Symmetry breaking and generational mixing in top-color-assisted technicolor

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Top-color-assisted technicolor provides a dynanamical explanation for electroweak and flavor symmetry breaking and for the large mass of the top quark without unnatural fine-tuning. A major challenge is to generate the observed mixing between heavy and light generations while breaking the strong top-color interactions near 1 TeV. I argue that these phenomena, as well as electroweak symmetry breaking, are intimately connected and I present a scenario for them based on nontrivial patterns of technifermion condensation. I also exhibit a class of models realizing this scenario. This picture leads to a rich phenomenology, especially in hadron and lepton collider experiments in the few hundred GeV to few TeV region and in precision electroweak tests at the Z^0, atomic parity violation, and polarized Møller scattering. [S0556-2821(96)05813-4]

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I. INTRODUCTION

Top-color-assisted technicolor (TC2) was proposed by Hill [1] to overcome major shortcomings of top-quark-condensate models of electroweak symmetry breaking [2,3] and of technicolor models of dynamical electroweak and flavor symmetry breaking [4,5]. Technicolor and extended technicolor (ETC) have been unable to provide a natural and plausible understanding of why the top quark mass is so large [6]. On the other hand, models in which strong top-color interactions drive top-quark condensation and electroweak symmetry breaking are unnatural. To reproduce the one-Higgs-doublet standard model consistent with precision electroweak measurements (especially of the parameter $\rho = M_W^2/M_Z^2\cos^2\theta_W \simeq 1$), the top-color energy scale must be much greater than the electroweak scale of ~ 1 TeV. This requires severe fine-tuning of the top-color coupling.

Hill's combination of top-color and technicolor keeps the best of both schemes. In TC2, technicolor interactions at the scale $\Lambda_{\rm TC} \simeq \Lambda_{\rm EW} \simeq 1$ TeV are mainly responsible for electroweak symmetry breaking. Extended technicolor is still required for the hard masses of all quarks and leptons except the top quark. Top color produces a large top condensate, $\langle \bar{t}t \rangle$, and all but a few GeV of[1] $m_t \simeq 175$ GeV. However, it contributes comparatively little to electroweak symmetry breaking. Thus, the topcolor scale can be lowered to near 1 TeV and the interaction requires little or no fine-tuning.

In the simplest example of Hill's TC2, there are separate color and weak hypercharge gauge groups for the heavy third generation of quarks and leptons and for the two light generations. The third generation transforms under strongly coupled $\mathrm{SU}(3)_1 \otimes \mathrm{U}(1)_1$ with the usual charges, while the light generations transform conventionally under weakly coupled $\mathrm{SU}(3)_2 \otimes \mathrm{U}(1)_2$. Near 1 TeV, these four groups are broken to the diagonal subgroup of ordinary color and hypercharge, $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y$. The desired pattern of condensation occurs because $\mathrm{U}(1)_1$ couplings are such that the spontaneously broken $\mathrm{SU}(3)_1 \otimes \mathrm{U}(1)_1$ interactions are supercritical only for the top quark.

Two important constraints were imposed on TC2 soon after Hill's proposal was made. The first is due to Chivukula, Dobrescu, and Terning (CDT) [7] who claimed that the technifermions required to break top and bottom quark chiral symmetries are likely to have custodial-isospin violating couplings to the strong $\mathrm{U}(1)_1$. To keep $\rho \simeq 1$, they argued, the $\mathrm{U}(1)_1$ interaction must be so weak that it is necessary to fine-tune the $\mathrm{SU}(3)_1$ coupling to within 1% of its critical value for top condensation and to increase the top-color boson mass above 4.5 TeV. Thus, TC2 still seemed to be unnatural. CDT stated that their bounds could be relaxed if $\mathrm{U}(1)_1$ couplings did not violate isospin. However, they expected that this would be difficult to implement because of the requirements of canceling gauge anomalies and of allowing mixing between the third and first two generations.

The second constraint on TC2 is due to Kominis [8] who showed, presuming that the b-quark's top-color interactions are not far from critical, the existence of relatively light scalar bound states of $t_L b_R$ and $b_L b_R$ that couple strongly ($\propto m_t$) to third generation quarks. These scalars can induce excessive B_d-B_d mixing which is proportional to the product $D^d_{Lbd} D^d_{Rbd}$ of the elements of the unitary matrices which diagonalize the (generally non-Hermitian) $Q = -\frac{1}{3}$ quark mass matrix.

The question of isospin violation and naturalness raised by CDT was addressed in Ref. [9]. We proposed that different technifermion isodoublets, T^t and T^b, give ETC mass to the top and bottom quarks. These doublets then could have different $\mathrm{U}(1)_1$ charges which were, however, isospin conserving for the right as well as left-handed parts of each doublet.[2] In addition, we exhibited a TC2 prototype in which (i) all gauge anomalies cancel; (ii) there are no very light

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[1] A small part of m_t must be generated by ETC to give mass to the Goldstone bosons-top-pions-associated with top condensation. Hill has pointed out that some, perhaps all, of the bottom quark mass may arise from $\mathrm{SU}(3)_1$ instantons [1].

[2] While this eliminates the large $\rho - 1$ discussed by CDT, there remain small, $O(\alpha)$, contributions from the $\mathrm{U}(1)_1$ interaction.

pseudo Goldstone bosons (loosely speaking, ''axions'') because all spontaneously broken global technifermion symmetries are broken explicitly by ETC [10]; and (iii) a mechanism exists for mixing the heavy and light generations.

Although the problem of B_d-B_d mixing raised by Kominis was not considered in [9], the U(1) symmetries of the model presented there automatically allow just one of two ETC-induced transitions in the quark mass matrix: $d_L, s_L \leftrightarrow b_R$ or $d_R, s_R \leftrightarrow b_L$. Thus, only D^d_{Lbd} or D^d_{Rbd}, respectively, can be sizable and the B_d-B_d constraint is satisfied. It is easy to see that the phenomenologically preferred transition is $d_L, s_L \leftrightarrow b_R$: The known mixings between the third and the first two generations are in the Kobayashi-Maskawa matrix for left-handed quarks, $V = D^{u\dagger}_L D^d_L$. They are $|V_{cb}| \simeq |V_{ts}| \simeq 0.03-0.05 \sim m_s/m_b$ and $|V_{ub}| \simeq |V_{td}| \simeq 0.002-0.015 \sim \sin\theta_C m_s/m_b$ [11]. These elements must arise from D^d_L, hence from the $d_L, s_L \leftrightarrow b_R$ transitions, because the corresponding elements in D^u_L are smaller by a factor of $m_b/m_t \simeq 0.03$.

