Massive quasiparticle model of the SU(3) gluon plasma

A. Peshier,[1,2] B. Kämpfer,[1,2] O. P. Pavlenko,[1,3] and G. Soff[2]
[1]*Research Center Rossendorf Inc., PF 510119, 01314 Dresden, Germany*
[2]*Institut für Theoretische Physik, Technische Universität Dresden, Mommsenstraβe 13, 01062 Dresden, Germany*
[3]*Institute for Theoretical Physics, 252143 Kiev-143, Ukraine*
(Received 1 September 1995)

Recent SU(3) gauge field lattice data for the equation of state are interpreted by a quasiparticle model with effective thermal gluon masses. The model is motivated by lowest-order perturbative QCD and describes very well the data. The proposed quasiparticle approach can be applied to study color excitations in the nonperturbative regime. As an example we estimate the temperature dependence of the Debye screening mass and find that it declines sharply when approaching the confinement temperature from above, while the thermal mass continuously rises. [S0556-2821(96)02815-9]

PACS number(s): 12.38.Gc, 11.15.Ha

I. INTRODUCTION

The recent progress in calculating properties of quarks and gluons by numerical methods on space-time lattices also provides new information on the equation of state. It is generally believed that at sufficiently high temperature the strongly interacting matter appears as plasma of quarks and gluons, while at low temperatures the matter constituents are represented by hadrons (we consider here charge symmetric matter). Because of problems in incorporating fermions on a lattice the most accurate information is available for the pure gluon plasma. There are extensive studies of the SU(2) [1,2] and SU(3) [3,4] gluon plasma. An intriguing question concerns finite-size effects and the extrapolation to the continuum limit, which has been investigated recently [2,4].

Even if precision data for the equation of state are available, one must ask whether it allows for an interpretation in terms of physical quantities. Indeed, there are various attempts to find suggestive interpretations of the lattice numerology. In an early approach the SU(2) data [1] are described by a low-momentum cutoff model. In Ref. [5] a finite gluon mass and a vacuum pressure are fitted to previous SU(3) data. More accurate SU(3) data [3] are described in Ref. [6] by a modified cutoff model with perturbative corrections and a bag constant, while in Refs. [7,8] a thermal mass alone is found to be sufficient for describing the data. The latter approach has also proven to be successful for the SU(2) data [9]. Despite the accuracy of the SU(3) data on a $16^3\times 4$ lattice [3], by now there are data on larger lattices available [4]. These new data seem to permit a safe extrapolation to the continuum limit and are worth to be interpreted.

The aim of our note is to present an interpretation of the recent SU(3) data [4] in terms of an ideal gas model of quasiparticles with thermal masses $m(T)$. This model can be applied for studying various physical quantities (such as Debye or screening mass of heavy quark potential, transport coefficients, dilepton and photon rates) at physically relevant, low temperatures near the confinement temperature T_c, where perturbative QCD cannot be utilized directly. The particular point we adopt is that the high temperature limit of our thermal mass follows essentially from perturbative QCD. Such a functional dependence of $m(T)$ turns out to reproduce quite well the newest SU(3) lattice data. We extend our model here to estimate the Debye mass in SU(3) gauge theory near T_c.

II. IDEAL QUASIPARTICLE GAS MODEL

Our goal is a quasiparticle model for the equation of state of a gluon plasma which is compatible with both the continuum extrapolation of lattice data, currently available up to $5T_c$, and the perturbative region, i.e., QCD at $T\to\infty$. We utilize the dispersion relation

$$\omega^2=k^2+m^2(T) \tag{1}$$

(ω and k are the quasiparticle energy and momentum). With the distribution function $f(k)=[\exp\{\sqrt{k^2+m^2(T)}/T\}-1]^{-1}$ the entropy density takes, then, the ideal gas form

$$s(T)=\frac{d}{2\pi^2 T}\int_0^\infty dk f(k) k^2 \frac{\frac{4}{3}k^2+m^2(T)}{\sqrt{k^2+m^2(T)}}, \tag{2}$$

