi^4/4$, H is about 10^{14} GeV for $\lambda\sim 10^{-13}$, which is required to produce the anisotropies of the cosmic microwave background (CMB) observed by the cosmic background explorer (COBE) Differential Microwave Radiometer (DMR) [11]. Then, from Eq. (1), the fluctuations of the phase θ_a become of order 1 for $F_a \lesssim 10^{13}$ GeV, which means that the phase is quite random. Therefore, when the universe cools down to about 1 GeV and the axion potential is formed, the axion sits at a different position of the potential in different regions of the universe. Since the axion potential has N discrete minima (N is the color anomaly), domain walls are produced [12]. The domain wall with $N>2$ is disastrous because it quickly dominates the density of the universe.

The second problem is that quantum fluctuations for the axion cause anisotropies of the CMB [13–15] which are too large. Since the axion is massless during inflation, the axion fluctuations do not contribute to the fluctuations of the total density of the universe. In that sense, the axion fluctuations are isocurvature, i.e., zero curvature fluctuations and the constant entropy perturbations between the axion and the photon: $S_{a\gamma} = \delta_a - \frac{3}{4}\delta_\gamma = \text{const}$, where δ_a and δ_γ are density perturbations of the axion and the photon, respectively. On the contrary, the adiabatic perturbations require $S_{a\gamma}=0$ with constant curvature perturbations.

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After the axion acquires a mass m_a, the axion fluctuations become density fluctuations given by $\delta\rho_a/\rho_a \sim \delta\theta_a/\theta_a$, which cause CMB temperature fluctuations $\delta T/T \sim \delta\theta_a/\theta_a$. From Eq. (1), the CMB anisotropies produced are of order 1, which contradicts observations.

It has been pointed out in Ref. [15] that the above two problems are simultaneously solved if the potential of the Peccei-Quinn scalar is very flat. For a flat potential the Peccei-Quinn scalar Φ_a can have a large expectation value $\sim M_{\rm Pl}$ at the epoch of inflation. Then we should take $\langle \Phi_a \rangle$ as the effective Peccei-Quinn scale instead of F_a, and the phase fluctuations are suppressed. Therefore, the production of domain walls is suppressed and isocurvature fluctuations decrease. However, the isocurvature fluctuations may not always be negligible relative to the total density fluctuations and hence to the CMB anisotropies. In fact, as is shown in the next section, the isocurvature fluctuations can be comparable with the adiabatic ones for a large region of parameter space. Therefore, it is natural for the axion to have both types of fluctuations in the chaotic inflation scenario.

Since isocurvature fluctuations give a 6 times larger contribution to CMB anisotropies at COBE scales [16] than adiabatic fluctuations, the mixture of isocurvature and adiabatic fluctuations tends to decrease the amplitude of the matter fluctuations if the amplitude is normalized to the COBE-DMR data. This means that a relatively high bias parameter, which is defined as the ratio between the density perturbations of galaxies and the total density perturbations including the contributions from dark matter, is necessary compared with the purely adiabatic case. In the standard adiabatic CDM scenario, i.e., density parameter $\Omega = 1$, Hubble constant $H \gtrsim 50$ km/s Mpc with the Harrison-Zeldovich initial power spectrum, the COBE normalization results in a bias parameter less than 1, which is quite unphysical and also contradicts observations [17–19]. Therefore, the high bias parameter predicted by a model with both adiabatic and isocurvature fluctuations is favored.[1]

In this paper, we consider axionic isocurvature fluctuations generated by chaotic inflation and investigate their cosmological effects on CMB anisotropies and the large-scale structure of the universe.

II. AXION FLUCTUATIONS

Let us first estimate the amplitude of the isocurvature and adiabatic fluctuations generated in the chaotic inflationary scenario. For a demonstration of our point, we consider a model where the Peccei-Quinn scalar plays the role of an inflaton. The potential for the Peccei-Quinn scalar is given by

$$V(\Phi_a) = \frac{\lambda}{4}(|\Phi_a|^2 - F_a)^2, \tag{2}$$

with $\lambda \sim 10^{-13}$. Here the axion field $a(x)$ is the phase of Φ_a, namely, $\Phi_a = |\Phi_a| e^{ia(x)/F_a}$. We also assume that the axion is dark matter and that the density of the axion is equal to the critical density. After the axion acquires a mass, the isocurvature fluctuations with comoving wave number k are given by