In the model of Ref. [9], the mechanism of top-color breaking was left unspecified and all technifermions were taken to be $SU(3)_1 \otimes SU(3)_2$ singlets. Thus, the transition $d_L, s_L \leftrightarrow b_R$ had to be generated by an externally induced term $\delta M_{\rm ETC}$ in the ETC mass matrix which transforms as (3,3) under the color groups. We then estimated

$$|V_{cb}| \simeq |D^d_{Lsb}| \simeq \frac{\delta m_{sb}}{m_b} \lesssim \frac{\delta m_{sb}}{m_b^{\rm ETC}} \simeq \frac{\delta M^2_{\rm ETC}}{M_s^2}, \qquad (1.1)$$

where δm_{sb} is the mixing term in the $Q = -\frac{1}{3}$ mass matrix, m_b is the mass of the b quark, and M_s is the mass of the ETC boson that generates the strange-quark mass, m_s. In a walking technicolor theory [12], $M_s \gtrsim 100$ TeV. However, we expect $\delta M_{\rm ETC} \simeq 1$ TeV because that is the scale at which top-color breaking naturally occurs. This gives s-b mixing that is at least 300 times too small. We stated in [9] that providing mixing of the observed size between the heavy and light generations is one of the great challenges to top-color-assisted technicolor.

This problem is addressed in the rest of this paper. I shall argue that generational mixing is intimately connected to top-color and electroweak symmetry breaking and that all these phenomena occur through technifermion condensation. In Secs. II–IV, I specify the gauge groups and describe the patterns of gauge symmetry breaking needed for standard model phenomenology. Nontrivial patterns of vacuum alignment play a central role in this. In Sec. V, I present a class of models which illustrate this scenario. The phenomenology of these models is sketched in Sec. VI. Special attention is placed on the Z' boson of the broken $U(1)_1$ symmetry. Its effects may be noticable in hadron collider production of jets and dileptons, e^+e^- collisions, atomic parity violation, polarized Møller scattering, and other precision electroweak measurements. I also emphasize observational consequences of vacuum alignment, especially technirho vector mesons and their decay to pairs of technipions and, possibly, CP violation.

II. GAUGE GROUPS

The gauge groups of immediate interest to us are

$$SU(N) \otimes SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2 \otimes SU(2), \qquad (2.1)$$

where for definiteness, I have assumed that the technicolor gauge group is $SU(N)$. To avoid light ''axions,'' all of these groups [except for the electroweak SU(2) and, possibly, parts of the U(1)'s] must be embedded in an extended technicolor group, $G_{\rm ETC}$. I will not specify $G_{\rm ETC}$. This difficult problem is reserved for the future. However, as in Ref. [9], I shall assume the existence of ETC-induced four-fermion operators which are needed to break quark, lepton, and technifermion chiral symmetries. Of course, these operators must be invariant under the groups in Eq. (2.1).

The coupling constants of $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ are denoted by g_1, g_2, g_1', g_2', where, nominally, the couplings satisfy $g_1 \gg g_2$ and $g_1' \gg g_2'$. When these gauge symmetries break, $SU(3)_1 \otimes SU(3)_2 \to SU(3)_C$ and $U(1)_1 \otimes U(1)_2 \to U(1)_Y$. We shall see that the breaking to $U(1)_Y$ must occur at an energy higher than the $SU(2) \otimes U(1)_Y$ breaking scale $\Lambda_{\rm EW}$. Then, the usual color and weak hypercharge couplings are

$$g_S = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \simeq g_2, \quad g' = \frac{g_1' g_2'}{\sqrt{g_1'^2 + g_2'^2}} \simeq g_2'. \qquad (2.2)$$

These symmetry breakings give rise to eight color-octet ''coloron'' (V_8) vector bosons and one neutral Z', all of which have mass of ~ 1 TeV [13,1].

Third-generation quarks $q^h = (t,b)$ will transform as (3,1) under $SU(3)_1 \otimes SU(3)_2$, while the first two generation quarks $q^l = (u,d)$, (c,s) transform as (1,3). Unlike the situation in the simple models of Refs. [1] and [9], we shall assume that *all* quarks and leptons carry both $U(1)_1$ and $U(1)_2$ charges. These hypercharge assignments must be such that the gauge interactions are supercritical only for the top quark. This new situation has important phenomenological consequences, outlined in Sec. VI.

III. $U(1)_1 \otimes U(1)_2$ BREAKING

In the scenario I describe, the extra Z' resulting from $U(1)_1 \otimes U(1)_2$ breaking has a mass of at most a few TeV and couples strongly to light, as well as heavy, quarks and leptons. Then, two conditions are necessary to prevent conflict with neutral current experiments. First, there must be a Z^0 boson with standard electroweak couplings to all quarks and leptons. To arrange this, there will be a hierarchy of symmetry breaking scales, with $U(1)_1 \otimes U(1)_2 \to U(1)_Y$ at 1–2 TeV, followed by $SU(2) \otimes U(1)_Y \to U(1)_{\rm EM}$ at the lower scale $\Lambda_{\rm EW}$. Assuming that technicolor interactions induce both symmetry breakdowns, the technifermions responsible for $U(1)_1 \otimes U(1)_2 \to U(1)_Y$—call them ψ_L and ψ_R—must belong to a *vectorial* representation of SU(2). To simplify the analysis, I make the minimal assumption that the $\psi_{L,R}$ are electrically neutral SU(2) singlets.

To produce this hierarchy of symmetry breaking scales, and yet maintain an asymptotically free technicolor, the $\psi_{L,R}$ should belong to a *higher-dimensional* representation of

SU(N), while the technifermions responsible for $SU(2)\otimes U(1)_Y$ breaking must belong to fundamental representations. This is reminiscent of multiscale technicolor [14], but there both the higher and fundamental representations participate in electroweak symmetry breaking. In the present model, I shall assume that $\psi_{L,R}$ belong to the $\frac{1}{2}N(N-1)$-dimensional antisymmetric tensor representation. I assume that this set of technifermions is large enough to ensure that the technicolor coupling ''walks'' for a large range of momenta [12].

The second constraint is that the Z' should not induce large flavor-changing interactions. This can be achieved if the $U(1)_1$ couplings of the two light generations are GIM symmetric. Then flavor-changing effects will nominally be of order $|V_{ub}|^2/M_{Z'}^2$ for $\Delta B_d=2$ processes, $|V_{cb}|^2/M_{Z'}^2$ for $\Delta B_s=2$, and negligibly small for $\Delta S=2$. These should be within experimental limits.[3] Nevertheless, a variety of interesting, and potentially dangerous, Z' phenomena are expected. These are discussed in Sec. VI.

