Nucleon spin structure and quark helicity decomposition

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Using the results for the first moments of the proton and deuteron spin structure functions $g_1^{p,d}$ measured at the SLAC experiment E143, we compare the conventional method of obtaining from those moments the quark helicity contributions to the proton spin assuming $\mathrm{SU}(3)_{\mathrm{flavor}}$ symmetry (which implies a strange quark sea helicity $\Delta s \neq 0$) to the results obtained assuming $\Delta s = 0$. We conclude that current experimental uncertainties cannot rule out the latter. Using $\Delta s = 0$ we extract the $\mathrm{SU}(3)_{\mathrm{flavor}}$ F and D terms directly from the first moments, which give an estimate of possible symmetry breaking. [S0556-2821(96)05115-6]

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Recent measurements [1–5] have contributed considerable new knowledge to our understanding of the nucleon spin structure. In this paper we will use the results of the most precise of those experiments, SLAC E143, and discuss the implications for the origin of nucleon spin. In particular, we will examine the conventional use of the approximate symmetry $\mathrm{SU}(3)_{\mathrm{flavor}}$ to determine the different quark flavors contributions to the nucleon spin. We will show that given the current experimental uncertainties, it is still possible to account for the nucleon spin as originating only from u and d valence and sea quarks, with no significant strange quark contribution. The total quark contribution, however, can be obtained without resort to $\mathrm{SU}(3)_{\mathrm{flavor}}$, and is very insensitive to models of breaking this symmetry [6]. In view of the current limited understanding of the nature and magnitude of $\mathrm{SU}(3)_{\mathrm{flavor}}$ symmetry breaking, we conclude that the use of $\mathrm{SU}(3)_{\mathrm{flavor}}$ underestimates the uncertainties of the flavor-specific quark helicity contributions to the nucleon spin.

Experiment E143 counted electrons scattered from polarized ammonia ($^{15}\mathrm{NH}_3$ and $^{15}\mathrm{ND}_3$) targets into two spectrometers aligned at 4.5° and 7° with respect to the beam for parallel and antiparallel beam and target spin configurations at three beam energies (29.1, 16.2, and 9.7 GeV), and for opposite orientations of the target spin transversal to the beam helicity, at 29.1 GeV. The structure functions $g_1^p(x,Q^2)$, $g_1^d(x,Q^2)$, and $g_1^n(x,Q^2)$ have been extracted for the 29.1 GeV data, over the range $0.029 < x < 0.8$ of the Bjorken x scaling variable and the four-momentum squared range $1.3 < Q^2 < 10$ $(\mathrm{GeV}/c)^2$. The average beam polarization was 85%, and the NH_3 and ND_3 average polarizations were 60% and 30%, respectively. The first moments of these structure functions have been computed and compared with theoretical predictions for the corresponding sum rules. The first results have been published [4,5,7,8].

One of the most significant conclusions that can be derived from the results of experiment E143 is a precise measurement of the quark contribution to the spin of the nucleon, which has been computed for both protons and neutrons. The starting point is the first moments of the structure functions

$$\Gamma_1^i(Q_0^2) = \int_0^1 g_1^i(x,Q_0^2)\,dx, \quad i = p,d,n, \tag{1}$$

which are computed from the measured structure functions evaluated at a common value of the four-momentum transfer $Q_0^2 = 3\ \mathrm{GeV}^2$. The conventional way to do this evaluation is to assume that either the ratio g_1/F_1 or the spin asymmetries A_1 and A_2 are independent of Q^2. Both assumptions are supported by the data to very good accuracy from $Q^2 = 1$ GeV^2 up to $Q^2 = 50\ \mathrm{GeV}^2$ [7]. Fits to the experimental values of g_1/F_1 or A_1 with and without Q^2-dependent terms are very similar.

The results are best summarized in a tabular form, shown in Table I. The numbers in the table are the values of the sum rules, followed by their statistical and systematic uncertainties (where present). The second and fourth columns show the published values for the two assumptions about the Q^2 dependence of the spin structure functions g_1/F_1 or A_1 and A_2 independent of Q^2. The "Theory" values are evaluated using the QCD corrections of Ref. [9] with $\alpha_s = 0.35 \pm 0.05$

TABLE I. Summary of E143 sum rules.

