# Analysis of two-body decays of charmed baryons using the quark-diagram scheme

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We give a general formulation of the quark-diagram scheme for the nonleptonic weak decays of baryons. We apply it to all the decays of the antitriplet and sextet charmed baryons and express their decay amplitudes in terms of the quark-diagram amplitudes. We have also given parametrizations for the effects of final-state interactions. For SU(3) violation effects, we only parametrize those in the horizontal *W*-loop quark diagrams whose contributions are solely due to SU(3)-violation effects. In the absence of all these effects, there are many relations among various decay modes. Some of the relations are valid even in the presence of final-state interactions when each decay amplitude in the relation contains only a single phase shift. All these relations provide useful frameworks to compare with future experiments and to find the effects of final-state interactions and SU(3) symmetry violations. [S0556-2821(96)02013-9]

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# I. INTRODUCTION

The study of charmed baryon physics is of current interest [1]. Many nonleptonic weak decay modes of the charmed baryons  $\Lambda_c^+$ ,  $\Xi_c^{0A}$ , and  $\Xi_c^{+A}$  have been measured [2] and more data are expected in the near future. Apart from model calculations [3-5], it is useful to study the nonleptonic weak decays in a way which is as model independent as possible. The two-body nonleptonic decays of charmed baryons have been analyzed in terms of SU(3)-irreducible-representation [SU(3)IR] amplitudes [6,7]. However, the quark-diagram scheme (i.e., analyzing the decays in terms of quark-diagram amplitudes) has the advantage that it is more intuitive and easier for implementing model calculations. It has been successfully applied to the hadronic weak decays of charmed and bottom mesons [8,9]. It has provided a framework with which we cannot only do the least model-dependent data analysis and give predictions but also make evaluations of theoretical model calculations. In this paper we give a general and unified formulation always using the orthonormal quark states and independent of SU(6) symmetry] of the quark-diagram scheme for the nonleptonic weak decays of baryons, which can be useful for all baryon (charm and bottom) nonleptonic decays. Here we apply it to all the twobody hadronic decays (quark mixing allowed, suppressed, and doubly suppressed) of the antitriplet and sextet charmed baryons and express them in terms of the quark-diagram amplitudes. We find consistent comparisons with the SU(3)IR results of Ref. [6]. In addition, the symmetry properties of the initial and final baryon wave functions in conjunction with the Pati-Woo theorem [11] for weak decays enable us to obtain more specific results than those from the SU(3)IR scheme.

We have also given parametrizations for effects of final-

state interactions. For SU(3) violations we indicate only those arising in the horizontal *W*-loop quark diagrams whose contributions are solely due to SU(3)-violation effects. (See the more detailed discussion about this at the end of Sec. III A.) In the absence of these effects, there are many relations among various decay modes. Some of the relations are valid even in the presence of final-state interactions when each decay amplitude in the relation contains only a single phase shift. It will be interesting to compare these relations with future experimental data. They provide a useful framework to find the effects of final-state interactions and SU(3) violations.

Earlier, Kohara gave a quark-diagram formulation for the quark-mixing-allowed decays of the antitriplet charmed baryons [10], with which our results agree. However, for the decay products containing an octet baryon he used a basis of quark states which is not orthonormal (which is all right though more complicated) and its choice seems to be based upon exact SU(6) symmetry (which is not all right since SU(6) is not an exact symmetry). The fact that our formulation demonstrates that the quark-diagram results are independent of whether one uses exact SU(6) or not helps to clarify this point for his formulation and explains why our results can possibly agree. [For detailed comments see Sec. II after Eq. (28) and for exact relations between amplitudes see Eq. (63) in Sec. IV B.]

In the framework of the quark-diagram scheme, all nonleptonic meson decays can be expressed in terms of six quark-diagram amplitudes [8]:  $\mathcal{A}$ , the external *W*-emission diagram;  $\mathcal{B}$ , the internal *W*-emission diagram;  $\mathcal{C}$ , the *W*-exchange diagram;  $\mathcal{D}$ , the *W*-annihilation diagram;  $\mathcal{E}$ , the horizontal *W*-loop diagram; and  $\mathcal{F}$ , the vertical *W*-loop diagram. These quark diagrams are specific and well-defined physical quantities. They are classified according to the topology of first-order weak interactions, with all QCD stronginteraction effects included. It is important to emphasize that strong interactions do not alter the classification of these diagrams. These quark diagrams have a one-to-one correspon-

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dence to those amplitudes classified according to SU(3) irreducible representations.

For the baryon decays, we can easily show by diagram drawing that the  $\mathcal{D}$  and  $\mathcal{F}$  types of amplitudes do not contribute. However, there are more possibilities in drawing the C and E types of amplitudes. More importantly, baryons, being made out of three quarks in contrast to two quarks for the mesons, bring along many essential complications. Though many textbooks [12] have discussed the baryon wave functions, we need to carefully develop the proper formulation suitable for the construction of the quark-diagram scheme for baryon decays. This is what we discuss in Sec. II, where the relations between the quark states and the baryon states are derived. We then apply these general results to the specific decays of the charmed baryons. In Secs. III and IV we give the quark-diagram formulation for the two-body decays of antitriplet charmed baryons into a pseudoscalar meson and a baryon (decuplet and octet); first for the case without effects of final-state interactions and SU(3) violations, and then for the case with these effects. We discuss their experimental implications and comment on previous related theoretical work. Section V is devoted to studying the nonleptonic weak decays of sextet charmed baryons. In Sec. VI we give a few concluding remarks.

# **II. QUARK STATES AND PARTICLE STATES**

To develop a quark-diagram scheme we need to fully understand the relation between the quark states and the particle states. Baryons are made out of three  $\frac{1}{2}$ -spin quarks. The baryon states form irreducible representations of SU(3) flavor and SU(2) spin from the tensor-product states of flavor and spin of three quarks which are written as the following orthonormalized states:

$$|q_1, S_{1z}; q_2, S_{2z}; q_3, S_{3z}\rangle = |q_1q_2q_3\rangle |S_{1z}S_{2z}S_{3z}\rangle.$$
(1)

There are  $3 \times 3 \times 3 = 27$  flavor states  $|q_1q_2q_3\rangle$  and  $2 \times 2 \times 2 = 8$  spin states  $|S_{1z}S_{2z}S_{3z}\rangle$ .

Let us first discuss the flavor irreducible representation states of the three quarks. The 27 triquark states can be decomposed into  $[8]_A$ ,  $[8]_S$ ,  $[1]_A$ , and  $[10]_{S_t}$  irreducible representations, denoted by the orthonormalized states

$$|\psi(8)_A\rangle, |\psi(8)_S\rangle, |\psi(1)_A\rangle, \text{ and } |\psi(10)_{S_I}\rangle.$$
 (2)

The transformation between the two bases, Eqs. (1) and (2), can be written in a  $27 \times 27$  matrix which is block diagonalized into the following submatrix transformations:

$$\begin{pmatrix} |\psi^{k}(8)_{S}\rangle \\ |\psi^{k}(8)_{A}\rangle \\ |\psi^{k}(10)_{S_{t}}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$
$$\times \begin{pmatrix} |q_{a}q_{b}q_{a}\rangle \\ |q_{b}q_{a}q_{a}\rangle \\ |q_{a}q_{a}q_{b}\rangle \end{pmatrix}, \qquad (3)$$

where *k* can be the proton, neutron,  $\Sigma^+, \Sigma^-, \Xi^0$ , and  $\Xi^-$  types, all of which have two identical quarks. There are six of such  $3 \times 3$  matrix equations totalling the transformations of 18 states out of the 27. Note that the subscripts *A* and *S* signify the antisymmetry and symmetry, respectively, between the first two quarks; the subscript *S<sub>i</sub>* denotes the total symmetry among the three quarks. Then there are the following transformations of the six states with all three quarks being different:

$$\begin{pmatrix} |\psi^{\Sigma}(8)_{S}\rangle \\ |\psi^{\Sigma}(8)_{A}\rangle \\ |\psi^{\Lambda}(8)_{S}\rangle \\ |\psi^{\Lambda}(8)_{A}\rangle \\ |\psi^{\Lambda}(1)_{A}\rangle \\ |\psi^{\Sigma}(10)_{S_{t}}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} & -2/\sqrt{12} & -2/\sqrt{12} \\ -1/2 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & -1/2 & -1/2 & 0 & 0 \\ 1/\sqrt{12} & -1/\sqrt{12} & -1/\sqrt{12} & -2/\sqrt{12} & 2/\sqrt{12} \\ 1/\sqrt{12} & -1/\sqrt{12} & -1/\sqrt{12} & 1/\sqrt{12} & -2/\sqrt{12} & 2/\sqrt{12} \\ 1/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & -1/\sqrt{6} \\ 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \\ \end{pmatrix} \begin{pmatrix} |sdu\rangle \\ |dsu\rangle \\ |usd\rangle \\ |uds\rangle \\ |uds\rangle \end{pmatrix} .$$
(4)

Finally, there are the three states with all three identical quarks:

$$|\Delta^{++}\rangle = |uuu\rangle, \tag{5}$$

$$|\Delta^{-}\rangle = |ddd\rangle, \tag{6}$$

$$|\Omega^{-}\rangle = |sss\rangle. \tag{7}$$

They give three diagonal transformations. These 27 equations, Eqs. (3) to (7), are actually equivalent to the following 27 equations:

$$|\psi^{k}(8)_{A}\rangle = \sum_{q_{i}=u,s,d} |q_{1}q_{2}q_{3}\rangle\langle q_{1}q_{2}q_{3}|\psi^{k}(8)_{A}\rangle,$$
 (8)

$$|\psi^{k}(8)_{S}\rangle = \sum_{q_{i}=u,s,d} |q_{1}q_{2}q_{3}\rangle\langle q_{1}q_{2}q_{3}|\psi^{k}(8)_{S}\rangle,$$
 (9)

$$\psi^{k}(1)_{A}\rangle = \sum_{q_{i}=u,s,d} |q_{1}q_{2}q_{3}\rangle \langle q_{1}q_{2}q_{3}|\psi^{k}(1)_{A}\rangle, \quad (10)$$

$$|\psi^{k}(10)_{S_{t}}\rangle = \sum_{q_{i}=u,s,d} |q_{1}q_{2}q_{3}\rangle \langle q_{1}q_{2}q_{3}|\psi^{k}(10)_{S_{t}}\rangle, \quad (11)$$

where the superscript k stands for the particles in the multiplets. These equations are obtained simply by multiplying the left-hand side (LHS) of these equations by the identity operator

$$\hat{I} = \sum_{q_i = u, d, s} |q_1 q_2 q_3\rangle \langle q_1 q_2 q_3|, \qquad (12)$$

which is the completeness of the orthonormal  $|q_1q_2q_3\rangle$  basis in the triquark vector space. The  $\langle q_1q_2q_3 | \psi^k(\cdots) \rangle$  numbers in Eqs. (8) to (11) are precisely those matrix elements in Eqs. (3) to (7).

Since the transformations Eqs. (3) to (7) are between two sets of orthonormal bases, the transformation matrix formed by these elements  $\langle q_1 q_2 q_3 | \psi^k(\cdots) \rangle$  is orthogonal. We can easily take the inverse of the transformation (i.e., take the transpose of the matrix) and express the quark states in terms of the irreducible representation states, i.e., the particle states. If the basis vectors are not orthonormal, not only will such inverse be harder to find, but also the completeness equation will be more complicated than that given by Eq. (12). Therefore, whenever possible, it is better to use the orthonormal basis.

Alternatively, we can also use the basis composed of the quark states that are symmetric and antisymmetric in the first two quarks: i.e.,

$$\begin{pmatrix} |\psi^{\Sigma}(8)_{S}\rangle \\ |\psi^{\Sigma}(8)_{A}\rangle \\ |\psi^{A}(8)_{S}\rangle \\ |\psi^{A}(8)_{A}\rangle \\ |\psi^{A}(1)_{A}\rangle \\ |\psi^{\Sigma}(10)_{S_{t}}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{6} & 0 & 1/\sqrt{6} \\ 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 & 1/\sqrt{3} \\ \end{pmatrix}$$

Likewise, in this basis the identity matrix becomes

$$\hat{I} = \sum_{q_a, q_b, q_c} \left( \left| \{q_a q_b\} q_c \right\rangle \langle \{q_a q_b\} q_c \right| + \left| [q_a q_b] q_c \right\rangle \langle [q_a q_b] q_c | \right).$$
(18)

Then Eqs. (8)–(11) can be recast into the form

$$|\psi^{k}(8)_{A}\rangle = \sum_{q_{a},q_{b},q_{c}} |[q_{a}q_{b}]q_{c}\rangle\langle [q_{a}q_{b}]q_{c}|\psi^{k}(8)_{A}\rangle, \qquad (19)$$

$$|\psi^{k}(8)_{S}\rangle = \sum_{q_{a},q_{b},q_{c}} |\{q_{a}q_{b}\}q_{c}\rangle\langle\{q_{a}q_{b}\}q_{c}|\psi^{k}(8)_{S}\rangle, \qquad (20)$$

$$|\psi^{k}(1)_{A}\rangle = \sum_{q_{a},q_{b},q_{c}} |[q_{a}q_{b}]q_{c}\rangle\langle [q_{a}q_{b}]q_{c}|\psi^{k}(1)_{A}\rangle, \qquad (21)$$

$$|\psi^{k}(10)_{S_{t}}\rangle = \sum_{q_{a},q_{b},q_{c}} |\{q_{a}q_{b}\}q_{c}\rangle\langle\{q_{a}q_{b}\}q_{c}|\psi^{k}(10)_{S_{t}}\rangle, \quad (22)$$

where we have used  $\langle [q_a q_b] q_c | \psi^k(8)_S \rangle = 0$  and  $\langle \{q_a q_b\} q_c | \psi^k(8)_A \rangle = 0$ . The coefficients on the right-hand side (RHS) of Eqs. (19)–(22) are the matrix elements in Eqs. (16) and (17).