IV. $SU(3)_1\otimes SU(3)_2$ AND ELECTROWEAK BREAKING AND GENERATIONAL MIXING

Turn now to symmetry breaking at lower energy scales. I recounted above that s-b mixing is too small by a factor of 300 if $SU(3)_1\otimes SU(3)_2$ breaking is introduced to the quark sector only by a mixing term in the ETC boson mass matrix. Since b_R transforms as $(3,1,1;-\frac{1}{3})$ under $SU(3)_1\otimes SU(3)_2\otimes SU(2)\otimes U(1)_Y$ and d_L, s_L as $(1,3,2;\frac{1}{6})$, it is tempting to suppose that the mechanism connecting d_L, s_L to b_R is at the same time responsible for breaking $SU(3)_1\otimes SU(3)_2\to SU(3)_C$ *and* $SU(2)\otimes U(1)_Y\to U(1)_{\rm EM}$. The generational mixing term transforms as (3,3) under the color groups. Therefore, I introduce colored technifermion isodoublets transforming under $SU(N)\otimes SU(3)_1\otimes SU(3)_2\otimes SU(2)$ as

$$T^1_{L(R)}=\begin{pmatrix}U^1\\D^1\end{pmatrix}_{L(R)}\in(N,3,1,2(1))$$

$$T^2_{L(R)}=\begin{pmatrix}U^2\\D^2\end{pmatrix}_{L(R)}\in(N,1,3,2(1)). \tag{4.1}$$

The transition d_L, $s_L\leftrightarrow D^2_L\leftrightarrow D^1_R\leftrightarrow b_R$ occurs if the appropriate ETC operator exists *and* if the condensate $\langle T^1_L T^2_R\rangle$ forms.

The patterns of condensation, $\langle\overline{T^i_L}T^j_R\rangle$, that occur depend on the strength of the interactions driving them and on explicit chiral symmetry breaking ($4T$) interactions that determine the correct chiral-perturbative ground state, i.e., ''align the vacuum'' [15]. The strong interactions driving technifermion condensation are SU(N), $SU(3)_1$, and $U(1)_1$. The technicolor interactions do not prefer any particular form for $\langle\overline{T^i_L}T^j_R\rangle$; $SU(3)_1$ drives $\langle T^1_L T^1_R\rangle\neq 0$; $U(1)_1$ drives $\langle T^1_L T^1_R\rangle$, $\langle T^2_L T^2_R\rangle\neq 0$ *or* $\langle T^1_L T^2_R\rangle\neq 0$, depending on the strong hypercharge assignments.

In the approximation that technicolor interactions dominate condensate formation, so that

$$\langle\overline{T^i_L}T^j_R\rangle=-\tfrac{1}{2}\Delta_T U_{ij}\quad(i,j=1,2), \tag{4.2}$$

it is easy to prove the following: If $T^1\in(3,1)$ and $T^2\in(1,3)$ are the only technifermions and if the vacuum-aligning interactions are $SU(3)_1\otimes SU(3)_2$ symmetric then, in each charge sector, the unitary matrix $U_{ij}=\delta_{ij}$ or $U_{ij}=(i\sigma_2)_{ij}$, but *not* a nontrivial combination of the two. Therefore, in order that $SU(3)_1\otimes SU(3)_2$ invariant direct mass terms, d_L, $s_L\leftrightarrow d_R$, s_R, and $b_L\leftrightarrow b_R$, occur as well as the mixing d_L, $s_L\leftrightarrow b_R$, it is necessary to introduce still other technifermions. The least number of additional technifermions involves $SU(3)_1\otimes SU(3)_2$ singlets. In the model described below, these will consist of three isodoublets: T^l giving direct mass terms to the light quarks and leptons; T^t giving the top quark its ETC mass; and T^b giving the bottom quark its ETC mass. These are the same technifermions used in the model of Ref. [9]. Introducing them enlarges the chiral symmetry—and the number of Goldstone bosons—of the model. Giving mass to all these bosons will require, among other things, a nontrivial pattern of T^1-T^2 condensation, $U=a_0 1+ia_2\sigma_2$. This simultaneously breaks the color and electroweak symmetries to $SU(3)_C\otimes U(1)_{\rm EM}$ and provides large generational mixing, e.g., $\delta m_{sb}\sim\langle T^1T^2\rangle_{Ms}/M_s^2\sim m_s$. The color-singlet technifermions help align the vacuum in this nontrivial way as well as contribute to electroweak symmetry breaking.

V. A MODEL

In this section I follow the format of Ref. [9] to construct a TC2 model with the symmetry breaking just outlined. First, I list hypercharge assignments for all the fermions and explain certain general constraints on them. Then I derive a condition on the hypercharges that must be satisfied in order that colored technifermions condense to break top-color SU(3). I conclude by discussing other conditions available to fix the hypercharges. Among these are the gauge anomaly constraints, given in the Appendix. The rest follow from specifying the ETC four-fermion operators necessary to give masses to quarks and leptons and to the Goldstone bosons associated with global symmetries. A family of solutions for the hypercharges satisfying all these constraints is obtained in the Appendix.

The fermions in the model, their color representations and U(1) charges are listed in Table I. A number of choices have been made at the outset to limit and simplify the charges and to achieve the scenario's objective.

(1) In order that technifermion condensates conserve electric charge, $u_1+u_2=v_1+v_2$, $x_1+x_2=x_1'+x_2'$, $y_1+y_2=y_1'+y_2'$, and $z_1+z_2=z_1'+z_2'$.

(2) The $U(1)_1$ charges of technifermions respect custodial isospin.

(3) The most important choice for our scenario is that of the $U(1)_1$ charges of T^1 and T^2. So long as $u_1\neq v_1$, the broken $U(1)_1$ interactions favor condensation of T^1 with T^2. If this interaction is stronger than the $SU(3)_1$ attraction for T^1 with itself and if we neglect other vacuum-aligning ETC interactions, then $\langle T^i_L T^j_R\rangle\propto(i\sigma_2)_{ij}$ in each charge sector. This alignment is discussed below.

[3]The most stringent constraint may come from $\Delta M_{B_d}/M_{B_d}$. In the model of Sec. V, this ratio depends in a complicated way on the $U(1)_1$ hypercharges b, b', d, d' and the magnitudes and phases of V_{ub} and V_{td}.

(4) We shall see that $u_1 \neq v_1$ implies $Y_{1i} \neq Y'_{1i}$ for the various fermions. Purely for simplicity, I have chosen $Y_1 = b'$ for all right-handed light quarks. I must choose $Y_1(t_R) \neq Y_1(b_R)$ to prevent strong b condensation. Again for simplicity, I put $Y_1(t_R) = -Y_1(b_R) = d'$. We shall see that dd' is positive, as it must be for t condensation.

(5) For the SU(N) antisymmetric tensor ψ, $\xi' \neq \xi$ guarantees $U(1)_1 \otimes U(1)_2 \to U(1)_Y$ when $\langle \psi_L \psi_R \rangle$ forms. Note that, if $N=4$, a single real ψ_L is sufficient to break the U(1)'s. Otherwise, to limit the parameters, $\xi' = -\xi$ may be assumed.