Sum rule	E143 g_1/F_1	Theory	E143 alternate A_1, A_2
Proton Γ_1^p	$0.127 \pm 0.004 \pm 0.010$	0.160 ± 0.006	$0.121 \pm 0.004 \pm 0.010$
Deuteron Γ_1^d	$0.042 \pm 0.003 \pm 0.004$	0.068 ± 0.005	$0.040 \pm 0.003 \pm 0.004$
Neutron $2\Gamma_1^d/\gamma_D - \Gamma_1^p$	$-0.037 \pm 0.008 \pm 0.011$	-0.011 ± 0.009	$-0.035 \pm 0.008 \pm 0.011$
Bjorken $2(\Gamma_1^p - \Gamma_1^d/\gamma_D)$	$0.163 \pm 0.010 \pm 0.016$	0.171 ± 0.008	$0.156 \pm 0.010 \pm 0.016$

at $Q^2=3$ (GeV/c)^2 [10]. The deuteron sum rule is shown in the standard form as per-nucleon average. The $\gamma_D=0.925$ factor is a correction for the deuteron's virtual D-state probability $w_D=0.05\pm0.02$, which represents a reduction in the measured nucleon asymmetry.

One can see that the theoretical Ellis-Jaffe values [11] for the proton, deuteron, and neutron sum rules are in strong disagreement with the measured quantities, in particular the deuteron result which differs by more than four standard deviations. The Bjorken sum rule [12] seems to be satisfied, within errors, for the g_1/F_1 independent of the Q^2 assumption. The other assumption (A_1, A_2 independent of Q^2) is further from the theoretical value.

From the expression for g_1/F_1 in terms of A_1 and A_2 given below, we can see that only one assumption (or may be neither) is valid, since the kinematic factor $\gamma^2=Q^2/\nu^2$ (where $\nu=E-E'$ is the difference between the beam energy E and the scattered lepton energy E') introduces a dependence that affects either side:

$$\frac{g_1(x,Q^2)}{F_1(x,Q^2)}=\frac{A_1(x,Q^2)+\gamma(Q^2)A_2(x,Q^2)}{1+\gamma^2(Q^2)}. \tag{2}$$

Moreover, if the left-hand side is independent of Q^2, the Q^2 dependence of the combination $A_1+\gamma A_2$ is just $1+\gamma^2$.

There is a kinematic correspondence between both assumptions. $g_1(x_0,Q^2)$ evaluated at fixed Q_0^2 under the assumption of g_1/F_1 independent of Q^2 is related to $g_1'(x_0,Q^2)$ from the other assumption at the same Q_0^2 by

$$g_1/g_1'=\frac{[A_1(x_0)+\gamma(Q^2)A_2(x_0)][1+\gamma^2(Q_0^2)]}{[A_1(x_0)+\gamma(Q_0^2)A_2(x_0)][1+\gamma^2(Q^2)]}, \tag{3}$$

where Q^2 is the experimental data's value of the four-momentum transfer at x_0, Q_0^2 is the chosen fixed Q^2 (e.g., 3 GeV^2), and A_1 and A_2 are averaged at x_0 over the measured Q^2 range. Current experimental precision is insufficient to determine which assumption is more valid.

The next step in computing the quark contribution to the nucleon spin is to connect the sum rules with quark helicity distributions in the nucleon. The spin structure functions in the parton model are interpreted as helicity densities of the different quark flavors:

$$g_1(x)=\frac{1}{2}\sum_i e_i^2[q_i^{\uparrow}(x)-q_i^{\downarrow}(x)], \quad i=u,\overline{u},d,\overline{d},s,\overline{s},\ldots . \tag{4}$$

Then, the integral of the structure function is

$$\Gamma_1^p=\frac{1}{2}\sum_i e_i^2\int_0^1[q_i^{\uparrow}(x)-q_i^{\downarrow}(x)]dx=\frac{1}{2}\sum_i e_i^2\Delta q_i. \tag{5}$$

