Isospin corrections to charmless semileptonic $B\to V$ transitions

J. L. Díaz-Cruz
Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, 72500 Puebla, Puebla, Mexico

G. López Castro
Departamento de Física, Cinvestav del IPN, Apartado Postal 14-740, 07000 México, D.F. Mexico

J. H. Muñoz
Departamento de Física, Cinvestav del IPN, Apartado Postal 14-740, 07000 México, D.F. Mexico and Departamento de Física, Universidad del Tolima, A. A. 546, Ibagué, Colombia

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We compute isospin corrections to the charmless semileptonic $B\to V$ transitions arising from ρ-ω mixing and discuss its relevance in the determination of V_{ub}. [S0556-2821(96)02715-4]

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The first measurement of an exclusive charmless semileptonic B decay has been reported recently by the CLEO Collaboration [1,4]. The yields for $B^+\to\pi^0 l^+\nu_l$ reported in Refs. [1,4] turn out to be strongly dependent on the theoretical models [2,3] used for the detection efficiencies, as it does the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) parameter $|V_{ub}|$ from these results.

The $|V_{ub}|$ parameter can be determined from exclusive charmless semileptonic B decays and also from the end-point region of the lepton spectrum in inclusive semileptonic B decays. Although, one does not expect that exclusive measurements will provide a better determination of $|V_{ub}|$ than what the inclusive measurements do [4], the former are important for several reasons. In addition to the complementary determination of $|V_{ub}|$ provided by exclusive B decays, these measurements will allow to test available theoretical models for form factors (relativistic [2] and nonrelativistic [3] quark models, and QCD sum rules [5]). In this paper we are concerned with important isospin-breaking corrections to $B\to Vl\nu (V=\rho^+,\ \rho^0,$ and $\omega)$ decays which have not been considered in previous analyses.

There are several motivations to consider all sources of theoretical corrections in the calculation of charmless $B\to V$ semileptonic decays. From the experimental point of view, the study of these decays can sensibly be improved at the B factory. In fact, actual measurements by the CLEO Collaboration [6] provide the upper limit $B(B^-\to\rho^0 l^-\overline{\nu_l})<(1.6-2.7)\times 10^{-4}$, which lies at the verge of theoretical predictions.

Among the theoretical motivations we can mention the following: (a) the decay rate of $B\to\rho l\nu$ provides 3–14% [2,3,5] of the inclusive $B\to X_u l\nu_l$ decays, (b) the ratio of decay widths for $(B\to Vl\nu)/(B\to\pi l\nu)$ and the polarization of the daughter vector mesons can be used to discriminate between form factor models and, (c) recent proposals [7] indicate that we could achieve a model-independent determination of $|V_{ub}|$ from $B\to\rho l\nu$, at the level of 10%, by using heavy quark effective theory (HQET) techniques and SU(3) flavor symmetry.

Let us now discuss the relevance of isospin-breaking corrections to semileptonic B decays. The experimental measurements reported by the CLEO Collaboration on $B\to\pi l\nu$ [1,4] and the upper limit set on $B\to\rho l\nu$ [6] rely on the assumptions

$$\Gamma(\overline{B^0}\to\rho^+ l^-\overline{\nu})=2\Gamma(B^-\to\rho^0 l^-\overline{\nu}) \approx 2\Gamma(B^-\to\omega l^-\overline{\nu}) \qquad (1)$$

and

$$\Gamma(\overline{B^0}\to\pi^+ l^-\overline{\nu})=2\Gamma(B^-\to\pi^0 l^-\overline{\nu}). \qquad (2)$$

The first rows in Eqs. (1) and (2) are valid in the limit of exact isospin symmetry, while the second equality in Eq. (1) follows from the $u\overline{u}$ content of ρ^0 and ω mesons in the limit of exact isospin symmetry. Corrections to these relations arise from electromagnetic radiative corrections and from the $u-d$ quark mass difference. Electromagnetic radiative corrections [8] to $B\to\rho l\nu$ and phase space corrections because of the physical masses of the mesons are expected to be negligible because of the large B-meson mass. Isospin corrections to Eq. (1) induced by ρ-ω mixing (which arise from the $u-d$ quark mass difference) are the subject of this paper. This correction turns out to be large because of the small difference in the vector-meson masses $(m_\omega-m_\rho\approx 12$ MeV) and the large difference in their decay widths $(\Gamma_\omega\ll\Gamma_\rho)$.[1]