I now show that, in the absence of other ETC operators, the $U(1)_1$ interactions can overwhelm $SU(3)_1$ to produce the alignment pattern $\langle \overline{T^i_L} T^j_R \rangle \propto (i\sigma_2)_{ij}$. The coupling of the Z' boson to a generic fermion χ with weak hypercharge $Y = Y_1 + Y_2$ and electric charge $Q = Y'_1 + Y'_2$ is

$$\mathcal{L}_{\overline{\chi} Z' \chi} = g_{Z'} Z'^{\mu} [(Y_1 - rY) \overline{\chi_L} \gamma_\mu \chi_L + (Y'_1 - rQ) \overline{\chi_R} \gamma_\mu \chi_R], \tag{5.1}$$

where $g_{Z'} = \sqrt{g'^2_1 + g'^2_2} \simeq g'_1$ and $r = g'^2_2 / g^2_{Z'} \ll 1$. Small mixing terms induced by electroweak symmetry breaking have been neglected in this expression. A similar interaction can be written for the massive V_8 bosons of broken $SU(3)_1 \otimes SU(3)_2$. Ignoring small terms in the Z' and V_8 couplings, the four-fermion interaction these bosons generate for T^1 and T^2 is

$$\begin{aligned} \mathcal{L}_{T^1 T^2} = -2\pi \Bigg\{ & \frac{\alpha_{Z'}}{M^2_{Z'}} [u_1 (\overline{T^1_L} \gamma_\mu T^1_L + \overline{T^2_R} \gamma_\mu T^2_R) \\ & + v_1 (\overline{T^1_R} \gamma_\mu T^1_R + \overline{T^2_L} \gamma_\mu T^2_L)]^2_{M_{Z'}} \\ & + \frac{\alpha_{V_8}}{M^2_{V_8}} \sum_a (\overline{T^1_L} \gamma_\mu t_a T^1_L + \overline{T^1_R} \gamma_\mu t_a T^1_R)^2_{M_{V_8}} \Bigg\}, \end{aligned} \tag{5.2}$$

where $\alpha_{Z'} = g^2_{Z'}/4\pi$ and the t_a are SU(3) matrices in the 3-representation. All the currents are $SU(N) \otimes SU(2)$ singlets, and the current $\times$ current products are renormalized at the corresponding massive boson masses.[4] Fierzing this interaction and retaining only the dominant $SU(3) \otimes SU(N) \otimes SU(2)$-singlet operators involved in condensate formation gives

$$\begin{aligned} \mathcal{L}_{T^1 T^2} = \frac{4\pi}{3NM^2_{V_8}} \Bigg[& \frac{u_1 v_1 \alpha_{Z'} M^2_{V_8}}{M^2_{Z'}} (\overline{T^1_L} T^1_R \overline{T^1_R} T^1_L \\ & + \overline{T^2_L} T^2_R \overline{T^2_R} T^2_L)_{M_{Z'}} \\ & + \frac{\alpha_{Z'} M^2_{V_8}}{M^2_{Z'}} (u^2_1 \overline{T^1_L} T^2_R \overline{T^2_R} T^1_L \\ & + v^2_1 \overline{T^2_L} T^1_R \overline{T^1_R} T^2_L)_{M_{Z'}} + \frac{4\alpha_{V_8}}{3} (\overline{T^1_L} T^1_R \overline{T^1_R} T^1_L)_{M_{V_8}}. \end{aligned} \tag{5.3}$$

To determine which of the operators in Eq. (5.3) is dominant, I make the large-N approximation that the anomalous dimensions of the $4T$ operators are given by the sum of the anomalous dimensions $\gamma_{m_{ij}}$ of their constituent bilinears $\overline{T^i} T^j$. Then, the condition that the vacuum energy $E = -\langle \mathcal{L}_{T^1 T^2} \rangle$ is minimized by $\langle \overline{T^i_L} T^j_R \rangle \propto (i\sigma_2)_{ij}$ is

$$\begin{aligned} & \frac{\alpha_{Z'} (u^2_1 + v^2_1) M^2_{V_8}}{M^2_{Z'}} \frac{Z^2_{12}(M_{Z'})}{Z^2_{11}(M_{V_8})} \\ & > \frac{4\alpha_{V_8}}{3} + \frac{u_1 v_1 \alpha_{Z'} M^2_{V_8}}{M^2_{Z'}} \frac{Z^2_{11}(M_{Z'}) + Z^2_{22}(M_{Z'})}{Z^2_{11}(M_{V_8})}, \end{aligned} \tag{5.4}$$

where

TABLE I. Lepton, quark, and technifermion colors and hypercharges.

Particle	$SU(3)_1$	$SU(3)_2$	Y_1	Y_2
l^l_L	1	1	a	$-\frac{1}{2}-a$
e_R, μ_R	1	1	a'	$-1-a'$
q^l_L	1	3	b	$\frac{1}{6}-b$
u_R, c_R	1	3	b'	$\frac{2}{3}-b'$
d_R, s_R	1	3	b'	$-\frac{1}{3}-b'$
l^h_L	1	1	c	$-\frac{1}{2}-c$
τ_R	1	1	c'	$-1-c'$
q^h_L	3	1	d	$\frac{1}{6}-d$
t_R	3	1	d'	$\frac{2}{3}-d'$
b_R	3	1	$-d'$	$-\frac{1}{3}+d'$
T^1_L	3	1	u_1	u_2
U^1_R	3	1	v_1	$v_2+\frac{1}{2}$
D^1_R	3	1	v_1	$v_2-\frac{1}{2}$
T^2_L	1	3	v_1	v_2
U^2_R	1	3	u_1	$u_2+\frac{1}{2}$
D^2_R	1	3	u_1	$u_2-\frac{1}{2}$
T^l_L	1	1	x_1	x_2
U^l_R	1	1	x'_1	$x'_2+\frac{1}{2}$
D^l_R	1	1	x'_1	$x'_2-\frac{1}{2}$
T^t_L	1	1	y_1	y_2
U^t_R	1	1	y'_1	$y'_2+\frac{1}{2}$
D^t_R	1	1	y'_1	$y'_2-\frac{1}{2}$
T^b_L	1	1	z_1	z_2
U^b_R	1	1	z'_1	$z'_2+\frac{1}{2}$
D^b_R	1	1	z'_1	$z'_2-\frac{1}{2}$
ψ_L	1	1	ξ	$-\xi$
ψ_R	1	1	ξ'	$-\xi'$

[4] Strictly speaking, the V_8 exchange terms in (5.2) should be written as integrals over current products with massless V_8 propagators. Since these integrals are dominated by momenta or ~ 1 TeV $\sim M_{V_8}$, the approximation used here is reasonable.

$$Z_{ij}(M)=\exp\left[\int_{\Lambda_{\rm TC}}^{M}\frac{d\mu}{\mu}\,\gamma_{m_{ij}}(\mu)\right]. \tag{5.5}$$

Since the U(1) symmetries are broken at a higher scale than the SU(3) and electroweak symmetries, $M_{Z'}$ may be several times larger than M_{V_8}. However, the energy range from M_{V_8} to $M_{Z'}$ overlaps the region in which T condensates form. Thus, the anomalous dimensions $\gamma_{m_{ij}}\simeq 1$ there [12]. In this limit, the condition (5.4) becomes $(u_1-v_1)^2 > 4\alpha_{V_8}/3\alpha_{Z'}$.