For the proton, for three quark flavors, one has

$$\Gamma_1^p=\frac{1}{2}\left(\frac{4}{9}\Delta u+\frac{1}{9}\Delta d+\frac{1}{9}\Delta s\right). \tag{6}$$

These relations hold true in the asymptotic limit, where the structure functions depend only on x. At finite Q^2, they are modified by QCD corrections, as mentioned above. Kodaira [13] derived the expression

$$\Gamma_1^p=\frac{1}{36}[3E_3a_3+E_8a_8+4E_0a_0], \tag{7}$$

where E_i are coefficient functions representing QCD corrections for the finite experimental Q^2. a_3 and a_8 are the nonsinglet and a_0 is the singlet quark axial-vector current matrix elements. The corresponding expressions for the neutron, the deuteron, and the Bjorken sum rule are

$$\Gamma_1^n=\frac{1}{36}[-3E_3a_3+E_8a_8+4E_0a_0],$$

$$\Gamma_1^d=\frac{1}{36}[E_8a_8+4E_0a_0]\gamma_D, \tag{8}$$

$$\Gamma_1^p-\Gamma_1^n=\frac{6E_3a_3}{36}=\frac{g_A}{6}E_3,$$

the last one being the well-known form of this sum rule, with QCD corrections.

The matrix elements can be expressed in terms of the quark helicity densities for each quark flavor:

$$a_0=\Delta u+\Delta d+\Delta s=\Sigma q,$$

$$a_3=\Delta u-\Delta d=F+D, \tag{9}$$

$$a_8=\Delta u+\Delta d-2\Delta s=3F-D.$$

One sees that $a_0=\Sigma q$, i.e., the quark spin. Also, the nonsinglet matrix element are related to the weak decays of the baryon octet. The flavor SU(3) parameters F and D relate the axial vector coupling constants of those decays, $F+D=g_A$ for neutron decay, and $3F-D$ can be computed from the hyperons' decay constants. The latest values [14] $F=0.459\pm0.008$ and $D=0.798\pm0.008$ were used.

The QCD corrections depend on the running strong coupling constant $\alpha_s(Q^2)$ and have been estimated for three flavors to the fourth order in α_s [15]:

$$E_0=(1-0.333a-0.550a^2-2a^3),$$

$$E_3=E_8=(1-a-3.583a^2-20.215a^3-130a^4), \tag{10}$$

where $a=\alpha_s(Q_0^2=3\ \mathrm{GeV}^2)/\pi$. The importance of applying these corrections to the experimental results at finite Q^2 was pointed out by Ellis and Karliner [16].

It is a straightforward matter then to solve for a_0, in terms of Γ_1, $a_3=g_A$, and $a_8=3F-D$. The QCD corrections ensure that all quantities are evaluated at the same scale as the experiment. The results are again best displayed in Table II, including the neutron values for the quark helicities, which are reported here for the first time in print, in terms of equivalent proton quark densities.

The uncertainties are combined statistical and systematic contributions. However, we must point out that the value and error on Δs are derived only from the values and experimen-

TABLE II. Summary of quark helicity components.

	Proton	Deuteron	Neutron
Δu	0.81 ± 0.04	0.83 ± 0.02	0.83 ± 0.04
Δd	-0.44 ± 0.04	-0.43 ± 0.02	-0.42 ± 0.04
Δs	-0.10 ± 0.04	-0.09 ± 0.02	-0.09 ± 0.04
Σq	0.27 ± 0.10	0.30 ± 0.06	0.32 ± 0.12

tal errors quoted on Γ_1^p, Γ_1^d, and F and D extracted from hyperon decays. For the latter, perfect SU(3)$_{\text{flavor}}$ symmetry is assumed, and symmetry-breaking or model-dependent uncertainties are not included. The deuteron measurement yields the most precise result.