Here we would like to emphasize that it is conceptually and practically simpler to consistently use the orthonormal quark states as the basis so that the identity operator has the

$$|\{q_a q_b\}q_c\rangle \equiv \frac{1}{\sqrt{2}(1-\delta_{ab})+2\delta_{ab}}(|q_a q_b q_c\rangle+|q_b q_a q_c\rangle),$$
(13)

$$[q_a q_b]q_c \rangle \equiv \frac{1}{\sqrt{2}} (|q_a q_b q_c \rangle - |q_b q_a q_c \rangle), \qquad (14)$$

or, inversely,

$$|q_{a}q_{b}q_{c}\rangle = \frac{\sqrt{2}(1-\delta_{ab})+2\,\delta_{ab}}{2}(|\{q_{a}q_{b}\}q_{c}\rangle+|[q_{a}q_{b}]q_{c}\rangle).$$
(15)

In this basis, Eqs. (3) and (4) become

$$\begin{pmatrix} |\psi^{k}(8)_{S}\rangle \\ |\psi^{k}(8)_{A}\rangle \\ |\psi^{k}(10)_{S_{t}}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 0 & 1 & 0 \\ \sqrt{2}/\sqrt{3} & 0 & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} |\{q_{a}q_{b}\}q_{a}\rangle \\ |[q_{a}q_{b}]q_{a}\rangle \\ |q_{a}q_{a}q_{b}\rangle \end{pmatrix}$$
(16)

and

simple expression of Eq. (12) or Eq. (18). They provide the proper transformation from the particle states to the quark states and vice versa as given by Eqs. (3) and (4), or equivalently by Eqs. (8) to (11); or by Eqs. (16) and (17), or equivalently Eqs. (19) to (22). These are the crucial relations we shall use in converting decay amplitudes in terms of particles to decay amplitudes in terms of quarks, i.e., the quark-diagram amplitudes.

Next we can form irreducible representations for the spin part of the particle from the tri- $\frac{1}{2}$ -spin states,

$$\begin{pmatrix} |\chi^{\pm 1/2}(\frac{1}{2})_{S}\rangle \\ |\chi^{\pm 1/2}(\frac{1}{2})_{A}\rangle \\ |\chi^{\pm 1/2}(\frac{3}{2})_{S_{t}}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \\ \times \begin{pmatrix} |\pm \frac{1}{2} & \pm \frac{1}{2} & \pm \frac{1}{2}\rangle \\ |\pm \frac{1}{2} & \pm \frac{1}{2} & \pm \frac{1}{2}\rangle \\ |\pm \frac{1}{2} & \pm \frac{1}{2} & \pm \frac{1}{2}\rangle \\ |\pm \frac{1}{2} & \pm \frac{1}{2} & \pm \frac{1}{2}\rangle \end{pmatrix}, \quad (23)$$

giving six equations, and

$$|\chi^{\pm 3/2}(\frac{3}{2})_{S_t}\rangle = |\pm \frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2}\rangle, \qquad (24)$$

giving two diagonal ones, totaling eight equations. The inverse of these equations is also easy to write out.

The baryon states must be totally antisymmetric in interchanging the composing quarks. Since the color part (which we do not discuss here; see e.g., Ref. [12]) is antisymmetric, the product of the flavor and the spin parts must be symmetric as the spatial wave function is symmetric for low-lying baryons. The decuplet baryons are made out of

$$|B^{m,k}(10)\rangle = |\chi^{m}(\frac{3}{2})_{S_{t}}\rangle|\psi^{k}(10)_{S_{t}}\rangle,$$
  
$$m = \pm \frac{1}{2}, \pm \frac{3}{2}, \text{ and } k = 1 \text{ to } 10.$$
(25)

The octet baryon is a combination of two parts:

$$|B^{m,k}(8)\rangle = a|B^{m,k}_{A}(8)\rangle + b|B^{m,k}_{S}(8)\rangle$$
$$= a|\chi^{m}(\frac{1}{2})_{A}\rangle|\psi^{k}(8)_{A}\rangle + b|\chi^{m}(\frac{1}{2})_{S}\rangle|\psi^{k}(8)_{S}\rangle, \quad (26)$$

where

$$|a|^2 + |b|^2 = 1. (27)$$

The precise values of a and b are not known. Importantly, our formalism does not need such information. We clearly show that the quark-diagram scheme is independent of the values of a and b.

If one assumes the SU(6) symmetry, then  $a=b=1/\sqrt{2}$ and the octet baryon wave function can be rewritten as

$$|B^{m,k}(8)\rangle = \frac{\sqrt{2}}{3} [|\chi^{m}_{A_{12}}(\frac{1}{2})\rangle|\psi^{k}_{A_{12}(8)}\rangle + |\chi^{m}_{A_{13}}(\frac{1}{2})\rangle|\psi^{k}_{A_{13}(8)}\rangle + |\chi^{m}_{A_{23}}(\frac{1}{2})\rangle|\psi^{k}_{A_{23}(8)}\rangle], \qquad (28)$$

where the subscripts ij to A and S indicate the pair that is antisymmetric or symmetric. However, SU(6) is not an exact symmetry. It is important to show that the quark-diagram scheme does not depend on it. Our formulation indeed confirms this requirement.

Earlier, Kohara gave a quark-diagram formulation for the quark-mixing-allowed decays of the antitriplet charmed baryons [10]. For  $B_c(\overline{3}) \rightarrow B(8) + M(8)$  decays (of which he

considered only the quark-mixing-allowed decays), it seems that Kohara [10] was motivated by Eq. (28), which is true only if SU(6) symmetry is exact, and chose the octet baryon state to be an equal combination of two different pairs of quarks both antisymmetric in flavor and spin, i.e., two of the terms in Eq. (28). One can easily check that the two terms are not orthonormal and they are not the conventional way of making the octet baryon wave function. Choosing a nonorthonormal basis for the quark states, if done correctly, is all right though complicated. (Kohara's paper did not provide enough detailed information for us to check directly his calculations without redoing all his formulation in the nonorthonormal basis he used. However, we can compare our results with his for the part he had calculated. Indeed, our results agree [see detailed comparisons given later by Eq. (63)].) However, we would still like to emphasize that the use of an orthonormal basis is much more convenient and, very importantly, that we have made sure that the quarkdiagram scheme does not depend upon SU(6) symmetry at all. For the  $B_c(3) \rightarrow B(10) + M(8)$  decays (of which Kohara also only considered the mixing-matrix-allowed decays), Kohara used the same basis as we use and our results agree.

In addition to the  $|B^{m,k}(8)\rangle$  states as given by Eq. (26), there are the states orthogonal to them, which are denoted by

$$|B_{\perp}^{m,k}(8)\rangle = b^* |\chi^m(\frac{1}{2})_A\rangle |\psi^k(8)_A\rangle - a^* |\chi^m(\frac{1}{2})_S\rangle |\psi^k(8)_S\rangle$$
(29)

and  $\langle B_{\perp}^{m,k}(8) | B^{m,k}(8) \rangle = 0$ . Nature does not realize these states, but they are there in the formalism and hence must be considered when completeness of these states is used.

Likewise, we can formulate the meson case, which is much simpler than the baryon case. We discuss it here for completeness and for comparison. Mesons are made out of  $\frac{1}{2}$ -spin quark-antiquark  $q'\bar{q}$  pairs belonging to the flavor  $[3]\times[\bar{3}]$  representation. They form flavor irreducible representations of  $3\times\bar{3} = 9 = 8 + 1$ , i.e., the nine quarkantiquark states can be decomposed into flavor [8] and [1] irreducible states denoted by  $|\phi^{j}(8)\rangle$  and  $|\phi(1)\rangle$ , respectively, where the superscript "j" denotes the eight particles in the [8] irreducible representations.

The transformation between the two bases, the quark basis and the irreducible-representation particle basis (both are orthonormal), can be written in a  $9 \times 9$  orthogonal matrix

	$ \phi^{\pi^+} angle$		1	0	0	0	0	0	0	0	0	$ u\bar{d}$	Ā	
	$\ket{\phi^{\scriptscriptstyle K^+}}$		0	1	0	0	0	0	0	0	0	us	>	
	$ \phi^{\pi^{-}} angle$		0	0	1	0	0	0	0	0	0	$ d\bar{u}$	>	
l	$\ket{\phi^{K^0}}$		0	0	0	1	0	0	0	0	0	$ d\bar{s} $	)	
	$ \phi^{K^{-}} angle$	=	0	0	0	0	1	0	0	0	0	$ s\bar{u} $	λ.	(30)
l	$ \mathcal{A}^{\overline{K}^0}\rangle$		0	0	0	0	0	1	0	0	0	$ s\bar{d} $	$\overline{\langle}$	
	$ \varphi\rangle$		0	0	0	0	0	0	$1/\sqrt{2}$	$-1/\sqrt{2}$	0	<i>u</i> ū	>	
	$ \phi\rangle$		0	0	0	0	0	0	$1/\sqrt{6}$	$1/\sqrt{6}$	$-2/\sqrt{6}$	$ d\bar{d}$	$\overline{\mathbf{b}}$	
	$ \psi^{\prime 0}\rangle$		0	0	0	0	0	0	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$	$ s\bar{s} $	>	
	$  \psi''\rangle $		1								1	1	1	

The nine equations given by the matrix equation can also be written out as

$$|M^{j}(8)\rangle = \sum_{\bar{q},q'} |\bar{q}q'\rangle \langle \bar{q}q'|M^{j}(8)\rangle$$
(31)

and

$$|M(1)\rangle = \sum_{\bar{q},q'} |\bar{q}q'\rangle \langle \bar{q}q'|M(1)\rangle, \qquad (32)$$

where the summation is for  $\overline{q} = \overline{u}, \overline{d}, \overline{s}$  and q' = u, d, s. These equations are obtained simply by multiplying the left-hand sides of (31) and (32) by

$$\hat{I} = \sum_{\bar{q},q'} |\bar{q}q'\rangle \langle \bar{q}q'|, \qquad (33)$$

which is the completeness relation of the orthonormal  $|q'\bar{q}\rangle$  basis in the quark-antiquark vector space.

The irreducible-representation states in spin are related to the spin-product space by

$$\begin{pmatrix} |\chi^{+}(1)\rangle \\ |\chi^{-}(1)\rangle \\ |\chi^{0}(1)\rangle \\ |\chi(0)\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} |+1/2+1/2\rangle \\ |-1/2-1/2\rangle \\ |-1/2+1/2\rangle \\ |+1/2-1/2\rangle \end{pmatrix}$$
(34)

and its inverse is trivially obtained (by using the transpose of the matrix):

$$\begin{pmatrix} |+1/2+1/2\rangle \\ |-1/2-1/2\rangle \\ |-1/2+1/2\rangle \\ |+1/2-1/2\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} |\chi^{+}(1)\rangle \\ |\chi^{-}(1)\rangle \\ |\chi^{0}(1)\rangle \\ |\chi(0)\rangle \end{pmatrix}.$$
(35)

For pseudoscalar mesons, the wave functions are simply given by

$$|M(1)\rangle = |\chi(0)\rangle |\phi(8)\rangle,$$
$$|M^{j}(8)\rangle = |\chi(0)\rangle |\phi^{j}(8)\rangle, \qquad (36)$$

where the superscript j indicates the eight different particles given in Eq. (30).

# III. QUARK-DIAGRAM SCHEME FOR $B_c(\overline{3}) \rightarrow B(10) + M(8)$

The light quarks of the charmed baryons belong to either a [ $\overline{3}$ ] or a [6] representation of the flavor SU(3). The  $\Lambda_c^+$ ,  $\Xi_c^{+A}$ , and  $\Xi_c^{0A}$  form a [ $\overline{3}$ ] representation. They all decay weakly. The  $\Omega_c^0$ ,  $\Xi_c^{+S}$ ,  $\Xi_c^{0S}$ ,  $\Sigma_c^{++}$ ,  $\Sigma_c^+$ , and  $\Sigma_c^0$  form a [6] representation; among them, however, only  $\Omega_c^0$  decays weakly (the  $\Sigma_c^{++,+,0}$  decay strongly to the  $\Lambda_c^+$  of the [ $\overline{3}$ ] representation and the  $\Xi_c^{+,0}$  decay electromagnetically).

We shall first discuss the simpler case of the decuplet baryon being in the decay products.