while the primary thermodynamical potential pressure p and the energy density e read

$$p(T)=\frac{d}{6\pi^2}\int_0^\infty dk f(k)\frac{k^4}{\sqrt{k^2+m^2(T)}}-B(T), \tag{3}$$

$$e(T)=\frac{d}{2\pi^2}\int_0^\infty dk f(k) k^2\sqrt{k^2+m^2(T)}+B(T) \tag{4}$$

(d is the gluon degeneracy factor). These relations are thermodynamically self-consistent; i.e., they satisfy $e+p=sT$ and $s=\partial p/\partial T$. The function $B(T)$ is a necessary quantity when allowing for a temperature-dependent quasiparticle energy $\omega(k,T)$ [5,10]. $B(T)$ is not a second independent function, but related to the thermal mass, because of self-consistency, via

$$B(T)=B_0-\frac{d}{4\pi^2}\int_{T_0}^{T} d\tau \frac{dm^2(\tau)}{d\tau}\int_0^\infty \frac{dk k^2 f(k)}{\sqrt{k^2+m^2(\tau)}}. \tag{5}$$

The integration constant B_0 resembles somewhat the bag constant. Note that the previous approaches [8,9] used $p(T)$ with $B(T)\equiv 0$, and that, consequently, neither the entropy density nor the energy density take the structure of an ideal gas.

To determine the functional dependence of $m(T)$ on temperature let us first consider the perturbative regime. The thermodynamical properties of the gluon plasma depend predominantly on the transverse part of the gluon self-energy [11,12]. In the weak coupling regime the transverse gluon self-energy in a gluon plasma with N_c colors results in a dispersion relation which can be approximated [11,12] by $\omega^2=\alpha k^2+\beta\omega_0^2$, with $\alpha = 1$ $(\frac{6}{5})$ and $\beta = \frac{3}{2}$ (1) at large (small) momenta and gauge-invariant plasma frequency $\omega_0^2=(N_c/9)g^2T^2$ (here, g^2 denotes the perturbative QCD coupling constant). By studying numerically the known integral representation of the polarization operators, we find that the large momentum approximation to the full transverse one-loop dispersion relation [13] holds at $k/T>2\sqrt{N_c/9}g$; longitudinal excitations are there overdamped. Otherwise, the large momentum region dominates the statistical integrals in Eqs. (2)–(4), e.g., more than 96.5% of the contribution to the energy density comes from $k/T\geqslant 1$. The error caused by the use of Eq. (1), instead of the exact one-loop dispersion relation, for $k<gT$ can be estimated as $\propto g^4$. Therefore, Eq. (1) represents an excellent approximation of QCD properties, relevant for evaluating Eqs. (2)–(4), and $m^2(T)=\beta\omega_0^2$ with $\beta=\frac{3}{2}$ is supported within this approximation. Hence, $m^2(T)=(1/\Gamma)g^2(T)T^2$, with $\Gamma=6/N_c$, emerges approximately from perturbative QCD.

Let us now compare the obtained pressure potential (3) at high temperature with the corresponding pressure obtained within first-order QCD. The high-temperature expansion (i.e., $m/T\ll 1$) of Eq. (3) with $B(T) = -p_{\mathrm{SB}}(15/8\pi^2)[m(T)/T]^2+\cdots$ reads

$$p=p_{\mathrm{SB}}\left[1-\frac{15}{8\pi^2}\left(\frac{m(T)}{T}\right)^2+\cdots\right], \tag{6}$$

where $p_{\mathrm{SB}}=(d\pi^2/90)T^4$. From QCD it is known [11] that the perturbative pressure is

$$p_{\mathrm{PQCD}}=\frac{2(N_c^2-1)\pi^2}{90}T^4\left[1-\frac{5N_c}{16\pi^2}g^2+\cdots\right]. \tag{7}$$

Comparing the leading terms in Eqs. (6) and (7), one reveals that, despite massive quasiparticles, one needs to include only the two transverse degrees of freedom, i.e., $d=2(N_c^2-1)$. The next-to-leading order terms in the parentheses confirm our above ansatz for $m^2(T)$.