It is clear from Table II that at most 36% of the proton helicity can be attributed to the quarks. The current explanation is that the remainder is carried by the gluons. Also, the valence quark contribution is suppressed if one assumes that the sea quark is polarized for all flavors at the same level as the strange sea, $\Delta s\approx-0.1$. Writing $\Delta q=\Delta q^{\text{valence}}+\Delta q^{\text{sea}}$ for u and d quarks, we get $\Delta u^v=0.83–(-0.1)\approx0.93$ $\Delta d^v=-0.43–(-0.1)\approx-0.33$, and $\Sigma q\approx0.62$, which agrees with the value expected from the relativistic constituent quark model [17]. This result is obtained by computing

$$a_0=\Delta u+\Delta d=\Sigma q=a_8=3F-D, \tag{11}$$

where the equality between a_0 and a_8 is based on $\Delta s=0$. Experimental evidence indicating that the strange sea is about half of the nonstrange sea [18] constrains the positivity bound on Δs (pointed out by Preparata and Soffer [19] back in 1988) to [20]

$$\left|\int_0^1\Delta s(x)dx\right|\leqslant0.07\pm0.03. \tag{12}$$

This bound poses an independent experimental constraint on the magnitude of a nonzero Δs.

If we assume $\Delta s=0$, the proton sum rule (ignoring QCD corrections) simplifies to $\Gamma_1^p=(4\Delta u+\Delta d)/18$, and since we have measured the Bjorken sum rule $\Gamma_1^p-\Gamma_1^n=(\Delta u-\Delta d)/6$, we can easily solve for Δu, Δd, and $\Sigma q=\Delta u+\Delta d$. A more accurate procedure would include QCD corrections, in which case the resulting quark helicities are given by

$$\Delta u=\frac{6\Gamma_1^p}{E_8}+\frac{12(E_8-2E_0)\Gamma_1^d}{E_8(E_8+4E_0)\gamma_D}=0.78\pm0.08,$$

$$\Delta d=-\frac{6\Gamma_1^p}{E_8}+\frac{24(E_8+E_0)\Gamma_1^d}{E_8(E_8+4E_0)\gamma_D}=-0.43\pm0.10, \tag{13}$$

$$\Sigma q=\frac{36\Gamma_1^d}{(E_8+4E_0)\gamma_D}=0.35\pm0.04.$$

Again, the deuteron sum rule result yields the best value for Σq. For the preceding calculation we have used as inputs the values shown under the g_1/F_1 column heading in Table I. (The result is rather insensitive to the choice of using the A_1, A_2 values.) Since no input other than the spin structure integrals is involved, the solution indicates that it is unnecessary to invoke a polarized strange sea contribution. If the strange sea is not polarized and we assume isospin symmetry holds, a more accurate value for the quark helicities can be obtained from

$$\Delta u-\Delta d=g_A=F+D=1.2573\pm0.0028,$$

$$\Delta u+\Delta d=\Sigma q=\frac{36\Gamma_1^d}{(E_8+4E_0)\gamma_D}=0.354\pm0.039. \tag{14}$$

This is a very simple system which involves no input other than the experimentally measured g_A and Γ_1^d, and no assumptions about SU(3) flavor symmetry. We obtain $\Delta u=0.804\pm0.020$ and $\Delta d=-0.453\pm0.020$. The small errors come from the form of $\Delta u(\Delta d)=[(-)g_A+\Sigma q]/2$.

If we decompose the experimental quark helicities into valence and sea contributions as we did earlier in the paper, we can estimate the sea contributions. For the valence contributions we take, without any further assumptions, the naive quark model (NQM) predictions $\Delta u^v=4/3$ and $\Delta d^v=-1/3$. The solutions are

$$\Delta u^s=-0.529,$$

$$\Delta d^s=-0.120. \tag{15}$$

Both sea contributions are negative, which is consistent with all flavors in the sea being polarized by the same mechanism. We notice that the ratio $\Delta u^s/\Delta d^s=4.5\pm0.1$. This type of decomposition is not entirely valid if one assumes $\Delta s\neq0$. A significantly different $\Delta u^s/\Delta d^s=5.2\pm0.2$ is obtained using Table II.