#### A. Formalism

Consider a particular charmed baryon  $B_c^{m,i_0}$  decaying into an octet meson  $M^{j_0}(8)$  and a decuplet baryon  $B^{m,k_0}(10)$ , where the subscript "0" signifies that we are discussing a specific baryon and a specific meson. The amplitude with the spin projection m,m' summed over is

$$\begin{split} A(i_{0} \rightarrow j_{0}k_{0}) &= \sum_{m,m'} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | M^{j_{0}} \rangle | B^{m',k_{0}}(10) \rangle , \text{ using Eq. (25) for } | B^{m,k_{0}}(10) \rangle , \\ &= \sum_{m,m'} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{3}{2})_{S_{t}} \rangle | \psi^{k_{0}}(10)_{S_{t}} \rangle , \text{ inserting Eq. (12)}, \\ &= \sum_{m,m',q_{i}} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{3}{2})_{S_{t}} \rangle | q_{1}q_{2}q_{3} \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle , \text{ using Eq. (36) for } M^{j_{0}}(8) \rangle , \\ &= \sum_{m,m',q_{i}} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0) \rangle | \psi^{j_{0}}(8) \rangle | \chi(\frac{3}{2})_{S_{t}} \rangle | q_{1}q_{2}q_{3} \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle , \text{ inserting Eq. (33),} \\ &= \sum_{m,m',\bar{q}_{i}} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0) \rangle | \chi^{m'}(\frac{3}{2})_{S_{t}} \rangle | \bar{q}q' \rangle | q_{1}q_{2}q_{3} \rangle \langle \bar{q}q' | \psi^{j_{0}}(8) \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle , \\ &= \sum_{m,m',\bar{q}_{i}} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0) \rangle | \chi^{m'}(\frac{3}{2})_{S_{t}} \rangle | \bar{q}q' \rangle | q_{1}q_{2}q_{3} \rangle \langle \bar{q}q' | \psi^{j_{0}}(8) \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle , \\ &= \sum_{\bar{q},q',q_{t}} \langle A(i_{0} \rightarrow \bar{q}q'q_{1}q_{2}q_{3}) \langle \bar{q}q' | \psi^{j_{0}}(8) \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle , \end{split}$$

(37)

where

$$\begin{aligned} \mathbf{A}(i_0 \to \overline{q}q'q_1q_2q_3) \\ &= \sum_{m,m'} \left\langle B_c^{m,i_0} | \hat{H}_W | \chi(0) \right\rangle | \chi^{m'}(\frac{3}{2})_{S_i} \right\rangle | \overline{q}q'q_1q_2q_3 \rangle \end{aligned}$$

$$(38)$$

are the quark-diagram amplitudes. Therefore Eq. (37) gives the particle amplitudes  $A(i_0 \rightarrow j_0 k_0)$  of  $B_c^{i_0}$  decaying into particles  $M^{j_0}(8)$  and  $B^{k_0}(10)$  in terms of the quark amplitudes  $A(i_0 \rightarrow \bar{q}q'q_1q_2q_3)$  of  $B_c^{i_0}$  decaying into quarks  $\bar{q}q'q_1q_2q_3$ . The coefficients  $\langle \bar{q}q' | \phi^{j_0}(8) \rangle$  and  $\langle q_1q_2q_3 | \psi^{k_0}(10)_{S_t} \rangle$ , each set of which forms elements of an orthogonal matrix, are those given in Eq. (30) and Eqs. (3) to (7), respectively.

Using the orthonormality of the coefficients, we can easily convert Eq. (37) to express the quark amplitudes in terms of the particle amplitudes

$$A(i_{0} \rightarrow \overline{q}q'q_{1}q_{2}q_{3}) = \sum_{j_{0},k_{0}} A(i_{0} \rightarrow j_{0}k_{0})$$
$$\times \langle \phi^{j_{0}}(8) | \overline{q}q' \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(10)_{S_{t}} \rangle,$$
(39)

using the orthonormality condition of the coefficients, which is the result of the orthonormality of the states.

We can also formulate the relation (37) in the basis given by Eqs. (13) and (14), which is also more convenient to apply since  $|\psi^{k_0}(10)_{S_t}\rangle$  is totally symmetric. Replacing "inserting Eq. (12)" by "inserting Eq. (18)" in Eq. (37), we obtain

$$A(i_0 \rightarrow j_0 k_0) \equiv \sum_{\overline{q}, q', q_i} A(i_0 \rightarrow \overline{q}q' \{q_a q_b\} q_c) \\ \times \langle q' \overline{q} | \phi^{j_0}(8) \rangle \langle \{q_a q_b\} q_c | \psi^{k_0}(10)_{S_t} \rangle,$$

$$(40)$$

where

$$A(i_0 \rightarrow \overline{q}q', \{q_a q_b\}q_c) = \sum_{m,m'} \langle B_c^{m,i_0} | \hat{H}_W | \chi(0) \rangle | \chi^{m'}(\frac{3}{2})_{S_t} \rangle | \overline{q}q', \{q_a q_b\}q_c \rangle.$$

$$(41)$$

Let us look more carefully at the amplitudes. For  $q_a = q_b$ ,

$$A(i_0 \rightarrow \overline{q}q', \{q_a q_b\}q_c) = A(i_0 \rightarrow \overline{q}q', q_a q_a q_c)$$
  
$$\equiv A_S[B_c(\overline{3}) \rightarrow B(10)M(8)],$$
  
(42)

 $A(i_0 \rightarrow \overline{q}q', \{q_a q_b\}q_c) = \frac{1}{\sqrt{2}} [A(i_0 \rightarrow \overline{q}q', q_a q_b q_c) + A(i_0 \rightarrow \overline{q}q', q_b q_a q_c)]$  $\equiv \sqrt{2} A_s [B_c(\overline{3}) \rightarrow B(10)M(8)], \tag{43}$ 

where we have used

$$A(i_0 \rightarrow \overline{q}q', q_a q_b q_c) = \frac{1}{2} [A(i_0 \rightarrow \overline{q}q', q_a q_b q_c) + A(i_0 \rightarrow \overline{q}q', q_b q_a q_c)]_{q_a \neq q_b}$$
$$\equiv A_S [B_c(\overline{3}) \rightarrow B(10)M(8)]. \quad (44)$$

We shall see later that this assumption gives results consistent with those using the SU(3)IR amplitudes. Equations (42) and (43) can be combined into one equation:

$$\begin{aligned} A(i_0 \rightarrow \overline{q}q', \{q_a q_b\}q_c) &= \left[\sqrt{2}(1 - \delta_{q_a q_b}) + \delta_{q_a q_b}\right] \\ &\times A_{\mathcal{S}}[B_c(\overline{3}) \rightarrow B(10)M(8)], \end{aligned}$$

which we substitute into Eq. (40) and obtain

$$A(i_0 \rightarrow j_0 k_0) \equiv \sum_{\overline{q}, q', q_i} \left[ \sqrt{2} (1 - \delta_{q_a q_b}) + \delta_{q_a q_b} \right]$$
$$\times A_S[B_c(\overline{3}) \rightarrow B(10)M(8)]$$
$$\times \langle q' \overline{q} | \phi^{j_0}(8) \rangle \langle \{q_a q_b\} q_c | \psi^{k_0}(10)_{S_t} \rangle.$$
(45)

Here in Eq. (37) and in Eq. (45) we see the important use of Eq. (12) and of Eq. (18) to convert particle amplitudes to quark amplitudes.

One can easily show by diagram drawing that the  $B_c(\overline{3}) \rightarrow B(10) + M(8)$  decays have contributions only from the W-exchange and the horizontal W-loop diagrams, i.e., the C and E types of amplitudes. In the A and B amplitudes, the two spectator quarks that are antisymmetrized in the initial charmed baryon state remain antisymmetrized after the weak-interaction decay and cannot contribute to make a B(10) whose wave function is totally symmetric. In the C and  $\mathcal{E}$  types of amplitudes, an appropriate quark pair  $\overline{q}_0 q_0$  is created so that the  $\overline{q}_0$  will combine with one of the quarks originating from the initial quark to form the meson  $j_0$ . Depending upon where the pair  $\overline{q}_0 q_0$  can be inserted in the diagrams, we have different types  $C_S$  and  $\mathcal{E}_S$  of amplitudes:  $C_{1S}$  for  $\overline{q}_0$  forming a meson with a spectator quark [which does not contribute in this case of B(10) in the final state];  $C_{2S}$  for  $\overline{q}_0$  forming a meson with the weak-interacting noncharmed quark;  $C'_{S}$  for  $\overline{q}_{0}$  forming a meson with the quark decayed from the charmed quark;  $\mathcal{E}_{S}$  for  $\overline{q}_{0}$  forming a meson with a spectator quark; and  $\mathcal{E}'_{S}$  for  $\overline{q_0}$  forming a meson with the quark decayed from the charmed quark (see Fig. 1). The quark  $q_0$  from the pair creation will combine with the other two quarks to become the final baryon  $k_0$ . Thus in Eq. (45) only  $q_1$  and  $q_2$  are summed over and Eq. (45) becomes

and for  $q_a \neq q_b$ ,



FIG. 1. Quark diagrams for the decay  $B_c(\overline{3}) \rightarrow B(10) + M(8)$ .

$$A(i_{0} \rightarrow j_{0}k_{0}) = C_{2S}[B_{c}(\overline{3}) \rightarrow B(10)M(8)][\sqrt{2}(1 - \delta_{q_{1}q_{3}}) + \delta_{q_{1}q_{3}}]\langle \overline{q}_{0}q_{2}'|\phi^{j_{0}}(8)\rangle\langle\{q_{1}q_{3}\}q_{0}|\psi^{k_{0}}(10)_{S_{t}}\rangle + C_{S}'[B_{c}(\overline{3}) \rightarrow B(10)M(8)] \times [\sqrt{2}(1 - \delta_{q_{1}q_{2}'}) + \delta_{q_{1}q_{2}'}] \times \langle \overline{q}_{0}q_{3}|\phi^{j_{0}}(8)\rangle\langle\{q_{1}q_{2}'\}q_{0}|\psi^{k_{0}}(10)_{S_{t}}\rangle + \mathcal{E}_{S}[B_{c}(\overline{3}) \rightarrow B(10)M(8)][\sqrt{2}(1 - \delta_{q_{3}q_{1}}) + \delta_{q_{3}q_{1}}]\langle \overline{q}_{0}q_{2}|\phi^{j_{0}}(8)\rangle\langle\{q_{3}q_{1}\}q_{0}|\psi^{k_{0}}(10)_{S_{t}}\rangle.$$
(46)

Using (46) for  $B_c(\overline{3}) \rightarrow B(10) + M(8)$  decays, we obtain column 2 of Tables Ia, Ib, and Ic. [In these tables we have dropped the parentheses that specify the decay of  $B_c(\overline{3}) \rightarrow B(10)M(8)$ .] We see that all  $B_c(\overline{3}) \rightarrow B(10)M(8)$  decays, 55 of them, can be expressed in terms of the three unknown amplitudes  $C_{2S}, C'_S$ , and  $\mathcal{E}_S$ . Therefore we obtain many relations among the particle decay amplitudes as shown in the next section.

Next we make an attempt to include and parametrize the effects of final-state interactions and SU(3) symmetry breaking. For final-state interactions we introduce an explicit factor  $e^{i\delta}$  to each isospin partial-wave amplitude. These phase shifts  $\delta$  in general have both real and imaginary parts; the imaginary component indicates the inelastic effect. SU(3)-violation effects can manifest themselves in several places. For example, the quark diagrams with  $s\overline{s}$  insertion are *a priori* different from those arising from  $u\overline{u}$  or  $d\overline{d}$  insertion. However, for simplicity of the presentation of the tables in the present paper, we parametrize SU(3) symmetry breaking only in the  $\mathcal{E}$  type of quark diagrams whose presence (i.e., contributions to the quark-mixing-singly-suppressed decay modes) is solely due to SU(3)-violation effects.

#### **B.** Results and tables

In the absence of effects from SU(3) breaking and finalstate interactions, the following relations can be obtained from the second column of Tables Ia to Ic, namely,

. .

$$|A(\Lambda_{c}^{+} \to \Sigma^{*+} \eta_{8})|^{2} = |A(\Xi^{0A} \to \Xi^{*0} \eta_{8})|^{2},$$

$$|A(\Xi_{c}^{0A} \to \Omega^{-}K^{+})|^{2} = 3|A(\Xi_{c}^{0A} \to \Xi^{*-}\pi^{+})|^{2}$$

$$= 3|\Lambda_{c}^{+} \to \Xi^{*0}K^{+})|^{2}$$

$$= 6|A(\Xi_{c}^{0A} \to \Xi^{*0}\pi^{0})|^{2}$$

$$= 6|A(\Lambda_{c}^{+} \to \Sigma^{*+}\pi^{0})|^{2}$$

$$= 6|A(\Lambda_{c}^{+} \to \Sigma^{*0}\pi^{+})|^{2}, \quad (47)$$

$$|A(\Lambda_{c}^{+} \to \Delta^{++} K^{-})|^{2} = 3|A(\Lambda_{c}^{+} \to \Delta^{+} \overline{K}^{0})|^{2}$$
$$= 3|A(\Xi_{c}^{0A} \to \Sigma^{*+} K^{-})|^{2}$$
$$= 6|A(\Xi_{c}^{0A} \to \Sigma^{*0} \overline{K}^{0})|^{2}$$

for quark-mixing-allowed modes;

$$|A(\Xi_{c}^{0A} \to \Sigma^{*0} \pi^{0})|^{2} = 3|A(\Xi_{c}^{0A} \to \Sigma^{*0} \eta_{8})|^{2},$$

$$A(\Xi_{c}^{0A} \to \Sigma^{*-} \pi^{+})|^{2} = |A(\Xi_{c}^{0A} \to \Xi^{*-} K^{+})|^{2}$$