Finally, we specify the coupling constant in accordance with perturbative QCD as

$$G^2(T)=\frac{48\pi^2}{11N_c\ln\left(\lambda\frac{T}{T_c}+\frac{T_s}{T_c}\right)^2}, \tag{8}$$

with T_s/T_c as phenomenological regularization as in Ref. [8] and $\lim_{T\to\infty}G^2(T)\to g^2(T)$; T_c/λ represents the usual regularization scale parameter Λ. In the following we utilize in Eqs. (1)–(5) the thermal mass

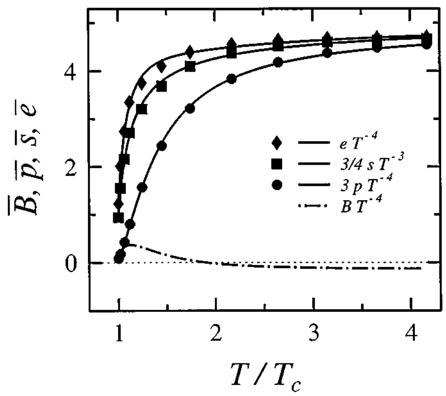

FIG. 1. Comparison of our model (thin lines) with continuum-extrapolated lattice data (symbols, from [4]) of scaled energy density $\overline{e}=e/T^4$, pressure $\overline{p}=3p/T^4$, and entropy density $\overline{s}=\frac{3}{4}s/T^3$. The dash-dotted curve depicts the function $\overline{B}=B(T)/T^4$.

$$m^2(T)=\frac{1}{\Gamma}G^2(T)T^2, \quad \Gamma=\frac{6}{N_c}. \tag{9}$$

III. ANALYSIS OF LATTICE DATA

We apply our model now to the SU(3) lattice data [4]. In addition to λ, T_s, and B_0, we also do not constrain the degeneracy d in order to get an optimum fit. In Fig. 1 we demonstrate that our model, defined by Eqs. (1)–(5), (8), and (9), describes very well the continuum-extrapolated data. As fit parameters we obtain $\lambda=4.17$, $T_s/T_c=-2.96$; $d=17.2$ is surprisingly near to the above anticipated value for the two transverse degrees of freedom of gluons. [Maybe this simple multiplicative deviation from 16 accounts for higher order corrections or some longitudinal contribution. Indeed, replacing Eq. (8) by the two-loop expression we find $d=16.6$ for the best fit.] $B_0=0.16T_c^4$ turns out as an optimum choice for the present data. As seen in Fig. 1 the function $B(T)$, which becomes small at $T>1.5T_c$, changes its sign at $2T_c$ (a similar observation was made in Ref. [10] for the older data [3]).

Figure 2 displays the interaction measure $(e-3p)T^{-4}$, which is a sensitive quantity related to the temperature dependence of the gluon condensate. One observes that for $T>1.2T_c$, the $32^3\times 6$ and $32^3\times 8$ lattice data are nicely re-

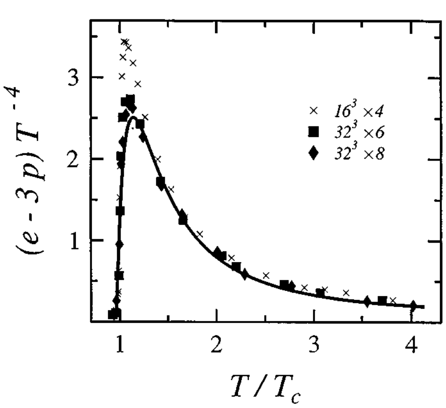

FIG. 2. The interaction measure as function of temperature (heavy full line: our model; symbols: lattice data [4].)

produced. In the region T_c–$1.2T_c$, the scaled energy density is a rapidly varying function. It might turn out that our quasiparticle model does not cover perfectly the very details of forthcoming high-precision lattice data in this region. However, it seems that the gross features of the equation of state in the physically relevant region are fairly well described. This gives some confidence in our quasiparticle interpretation.