The rest of my discussion of this model concerns how the $U(1)_1$ and $U(1)_2$ hypercharges are to be fixed. I start with the gauge anomaly conditions. The eight independent conditions are given in the Appendix. These constraints, together with the four equal-charge conditions, do not fix the 26 unknown $U(1)_i$ charges. Further limitations on the Y_i follow from requiring the presence of ETC-generated four-fermion operators breaking all but gauged symmetries. To give mass to quarks and leptons, I assume the ETC operators

$$\begin{aligned}
&\overline{l^l_{iL}}\gamma^\mu T^l_L\overline{D^l_R}\gamma_\mu e_{jR}\Rightarrow a-a'=x_1-x_1',\\
&\overline{q^l_{iL}}\gamma^\mu T^l_L\overline{T^l_R}\gamma_\mu q^l_{jR}\Rightarrow b-b'=x_1-x_1',\\
&\overline{l^h_L}\gamma^\mu T^l_L\overline{D^l_R}\gamma_\mu \tau_R\Rightarrow c-c'=x_1-x_1',\\
&\overline{q^h_L}\gamma^\mu T^t_L\overline{U^t_R}\gamma_\mu t_R\Rightarrow d-d'=y_1-y_1',\\
&\overline{q^h_L}\gamma^\mu T^b_L\overline{D^b_R}\gamma_\mu b_R\Rightarrow d+d'=z_1-z_1'.
\end{aligned} \tag{5.6}$$

To generate $d_L, s_L\leftrightarrow b_R$, I require the operator

$$\overline{q^l_{iL}}\gamma^\mu T^2_L\overline{D^1_R}\gamma_\mu b_R\Rightarrow b+d'=0. \tag{5.7}$$

To forbid $d_R, s_R\leftrightarrow b_L$, ETC interactions must not generate the operator $\overline{q^h_L}\gamma^\mu T^1_L\overline{D^2_R}\gamma_\mu d_{iR}$. This gives the constraint

$$d-b'\neq 0. \tag{5.8}$$

We shall see that this follows from requiring the existence of other four-fermion operators and also the anomaly constraints. Thus, this operator does not appear without the intervention of $U(1)_1$ breaking and so the transition $d_R, s_R\leftrightarrow b_L$ is automatically suppressed relative to $d_L, s_L\leftrightarrow b_R$ by a factor of $\delta M^2_{\rm ETC}/M_s^2\sim 10^{-4}$.

Next, I enumerate the chiral symmetries and Goldstone bosons of the model, to determine what $4T$ operators are needed to give them mass. The simplest way to do this is to imagine that all gauge interactions, including $SU(3)_1\otimes SU(3)_2\otimes U(1)_1$, may be neglected compared to technicolor. Then, grouping the technifermions into three triplet isodoublets, T^1, T^2 and $T^3=T^l,T^t,T^b$, the chiral symmetry group of these technifermions plus $\psi_{L,R}$ is

$$G_\chi=SU(18)_L\otimes SU(18)_R\otimes U(1)_A. \tag{5.9}$$

The $U(1)_A$ current involves all technifermions and has no technicolor anomaly. It is spontaneously broken principally by $\langle\psi_L\psi_R\rangle$. A linear combination of this current and generators of $SU(18)_A$ is exactly conserved and couples to the Goldstone boson eaten by the Z'. The orthogonal Goldstone boson gets mass from $SU(3)_1$ instantons and broken ETC interactions. We need not be further concerned with $U(1)_A$.

When T condensates break $SU(18)_L\otimes SU(18)_R$ to an SU(18) subgroup, there are 323 Goldstone bosons or technipions, π_T.[5] These may be conveniently classified according to the subgroup

$$H_\chi=SU(3)_1\otimes SU(3)_2\otimes SU(3)_3\otimes SU(2)\otimes U(1)_3\otimes U(1)_8, \tag{5.10}$$

where $SU(3)_i$ acts on the triplet T^i, SU(2) acts on the isodoublets within the triplets, and $U(1)_{3,8}$ are generated by the diagonal charges of the SU(3) defined on the triplet T^1,T^2,T^3:

$$\begin{aligned}
&T^1\in\left(3,1,1,2;\frac{1}{2},\sqrt{\tfrac{1}{12}}\right),\quad T^2\in(1,3,1,2;-\tfrac{1}{2},\sqrt{\tfrac{1}{12}}),\\
&T^3\in(1,1,3,2;0,-\sqrt{\tfrac{1}{3}}).
\end{aligned} \tag{5.11}$$

The 323 Goldstone bosons consist of: three SU(3)-singlet isotriplets, (1,1,1,3); three octet isotriplets plus three octet isosinglets; two singlets, (1,1,1,1); and three sets of $(3,3)\oplus(3,3)$ isotriplets and isosinglets.

The diagonal linear combination of the three (1,1,1,3)'s become $W^\pm_L$ and Z^0_L. Thus, ignoring the effects of color interactions, the decay constant of the technipions is $F_T=246\ \text{GeV}/\sqrt{9}=82$ GeV.[6] A linear combination of the (8,1,1,1) and(1,8,1,1) are absorbed in $SU(3)_1\oplus SU(3)_2\to SU(3)_C$, driven by $\langle T^1_L T^2_R\rangle$. Of the remaining 312 Goldstone bosons, all those which are $SU(3)_1\otimes SU(3)_2$ nonsinglets (there are 272 of these) acquire mass of at least $\sqrt{(\alpha_S\Lambda_T^4/F_T^2)}\simeq 250$ GeV from color interactions (see the papers by Peskin and Preskill in Ref. [15]).

This leaves 40 technipions whose mass must arise from ETC-generated 4T interactions. They transform as $(1,1,8,3)\oplus(1,1,8,1)\oplus(1,1,1,3)\oplus(1,1,1,3)\oplus(1,1,1,1)\oplus(1,1,1,1)$. Consider the two isotriplets (1,1,1,3) orthogonal to the longitudinal weak bosons. It is possible to form one linear combination of these states that contains no T^iT^i component for one of the values of $i=1,2,3$. Therefore, there must be a $4T$ term involving two technifermions of the form $T^i_L\gamma^\mu T^j_L T^j_R\gamma_\mu T^i_R$, with $i\neq j$, to insure that both isotriplets

[5] I do not know whether this is a record number of Goldstone bosons, as has been speculated. It certainly is a matter of concern whether they may make a large positive contribution to the S parameter. This is the case if they may be approximated as pseudo-Goldstone bosons [16]. As I have discussed elsewhere [17], this may be a poor approximation for the technipions in a walking technicolor model with its large anomalous dimensions. Furthermore, in such a model, there are additional, possibly negative, contributions to S which cannot be evaluated simply by scaling from QCD (see also Ref. [18]).