$$= 4|A(\Lambda_{c}^{+} \to \Delta^{0} \pi^{+})|^{2}$$

$$= 4|A(\Xi_{c}^{+A} \to \Xi^{*0} K^{+})|^{2}$$

$$= 8|A(\Lambda_{c}^{+} \to \Sigma^{*0} K^{+})|^{2}$$

$$= 8|A(\Xi_{c}^{+A} \to \Sigma^{*0} \pi^{+})|^{2}$$

$$= 8|A(\Xi_{c}^{+A} \to \Sigma^{*+} \pi^{0})|^{2}, \quad (48)$$

TABLE I. Quark-diagram am	plitudes for decays of $B_c(3) \rightarrow B(10) + h$	1(8). In Tables I–III SU(3) symmetry breaking is I	parametrized only in terms of the ${\mathcal E}$ type of quark diagrams.
Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
		(a) Quark-mixing-allowed modes	
$\Lambda_c^+ { ightarrow} \Delta^{++} K^-$	$\mathcal{C}_{S}^{\prime}$	$\mathcal{C}_{S}^{\prime}$	$\frac{1}{2} [C_{s}^{c} \exp(i \partial_{2}^{\mathrm{tK}}) + \mathcal{C}_{s}^{c} \exp(i \partial_{1}^{\mathrm{tK}})]$
$\rightarrow \Delta^+ \vec{K}^0$	$\frac{1}{\sqrt{3}}C_{\rm S}'$	$\frac{1}{\sqrt{3}C_{S}}$	$\frac{1}{2\sqrt{3}} \left[ 3\mathcal{C}_{s}^{exp}(i\delta_{2}^{dK}) - \mathcal{C}_{s}^{exp}(i\delta_{1}^{dK}) \right]$
$ ightarrow \Sigma^{*0} \pi^+$	$\frac{1}{\sqrt{6}}C_{2S}$	$\frac{1}{\sqrt{6}}c_{2S}$	$\frac{1}{\sqrt{6}}\mathcal{C}_{2S}\mathrm{exp}(i\delta_2^{\mathbb{X}\pi})$
${\rightarrow} \Sigma^{*+} \pi^0$	$\frac{1}{\sqrt{6}}C_{2S}$	$\frac{1}{\sqrt{6}}c_{2S}$	$\frac{1}{\sqrt{6}}\mathcal{C}_{2S}\mathrm{exp}(i\delta_2^{\mathrm{v}}\pi)$
${\rightarrow} \Sigma^{*+} \eta_0$	$\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$rac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$\frac{1}{3}(\mathcal{C}_{2S} + \mathcal{C}_{S}') \exp(i \delta_{1}^{\Sigma} \eta_{0})$
${\rightarrow} \Sigma^{*+} \eta_8$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S}{-}2\mathcal{C}_{S}')$	$rac{1}{3\sqrt{2}}(\mathcal{C}_{2S}\!-\!2\mathcal{C}_S')$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S}-2\mathcal{C}_{S}^{*})\exp(i\delta_{1}^{X}\eta_{8})$
$\rightarrow \Pi_{*0} X^+$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{\sqrt{3}}\mathcal{C}_{2,5}\mathrm{exp}(i\delta_1^{\Xi^{\ast}K})$
$\Xi_c^{+A} {\longrightarrow} \Sigma^{*+} \overline{K}^0$	0	0	0
$\rightarrow \Pi *^0 \pi^+$	0	0	0
$\Xi_c^{0A} { ightarrow} \Omega^- K^+$	$-\mathcal{C}_{2S}$	$-\mathcal{C}_{2S}$	$-\mathcal{C}_{2,\mathrm{Sexp}}(i\delta_{1/2}^{\Omega K})$
$ ightarrow \Sigma *^{0} ar{K}^{0}$	$-rac{1}{\sqrt{6}}C'_{S}$	$-rac{1}{\sqrt{6}}\mathcal{C}'_{S}$	$-\frac{1}{3\sqrt{6}}\left[4\mathcal{C}_{\mathrm{S}}^{\mathrm{e}}\mathrm{exp}(i\delta_{3,2}^{\mathrm{X}\ast K})-\mathcal{C}_{\mathrm{S}}^{\mathrm{e}}\mathrm{exp}(i\delta_{1,2}^{\mathrm{X}\ast K})\right]$
$\rightarrow \Sigma^{*+}K^{-}$	$-rac{1}{\sqrt{3}}\mathcal{C}_{S}'$	$-rac{1}{\sqrt{3}}\mathcal{C}'_{S}$	$-\frac{1}{3\sqrt{3}}\left[2\mathcal{C}_{S}^{\prime}\mathrm{exp}(i\delta_{3/2}^{\mathbb{X}}^{*}\kappa)+\mathcal{C}_{S}^{\prime}\mathrm{exp}(i\delta_{1/2}^{\mathbb{X}}^{*}\kappa)\right]$
$\stackrel{+}{\rightarrow} \pi^+$	$-rac{1}{\sqrt{3}}\mathcal{C}_{2S}$	$-\frac{1}{\sqrt{3}}\mathcal{C}_{2S}$	$-\frac{1}{3\sqrt{3}}\left[2\mathcal{C}_{23}\mathrm{exp}(i\delta_{32}^{\Xi^{*}\pi})+\mathcal{C}_{23}\mathrm{exp}(i\delta_{12}^{\Xi^{*}\pi})\right]$
$\to \Pi^{*0} \boldsymbol{\pi}^0$	$-rac{1}{\sqrt{6}}\mathcal{C}_{2S}$	$-\frac{1}{\sqrt{6}}c_{2S}$	$-\frac{1}{3\sqrt{6}} \left[ 4\mathcal{C}_{2S} \mathrm{exp}(i \delta_{3/2}^{\Xi^*\pi}) - \mathcal{C}_{2S} \mathrm{exp}(i \delta_{1/2}^{\Xi^*\pi}) \right]$
$ ightarrow \Xi^{st_0} \eta_0$	$-\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_{5})$	$-rac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_{S})$	$-\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_{S})\exp(i\delta_{1/2}^{\Xi^{*}\eta_{0}})$
$ ightarrow \Pi^{*0} \eta_8$	$-rac{1}{3\sqrt{2}}(\mathcal{C}_{2S}{-2\mathcal{C}'_S})$	$-rac{1}{3\sqrt{2}}(\mathcal{C}_{2S}\!-\!2\mathcal{C}_{S}')$	$-\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S}-2\mathcal{C}_{S}')\mathrm{exp}(i\delta_{1/2}^{\Xi^{*}\eta_{S}})$

		· · · · ·	Amplitudes with
	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	SU(3)-symmetry breaking and final-state interactions
a_	$-\mathcal{C}'_{\rm S}$	(b) Quark-mixing-suppressed modes $-C'_{s} + \mathcal{E}_{s}$	$\frac{1}{15}\{-3(\mathcal{C}_{2S}+\mathcal{E}_{S})\exp(i\partial_{x}^{\lambda}\pi)+[3(\mathcal{C}_{2S}+\mathcal{E}_{S})-5(\mathcal{C}_{S}'$
			$-\mathcal{E}_{S}]]\exp(i\delta_{1}^{\Delta\pi})-10(\mathcal{C}_{S}^{\prime}-\mathcal{E}_{S})\exp(i\delta_{0}^{\Delta\pi})\}$
	$-rac{1}{\sqrt{3}}\mathcal{C}_{2S}$	$-rac{1}{\sqrt{3}}(\mathcal{C}_{2S}+\mathcal{E}_{S})$	$\frac{1}{10\sqrt{3}} \bigg\{ -6(\mathcal{C}_{2S} + \mathcal{E}_{S}) \exp(i\partial_{2}^{\lambda}\pi) + \bigg[\frac{20}{3}(\mathcal{C}_{S}' - \mathcal{E}_{S}) - 4(\mathcal{C}_{2S}) \bigg] \bigg\}$
			$+ \mathcal{E}_{S} \bigg] \exp(i \delta_{1}^{A} \pi) - \frac{20}{3} (\mathcal{C}_{S}' - \mathcal{E}_{S}) \exp(i \delta_{0}^{A} \pi) \bigg\}$
0	$\frac{1}{\sqrt{6}}(\mathcal{C}'_{S}-\mathcal{C}_{2S})$	$\frac{1}{\sqrt{6}}\left(\mathcal{C}_{S}^{\prime}\!-\!\mathcal{C}_{2S}\!-\!2\mathcal{E}_{S}\right)$	$\frac{1}{5\sqrt{6}}\left[-6(\mathcal{C}_{2S}+\mathcal{E}_{S})\exp(i\partial_{2}^{\lambda}\pi)+\left(-\frac{5}{3}(\mathcal{C}_{S}'-\mathcal{E}_{S})+(\mathcal{C}_{2S}+\mathcal{E}_{S})\right)\right]$
			$\times \exp(i\delta_1^{\Delta^n}) + \frac{20}{3}(C_s' - \mathcal{E}_S) \exp(i\delta_0^{\Delta^n}) \bigg]$
	$-\frac{1}{3}(\mathcal{C}_{S}^{\prime}-\mathcal{C}_{2S})$	$-rac{1}{3}(\mathcal{C}'_{\mathrm{S}}-\mathcal{C}_{2S})$	$-\frac{1}{3}(\mathcal{C}_{s}^{2}-\mathcal{C}_{2s})\exp(i\delta_{2}^{\Delta}n_{0})$
~	$-rac{1}{3\sqrt{2}}(\mathcal{C}_S'+\mathcal{C}_{2S})$	$-rac{1}{3\sqrt{2}}(\mathcal{C}_{S}'+\mathcal{C}_{2S})$	$-\frac{1}{3\sqrt{2}}(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S})\exp(i\delta_{2}^{\Delta\eta_{S}})$
60 V0	$-rac{1}{\sqrt{3}}C_{ m S}^{\prime}$	$rac{1}{\sqrt{3}}(-\mathcal{C}_{ m s}^{2}+\mathcal{E}_{S})$	$-\frac{1}{3\sqrt{3}}\left[\left(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S}\right)\exp(i\delta_{2}^{\Sigma\ast K})+2\left(\mathcal{C}_{S}^{\prime}-\mathcal{C}_{2S}\right)\exp(i\delta_{1}^{\Sigma\ast K})\right]$
+	$-\frac{1}{\sqrt{6}}c_{2S}$	$-rac{1}{\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$-\frac{\sqrt{2}}{3\sqrt{3}}\left[\left(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S}\right)\exp(i\delta_{2}^{*}*^{\prime})-\left(\mathcal{C}_{S}^{\prime}-\mathcal{C}_{2S}\right)\exp(i\delta_{1}^{*}*^{\prime})\right]$
	${\cal C}_{S}^{\prime}$	$\mathcal{C}'_S + \mathcal{E}_S$	$\frac{1}{2}(\mathcal{C}'_{\rm S} + \mathcal{E}_{\rm S})[\exp(i\partial_2^{\rm AK}) + \exp(i\partial_1^{\rm AK})]$
£4	$\frac{1}{\sqrt{3}}C_s'$	$rac{1}{\sqrt{3}}(\mathcal{C}'_{S}+\mathcal{E}_{S})$	$\frac{1}{2\sqrt{3}}\left(\mathcal{C}_{S}^{\prime}+\mathcal{E}_{S}\right)\left[3\exp(i\partial_{2}^{\Delta K})-\exp(i\partial_{1}^{\Delta K})\right]$
<b>μ</b> +	$\frac{1}{\sqrt{6}c_{2S}}$	$rac{1}{\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$\frac{1}{\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_S)\mathrm{exp}(i\delta_2^{\mathbf{X}}{}^*\pi)$

TABLE I. (Continued).

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1	(Continued).
	TABLE I.