As a matter of fact, we mention three obvious aspects of our phenomenological approach. (i) The flexibility, introduced by the definition of $G(T)$ in Eq. (8), allows to some extent for the description of the data. (ii) This flexibility is sufficient to describe the data on the basis of a different model [14], wherein Eqs. (6) and (7) would be partially identified [15] and in Eq. (9) a different numerical factor would appear. (iii) Higher order agreement of Eqs. (6) and (7) is not fully achievable, therefore, Eq. (3) cannot be considered as resumed expression.

IV. SCREENING MASS

Our model can be applied to study various collective properties of a colored quark-gluon system. Since even at comparatively low temperatures the gas of quasiparticles still remains weakly interacting, we have a chance to treat this gas in a perturbative way down to T_c. Here, we estimate as an example the temperature dependence of the Debye screening mass. The Debye mass reflects the fundamental property of a plasma medium to screen the static chromoelectric interactions. Following the standard definition, the Debye mass m_D for an electromagnetic plasma is given by the small momentum limit of the static longitudinal photon self-energy function $\Pi_{00}(\omega,k)$ [11]:

$$m_D^2=\lim_{k\to 0}\Pi_{00}(\omega=0,k). \tag{10}$$

It is connected with the longitudinal part of the plasma dielectric tensor $\epsilon_L(\omega,k)$ via $k^2+\Pi_{00}(0,k)=k^2\epsilon_L(0,k)$ [16]. At leading order in α_s the above definition is valid also for the QCD plasma [17]. In our model α_s is the coupling constant of the color interaction between quasiparticles. The chromoelectrical tensor $\epsilon_L(\omega,k)$ can be calculated in lowest order in α_s within the kinetic theory of collective color excitations [18] with the corresponding corrections related to the nonzero effective mass $m(T)$ of our quasiparticles. (The analogous approach has been employed for calculations within the cutoff model [19].)

For the gluon plasma the chromoelectrical tensor is

$$\epsilon_L=1+\frac{g^2N_c\gamma}{\omega k^2}\int\frac{d^3p}{(2\pi)^3}\,\frac{\vec{k}\cdot\vec{p}}{E\omega-\vec{k}\cdot\vec{p}+i\varepsilon}\left(\vec{k}\cdot\frac{\partial}{\partial\vec{p}}\right)f(p), \tag{11}$$

where $k^\mu=(\omega,\vec{k})$ is the wave four-vector, and $f(p)$ denotes the above distribution function of quasiparticles with four-momentum $p^\mu=(E,\vec{p})$ and the dispersion relation (1). The factor $\gamma=2$ accounts for the spin degrees of freedom of the quasiparticles with respect to the asymptotic limit above and to Ref. [5]. Solving Eq. (11) in the limit (10) yields

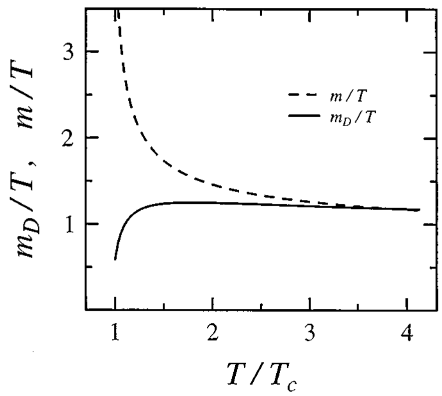

FIG. 3. The thermal mass (dashed line) and the estimated Debye screening mass (full line) in our model.

$$m_D^2=\frac{N_c}{\pi^2}g^2T^2J_g(T), \tag{12}$$

$$J_g(T)=T^{-3}\int_0^\infty dp\,p^2f^2(p)\exp\left\{\frac{\sqrt{p^2+m^2(T)}}{T}\right\}.$$