$$\delta_a^{\rm iso}(k) \equiv \left(\frac{\delta\rho_a}{\rho_a}(k)\right)_{\rm iso} = \frac{2\,\delta a}{a} = \frac{\sqrt{2}H}{\Phi_a \theta_a} k^{-3/2}, \tag{3}$$

where H is the Hubble constant when the comoving wavelength k^{-1} becomes equal to the Hubble radius H^{-1} during the inflation epoch. It should be noted that the above initial spectrum is the Harrison-Zeldovich type in case of isocurvature perturbations. Therefore, in our simple model, there is a negligible contribution from the gravity wave mode. Since the fluctuations for θ_a are much less than 1 for $\Phi \sim M_{\rm Pl}$, inflation can make θ_a homogeneous beyond the present-day horizon. Therefore, the domain wall problem might be solved [24].

On the other hand, the inflaton generates adiabatic fluctuations which amount to

$$\delta_a^{\rm ad}(k) \equiv \left(\frac{\delta\rho}{\rho}(k)\right)_{\rm ad} = \frac{2H^3}{3V'\tilde{H}^2 R(t)^2} k^{1/2}, \tag{4}$$

where $R(t)$ and $\tilde{H}$ are the scale factor and Hubble constant at some arbitrary time t. Here we assume that the universe is radiation dominated and that the wavelength $[R(t)/k]$ is larger that the horizon $(\sim \tilde{H}^{-1})$. To compare these two types of fluctuations, it is convenient to take the ratio of the power spectra $[P(k) \equiv \delta(k)^2]$ at horizon crossing, i.e., $k^{-1}R = \tilde{H}^{-1}$, which is written as

$$\alpha \equiv \left.\frac{P_{\rm iso}}{P_{\rm ad}}\right|_{k/R=\tilde{H}} = \frac{9(V')^2}{H^4 \Phi_a^2 \theta_a^2}. \tag{5}$$

Since the cosmologically interesting scales $(k^{-1} \sim 1\ \text{kpc} - 3000\ \text{Mpc})$ correspond to the Hubble radius for $\Phi_a \simeq 4M_{\rm Pl}$ at the inflation epoch, α is given by

$$\alpha \simeq 2 \times 10^{-3} \theta_a^{-2}. \tag{6}$$

Furthermore, since the axion is dark matter, θ_a is related to the Peccei-Quinn scale F_a by [6]

$$\theta_a \simeq 0.017 \left(\frac{\Omega_a h^2}{0.25}\right)^{1/2} \left(\frac{F_a}{10^{15}\ \text{GeV}}\right)^{-0.59}, \tag{7}$$

where Ω_a is the density parameter of the axion at the present epoch and h is the dimensionless Hubble constant normalized to 100 km/s Mpc. Then the ratio α is written as

$$\alpha \simeq 6.24 \left(\frac{F_a}{10^{15}\ \text{GeV}}\right)^{1.18} \left(\frac{\Omega_a h^2}{0.25}\right)^{-1}. \tag{8}$$

Therefore, the isocurvature fluctuations are comparable with the adiabatic ones for $F_a \gtrsim 10^{14}$ GeV.

It seems natural to take $\theta_a \sim 1$ (corresponding to $F_a \sim 10^{12}$ GeV). In this case we have $\alpha \sim 10^{-3}$ and adiabatic

[1] There are other alternatives of standard CDM models to explain the large-scale structure and the COBE normalization at once such as models with a tilted spectrum [20], with the cosmological constant [21,22], or with a low Hubble constant [23].

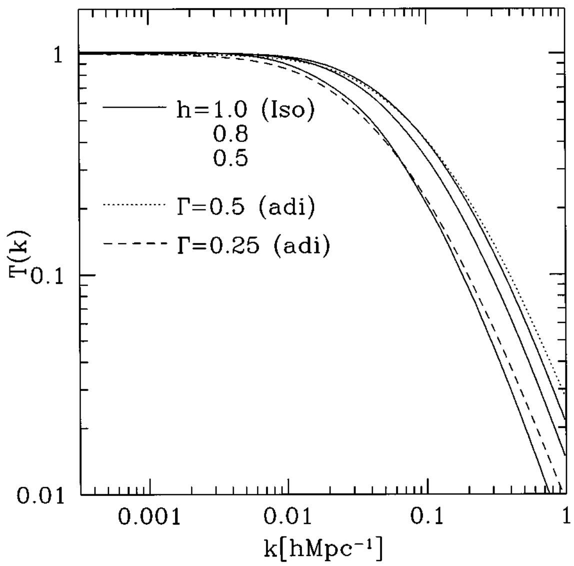

FIG. 1. The matter transfer function $T(k)$ for $\Omega=1$ pure isocurvature models with $h=0.5$, 0.8, and 1.0. Adiabatic CDM models with $\Gamma=0.5$ and 0.25 are also plotted.