[6] I am suppressing the role of the SU(2)⊗U(1) chiral symmetry of $(t,b)_L$ and t_R in this discussion. The three Goldstone top-pions, π_t, arising from its breakdown combine with the (1,1,1,3)'s to form the longitudinal weak bosons. In our normalization, Hill's estimate of the top-pion decay constant is $F_t\simeq 70$ GeV [1]. The uneaten component of the top-pions acquires its mass from the ETC part of the top quark mass: $M^2_{\pi_t}\simeq m_t^{\rm ETC}\langle\overline{tt}\rangle/F_t^2$.

get mass. The only operators consistent with $u_1-v_1\neq 0$ have $i=1$ or 2 and $j=3$, with $T^3=T^l$ or T^t or T^b. Finally, in order that such an interaction contributes to (1,1,1,3) technipion masses, it is necessary that the condensates $\langle \overline{T_L^1} T_R^1\rangle$ and $\langle T_L^2 T_R^2\rangle$ form, i.e., that the matrix U in Eq. (4.2) is a nontrivial combination of 1 and $i\sigma_2$. Any of these $4T$ operators, in concert with $\mathrm{SU}(N)\otimes \mathrm{SU}(3)_1\otimes \mathrm{U}(1)_1$ interactions, can lead to such a pattern of vacuum alignment. As a specific choice consistent with Eqs. (5.6) and (5.7), I assume the existence of the operator

$$\overline{T_L^1}\gamma^\mu T_L^t \overline{T_R^t}\gamma_\mu(a+b\sigma_3)T_R^1 \Rightarrow y_1-y_1'=u_1-v_1. \tag{5.12}$$

Equations (5.6), (5.7), (5.12), and the anomaly conditions for $\mathrm{U}(1)_{1,2}[\mathrm{SU}(N)]^2$ and $\mathrm{U}(1)_{1,2}[\mathrm{SU}(3)_{1,2}]^2$ lead to the relations

$$\begin{aligned} a-a'=b-b'=c-c'=x_1-x_1'&=\tfrac{1}{2}N(u_1-v_1),\\ d-d'=y_1-y_1'&=u_1-v_1,\\ d+d'=z_1-z_1'&=-(2N+1)(u_1-v_1),\\ d&=-N(u_1-v_1),\\ d'=-b&=-(N+1)(u_1-v_1),\\ b'&=\tfrac{1}{2}(N+2)(u_1-v_1),\\ (N-2)(\xi-\xi')&=3N(u_1-v_1). \end{aligned} \tag{5.13}$$

We see that the constraint $d-b'\neq 0$ forbidding $d_R, s_R\leftrightarrow s_L$ is satisfied. Also, $dd'>0$, just what is needed for top, but not bottom, quarks to condense. The condition $\xi-\xi'\neq 0$ that $\langle \psi_L\psi_R\rangle$ breaks $\mathrm{U}(1)_1\oplus \mathrm{U}(1)_2\rightarrow \mathrm{U}(1)_Y$ is equivalent to $u_1-v_1\neq 0$, necessary for $\langle T_L^1 T_R^2\rangle\neq 0$.

Finally, there are (1,1,8,3), (1,1,8,1), and (1,1,1,1) technipions composed of T^l and T^b that do not acquire mass from the operator in Eq. (5.12). Combinations of spontaneously broken currents such as $T^2\gamma_\mu\gamma_5 T^2-3T^b\gamma_\mu\gamma_5 T^b$ are also left conserved by this operator. Thus, we need a $4T$ operator involving both T^l and T^b. One choice (of several) that is consistent with all the operators assumed so far is

$$\begin{aligned} \overline{T_L^l}\gamma^\mu T_L^t \overline{T_R^b}\gamma_\mu(a+b\sigma_3)T_R^l \Rightarrow y_1-z_1'=z_2'-y_2&=x_1-x_1'\\ &=\tfrac{1}{2}N(u_1-v_1). \end{aligned} \tag{5.14}$$

Note that T^t and T^b must have the same electric charges, i.e., $y_1+y_2=z_1+z_2$.

We now have 18 linear plus three nonlinear conditions on the 26 hypercharges. In the Appendix, I exhibit solutions to these equations for which $|u_1-v_1|\sim 1$. The vacuum alignment program, including determination of the eigenvalues and eigenstates of the technipion mass matrix, is outlined in Sec. VI and then deferred to a later paper.

VI. THE PHENOMENOLOGY OF TOP-COLOR-ASSISTED TECHNICOLOR

The picture of top-color-assisted technicolor I have drawn in this paper leads to a wide variety of phenomena in the TeV energy region, many of which are likely to be accessible in Tevatron collider experiments and, possibly, in experiments at the CERN e^+e^- collider LEP 2. Here is a list of the more obvious issues.

(1) The Z' boson, with $M_{Z'}=1$–5 TeV.

(2) The V_8 colorons, with mass $M_{V_8}\lesssim 1$ TeV. Their phenomenology was discussed in Refs. [13] and [19].

(3) The quantum numbers, masses, and production and decay modes of technirhos, technipions, and top pions.

(4) A possible outcome of vacuum alignment is the appearance of CP-violating phases in the unitary matrices defining mass eigenstate quarks (see Eichten, Lane, and Preskill in Ref. [15]).

(5) Cosmological consequences of the ψ fermion which, apparently, must have a component that is stable against weak decay.

(6) If $|u_1-v_1|$ must be order 1, some of the hypercharges in Eq. (5.13) are $O(N)$. This raises the question of the triviality of the $\mathrm{U}(1)_1$ interaction: does it set in at an energy much lower than the one at which we can envisage $\mathrm{U}(1)_1$ being unified into an asymptotically free ETC group? Each of these topics requires extensive study. Here, I briefly discuss only the Z' and the aspects of vacuum alignment. Details are under investigation by others or postponed to later papers.

Z' physics

The mass of the Z' arises mainly from ψ condensation:

$$M_{Z'}\simeq g_{Z'}|\xi-\xi'|F_\psi, \tag{6.1}$$

where $\xi-\xi'=3N(u_1-v_1)/(N-2)\sim 1$, and $F_\psi\sim 1$ TeV is the π_ψ decay constant. This is the basis of my estimate of $M_{Z'}$. The Z' decays into technifermion, quark and lepton pairs, with large couplings to all. Thus, its width is large, probably several hundred GeV [13]. I emphasize that in this scenario the Z' necessarily couples strongly to the first two generations of quarks and leptons.

There are several precision electroweak studies that probe for the Z' [20]. Mixing of the Z' and Z^0 affects the latter's couplings to quark and lepton pairs. If the Z' width is not an issue, the magnitude of these mixing effects is

$$\phi_{ZZ'}\simeq \frac{g_{Z'}M_Z^2}{g_Z M_{Z'}^2}, \tag{6.2}$$

where $g_Z=\sqrt{g^2+g'^2}$. This mixing also affects the S parameter [16].