In the limit $m(T)/T\to 0$, one recovers the well-known perturbative QCD limit $m_D^2=\frac{1}{3}N_cg^2T^2$. The perturbative QCD coupling constant can be expressed by

$$\alpha_s=\frac{g^2}{4\pi}=\frac{12\pi}{11N_c\ln\frac{M^2}{\Lambda^2}}, \tag{13}$$

where the quantity M^2 is determined by averaging over the squared quasiparticle momenta [11]: i.e.,

$$M^2(T)=\frac{4}{3}\,\frac{\int_0^\infty dp f(p)p^4}{\int_0^\infty dp f(p)p^2}. \tag{14}$$

We choose the scale parameter Λ in accordance with the high-temperature limit of Eqs. (8) and (13). Since in this limit $M\approx 3.7T$, we find $\Lambda/T_c=3.7\lambda^{-1}$. For the temperature dependence of $m(T)$, extracted above from the lattice data, the coupling constant (13) remains as small as 0.32 at T_c. So, based on the perturbative ansatz for m_D^2, we get in our quasiparticle model the opportunity to describe the Debye mass in the nonperturbative region. Unfortunately, the direct comparison of the screening mass (12) with SU(3) lattice data [20,21] faces some difficulties since numerical precision measurements on large enough lattices and safe continuum extrapolations are not yet performed. Nevertheless, considering the data [20], one can extract two main features: (i) at temperatures of $T>2T_c$, the Debye mass depends weakly on the temperature, and (ii) $m_D(T)$ possesses a maximum around $1.5T_c$ and seems to drop abruptly when approaching T_c from above. Figure 3 shows that both of the above features are covered by our effective model. We find it especially important that our model describes in the nonperturbative region the sharp drop near T_c. Also, some measure of the typical scale of changes of m_D, say $m_D(T=1.4T_c)/m_D(T=1.1T_c)$, is in nice agreement with the lattice results [20]. At high temperatures of $T=11T_c$, our model results in $m_D/T\approx$ 1.06 which is near the data [20]. The difference of about 10% might be attributed to comparably large finite-size effects at high temperatures. Since such effects are expected to be not so important at smaller temperatures, we find the comparison of our model in this region with lattice data quite encouraging, despite the fact that the model parameters are adjusted by reproducing the equation of state on larger lattices.

As shown in Fig. 3 the effective gluon mass has, in the region near T_c, a completely different behavior as compared with the screening mass. Actually, the ratio m^2/m_D^2 can be used as a measure of nonperturbative effects since in the perturbative region $m^2/m_D^2\approx\frac{1}{2}$, while at $T\to T_c$ the ratio becomes $m^2/m_D^2\gg 1$, sharply rising when approaching T_c. Such a sharp decline of m_D near T_c might have quite interesting consequences for several deconfinement probes in ultrarelativistic heavy-ion collisions, e.g., for the jet unquenching [22], lepton and photon probes [8,23], strangeness enhancement via the heavy gluon decay $g^*\to s\overline{s}$ [5], and J/ψ suppression [24].

V. SUMMARY

In summary, we present an interpretation of new SU(3) gluon lattice data within a model of an ideal gas of quasiparticles with effective thermal masses, which is motivated by perturbative QCD. Such a functional dependence of the effective mass is found to reproduce rather perfectly the recent SU(3) lattice data of thermodynamical parameters. We utilize our model to estimate the behavior of the Debye screening mass near the confinement temperature and find a sharply dropping Debye mass, when approaching close to T_c from above, while the thermal mass continuously rises.

ACKNOWLEDGMENTS

Discussions with J. Engels, M. I. Gorenstein, U. Heinz, B. Müller, K. Redlich, D. H. Rischke, H. Satz, M. Thoma, G. M. Zinovjev, and the participants of the QCD workshop in ECT* Trento are gratefully acknowledged. F. Karsch is especially thanked for informing us on his lattice results prior to publication. O.P.P. is grateful for the warm hospitality of the Research Center, Rossendorf. The work was supported in part by BMBF Grant No. 06DR666, and the Grant No. U4D000 by International Scientific Foundation.