Amplitudes with SU(3)-symmetry breaking and final-state interactions	$rac{1}{\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_S)\mathrm{exp}(i\delta_2^{\mathbf{X}^*\pi})$	$\frac{1}{3}(\mathcal{C}_{S}'+\mathcal{C}_{2S}+2\mathcal{E}_{S})\exp(i\delta_{1}^{\mathbf{X}^{*}}\eta_{0})$	$rac{1}{2\sqrt{2}}(\mathcal{C}_{2S}-2\mathcal{C}_{S}^{\prime}-\mathcal{E}_{S})\mathrm{exp}(i\delta_{1}^{\Sigma^{*}\eta_{S}})$	$\frac{1}{\sqrt{3}}(\mathcal{C}_{2S} + \mathcal{E}_S) \exp(i \delta_{\Gamma}^{\Xi * K})$	$rac{1}{\sqrt{3}}(\mathcal{C}_{S}'+\mathcal{E}_{S})\exp(i\partial_{2}^{\Delta K})$	$rac{1}{\sqrt{3}}(\mathcal{C}_{S}'+\mathcal{E}_{S})\exp(i\partial_{2}^{\Delta K})$	$\frac{1}{6\sqrt{3}} [2(2\mathcal{C}_{2S} - \mathcal{C}_{5}') \exp(i\delta_{2}^{\Sigma^{*}}\pi) - 3(\mathcal{C}_{5}' + 2\mathcal{C}_{2S} + \mathcal{E}_{5}) \exp(i\delta_{1}^{\Sigma^{*}}\pi)$	$-(\mathcal{C}'_{s}-2\mathcal{C}_{2S}+3\mathcal{E}_{S})\mathrm{exp}(i\delta_{0}^{x^{*}\pi})]$	$\frac{1}{6\sqrt{3}} \left[ 2(2\mathcal{C}_{2S} - \mathcal{C}'_{S}) \exp(i\delta_{2}^{X*}\pi) + 3(\mathcal{C}'_{S} + 2\mathcal{C}_{2S} + \mathcal{E}_{S}) \exp(i\delta_{1}^{X*}\pi) \right]$	$-(\mathcal{C}_{3}^{\prime}-2\mathcal{C}_{2S}+3\mathcal{E}_{S})\exp(i\delta_{0}^{X^{*}\pi})]$	$\frac{1}{6\sqrt{3}} \left[ 4(2\mathcal{C}_{2S} - \mathcal{C}'_{S}) \exp(i\delta_{2}^{\mathbb{X}^{*}\pi}) + (\mathcal{C}'_{S} + 2\mathcal{C}_{2S} + 3\mathcal{E}_{S}) \exp(i\delta_{0}^{\mathbb{X}^{*}\pi}) \right]$	$\frac{\sqrt{2}}{3}(C'_{5}+\mathcal{C}_{2S}+\mathcal{E}_{S})\exp(i\delta_{1}^{2^{*}*}\eta_{0})$	$\frac{1}{6}(-\mathcal{C}_{S}'+2\mathcal{C}_{2S}-\mathcal{E}_{S})\exp(i\delta_{1}^{\Sigma^{*}\eta_{S}})$	$\frac{1}{2\sqrt{3}}\left[\left(-\mathcal{C}_{S}^{\prime}+2\mathcal{C}_{2S}-\mathcal{E}_{S}\right)\exp(i\delta_{1}^{\mathbb{Z}^{*}K})-\left(\mathcal{C}_{S}^{\prime}+2\mathcal{C}_{2S}+\mathcal{E}_{S}\right)\exp(i\delta_{0}^{\mathbb{Z}^{*}K})\right]$	$\frac{1}{2\sqrt{3}}\left[(-\mathcal{C}_{S}^{\prime}+2\mathcal{C}_{2S}-\mathcal{E}_{S})\exp(i\delta_{1}^{\overline{a}^{\ast}K})+(\mathcal{C}_{S}^{\prime}+2\mathcal{C}_{2S}+\mathcal{E}_{S})\exp(i\delta_{0}^{\overline{a}^{\ast}K})\right]$
Amplitudes with SU(3)-symmetry breaking	$\frac{1}{\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$rac{1}{3}(\mathcal{C}'_S+\mathcal{C}_{2S}+2\mathcal{E}_S)$	$rac{1}{3\sqrt{7}}(\mathcal{C}_{2S}\!-\!2\mathcal{C}_{S}'\!-\!\mathcal{E}_{S})$	$rac{1}{\sqrt{3}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$rac{1}{\sqrt{3}}\left(\mathcal{C}_{S}'+\mathcal{E}_{S} ight)$	$rac{1}{\sqrt{3}}\left(\mathcal{C}_{S}'+\mathcal{E}_{S} ight)$	$-rac{1}{\sqrt{3}}(\mathcal{C}'_{s}+\mathcal{E}_{s})$		$\frac{2}{\sqrt{3}}C_{2S}$		$rac{1}{2\sqrt{3}}(-\mathcal{C}_{S}'+2\mathcal{C}_{2S}-\mathcal{E}_{S})$	$rac{\sqrt{2}}{3}(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S}+\mathcal{E}_{S})$	$rac{1}{6}(-\mathcal{C}_{S}'+2\mathcal{C}_{2S}-\mathcal{E}_{S})$	$-rac{1}{\sqrt{3}}\left(\mathcal{C}_{S}'+\mathcal{E}_{S} ight)$	$\frac{2}{\sqrt{3}}C_{2S}$
Amplitudes with SU(3) symmetry	$\frac{1}{\sqrt{6}}c_{2S}$	$\frac{1}{3}(\mathcal{C}'_{s}+\mathcal{C}_{2,s})$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S}-2\mathcal{C}_{S}^{\prime})$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{\sqrt{3}C_S'}$	$\frac{1}{\sqrt{3}C_S}$	$-\frac{1}{\sqrt{3}}c_s'$		$\frac{2}{\sqrt{3}}C_{2S}$		$\frac{1}{2\sqrt{3}}(-\mathcal{C}_S'+2\mathcal{C}_{2S})$	$\frac{\sqrt{2}}{3}(\mathcal{C}'_{s}+\mathcal{C}_{2,s})$	$\frac{1}{6}(-\mathcal{C}'_S+2\mathcal{C}_{2S})$	$-\frac{1}{\sqrt{3}}C_{S}$	$\frac{2}{\sqrt{3}}C_{2S}$
Reaction	$ ightarrow \Sigma^{*+} \pi^0$	$ ightarrow \Sigma^{*+} \eta_0$	${ ightarrow}\Sigma^{*+}\eta_8$	${}^+\Pi^{*0}K^+$	$\Xi_c^{0A} \!  ightarrow \! \Delta^0 ec{K}^0$	$ ightarrow \Delta^+ K^-$	$\rightarrow \Sigma^{*+} \pi^{-}$		$ ightarrow \Sigma^{*-} \pi^+$		$ ightarrow \Sigma^{*0} \pi^0$	${\rightarrow} \Sigma^{*0} \eta_0$	$ ightarrow \Sigma^{*0} \eta_8$	• <sup>™</sup> <sup>™</sup> <sup>™</sup>	${\rightarrow} \Pi^{+}K^{+}$

Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
$\Lambda^+_+ \rightarrow \Delta^+ K^0$	0	(c) Quark-mixing-doubly-suppress	ed modes 0
$\rightarrow \Delta^0 K^+$	0	0	0
$\Xi_c^{+A} { ightarrow} \Delta^{++} \pi^-$	$\mathcal{C}_{S}'$	$\mathcal{C}_{S}'$	$\frac{1}{15} \big[ \left[ 3\mathcal{C}_{2S} \mathrm{exp}(i \delta_{3/2}^{\Delta,\pi}) + (5\mathcal{C}_{S}' - 3\mathcal{C}_{2S}) \mathrm{exp}(i \delta_{3/2}^{\Delta,\pi}) + 10\mathcal{C}_{S}' \mathrm{exp}(i \delta_{1/2}^{\Delta,\pi}) \big] \right]$
$ ightarrow \Delta^0 oldsymbol{\pi}^+$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{\sqrt{3}}C_{2S}$	$\frac{1}{30\sqrt{3}} \left[ 18\mathcal{C}_{2S} \exp(i\partial_{3/2}^{\Lambda}) - 4(5\mathcal{C}_{S}' - 3\mathcal{C}_{2S}) \exp(i\partial_{3/2}^{\Lambda}) + 20\mathcal{C}_{S}' \exp(i\partial_{1/2}^{\Lambda}) \right]$
$\rightarrow \Delta^+ \pi^0$	$\frac{1}{\sqrt{6}} \left(-\mathcal{C}_{S}' + \mathcal{C}_{2S}\right)$	$\frac{1}{\sqrt{6}}(-\mathcal{C}'_S+\mathcal{C}_{2S})$	$\frac{1}{15\sqrt{6}} \Big[ \big[ 3\mathcal{C}_{2s} \exp(i\delta_{5/2}^{\Delta\pi}) + (5\mathcal{C}_{s}' - 3\mathcal{C}_{2s}) \exp(i\delta_{3/2}^{\Delta\pi}) - 20\mathcal{C}_{s}' \exp(i\delta_{1/2}^{\Delta\pi}) \big] \Big]$
$ ightarrow \Delta^+ oldsymbol{\eta}_0$	$\frac{1}{3}(\mathcal{C}'_{S}+\mathcal{C}_{2S})$	$\frac{1}{3}(\mathcal{C}'_{S}+\mathcal{C}_{2S})$	$\frac{1}{3}(C'_S+\mathcal{C}_{2S})\exp(i\delta^{\Delta}_{3/2})$
$ ightarrow \Delta^+ \eta_8$	$\frac{1}{3\sqrt{2}}(\mathcal{C}'_{2}+\mathcal{C}_{2S})$	$\frac{1}{3\sqrt{2}}(\mathcal{C}'_S+\mathcal{C}_{2S})$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{3}^{4}+\mathcal{C}_{2S})\exp(i\delta_{3/2}^{\Delta}n_{8})$
$ ightarrow \Sigma *^0 K^+$	$\frac{1}{\sqrt{6}}c_{2S}$	$\frac{1}{\sqrt{6}}C_{2S}$	$\frac{1}{3\sqrt{6}} \left[ 2(\mathcal{C}_{2S} + \mathcal{C}'_{S}) \exp(i \delta_{3/2}^{**K}) - (2\mathcal{C}'_{S} - \mathcal{C}_{2S}) \exp(i \delta_{1/2}^{**K}) \right]$
$ ightarrow \Sigma^{*+} K^0$	$\frac{1}{\sqrt{3}}c_{\rm s}^{\prime}$	$\frac{1}{\sqrt{3}c_s}$	$\frac{1}{3\sqrt{3}} \left[ \left( \mathcal{C}_{2S} + \mathcal{C}'_{S} \right) \exp(i \delta_{3,2}^{\mathbb{X} * K}) + 2 \left( 2 \mathcal{C}'_{S} - \mathcal{C}_{2S} \right) \exp(i \delta_{1,2}^{\mathbb{X} * K}) \right]$
$\Pi_c^{0A} \!  ightarrow \! \Delta^+  \pi^-$	$\frac{1}{\sqrt{3}c_s}$	$\frac{1}{\sqrt{3}c_s}$	$\frac{1}{15\sqrt{3}} \left[9\mathcal{C}_{2S} \exp(i\delta_{5/2}^{\Delta\pi}) + 2(5\mathcal{C}_{S}' - 7\mathcal{C}_{2S}) \exp(i\delta_{3/2}^{\Delta\pi}) + 5(\mathcal{C}_{S}' + \mathcal{C}_{2S}) \exp(i\delta_{1/2}^{\Delta\pi})\right]$
$ ightarrow \Delta^- oldsymbol{\pi}^+$	$\mathcal{C}_{2S}$	$\mathcal{C}_{2S}$	$\frac{1}{15} [3\mathcal{C}_{2S} \exp(i\delta_{S/2}^{\Delta,\pi}) - (5\mathcal{C}_{S}' - 7\mathcal{C}_{2S}) \exp(i\delta_{S/2}^{\Delta,\pi}) + 5(\mathcal{C}_{S}' + \mathcal{C}_{2S}) \exp(i\delta_{1/2}^{\Delta,\pi})]$
$ ightarrow \Delta^0 \pi^0$	$\frac{1}{\sqrt{6}} \left(-\mathcal{C}_S' + \mathcal{C}_{2S}\right)$	$\frac{1}{\sqrt{6}}(-\mathcal{C}'_S+\mathcal{C}_{2S})$	$\frac{1}{15\sqrt{6}} \left[ 18\mathcal{C}_{2S} \exp(i\delta_{3/2}^{\Delta\pi}) - (5\mathcal{C}_{S}' - 7\mathcal{C}_{2S}) \exp(i\delta_{3/2}^{\Delta\pi}) - 10(\mathcal{C}_{S}' + \mathcal{C}_{2S}) \exp(i\delta_{1/2}^{\Delta\pi}) \right]$
$ ightarrow \Delta  oldsymbol{\eta}_0$	$\frac{1}{3}(\mathcal{C}'_{S}+\mathcal{C}_{2S})$	$\frac{1}{3}(\mathcal{C}'_{S}+\mathcal{C}_{2S})$	$\frac{1}{3}(C'_S + \mathcal{C}_{2S}) \exp(i\delta_{3/2}^{\Delta} \eta_0)$
$ ightarrow \Delta \eta_8$	$\frac{1}{3\sqrt{2}}(\mathcal{C}'_{5}+\mathcal{C}_{2S})$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S})$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{3}^{4}+\mathcal{C}_{2S})\exp(i\delta_{3/2}^{\Delta \eta_{8}})$
$ ightarrow \Sigma^{*0} K^0$	$\frac{1}{\sqrt{6}c_s}$	$\frac{1}{\sqrt{6}c_s}$	$\frac{1}{3\sqrt{6}} \left[ 2(C_{3}' + \mathcal{C}_{2S}) \exp(i\delta_{3/2}^{\mathbb{X}*K}) + (C_{3}' - 2\mathcal{C}_{2S}) \exp(i\delta_{1/2}^{\mathbb{X}*K}) \right]$
$\rightarrow \Sigma^{*-K^+}$	$\frac{1}{\sqrt{3}}c_{2s}$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{3\sqrt{3}}\left[\left(\mathcal{C}_{S}'+\mathcal{C}_{2S}\right)\exp(i\delta_{3/2}^{\Sigma *K})-\left(\mathcal{C}_{S}'-2\mathcal{C}_{2S}\right)\exp(i\delta_{1/2}^{\Sigma *K})\right]$

TABLE I. (Continued).

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(53)

$$\begin{split} |A(\Lambda_c^+ \to \Delta^{++} \pi^-)|^2 &= |A(\Xi_c^{+A} \to \Delta^{++} K^-)|^2 \\ &= 3|A(\Lambda_c^+ \to \Sigma^{*+} K^0)|^2 \\ &= 3|A(\Xi_c^{+A} \to \Delta^{+} \overline{K}^0)|^2 \\ &= 3|A(\Xi_c^{0A} \to \Delta^0 \overline{K}^0)|^2 \\ &= 3|A(\Xi_c^{0A} \to \Xi^{*0} K^0)|^2 \\ &= 3|A(\Xi_c^{0A} \to \Sigma^{*+} \pi^-)|^2 \\ &= 3|A(\Xi_c^{0A} \to \Delta^{+} K^-)|^2 \end{split}$$

for quark-mixing-suppressed modes;

$$\begin{aligned} |A(\Xi_{c}^{+A} \to \Delta^{+} \eta_{8})|^{2} &= |A(\Xi_{c}^{0A} \to \Delta^{0} \eta_{8})|^{2}, \\ |A(\Xi_{c}^{0A} \to \Delta^{0} \pi^{0})|^{2} &= 2|A(\Xi_{c}^{+A} \to \Delta^{+} \pi^{0})|^{2}, \\ |A(\Xi_{c}^{+A} \to \Delta^{++} \pi^{-})|^{2} &= 3|A(\Xi_{c}^{+A} \to \Sigma^{*+} K^{0})|^{2} \\ &= 3|A(\Xi_{c}^{0A} \to \Delta^{+} \pi^{-})|^{2} \\ &= 6|A(\Xi_{c}^{0A} \to \Delta^{+} \pi^{+})|^{2} \\ &= 6|A(\Xi_{c}^{0A} \to \Sigma^{*0} K^{0})|^{2}, \quad (49) \\ &= 3|A(\Xi_{c}^{0A} \to \Delta^{-} \pi^{+})|^{2} \\ &= 3|A(\Xi_{c}^{0A} \to \Sigma^{*-} K^{+})|^{2} \\ &= 6|A(\Xi_{c}^{+A} \to \Sigma^{*0} K^{+})|^{2} \end{aligned}$$

for quark-mixing-doubly-suppressed modes; and many relations between quark-mixing-allowed, -suppressed, and -doubly-suppressed decay modes, for example,

$$|A(\Lambda_{c}^{+} \to \Delta^{0} \pi^{+})|^{2} = 2s_{1}^{2}|A(\Lambda_{c}^{+} \to \Sigma^{*+} \pi^{0})|^{2},$$
  

$$|A(\Lambda_{c}^{+} \to \Delta^{++} \pi^{-})|^{2} = s_{1}^{2}|A(\Lambda_{c}^{+} \to \Delta^{++} K^{-})|^{2},$$
  

$$|A(\Xi_{c}^{+A} \to \Sigma^{*0} K^{+})|^{2} = s_{1}^{4}|A(\Lambda_{c}^{+} \to \Sigma^{*+} \pi^{0})|^{2},$$
 (50)  

$$|A(\Xi_{c}^{+A} \to \Delta^{++} \pi^{-})|^{2} = s_{1}^{4}|A(\Lambda_{c}^{+} \to \Delta^{++} K^{-})|^{2}.$$