Let us proceed with our calculation. In the limit of isospin symmetry, ρ^0 and ω are isospin eigenstates with flavor content: $\rho^I=(u\overline{u}-d\overline{d})/\sqrt{2}$ and $\omega^I=(u\overline{u}+d\overline{d})/\sqrt{2}$. Since the spectator quark in $B^-\to(\rho^0,\omega)l^-\overline{\nu_l}$ is $\overline{u}$, the ρ^0 and ω mesons are produced from their $u\overline{u}$ quark content. This provides the equality between hadronic matrix elements

$$\langle\rho^I|\overline{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle=\langle\omega^I|\overline{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle \qquad (3)$$

[1]Isospin corrections to Eq. (2), arising from π^0-η mixing, are negligible [9].

which leads to the second row in Eq. (1).

When we introduce isospin breaking, ρ^I and ω^I get mixed into physical states ρ^0 and ω: namely,

$$\omega=\omega^I-\epsilon'\rho^I=\frac{1}{\sqrt{2}}(1-\epsilon')u\overline{u}+\frac{1}{\sqrt{2}}(1+\epsilon')d\overline{d}, \quad (4)$$

$$\rho^0=\rho^I+\epsilon\omega^I=\frac{1}{\sqrt{2}}(1+\epsilon)u\overline{u}+\frac{1}{\sqrt{2}}(-1+\epsilon)d\overline{d}, \quad (5)$$

where $\epsilon(\epsilon')$ are the contributions of ρ-ω mixing, with the expressions

$$\epsilon=\frac{m_{\rho\omega}^2}{m_\rho^2-m_\omega^2+im_\omega\Gamma_\omega}, \quad (6)$$

$$\epsilon'=\frac{m_{\rho\omega}^2}{m_\omega^2-m_\rho^2+im_\rho\Gamma_\rho}, \quad (7)$$

where $m_{\rho\omega}^2=(-3.67\pm0.30)\times10^{-3}\mathrm{GeV}^2$ [10] is the strength of ρ-ω mixing.

After including ρ-ω mixing, Eq. (3) becomes

$$\langle\rho^0|\overline{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle=\frac{1+\epsilon}{1-\epsilon'}\langle\omega|\overline{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle. \quad (8)$$

From this equation we can obtain the ratio for ρ^0 and ω production in B^- decays. If we use $m_\rho=757.5$ MeV and $\Gamma_\rho=142.5$ MeV as obtained from a recent fit to the pion form factor [10] and the $\omega(782)$ parameters from [11], we get

$$\frac{\Gamma(B^-\to\rho^0l^-\overline{\nu})}{\Gamma(B^-\to\omega l^-\overline{\nu})}=\left|\frac{1+\epsilon}{1-\epsilon'}\right|^2\approx1.172, \quad (9)$$

whereas the use of PDG values [11] for the ρ^0 parameters gives

$$\frac{\Gamma(B^-\to\rho^0l^-\overline{\nu})}{\Gamma(B^-\to\omega l^-\overline{\nu})}\approx1.367, \quad (10)$$

which looks like a rather large correction.

Note that the ratio $2\Gamma(B^-\to\rho^0l^-\overline{\nu_l})/\Gamma(\overline{B^0}\to\rho^+l^-\overline{\nu_l})$ is modified by almost the same amount as Eq. (9) does, since ρ-ω mixing affects only the neutral vector mesons. We would like to stress that $B\to(\rho,\omega)l\nu$ are affected by this correction regardless of the model used to describe the form factors of the $B\to V$ transition. Let us comment that the calculation of the corresponding form factors in Refs. [2] and [3] assume explicitly $m_u=m_d$. Note that the ratio $\Gamma(B^-\to\rho^0l^-\overline{\nu})/\Gamma(\overline{B^0}\to\rho^+l^-\overline{\nu})$ plays for B decays the same role as $\Gamma(K^+\to\pi^0l^+\nu)/\Gamma(K_L\to\pi^-l^+\nu)$ [9] does for K decays in order to test the flavor-symmetry-breaking corrections to form factors at zero momentum transfer (which are essential for the determination of V_{us}).