[1] J. Engels, J. Fingberg, K. Redlich, H. Satz, and M. Weber, Z. Phys. C **42**, 341 (1989).
[2] J. Engels, F. Karsch, and K. Redlich, Nucl. Phys. **B435**, 295 (1995).
[3] Y. Deng, in *Lattice '88*, Proceedings of the International Symposium, Batavia, Illinois, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 334 (1989)]; F.R. Brown, N.H. Christ, Y. Deng, M. Gao, and T.J. Woch, Phys. Rev. Lett. **61**, 2058 (1988); J. Engels, J. Fingberg, F. Karsch, D. Miller, and M. Weber, Phys. Lett. B **252**, 625 (1990).
[4] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lütgemeier, and B. Petersson, Phys. Rev. Lett. **75**, 4169 (1995).
[5] T.S. Biro, P. Levai, and B. Müller, Phys. Rev. D **42**, 3078 (1990).
[6] D.H. Rischke, M.I. Gorenstein, A. Schäfer, H. Stöcker, and W. Greiner, Phys. Lett. B **278**, 19 (1992); Z. Phys. C **56**, 325 (1992).
[7] W. Greiner and D.H. Rischke, Columbia Report No. CU-TH-648, 1994 (unpublished).
[8] A. Peshier, B. Kämpfer, O.P. Pavlenko, and G. Soff, Phys. Lett. B **337**, 235 (1994).
[9] V. Goloviznin and H. Satz, Z. Phys. C **57**, 671 (1993); A. Peshier, B. Kämpfer, O.P. Pavlenko, and G. Soff, in *Hot Hadronic Matter: Theory and Experiment*, edited by H.H. Gutbrod and J. Rafelski, NATO ASI B Vol. 346 (Plenum, New York, 1995), p. 139.
[10] M.I. Gorenstein and S.N. Yang, Phys. Rev. D **52**, 5206 (1995).
[11] J.I. Kapusta, Nucl. Phys. **B148**, 461 (1979); *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, MA, 1989).
[12] V.V. Klimov, Sov. Phys. JETP **55**, 199 (1982); H.A. Weldon, Phys. Rev. D **26**, 1394 (1982).
[13] R.D. Pisarski, Physica A **158**, 146 (1989); M. Thoma, in *Quark-gluon plasma II*, edited by R. Hwa (World Scientific, Singapore, 1995).
[14] A. Peshier, B. Kämpfer, O.P. Pavlenko, and G. Soff, Report No. FZR-105, 1995 (unpublished).
[15] H.G. Dosch, H.J. Pirner, and Yu.A. Simonov, Phys. Lett. B **349**, 335 (1995).
[16] V.P. Silin, Sov. Phys. JETP **11**, 1136 (1960).
[17] A.K. Rebhan, Nucl. Phys. **430**, 319 (1994); E. Braaten and A. Nieto, Phys. Rev. Lett. **73**, 2402 (1994).
[18] H.Th. Elze and U. Heinz, Phys. Rep. **183**, 81 (1989); H.Th. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B **177**, 402 (1986); Nucl. Phys. **B276**, 706 (1986); S. Mrówczyński, Phys. Rev. D **39**, 1940 (1989).
[19] M.I. Gorenstein, O.A. Mogilevski, and S. Mrówczyński, Phys. Lett. B **246**, 200 (1990).
[20] M. Gao, Phys. Rev. D **41**, 626 (1990).
[21] L. Kärkkäinen *et al.*, Phys. Lett. B **282**, 121 (1992); Nucl. Phys. **B395**, 733 (1993); N. Attig, F. Karsch, B. Petersson, H. Satz, and M. Wolf, Phys. Lett. B **209**, 65 (1988).
[22] M. Gyulassy and M. Plümer, Phys. Lett. B **243**, 432 (1990); M. Plümer, M. Gyulassy, and X.N. Wang, Nucl. Phys. **A590**, 511c (1995).
[23] P.A. Henning and E. Quack, Phys. Rev. Lett. **75**, 2811 (1995).
[24] H. Satz, Nucl. Phys. **A590**, 63c (1995).