Several comments are in order. (i) Table Ia and relations (47) for quark-mixing-allowed decays  $B_c(\overline{3}) \rightarrow B(10) + M(8)$  were also obtained previously by Kohara [10] and his results are in agreement with ours. (ii) Though all the above quark-diagram relations are obtained in the absence of effects from SU(3) violations and final-state interactions, if each quark-diagram amplitude in the SU(3) relation contains only a single isospin phase shift, then such a relation holds

even in the presence of final-state interactions because of SU(3) symmetry for phase shifts. Examples are the first relation in Eqs. (47) and (49). The same is true for the relations obtained below. (iii) We note that the quark-mixing-allowed decays of an antitriplet charmed baryon into a decuplet baryon and a pseudoscalar meson can occur only through *W*-exchange diagrams. The experimental measurement of  $\Lambda_c^+ \rightarrow \Delta^{++} K^-$  [2] indicates that the *W*-exchange mechanism plays a significant role in charmed baryon decays. (iv) The quark-mixing-allowed decays of  $\Xi_c^{+A}$  and quark-mixing-doubly-suppressed decays of  $\Lambda_c^+$  into a decuplet baryon are prohibited in the quark-diagram scheme:

$$|A(\Xi_{c}^{+A} \to \Sigma^{*+} \overline{K}^{0})|^{2} = 0, | \quad A(\Xi_{c}^{+A} \to \Xi^{*0} \pi^{+})|^{2} = 0, \\ |A(\Lambda_{c}^{+} \to \Delta^{+} K^{0})|^{2} = 0, | \quad A(\Lambda_{c}^{+} \to \Delta^{0} K^{+})|^{2} = 0.$$
(51)

In the SU(3)IR approach of Savage and Springer (SS) [6], these decays are governed by the reduced matrix element  $\alpha$ defined in Eq. (17) of Ref. [6]. However, we see that they are forbidden in the quark-diagram scheme since they are given by the quark diagram  $\mathcal{A}$  or  $\mathcal{B}'$  and they give zero contribution, as we discussed before, because of the unmatching symmetry properties of the antitriplet charmed baryon and the decuplet baryon. Furthermore, we note that the SU(3)IR approach of SS will predict the above relations (48)–(51) only if the reduced matrix elements  $\alpha$  and  $\gamma$  make no contributions. As a consequence, there are only two independent SU(3) reduced matrix elements  $\beta$  and  $\delta$ . The quark-diagram amplitudes and the SU(3)-symmetry parameters are related by

$$\beta = \frac{1}{2} (C'_{S} + C_{2S}), \quad \delta = \frac{1}{2} (C'_{S} - C_{2S}), \quad \alpha = \gamma = 0.$$
(52)

# IV. QUARK-DIAGRAM SCHEME FOR $B_c(\overline{3}) \rightarrow B(8) + M(8)$

## A. The formalism

The formalism is very similar to that given in Sec. III A for the decuplet baryon in the final state except for the complication that the octet baryons are made up of two orthonormal parts, Eq. (26). We shall see that all it does is that each type of quark amplitude A will be made up of two independent ones, the symmetric and the antisymmetric. Following the similar procedure used in Eqs. (37) and (50), we derive

$$\begin{split} A(i_{0} \rightarrow j_{0}k_{0}) &= \sum_{m,m'} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | M^{j_{0}}(8) \rangle | B^{m',k_{0}}(8) \rangle \\ &= \sum_{m,m'} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | M^{j_{0}}(8) \rangle [a | \chi^{m'}(\frac{1}{2})_{A} \rangle | \psi^{k_{0}}(8)_{A} \rangle + b | \chi^{m'}(\frac{1}{2})_{S} \rangle | \psi^{k_{0}}(8)_{S} \rangle ] \\ &= \sum_{m,m',q_{i}} a \langle B_{c}^{m,i_{0}} | H_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{1}{2})_{A} \rangle | q_{1}q_{2}q_{3} \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(8)_{A} \rangle \\ &+ \sum_{m,m',q_{i}} b \langle B_{c}^{m,i_{0}} | H_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{1}{2})_{S} \rangle | q_{1}q_{2}q_{3} \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(8)_{S} \rangle \\ &= \sum_{m,m',q_{i}} a \langle B_{c}^{m,i_{0}} | H_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{1}{2})_{A} \rangle | [q_{1}q_{2}]q_{3} \rangle \langle [q_{1}q_{2}]q_{3} | \psi^{k_{0}}(8)_{A} \rangle \\ &+ \sum_{m,m',q_{i}} b \langle B_{c}^{m,i_{0}} | H_{W} | M^{j_{0}}(8) \rangle | \chi^{m'}(\frac{1}{2})_{S} \rangle | \{q_{1}q_{2}\}q_{3} \rangle \langle \{q_{1}q_{2}\}q_{3} | \psi^{k_{0}}(8)_{S} \rangle. \end{split}$$

1

To decompose the meson state into the  $q'\bar{q}$  state, we insert in Eq. (53) the completeness relation Eq. (33) and obtain

$$A(i_{0} \rightarrow j_{0}k_{0}) = \sum_{m,m',\bar{q},q',q_{i}} b^{*} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0^{-}) \rangle | \bar{q}q' \rangle | \chi^{m'}(\frac{1}{2})_{A} \rangle | [q_{1}q_{2}]q_{3} \rangle \langle \bar{q}q' | \phi^{j_{0}}(8) \rangle \langle [q_{1}q_{2}]q_{3} | \psi^{k_{0}}(8)_{A} \rangle$$

$$- \sum_{m,m',\bar{q},q',q_{i}} a^{*} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0^{-}) \rangle | \bar{q}q' \rangle | \chi^{m'}(\frac{1}{2})_{S} \rangle | \{q_{1}q_{2}\}q_{3} \rangle \langle \bar{q}q' | \phi^{j_{0}}(8) \rangle \langle q_{1}q_{2}q_{3} | \psi^{k_{0}}(8)_{S} \rangle$$

$$\equiv \sum_{\bar{q},q',q_{i}} A(i_{0} \rightarrow \bar{q}q' [q_{1}q_{2}]q_{3}) \langle \bar{q}q' | \phi^{j_{0}}(8) \rangle \langle [q_{1}q_{2}]q_{3} | \psi^{k_{0}}(8)_{A} \rangle$$

$$+ \sum_{\bar{q},q',q_{i}} A(i_{0} \rightarrow \bar{q}q' \{q_{1}q_{2}\}q_{3}) \langle \bar{q}q' | \phi^{j_{0}}(8) \rangle \langle \{q_{1}q_{2}\}q_{3} | \psi^{k_{0}}(8)_{S} \rangle, \qquad (54)$$

where

$$A(i_{0} \to \bar{q}q'[q_{1}q_{2}]q_{3}) \equiv \sum_{m,m'} b^{*} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0^{-}) \rangle | \bar{q}q' \rangle | \chi^{m'}(\frac{1}{2})_{A} \rangle | [q_{1}q_{2}]q_{3} \rangle = A_{A} [B_{c}(\overline{3}) \to B(8)M(8)],$$
(55)

and

$$A(i_{0} \rightarrow \overline{q}q'\{q_{1}q_{2}\}q_{3}) \equiv \sum_{m,m'} b^{*} \langle B_{c}^{m,i_{0}} | \hat{H}_{W} | \chi(0^{-}) \rangle | \overline{q}q' \rangle | \chi^{m'}(\frac{1}{2})_{A} \rangle | \{q_{1}q_{2}\}q_{3} \rangle$$
  
=  $[\sqrt{2}(1 - \delta_{q_{1}q_{2}}) + \delta_{q_{1}q_{2}}] A_{S} [B_{c}(\overline{3}) \rightarrow B(8)M(8)].$  (56)

Now the decay amplitudes into particles are related to the decay amplitudes into quarks.

Therefore the important result we have established is that for the decays into the B(8), the quark diagrams have two independent types: the symmetric and the antisymmetric,  $A_A$  and  $A_S$ . This result is independent of what particles the B(8)'s decay from or are associated with.

Let us discuss now specifically what types of quark-diagram amplitudes will contribute. For  $B_c(\overline{3}) \rightarrow B(8) + M(8)$ , the two initial noncharmed quarks, say,  $q_1$  and  $q_2$ , are antisymmetric in flavor. In diagram  $\mathcal{A}$ ,  $q_1$  and  $q_2$  are spectators; therefore they stay antisymmetric in the final state. We denote the quark arising from the charmed quark decay as  $q_3$ , and the quark-antiquark pair from the W as  $\overline{q}_0 q'_0$ . In diagram  $\mathcal{B}'$  (the prime signifies that the quark  $q_3$  coming from the charmed quark decay contributes to the final meson formation rather than the final baryon formation),  $q_1$  and  $q_2$  are also spectators; therefore they stay antisymmetric in the final product. In diagram  $\mathcal{B}$ ,  $q_3$  and  $q'_0$  are forced to be flavor antisymmetric due to the Pati-Woo theorem [11]; so are the quark pair  $q'_1q_3$  in diagram  $\mathcal{C}_1$ . Note that the quark-diagram amplitudes  $\mathcal{B}'_S$  and  $\mathcal{C}_{1S}$  vanish because of the Pati-Woo theorem which results from the facts that the  $(V-A) \times (V-A)$  structure of weak interactions is invariant under the Fierz transformation and that the baryon wave function is color antisymmetric. This theorem requires that the quark pair in a baryon produced by weak interactions be antisymmetric in flavor. Putting together all this information and referring to Fig. 2, we find a detailed expression for Eq. (54):

$$\begin{aligned} A(i_{0} \rightarrow j_{0}k_{0}) &= \mathcal{A}_{A}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{0}^{\prime}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{2}]q_{3}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{B}_{A}^{\prime}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{3}]q_{0}^{\prime}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{B}_{A}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{3}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{C}_{1A}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{3}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{C}_{2A}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}^{\prime}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{3}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{C}_{2S}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}^{\prime}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{3}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{C}_{A}^{\prime}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{3}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{2}^{\prime}]q_{3}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{C}_{S}^{\prime}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{3}| \phi^{j_{0}}(8)\rangle \langle [q_{1}q_{2}^{\prime}]q_{0}| \psi^{k_{0}}(8)_{S}\rangle [\sqrt{2}(1 - \delta_{q_{1}q_{2}^{\prime}}) + \delta_{q_{1}q_{2}^{\prime}}] \\ &+ \mathcal{E}_{A}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{3}q_{1}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{E}_{S}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{3}q_{1}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle \\ &+ \mathcal{E}_{S}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{3}q_{1}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle [\sqrt{2}(1 - \delta_{q_{3}q_{1}}) + \delta_{q_{3}q_{1}}] \\ &+ \mathcal{E}_{A}^{\prime}[B_{c}(\overline{3}) \rightarrow B(8)M(8)]\langle \overline{q}_{0}q_{2}| \phi^{j_{0}}(8)\rangle \langle [q_{3}q_{1}]q_{0}| \psi^{k_{0}}(8)_{A}\rangle$$



FIG. 2. Quark diagrams for the decay  $B_c(\overline{3}) \rightarrow B(8) + M(8)$ .

Applying this to all the  $B_c(\overline{3}) \rightarrow B(8)M(8)$  decays, we can express all the 58 decays in terms of the 11 unknown amplitudes in (57) (see also Table II).