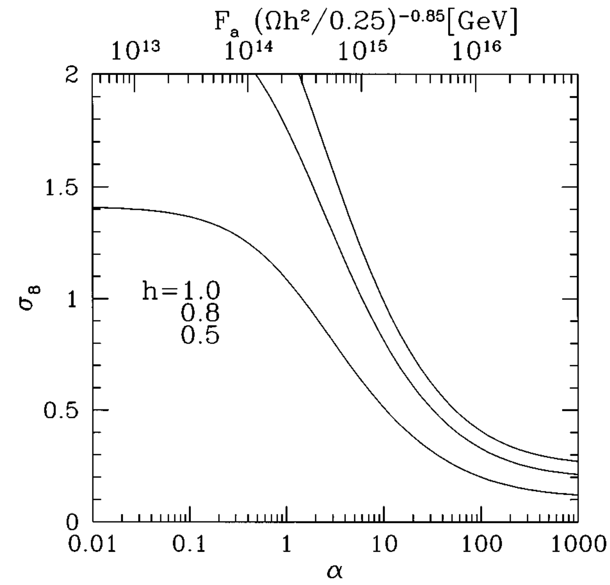

FIG. 2. σ_8 for models with adiabatic and isocurvature fluctuations as a function of α (lower x axis) or $F_a(\Omega h^2/0.25)^{-0.85}$ (upper x axis). We take $h=0.5$, 0.8, and 1.0.

fluctuations dominate. However, since θ_a is homogeneous over the entire universe, we can take θ_a as a free parameter and we are allowed to take $\theta_a \sim 10^{-2}$, leading to $\alpha \sim 10$. This contrasts with the standard cosmology where θ_a is random in space and the averaged value $\pi/\sqrt{3}$ should be taken. Furthermore, if the Peccei-Quinn scalar is independent of the inflaton, the expectation value of Φ_a at the inflation epoch can be less than the Planck mass, depending on the coupling constant of $|\Phi|^4$. In this model we have found that the isocurvature fluctuations comparable with the adiabatic ones, i.e., $\alpha \sim 1$, are produced even for $F_a \simeq 10^{12}$ GeV (equivalently $\theta_a \sim 1$).

III. OBSERVATIONAL CONSTRAINTS

A. Pure isocurvature fluctuations

First we examine models with pure isocurvature fluctuations. Throughout this paper, we only consider models with total density parameter $\Omega=1$ and baryon density parameter $\Omega_b=0.0125h^{-2}$ derived from primordial nucleosynthesis [25]. The matter transfer functions $T(k)$ for different h's are shown in Fig. 1, together with those for adiabatic cold dark matter models. The transfer function is defined as $T(k)\equiv\widetilde{\delta}(k)/\widetilde{\delta}(0)$, where $\widetilde{\delta}(k)$ is $k^{3/2}\delta_a^{\rm iso}(k)$ for isocurvature and $k^{-1/2}\delta_a^{\rm ad}(k)$ for adiabatic fluctuations. It is well known that the transfer functions for adiabatic CDM models are controlled by a single parameter $\Gamma\equiv\Omega h$ for low baryon density [26]. Recent large-scale structure observations suggest the best-fit value of $\Gamma\approx0.25$ [18,21]. In Fig. 1, it should be noticed that the transfer function of the isocurvature model with $h=0.5$ is very similar to the best-fit adiabatic one. However, as we pointed out before, there is a problem of overproducing temperature fluctuations on large scales for isocurvature perturbations. Let us consider next the amplitude of mass fluctuations at $8h^{-1}$ Mpc, i.e., σ_8, which is defined as

$$\sigma_8^2 \equiv \frac{1}{2\pi^2}\int\frac{dk}{k}k^3P(k)\left(\frac{3j_1(kR)}{kR}\right)^2\bigg|_{R=8h^{-1}\ \mathrm{Mpc}}, \tag{9}$$

where j_1 is the first order spherical Bessel function. The bias parameter b is the inverse of σ_8: $b=\sigma_8^{-1}$. If we normalize the amplitude of fluctuations to the COBE-DMR data, the values of σ_8 are 0.11, 0.20, and 0.25 for $h=0.5$, 0.8, and 1.0, respectively. Here we take the normalization scheme proposed by White and Bunn [27] for the COBE normalization. The observed values of σ_8 are 0.57 from galaxy cluster surveys [17], 0.75 from galaxy and cluster correlations [18], and 0.5–1.3 from peculiar velocity fields [19] if $\Omega=1$ is assumed. Therefore we can reject pure isocurvature models using these observations.