Mixing and direct Z' interactions together influence other, very low-energy measurements. For example, in the class of models outlined above, the electron has an axial-vector coupling to the Z'. This is probed in atomic parity violation experiments, which are especially sensitive to the product of this coupling with the vector part of the isoscalar nuclear current [21]. The effective interaction in our model is

$$\mathcal{L}_{\mathrm{APV}}=-\frac{g_{Z'}^2(a'-a)(b'+b)}{4M_{Z'}^2}\,\overline{e}\gamma^\mu\gamma_5 e(\overline{u}\gamma_\mu u+\overline{d}\gamma_\mu d). \tag{6.3}$$

The product $(a'-a)(b'+b)/4=-N(3N+4)(u_1-v_1)^2/16$ can be large in this model. Out of concern for this, I have tried to construct models within the present framework in which the electron's coupling to Z' is purely vectorial. So far, I have not found one that has a nontrivial $(u_1-v_1\neq 0)$ solution to the anomaly conditions.

As a second example, the polarized Møller scattering experiment recently proposed by Kumar and his collaborators [22] is sensitive to the combination a'^2-a^2 of electron couplings to the Z'. The effective interaction is (apart from mixing effects)

$$\mathcal{L}_{\text{Møller}}=-\frac{g_{Z'}^2}{2M_{Z'}^2}\,[a^2(\overline{e_L}\gamma_\mu e_L)^2+a'^2(\overline{e_R}\gamma_\mu e_R)^2]. \tag{6.4}$$

The Z' will also be visible in current and planned high-energy collider experiments. At subprocess energies well below the Z' mass, its effects are still well-approximated by four-fermion ''contact'' interactions, similar to those expected for composite quarks and leptons [23]. Thus, at the Tevatron collider, the $Z's$ strong couplings to quarks produce an excess of high-E_T jets[7,8] and high-mass dileptons. The effective interactions are

$$\mathcal{L}_{qq}=-\frac{g_{Z'}^2}{2M_{Z'}^2}\left[\sum_{q=u,d,c,s}(b\overline{q_L}\gamma_\mu q_L+b'\overline{q_R}\gamma_\mu q_R)\right.$$
$$\left.+d(\overline{t_L}\gamma_\mu t_L+\overline{b_L}\gamma_\mu b_L)+d'(\overline{t_R}\gamma_\mu t_R-\overline{b_R}\gamma_\mu b_R)\right]^2, \tag{6.5}$$

$$\mathcal{L}_{ql}=-\frac{g_{Z'}^2}{M_{Z'}^2}\sum_{q=u,d,c,s}(b\overline{q_L}\gamma^\mu q_L+b'\overline{q_R}\gamma^\mu q_R)$$
$$\times\sum_{l=e,\mu}(a\overline{l_L}\gamma_\mu l_L+a'\overline{l_R}\gamma_\mu l_R).$$

In these expressions, we have ignored small effects of mixing among quark generations. Note that there are simplications of the couplings such as $g_{Z'}^2 b^2/M_{Z'}^2\simeq[(N+1)(N-2)/3NF_\psi]^2$. The Z' interaction affecting Bhabha scattering and muon-pair production in e^+e^- collisions is

$$\mathcal{L}_{ll}=-\frac{g_{Z'}^2}{2M_{Z'}^2}\left[\sum_{l=e,\mu}(a\overline{l_L}\gamma_\mu l_L+a'\overline{l_R}\gamma_\mu l_R)\right]^2. \tag{6.6}$$

Jet production in e^+e^- collisions is modified by $\mathcal{L}_{ql}$. Corresponding interactions influence tau-pair production. At the CERN Large Hadron Collider (LHC), the excess of high-E_T jets will be enormous and the Z' shape should be observable as a resonance in dileptons if not in dijets. A high luminosity e^+e^- collider with $\sqrt{s}\simeq M_{Z'}$ can make detailed studies of the Z' couplings. One with $\sqrt{s}\simeq 500$ GeV may be able to detect signs of γ-Z-Z' interference.

Vacuum alignment and technihadron physics

The spectrum of technirhos ρ_T in this model is the same as that given above for the technipions. Determining the mass-eigenstate π_T and ρ_T is the problem of vacuum alignment in the technifermion sector. This is essentially the same as diagonalizing the technifermion mass matrix (see, however, footnote 4 for a caveat on the use of chiral perturbation theory.) The top-pions π_t formed from $(t,b)_L$ and t_R must be added to this large π_T-diagonalization calculation. Once mass eigenstates are determined, the $\rho_T\to\pi_T\pi_T$ couplings can be determined by symmetry (see, e.g., [14]). Note that the ρ_T decay modes may include one or two weak bosons, $W_L^\pm$ and Z_L^0. Vacuum alignment also determines the pattern of technifermion condensation, relevant for mixing between heavy and light quarks, and feeds into the Kobayashi-Maskawa matrix and other quark mixing angles and phases.

Vacuum alignment is carried out by minimizing the ground-state energy of broken ETC and $\mathrm{SU}(3)_1\otimes\mathrm{U}(1)_1$ four-fermion operators and of second-order QCD interactions [15]. In the absence of a concrete ETC model, the most that can be done is to make ''reasonable'' guesses for the coefficients of allowed operators—those already assumed plus others consistent with symmetries. Different assumptions for the relative strengths and signs of the operators will lead to different vacua, patterns of condensation, and π_T and ρ_T spectroscopies. Such studies should give us a plausible range of expectations for this aspect of TC2 phenomenology. Some issues of immediate concern are the following.

(i) Typical masses of the charged top-pion and its mixing with technipions. The concern here is that the decay rates for $t\to\pi_t b$ or $\pi_t b$ may be too large [25].

(ii) Masses of the π_T and ρ_T. Technipion decays are mediated by ETC interactions connecting technifermions to quarks and leptons. Thus, the π_T are expected to decay to heavy quark and lepton pairs. The existence of ''leptoquark'' decay modes such as $\pi_T\to\underline{b}\tau$ depends on whether ETC operators such as $b_R\gamma^\mu D_R^1\overline{D}_L^l\gamma_\mu l_L^h$ are allowed. Experiments at the LEP collider will soon be able to set limits in excess of 75 GeV for charged π_T. Mixing between gluons and color-octet ρ_T leads to copious production of colored π_T; Tevatron collider searches should be able to discover them with masses up to several hundred GeV. Production of color-singlet $\rho_T\to\pi_T\pi_T$, $W_L\pi_T$, $Z_L\pi_T$, W_LW_L, and W_LZ_L should be accessible at the Tevatron for π_T masses of 100–200 GeV [14]. Another process to be searched for at the Tevatron is $gg\to\pi_T^0\to b\overline{b}$ or $t\overline{t}$, if $M_{\pi_T^0}>2m_t$. For the longer term, ρ_T and π_T masses are needed for LHC and large e^+e^- collider studies.

(iii) Vacuum alignment may produce phases in quark (and technifermion) mixing matrices that induce detectable CP violation in the neutral K- and B-meson systems, in the neutron electric dipole moment, and so on. If this happens, it will be important to determine whether strong CP violation can be avoided.