We can turn the analysis around and try to estimate what is the amount of symmetry breaking that $\Delta s=0$ represents. The procedure is straightforward: We solve the expressions for a_3 and a_8 [Eq. (9)] for F and D, using our results for Δu and Δd [Eq. (13)]. We find

$$F=0.389\pm0.044,$$

$$D=0.814\pm0.146,$$

$$F+D=1.203\pm0.152, \tag{16}$$

$$F/D=0.477\pm0.037.$$

The errors include the contributions of both Γ_1^p and Γ_1^d and the errors in the QCD coefficient functions E_0 and E_8, which come from the uncertainty in α_s. The result for $F+D$ is less than one standard deviation smaller than the accepted value for this quantity. This important result shows that the experimental verification of the Bjorken sum rule needs no additional inputs, such as assumptions about the sea quark polarization.

Using the improved combined systematic and statistical errors on $\Gamma_1^p=0.125\pm0.008$ of Ref. [21], we obtain $F=0.380\pm0.035$, $D=0.784\pm0.121$, $F+D=1.164\pm0.13$, and $F/D=0.484\pm0.036$, in good agreement with the preceding values.

Combining our extracted F and D to compare with the measured values of the hyperons axial to vector current ra-

TABLE III. Hyperon decay axial vector constants.

Hyperon decays		Extracted F, D	Experiment
$F+D$	$n\to p$	$1.203+/-0.152$	$1.2753+/-0.003$
$F+D/3$	$\Lambda\to p$	$0.660+/-0.066$	$0.718+/-0.015$
$-F+D$*	$\Sigma^-\to n$	$0.426+/-0.152$	$0.340+/-0.017$
$-F+D/3$ [a]	$\Xi^-\to\Lambda$	$-0.117+/-0.066$	$-0.250+/-0.050$

[a]We use the PDG sign convention [22].

tios g_A/g_V, we find a general agreement with the measured values [22], as shown in Table III.

The χ^2 probability for the four cases is 0.42, and for the first three cases, 0.73. Obviously we do not claim that we are extracting the hyperon g_A/g_V ratios from the spin structure sum rules, only that the $\Delta s=0$ assumption is indeed not inconsistent with any experimental data, at the current level of precision of the spin structure data. With the F and D values obtained using the proton integral of Ref. [21], the χ^2 probability for the four cases is 0.18, and 0.39 for the first three cases.

In a recent work, Ehrnsperger and Schäefer [23] argue that, in fact, flavor SU(3)-symmetry-breaking effects can be of significant size, reducing the magnitude of the ratio F/D. In their model of $\mathrm{SU}(3)_{\mathrm{flavor}}$ breaking, this ratio is found to be $F/D=0.492\pm0.083$, a value that agrees with our result above. (We should mention that a weighted fit of the F/D ratios computed in Ref. [23] yields $F/D_{n\to p}=0.496$.) This result gives further indication that the nucleon spin structure measurements are not sufficiently precise to lead to the conclusion that the strange sea in the nucleon is polarized.

On a closely related aspect of the proton's spin, the contribution of non-SU(6) configurations in the proton wave function, which Lipkin [24] argued would be required to explain the difference between the NQM value of $g_A=5/3=1.66$ and the experimental value, is substantially reduced if we use our $\Delta s=0$ result for $\Sigma q=0.354\pm0.039$, instead of the old European Muon Collaboration (EMC) result $\Sigma q\simeq 0$ [25] that Lipkin and others [20] have used. The contribution of non-SU(6) states to the proton spin is given by

$$(S_z)_\psi=\frac{(1-\Sigma q)}{2\sin^2\theta}-\frac{1}{2}=\frac{0.32}{\sin^2\theta}-0.5. \qquad (17)$$

$\sin^2\theta\leqslant 3/16$ is the mixing angle between the SU(6) and non-SU(6) components in the proton wave function. Taking θ at its maximum one has $(S_z)_\psi=1.22\pm0.10$, which is not inconsistent with orbital angular momentum of the partons contributing to the proton spin.

In summary, there are numerous indications that a properly understood application of the constituent quark model may yet explain most of the features of the nucleon spin.

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