# **B.** Results and tables

From the second column of Tables IIa–IIc, which gives the results in the absence of effects from SU(3) violations and final-state interactions, we have the relations

$$|A(\Xi_{c}^{0A} \to \Sigma^{-} \pi^{+})|^{2} = |A(\Xi_{c}^{0A} \to \Xi^{-} K^{+})|^{2},$$

$$|A(\Xi_{c}^{0A} \to n\bar{K}^{0})|^{2} = |A(\Xi_{c}^{0A} \to \Xi^{0}K^{0})|^{2},$$

$$|A(\Xi_{c}^{0A} \to \Sigma^{+} \pi^{-})|^{2} = |A(\Xi_{c}^{0A} \to pK^{-})|^{2},$$

$$|A(\Xi_{c}^{+A} \to p\bar{K}^{0})|^{2} = |A(\Lambda_{c}^{+} \to \Sigma^{+}K^{0})|^{2},$$

$$|A(\Xi_{c}^{+A} \to \Xi^{0}K^{+})|^{2} = |A(\Lambda_{c}^{+} \to n\pi^{+})|^{2},$$

$$|A(\Xi_{c}^{0A} \to \Lambda^{0} \eta_{8})|^{2} = |A(\Xi_{c}^{0A} \to \Sigma^{0} \pi^{0})|^{2},$$

$$(58)$$

Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
		(a) Quark-mixing-allowed modes	
$\Lambda_c^+ { ightarrow} p ec K^0$	$rac{1}{2} \mathcal{B}'_A - rac{1}{\sqrt{3}} \mathcal{C}'_S$	$rac{1}{2}{\mathcal B}'_A-rac{1}{\sqrt{3}}{\mathcal C}'_S$	$\left(rac{1}{2}{\cal B}_A'-rac{1}{\sqrt{3}}{\cal C}_S' ight)\exp(i{\cal B}_{ m I}^{Var{K}})$
$ ightarrow \Lambda^+$	$\frac{1}{2\sqrt{6}}(2\mathcal{A}_A+\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{2}}\mathcal{C}_{2S}$	$\frac{1}{2\sqrt{6}}(2\mathcal{A}_A+\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{2}}\mathcal{C}_{2S}$	$\left[\frac{1}{2\sqrt{6}}(2\mathcal{A}_A + \mathcal{B}_A - \mathcal{C}_{1A} - \mathcal{C}_{2A}) - \frac{1}{2\sqrt{2}}\mathcal{C}_{2S}\right] \exp(i\delta_1^{\Lambda \pi})$
$ ightarrow \Sigma^0 \pi^+$	$\frac{1}{2\sqrt{2}}(B_A - {\cal C}_{1A} + {\cal C}_{2A}) + \frac{1}{2\sqrt{6}}{\cal C}_{2S}$	$\frac{1}{2\sqrt{2}}(\mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{2\sqrt{6}}\mathcal{C}_{2S}$	$\left[\frac{1}{2\sqrt{2}}(B_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{2\sqrt{6}}\mathcal{C}_{2S}\right] \exp(i\partial_2^{\Sigma}\pi)$
$\rightarrow \Sigma^+ \pi^0$	$\frac{1}{2\sqrt{2}}(B_A - {\cal C}_{1A} + {\cal C}_{2A}) + \frac{1}{2\sqrt{6}}{\cal C}_{2S}$	$\frac{1}{2\sqrt{2}}(\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{2\sqrt{6}}\mathcal{C}_{2S}$	$\left[\frac{1}{2\sqrt{2}}(B_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{2\sqrt{6}}\mathcal{C}_{2S}\right] \exp(i\delta_2^{\mathbb{Z}\pi})$
${\rightarrow}\Sigma^+ \eta_0$	$\frac{1}{2\sqrt{3}}(-\mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6}\mathcal{C}_{2S} - \frac{1}{3}\mathcal{C}_S'$	$rac{1}{2\sqrt{3}}(-\mathcal{B}_{A}-\mathcal{C}_{1A}+\mathcal{C}_{2A})+rac{1}{6}\mathcal{C}_{2S}-rac{1}{3}\mathcal{C}_{S}'$	$\left[\frac{1}{2\sqrt{3}}(-\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{6}\mathcal{C}_{2S}-\frac{1}{3}\mathcal{C}_S'\right]\exp(i\delta_1^{\Sigma\eta_0})$
${ ightarrow}\Sigma^+\eta_8$	$\frac{1}{2\sqrt{6}}(-\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{6\sqrt{2}}\mathcal{C}_{2S}+\frac{\sqrt{2}}{3}\mathcal{C}_S'$	$\frac{1}{2\sqrt{6}}(-\mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6\sqrt{2}}\mathcal{C}_{2S} + \frac{\sqrt{2}}{3}\mathcal{C}_{S}'$	$\left[\frac{1}{2\sqrt{6}}(-\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{6\sqrt{2}}\mathcal{C}_{2S}+\frac{\sqrt{2}}{3}\mathcal{C}_{S}'\right]\exp(i\delta_1^{2^{N}}h)$
$ ightarrow \Xi_0 K^+$	$\frac{1}{2}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$rac{1}{2}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})-rac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$\left[\frac{1}{2}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{3}}\mathcal{C}_{2S}\right]\exp(i\partial_{1}^{\Xi}K)$
$\Xi_c^{+A} {\to} \Sigma^+ \overline{K}^0$	$\frac{1}{2}(-\mathcal{B}_A+\mathcal{B}_A')$	$\frac{1}{2}(-\mathcal{B}_A+\mathcal{B}_A')$	$rac{1}{2}(-B_A+B_A')\exp(i\delta_{3/2}^{\Sigma}ar{K})$
$\rightarrow \Pi^0 \pi^+$	$\frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A)$	$\frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A)$	$\frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A) \exp(i\partial_{3/2}^{\Xi\pi})$
$\Xi_c^{0A}\! ightarrow\!\Lambda ar{K}^0$	$rac{1}{2\sqrt{6}}(-\mathcal{B}_A-\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A+\sqrt{3}\mathcal{C}'_S)$	$rac{1}{2\sqrt{6}}\left(-\mathcal{B}_{A}-\mathcal{B}_{A}^{\prime}+\mathcal{C}_{1A}+\mathcal{C}_{A}^{\prime}+\sqrt{3}\mathcal{C}_{S}^{\prime} ight)$	$\frac{1}{2\sqrt{6}}(-\mathcal{B}_A-\mathcal{B}_A'+\mathcal{C}_{1A}+\mathcal{C}_A'+\sqrt{3}\mathcal{C}_S')\exp(i\delta_{1/2}^{\Lambda\bar{K}})$
$ ightarrow \Sigma^0 ec{K}^0$	$\frac{1}{2\sqrt{2}}(-\mathcal{B}_A+\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A)-\frac{1}{2\sqrt{6}}\mathcal{C}'_S$	$\frac{1}{2\sqrt{2}}(-\mathcal{B}_A+\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A)-\frac{1}{2\sqrt{6}}\mathcal{C}'_S$	$\frac{1}{3\sqrt{2}}\left[\left(-\mathcal{B}_A+\mathcal{B}'_A+2\mathcal{C}_{1A}+2\mathcal{C}'_A-\frac{2}{\sqrt{3}}\mathcal{C}'_S\right)\exp(i\delta_{3/2}^{\mathcal{S}})\right]$
			$-\frac{1}{2}\left(\mathcal{B}_{A}-\mathcal{B}_{A}^{\prime}+\mathcal{C}_{1A}+\mathcal{C}_{A}^{\prime}-\frac{1}{\sqrt{3}}\mathcal{C}_{S}^{\prime}\right)\exp(i\delta_{1,2}^{\Sigma})$
$\rightarrow \Sigma^+ K^-$	$\frac{1}{2}(\mathcal{C}_{1A}+\mathcal{C}_A')-\frac{1}{2\sqrt{3}}\mathcal{C}_S'$	$rac{1}{2}(\mathcal{C}_{1A}+\mathcal{C}'_A)-rac{1}{2\sqrt{3}}\mathcal{C}'_S$	$\frac{1}{6}\left[\left(-\mathcal{B}_A+\mathcal{B}'_A+2\mathcal{C}_{1A}+2\mathcal{C}'_A-\frac{2}{\sqrt{3}}\mathcal{C}'_S\right)\exp(i\delta_{3\mathcal{L}}^{\Sigma}\overline{K})\right]$
			$+ \left( \mathcal{B}_A - \mathcal{B}'_A + \mathcal{C}_{1A} + \mathcal{C}'_A - \frac{1}{\sqrt{3}} \mathcal{C}'_S \right) \exp(i  \delta^X_{D,2}) \right]$

TABLE II. Quark-diagram amplitudes for decays of  $B_c(\overline{3}) \rightarrow B(8) + M(8)$ .

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Amplitudes with SU(3)-symmetry breaking and final-state interactions	$\frac{1}{3\sqrt{2}} \left[ \left( \mathcal{A}_A + \sqrt{2}\mathcal{B}_A - \frac{4}{\sqrt{6}}\mathcal{C}_{2S} \right) \exp(i\delta_{3/2}^{\Xi\pi}) \right.$	$+2\left(\mathcal{A}_{A}-\frac{1}{\sqrt{2}}\mathcal{B}_{A}-\frac{1}{\sqrt{6}}\mathcal{C}_{2S}\right)\exp(i\delta_{12}^{\Xi\pi})$	$\frac{1}{3}\left[\left(\mathcal{A}_{A}+\sqrt{2}B_{A}-\frac{4}{\sqrt{6}}\mathcal{C}_{2S}\right)\exp(i\delta_{3/2}^{\Xi})-\left(\mathcal{A}_{A}-\frac{1}{\sqrt{2}}B_{A}-\frac{1}{\sqrt{6}}\mathcal{C}_{2S}\right)\exp(i\delta_{1/2}^{\Xi})\right]$	$\left[-\frac{1}{\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}_A')+\frac{1}{6}(2\mathcal{C}_{2S}-\mathcal{C}_S')\right]\exp(i\delta_{12}^{\Xi}\eta_0)$	$\left[\frac{1}{2\sqrt{3}}(-\mathcal{B}_{A}+2\mathcal{C}_{1A}+2\mathcal{C}_{A})+\frac{1}{3}(\mathcal{C}_{3}^{\prime}+\mathcal{C}_{2S})\right]\exp(i\delta_{3/2}^{\Xi}\eta_{8})$	$\frac{1}{3\sqrt{6}} \left[ (2\sqrt{3}\mathcal{B}_A + 2\mathcal{C}_S' + 2\mathcal{C}_{2S} - (\sqrt{2} + 2)\mathcal{E}_S)\exp(i\delta_{32}^{\Sigma K}) \right]$	$-\left(\frac{\sqrt{3}}{2}\mathcal{B}_A + 2\mathcal{C}_S' - \mathcal{C}_{2S} + (1 - \sqrt{2})\mathcal{E}_S\right) \exp(i\delta_{12}^{\mathcal{K}K})$ $\frac{1}{3\sqrt{6}} \left[\left(\sqrt{6}\mathcal{B}_A + \sqrt{2}\mathcal{C}_S' + \sqrt{2}\mathcal{C}_{2S} - (1 + \sqrt{2})\mathcal{E}_S\right) \exp(i\delta_{32}^{\mathcal{K}K})$	$-\left(\frac{\sqrt{6}}{2}B_A + 2\sqrt{2}C_S' - \sqrt{2}C_{2S} + (\sqrt{2}-2)\mathcal{E}_S\right)\exp(i\delta_{1/2}^{NK})$ $\frac{1}{2\sqrt{2}}(2\mathcal{A}_A + B_A + 2\mathcal{C}_{1A} - 2\mathcal{C}_{2A} - 2\mathcal{E}_A + 2\mathcal{E}_A')\exp(i\delta_{1/2}^{NK})$	$\frac{1}{6\sqrt{3}}\left[\left(-\sqrt{3}\left(\mathcal{A}_{A}-\mathcal{B}_{A}^{\prime}\right)-2\left(\mathcal{C}_{S}^{\prime}-\mathcal{E}_{S}\right)\exp(i\beta_{3/2}^{V\pi})\right)+\left(\frac{\sqrt{3}}{2}\left(-4\mathcal{A}_{A}+6\mathcal{B}_{A}-2\mathcal{B}_{A}^{\prime}\right)\right]$	$-3\mathcal{C}_{1A}+3\mathcal{C}_{2A}+3\mathcal{E}_{A}-3\mathcal{E}_{A})+2(\mathcal{C}_{S}-\mathcal{E}_{S})+3(\mathcal{C}_{2S}-\mathcal{E}_{S})\left(\exp(i\partial_{1/2}^{V,\pi})\right)$
Amplitudes with SU(3)-symmetry breaking	$\frac{1}{2}\mathcal{A}_A-\frac{1}{\sqrt{3}}\mathcal{C}_{2S}$	-	$rac{1}{\sqrt{2}}\mathcal{B}_A + rac{1}{\sqrt{6}}\mathcal{C}_{2S}$	$-rac{1}{\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}'_A)+rac{1}{6}(2\mathcal{C}_{2S}-\mathcal{C}'_S)$	$\frac{1}{2\sqrt{3}}(-\mathcal{B}_{A}+2\mathcal{C}_{1A}+2\mathcal{C}'_{A})+\frac{1}{3}(\mathcal{C}'_{S}+\mathcal{C}_{2S})$	(b) Quark-intring-suppressed inou $\frac{1}{2\sqrt{2}}\mathcal{B}_{A} + \frac{1}{\sqrt{6}}\left(\mathcal{C}_{2S} - \mathcal{E}_{S}\right)$	$-rac{1}{2}B_{ m A}+rac{1}{\sqrt{3}}(\mathcal{C}_{ m S}^{\prime}-\mathcal{E}_{ m S})$	$rac{1}{\sqrt{\mathcal{F}}}(2\mathcal{A}_A+\mathcal{B}_A+2\mathcal{C}_{1A}-2\mathcal{C}_{2A}-2\mathcal{E}_A+2\mathcal{E}_A')$	$\mathcal{L}_{VO}$ $rac{1}{2}(\mathcal{A}_{A}+\mathcal{B}_{A}-\mathcal{C}_{1A}+\mathcal{C}_{2A}+\mathcal{E}_{A}-\mathcal{E}_{A})$	$-rac{1}{2\sqrt{3}}(\mathcal{C}_{2S}+\mathcal{E}_{S})$
Amplitudes with SU(3) symmetry	$rac{1}{2}\mathcal{A}_A - rac{1}{\sqrt{3}}\mathcal{C}_{2S}$	-	$\frac{1}{\sqrt{2}}B_A + \frac{1}{\sqrt{6}}C_{2S}$	$-rac{1}{\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}_A')+rac{1}{6}(2\mathcal{C}_{2S}-\mathcal{C}_S')$	$\frac{1}{2\sqrt{3}}(-\mathcal{B}_{A}+2\mathcal{C}_{1A}+2\mathcal{C}_{A}')+\frac{1}{3}(\mathcal{C}_{S}'+\mathcal{C}_{2S})$	$\frac{1}{2\sqrt{2}}\mathcal{B}_A + \frac{1}{\sqrt{6}}\mathcal{C}_{2S}$	$-rac{1}{2}\mathcal{B}_A+rac{1}{\sqrt{3}}\mathcal{C}'_S$	$rac{1}{2}(2\mathcal{A}_A+\mathcal{B}_A+2\mathcal{C}_{1A}-2\mathcal{C}_{2A})$	$rac{2\sqrt{6}}{2}(\mathcal{A}_A+\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$-rac{1}{2\sqrt{3}}\mathcal{C}_{2S}$
Reaction	ש+ דע ל		$\rightarrow \Pi^{0} \boldsymbol{\pi}^{0}$	$\to \Pi^0 \boldsymbol{\eta}_0$	$\to \Pi^0  \boldsymbol{\eta}_8$	$\Lambda_c^+\!\!\rightarrow\!\Sigma^0 K^+$	${\rightarrow} \Sigma^+ K^0$	$ ightarrow \Lambda K^+$	$ ightarrow n  \pi^+$	

TABLE II. (Continued).