These brief remarks only scratch the surface of the phenomenological aspects of the scenario I have presented. I do hope they give a flavor of the richness of top-color-assisted

[7]The V_8 colorons enhance only $t\overline{t}$ and $b\overline{b}$ production.

[8]As this paper was being completed, I received two preprints discussing the possibility that a TeV-mass Z' boson affects high-E_T jet production and the branching ratios for Z^0 decay to $b\overline{b}$ and $c\overline{c}$ [24].

technicolor. I do not expect the specific class of models described here to pass all the tests it faces. But, in facing them, I expect we will learn how to build more complete and more successful models.

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APPENDIX: ANOMALY CONDITIONS AND HYPERCHARGE SOLUTIONS

There are five linear and four cubic equations for the hypercharges in Table I arising from the requirement that $\mathrm{U}(1)_i$ gauge anomalies cancel:

$$U(1)_{1,2}[\mathrm{SU}(N)]^2:\ x_1-x_1'+y_1-y_1'+z_1-z_1'\equiv x_2'-x_2+y_2'-y_2+z_2'-z_2=-\tfrac{1}{2}(N-2)(\xi-\xi'),$$

$$\mathrm{U}(1)_{1,2}[\mathrm{SU}(3)_1]^2:\ d=-N(u_1-v_1),$$

$$\mathrm{U}(1)_{1,2}[\mathrm{SU}(3)_2]^2:\ b-b'=\tfrac{1}{2}N(u_1-v_1),$$

$$\mathrm{U}(1)_{1,2}[\mathrm{SU}(2)]^2:\ 2(a+3b)+(c+3d)=-N[3(u_1+v_1)+x_1+y_1+z_1]=N[3(u_2+v_2)+x_2+y_2+z_2],$$

$$[\mathrm{U}(1)_1]^3:\ 0=\tfrac{1}{2}N(N-1)(\xi^3-\xi'^3)+2[2a^3-a'^3+6(b^3-b'^3)]+2c^3-c'^3+6d^3+2N(x_1^3-x_1'^3+y_1^3-y_1'^3+z_1^3-z_1'^3),$$

$$\begin{aligned}[\mathrm{U}(1)_2]^3:\ 0=&-\tfrac{1}{2}N(N-1)(\xi^3-\xi'^3)-2[2a^3-a'^3+6(b^3-b'^3)]-(2c^3-c'^3+6d^3)+2N[x_2^3-x_2'^3+y_2^3-y_2'^3+z_2^3-z_2'^3\\&-\tfrac{9}{4}(u_2+v_2)-\tfrac{3}{4}(x_2'+y_2'+z_2')]+2[3(a'^2-a^2)+3(a'-\tfrac{1}{2}a)+3(b^2-b'^2)+5b'-\tfrac{1}{2}b]+3(c'^2-c^2)\\&+3(c'-\tfrac{1}{2}c)+3(d^2-d'^2)+3d'-\tfrac{1}{2}d,\end{aligned}$$

$$\begin{aligned}[\mathrm{U}(1)_1]^2\mathrm{U}(1)_2:\ 0=&-\tfrac{1}{2}N(N-1)(\xi^3-\xi'^3)-2[2a^3-a'^3+a^2-a'^2+6(b^3-b'^3)+b'^2-b^2]\\&-(2c^3-c'^3+c^2-c'^2+6d^3+d'^2-d^2)+2N(x_1^2x_2-x_1'^2x_2'+y_1^2y_2-y_1'^2y_2'+z_1^2z_2-z_1'^2z_2'),\end{aligned}$$

$$\begin{aligned}[\mathrm{U}(1)_2]^2\mathrm{U}(1)_1:\ 0=&\tfrac{1}{2}N(N-1)(\xi^3-\xi'^3)+2[2a^3-a'^3+2(a^2-a'^2)+6(b^3-b'^3)+2(b'^2-b^2)]+2c^3-c'^3+2(c^2-c'^2)\\&+6d^3+2(d'^2-d^2)+2N[x_2^2x_1-x_2'^2x_1'+y_2^2y_1-y_2'^2y_1'+z_2^2z_1-z_2'^2z_1'-\tfrac{3}{4}(u_1+v_1)-\tfrac{1}{4}(x_1'+y_1'+z_1')]\\&+2(\tfrac{1}{2}a-a'+\tfrac{1}{6}b-\tfrac{5}{3}b')+\tfrac{1}{2}c-c'+\tfrac{1}{6}d-d'.\end{aligned}\tag{A1}$$

These 4 cubic equations are not independent because the $[\mathrm{U}(1)_Y]^3=[\mathrm{U}(1)_1+\mathrm{U}(1)_2]^3$ anomaly cancellation is guaranteed by the $\mathrm{U}(1)_Y[\mathrm{SU}(2)]^2$ condition. A convenient set of three independent cubic equations consists of $[\mathrm{U}(1)_1]^3$ plus $[\mathrm{U}(1)_1]^2\mathrm{U}(1)_Y$ and $[\mathrm{U}(1)_1]^3+[\mathrm{U}(1)_2]^3-3[\mathrm{U}(1)_1]^2\mathrm{U}(1)_Y$:

$$\begin{aligned}[\mathrm{U}(1)_1]^2\mathrm{U}(1)_Y:\ 0=&2(a'^2-a^2+b^2-b'^2)+c'^2-c^2+d^2-d'^2\\&+2N[(x_1+x_2)(x_1^2-x_1'^2)+(y_1+y_2)(y_1^2-y_1'^2)+(z_1+z_2)(z_1^2-z_1'^2)],\end{aligned}$$

$$[\mathrm{U}(1)_1]^3+[\mathrm{U}(1)_2]^3-3[\mathrm{U}(1)_1]^2\mathrm{U}(1)_Y:\ 0=(u_1-v_1)\{2N^2[4(y_1+y_2)^2-(x_1+x_2)^2+\tfrac{3}{4}]-(5N+2)\}.\tag{A2}$$

In the last equation, I used results from Eq. (5.13).

The 18 linear and three nonlinear equations satisfied by the 26 hypercharges do not determine them uniquely. I sought numerical solutions to them that have $u=\frac{1}{2}(u_1-v_1)\neq 0$ as follows: First, I set $\xi'=-\xi$ and $c=a$. Then I chose values for x_1, y_1 and y_1+y_2, and solved for u, a and x_1+x_2. To obtain $u_1-v_1\sim 1$, I input x_1, $y_1=O(Nu)$. For $N=4$ and $y_1+y_2=0$ (which implies $x_1+x_2=\pm\frac{1}{4}$) and $x_1=y_1=10$, I obtained

$$u=1.075,\ \ a=1.040\quad(\text{for } x_1+x_2=-\tfrac{1}{4}),$$

$$u=1.197,\ \ a=12.054\quad(\text{for } x_1+x_2=\tfrac{1}{4}).\tag{A3}$$

As is apparent from Eqs. (A2), these solutions scale linearly with the input values of x_1 and y_1. Values of a as large as 12 are doubtless ruled out.

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