B. Adiabatic and isocurvature fluctuations

Next we investigate models with an admixture of isocurvature and adiabatic perturbations. As we have shown in Sec. II, the amplitudes of the isocurvature and adiabatic fluctuations are comparable in chaotic inflation, for a certain range of initial values of θ_a. Thus it may be interesting to study the cosmological effects of these admixture fluctuations. In linear perturbation theory, isocurvature and adiabatic perturbations are independent solutions. Therefore there is no correlation between these two modes. We simply add two power spectra in order to get the total one. In Fig. 2, we show the values of σ_8 as a function of α (or F_a) for models with $h=0.5$, 0.8, and 1.0. As is shown in this figure, we can easily overcome the antibias problem of the standard purely adiabatic CDM model by employing admixture models. Assuming the COBE normalization, the value of σ_8 for standard CDM ($\Omega=1$, $h=0.5$, and $\Omega_b=0.05$ with adiabatic perturbations) is 1.4. For admixture models, we can obtain desirable values of $\sigma_8\simeq$ 0.5–0.8 for the range of $\alpha\approx$ 1–10

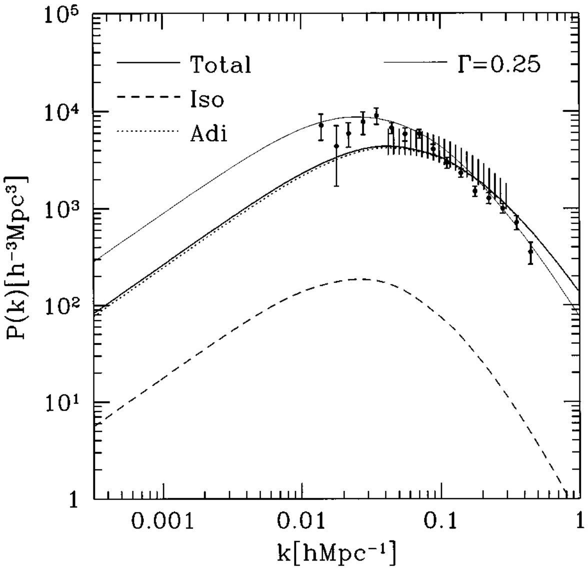

FIG. 3. Matter power spectrum $P(k)$ for the $\Omega=1$, $h=0.5$, and $\Omega_b=0.05$ models with adiabatic and isocurvature fluctuations. We take $\alpha=3.75$, i.e., $\sigma_8=0.75$. Contributions from isocurvature fluctuations and adiabatic fluctuations are plotted, together with the total power spectrum. An adiabatic CDM model with $\Gamma=0.25$ and $\sigma_8=0.75$ is also plotted. The observational data are taken from Peacock and Dodds [18]. Shaded regions are the best-fit value of the Mark III catalog of peculiar velocities of galaxies by Zaroubi *et al.* [28] with 30% errors.

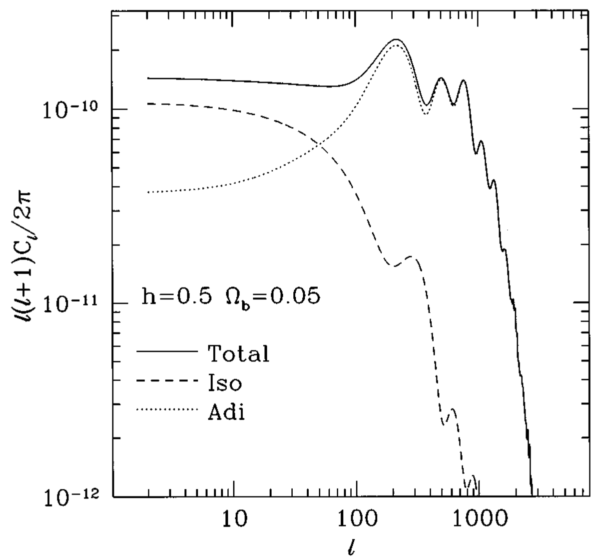

FIG. 4. Power spectrum of CMB anisotropies C_l for the $\Omega=1$, $h=0.5$, and $\Omega_b=0.05$ models with adiabatic and isocurvature fluctuations as a function of multipole moments l. We take $\alpha=3.75$, i.e., $\sigma_8=0.75$. Contributions from isocurvature fluctuations and adiabatic fluctuations are plotted, together with total power spectrum.

or $F_a \approx 10^{14-15}$ GeV. For the model with $h=0.5$, $\sigma_8=0.57$ for $\alpha=7.7$ and $\sigma_8=0.75$ for $\alpha=3.8$.