The individual decay rates for B^- semileptonic decays are affected by ρ-ω mixing as

$$\Gamma(B^-\to\rho^0l^-\overline{\nu_l})=|1+\epsilon|^2\Gamma^0(B^-\to\rho^0l^-\overline{\nu_l}), \quad (11)$$

$$\Gamma(B^-\to\omega l^-\overline{\nu_l})=|1-\epsilon'|^2\Gamma^0(B^-\to\omega l^-\overline{\nu_l}), \quad (12)$$

where Γ^0 denotes the decay rate without ρ-ω mixing. Since $|1+\epsilon|\approx1.095$ and $|1-\epsilon'|\approx1.011$ (1.18 and 1.005, respectively, if the ρ^0 parameters of [11] are used), the values of $|V_{ub}|$ as extracted from $B^-\to\rho^0l^-\overline{\nu}$ and $B\to\omega l^-\overline{\nu}$ would be, respectively, 10% and 1% (18% and 0.5%) higher if ρ-ω mixing were not included.

It is straightforward to extend this analysis to strangeness-conserving $D^+\to(\rho^0,\omega)$ semileptonic transitions. Since the $D^+\to(\rho^0,\omega)l^+\nu$ decays proceed through the elementary transition $c\to dl^+\nu$, the spectator $\overline{d}$ quark gets combined with the daugther d quark to produce the ρ^0 and ω mesons. Using the ρ^0 parameters from Ref. [10], we get, after including ρ-ω mixing, the ratio

$$\frac{\Gamma(D^+\to\rho^0l^+\nu)}{\Gamma(D^+\to\omega l^+\nu)}=\left|\frac{-1+\epsilon}{1+\epsilon'}\right|^2\approx0.837\ \ (0.692); \quad (13)$$

i.e., the correction because of ρ-ω mixing is similar as in B decays but it goes in the opposite direction (the number in brackets is obtained for the ρ,ω parameters of Ref. [11]). The corresponding experimental information [11] available for semileptonic D decays is not precise enough to allow a test of Eq. (13).

Finally, in order to trust our calculations, we can compute the ratio of decay widths for radiative decays of ρ mesons, namely, $R\equiv\Gamma(\rho^0\to\pi^0\gamma)/\Gamma(\rho^+\to\pi^+\gamma)$. In this case, the ρ^0 decay receives an additional contribution from ρ-ω mixing ($\rho^0\to\omega\to\pi^0\gamma$). The ratio R is modified to become [10]

$$R=\left|1+\frac{f_\omega}{f_\rho}\cdot\epsilon\right|^2\left|\frac{\vec{k}_{\pi^0}}{\vec{k}_{\pi^+}}\right|^3, \quad (14)$$

where $\vec{k}_\pi$ is the pion momentum in the ρ rest frame, and em_V^2/f_V defines the vector-meson–photon coupling.

Using $f_\rho=5.0$ and $f_\omega=17.0$ from $(\rho^0,\omega)\to e^+e^-$ decays, we obtain [10]

$$R=1.77\ \ (2.40), \quad (15)$$

where the number in brackets corresponds to the ρ^0 parameters of Ref. [11]. The above result is in good agreement with the experimental value $R^{\mathrm{expt}}=1.78\pm0.49$ [11].

In conclusion, the ρ-ω mixing induces a sizable correction to the isospin symmetry relations given in Eq. (1). This overall correction to $B\to(\rho,\omega)l^-\overline{\nu}$ is present regardless of the specific form factor model [2,3,5] used to describe the hadronic weak transition. The values of V_{ub} as extracted from $B^-\to(\rho^0,\omega)l^-\overline{\nu}$ would be overestimated by 10% and 1% if ρ-ω mixing is not included.

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