The matter power spectrum of the model with $h=0.5$ and $\alpha=3.8$, i.e., $\sigma_8=0.75$, is shown in Fig. 3 If $\sigma_8 \gtrsim 0.5$, the difference between purely adiabatic and an admixture of adiabatic and isocurvature fluctuations is very small. Therefore, the same problem with the purely adiabatic CDM models arises. Namely, it is difficult to obtain the shape which fits the observations if we employ $h \gtrsim 0.5$. However, a recent analysis of velocity fields [28], which have much sensitivity on larger scales, suggests that the turnover point of the power spectrum is smaller than previously thought. Their best-fit shape is $\Gamma \simeq 0.5$ (see shaded regions of Fig. 3). Therefore, it might be premature to rule out the model merely from the shape of the power spectrum.

Figure 4 shows CMB anisotropy multipole moments $C_l = \langle |a_{lm}|^2 \rangle$. Here $\delta T/T = \Sigma_{lm} a_{lm} Y_{lm}$, with Y_{lm} being spherical harmonics. There is a clear distinction between pure adiabatic and admixture spectra. The admixture spectrum has very low peaks against the Sachs-Wolfe plateau on small l's. It might be possible to determine α from future experiments using a new satellite or long-duration balloon flights. In order to see how general this feature of the temperature spectrum is, we plot C_l's for a desirable range of σ_8, e.g., 0.5–1.0 for $h=0.5$ and 0.8, in Fig. 5. In the case of $h=0.5$, there still remain high peaks for $\sigma_8 \simeq 1$ since the isocurvature and adiabatic fluctuations are comparable. These peaks disappear, however, if we consider the high Hubble constant and/or low σ_8 because isocurvature fluctuations dominate.

IV. CONCLUSION AND REMARKS

We have examined the cosmological implication of axions in the chaotic inflationary scenario. By assuming a very flat potential for the Peccei-Quinn scalar, we can solve the overproduction problems of domain walls and of CMB anisotropies. The simplest model, where the Peccei-Quinn scalar plays the role of an inflaton of chaotic inflation, has been investigated. This scalar field produces both adiabatic and isocurvature fluctuations. From recent observations of large-scale structure and CMB anisotropies, models with pure isocurvature fluctuations (or a negligible amount of

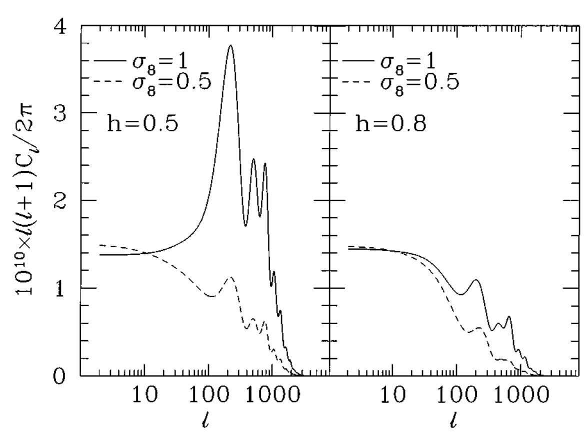

FIG. 5. Power spectra of CMB anisotropies C_l's for the $\Omega=1$ and $\Omega_b=0.05$ models with admixture fluctuations as a function of multipole moments l. Left panel: models with $h=0.5$, $\sigma_8=1.0$, i.e., $\alpha=1.45$ (solid line), and $\sigma_8=0.5$, i.e., $\alpha=10.6$ (dashed line), are plotted. Right panel: models with $h=0.8$, $\sigma_8=1.0$, i.e., $\alpha=6.04$ (solid line), and $\sigma_8=0.5$, i.e., $\alpha=32.0$ (dashed line), are plotted.

adiabatic fluctuations) are ruled out. The preferable value of F_a for the desired bias parameter ($b \sim 2$) is about 10^{15} GeV which happens to be the grand unified theory (GUT) scale.

Note added. After finishing this paper, we became aware of a paper by Stompor *et al.* [29] which also discusses the cosmological consequences of an admixture of fluctuations. However, explicit models for producing both adiabatic and isocurvature fluctuations were not considered in their paper.

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