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Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
→ <i>р</i> т <sup>0</sup>	$-rac{1}{2\sqrt{2}}(\mathcal{B}_A-\mathcal{B}_A'-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$-rac{1}{2\sqrt{2}}(\mathcal{B}_A-\mathcal{B}_A'-\mathcal{C}_{1A}+\mathcal{C}_{2A}+\mathcal{E}_A-\mathcal{E}_A')$	$-\frac{1}{3\sqrt{6}}\left[\left(-2\sqrt{3}(\mathcal{A}_A-\mathcal{B}_A')-2(\mathcal{C}_S'-\mathcal{E}_S)\exp(i\delta_{3/2}^{V,\pi})\right)\right.$
	$-rac{1}{2\sqrt{6}}c_{2S}-rac{1}{\sqrt{6}}c_{S}'$	$-rac{1}{2\sqrt{6}}(\mathcal{C}_{2S}\!-\!\mathcal{E}_S)-rac{1}{\sqrt{6}}(\mathcal{C}_S'\!-\!\mathcal{E}_S)$	$-\left(\frac{\sqrt{3}}{2}\left(-4\mathcal{A}_{A}+6\mathcal{B}_{A}-2\mathcal{B}_{A}^{\prime}-3\mathcal{C}_{1A}+3\mathcal{C}_{2A}+3\mathcal{E}_{A}-3\mathcal{E}_{A}^{\prime}\right)\right.$
			$+ \left. 2 \left( \mathcal{C}_{S}' - \mathcal{E}_{S} \right) + 3 \left( \mathcal{C}_{2S} - \mathcal{E}_{S} \right) \right) \exp(i  \delta_{1/2}^{N_{T}}) \right]$
$ o p  \eta_0$	$\frac{1}{2\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_{2A})$	$rac{1}{2\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_{2A}-\mathcal{E}_A+\mathcal{E}_A')$	$\left[\frac{1}{2\sqrt{3}}(\mathcal{B}_A + \mathcal{C}_{1A} - \mathcal{C}_{2A} - \mathcal{E}_A + \mathcal{E}'_A) - \frac{1}{6}(\mathcal{C}_{2S} - \mathcal{E}_S) + (\mathcal{C}'_S - \mathcal{E}_S)\right] \exp(i\mathcal{B}_{1/2}^{V\eta_8})$
	$-\frac{1}{6}c_{2S}+\frac{1}{3}c_{S}'$	$-rac{1}{6}(\mathcal{C}_{2S}\!-\!\mathcal{E}_S)+rac{1}{3}(\mathcal{C}'_S\!-\!\mathcal{E}_S)$	
$ ightarrow p\eta_8$	$-\frac{1}{2\sqrt{6}}(-\mathcal{B}_A+3\mathcal{B}_A'-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$-rac{1}{2\sqrt{6}}(-\mathcal{B}_A+3\mathcal{B}_A'-\mathcal{C}_{1A}+\mathcal{C}_{2A}+\mathcal{E}_A-\mathcal{E}_A')$	$\left[-\frac{1}{2\sqrt{6}}(-\mathcal{B}_A+3\mathcal{B}_A'-\mathcal{C}_{1A}+\mathcal{C}_{2A}+\mathcal{E}_A-\mathcal{E}_A')\right.$
	$+ rac{1}{3\sqrt{2}} \mathcal{C}'_{S} - rac{1}{6\sqrt{2}} \mathcal{C}_{2S}$	$+rac{1}{3\sqrt{2}}(\mathcal{C}'_{s}-\mathcal{E}_{S})-rac{1}{6\sqrt{2}}(\mathcal{C}_{2S}-\mathcal{E}_{S})$	$+\frac{1}{3\sqrt{2}}(\mathcal{C}'_{S}-\mathcal{E}_{S})-\frac{1}{6\sqrt{2}}(\mathcal{C}_{2S}-\mathcal{E}_{S})\left[\exp(i\delta^{V\eta_{S}}_{1/2})\right]$
$\Pi_c^{+A} \rightarrow \Sigma^0 \pi^+$	$\frac{1}{2\sqrt{2}}(-\mathcal{A}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$rac{1}{2\sqrt{2}}(-\mathcal{A}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A}+\mathcal{E}_A+\mathcal{E}_A')$	$\frac{1}{4\sqrt{2}} \left[ \left( -\mathcal{A}_A + \mathcal{B}_A - 2\mathcal{C}_{1A} + 2\mathcal{C}_{2A} + 2\mathcal{E}_A + 2\mathcal{E}_A \mathbf{i} + \frac{2}{\sqrt{3}} \mathcal{E}_S \right) \exp(i\delta_2^{\Sigma} \pi) \right]$
	$+\frac{1}{2\sqrt{6}}c_{2S}$	$+rac{1}{2\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_{S})$	$-\left(\mathcal{A}_{A}+\mathcal{B}_{A}+\frac{2}{\sqrt{3}}\mathcal{C}_{2S}\right)\exp(i\delta_{1}^{\mathcal{T}}\pi)\bigg]$
${\to} \Sigma^+ \pi^0$	$\frac{1}{2\sqrt{2}}(\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$rac{1}{2\sqrt{2}}(\mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_A + \mathcal{E}_A')$	$\frac{1}{4\sqrt{2}}\left[\left(-\mathcal{A}_A+\mathcal{B}_A-2\mathcal{C}_{1A}+2\mathcal{C}_{2A}+2\mathcal{E}_A+\frac{2}{\sqrt{3}}\mathcal{E}_S\right)\exp(i\partial_2^{\mathbb{X}\pi})\right.$
	$+\frac{1}{2\sqrt{6}}c_{2S}$	$+rac{1}{2\sqrt{6}}(\mathcal{C}_{2S}+\mathcal{E}_{S})$	$-\left(\mathcal{A}_A+\mathcal{B}_A+\frac{2}{\sqrt{3}}\mathcal{C}_{2S}\right)\exp(i\delta_1^{\mathcal{T}}\pi)\bigg]$
${\rightarrow}\Sigma^+\eta_0$	$\frac{1}{2\sqrt{3}}(\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})$	$rac{1}{2\sqrt{3}}(\mathcal{B}_A\!-\!\mathcal{C}_{1A}\!+\!\mathcal{C}_{2A}\!+\!\mathcal{E}_A\!+\!\mathcal{E}_A')$	$\left[\frac{1}{2\sqrt{3}}(\mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_A + \mathcal{E}_A') + \frac{1}{6}(\mathcal{C}_{2S} + \mathcal{E}_S) - \frac{1}{3}(\mathcal{C}_S' + \mathcal{E}_S)\right] \exp(i\delta_1^{\Sigma}\eta_0)$
	$+\frac{1}{6}\mathcal{C}_{2S}-\frac{1}{3}\mathcal{C}'_{S}$	$+\frac{1}{6}(\mathcal{C}_{2S}+\mathcal{E}_S)-\frac{1}{3}(\mathcal{C}'_S+\mathcal{E}_S)$	

(Continued)	
II.	
<b>FABLE</b>	

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Amplitudes with       itudes with       SU(3)-symmetry breaking       and final-state interactions	$-\mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_{A} + \mathcal{E}_{A}') \qquad \left[ \frac{1}{2\sqrt{6}} (2\mathcal{B}_{A} - 3\mathcal{B}_{A}' - \mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_{A} + \mathcal{E}_{A}') + \frac{1}{6\sqrt{2}} (\mathcal{C}_{2S} + \mathcal{E}_{S}) - \frac{\sqrt{2}}{3} (\mathcal{C}_{S}' - \mathcal{E}_{S}) \right] \exp(i\delta_{1}^{\mathcal{Z}} \eta_{S})$	$+\mathcal{E}_{s})-rac{\sqrt{2}}{3}(\mathcal{C}_{s}^{\prime}-\mathcal{E}_{s})$	$_{4}+\mathcal{C}_{1A}-\mathcal{C}_{2A}-\mathcal{E}_{A}-\mathcal{E}_{A}) \qquad \qquad \left[\frac{1}{2\sqrt{6}}(-\mathcal{A}_{A}-2\mathcal{B}_{A}+\mathcal{C}_{1A}-\mathcal{C}_{2A}-\mathcal{E}_{A}-\mathcal{E}_{A}')-\frac{1}{2\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{E}_{S})\right]\exp(i\partial_{1}^{A}\pi)$	$rac{1}{\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$\mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_{A} + \mathcal{E}_{A}') \qquad \left[ \frac{1}{2} (\mathcal{A}_{A} + \mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A} + \mathcal{E}_{A}' + \mathcal{E}_{A}') - \frac{1}{2\sqrt{3}} (\mathcal{C}_{2S} + \mathcal{E}_{S}) \right] \exp(i\delta_{1}^{\overline{=}K})$	$rac{1}{\sqrt{3}}(\mathcal{C}_{2S}+\mathcal{E}_S)$	$\frac{1}{\sqrt{3}}(\mathcal{C}_{S}' + \mathcal{E}_{S}) \left[ \frac{1}{2} \mathcal{B}_{A} - \frac{1}{\sqrt{3}}(\mathcal{C}_{S}' + \mathcal{E}_{S}) \right] \exp(i\beta_{1}^{V}\overline{\kappa})$	$\mathcal{E}_A) + \frac{1}{2\sqrt{3}}(\mathcal{C}'_S - \mathcal{E}_S) \qquad \qquad \frac{1}{12}\{(-\mathcal{A}_A + \mathcal{B}_A - 2\mathcal{C}_{1A} + 4\mathcal{C}_{2S} + 2\mathcal{E}'_A)\exp(i\partial_2^N \pi)$	$+ [3(\mathcal{A}_A - \mathcal{C}_{1A} - \mathcal{C}'_A - 2\mathcal{C}_{2S} + \mathcal{E}_A - \mathcal{E}'_A) + \sqrt{3}(\mathcal{C}'_S - \mathcal{E}_S)] \exp(i\delta_1^{\Sigma} \pi)$	$+[-(2\mathcal{A}_A + \mathcal{B}_A + \mathcal{C}_{1A} + 3\mathcal{C}_A' - 2\mathcal{C}_{2S} - 3\mathcal{E}_A - \mathcal{E}_A') + \sqrt{3}(\mathcal{C}_S' - \mathcal{E}_S)]\exp(i\delta_0^{\Delta}\pi)\}$	$+\frac{2}{\sqrt{3}}c_{2S}+\mathcal{E}'_A\right) \qquad \qquad \frac{1}{12}\{(-\mathcal{A}_A+\mathcal{B}_A-2\mathcal{C}_{1A}+4\mathcal{C}_{2S}+2\mathcal{E}'_A)\exp(i\partial_2^{\mathbb{N}\pi})$	$-\left[3(\mathcal{A}_{A}-\mathcal{C}_{1A}-\mathcal{C}_{1A}-\mathcal{C}_{A}^{\prime}-2\mathcal{C}_{2S}+\mathcal{E}_{A}-\mathcal{E}_{S}^{\prime})+\sqrt{3}(\mathcal{C}_{S}^{\prime}-\mathcal{E}_{S})\right]\exp(i\delta_{1}^{S}\pi)$	$+[-(2\mathcal{A}_A+\mathcal{B}_A+\mathcal{C}_{1A}+3\mathcal{C}_A'-2\mathcal{C}_{2S}-3\mathcal{E}_A-\mathcal{E}_A')+\sqrt{3}(\mathcal{C}_S'-\mathcal{E}_S)]\exp(i\delta_0^{\mathbb{Z}}\pi)\}$	$\frac{1}{12}\{2(-\mathcal{A}_{A}+\mathcal{B}_{A}-2\mathcal{C}_{1A}+4\mathcal{C}_{2S}+2\mathcal{E}_{A}')\exp(i\delta_{2}^{X}\pi)$	
Amplitudes with SU(3) symmetry SU(3)-sym	$\frac{1}{2\sqrt{6}}(2B_A - 3B'_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) \qquad \frac{1}{2\sqrt{6}}(2B_A - 3B'_A)$	$+\frac{1}{6\sqrt{2}}c_{2S}-\frac{\sqrt{2}}{3}c'_{S} +\frac{1}{6\sqrt{2}}(C_{2S})$	$\frac{1}{2\sqrt{6}}(-\mathcal{A}_A - 2\mathcal{B}_A + \mathcal{C}_{1A} - \mathcal{C}_{2A}) \qquad \frac{1}{2\sqrt{6}}(-\mathcal{A}_A - 2\mathcal{B}_A$	$-\frac{1}{2\sqrt{2}}c_{2S}$	$\frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) \qquad \qquad \frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A - \mathcal{C}_{2A})$	$-\frac{1}{2\sqrt{3}}c_{2S}$	$\frac{1}{2}\mathcal{B}_A - \frac{1}{\sqrt{3}}\mathcal{C}'_S \qquad \qquad \frac{1}{2}\mathcal{B}_A - \frac{1}{2$	$-\frac{1}{2}(\mathcal{C}_{1A} + \mathcal{C}'_{A}) + \frac{1}{2\sqrt{3}}\mathcal{C}'_{S} \qquad -\frac{1}{2}(\mathcal{C}_{1A} + \mathcal{C}'_{A} -$			$rac{1}{2}\left(-\mathcal{A}_A+rac{2}{\sqrt{3}}\mathcal{C}_{2S} ight) \qquad \qquad$			$\frac{1}{4}(\mathcal{B}'_A + \mathcal{C}_{1A} + \mathcal{C}'_A) \qquad \qquad \frac{1}{4}(\mathcal{B}'_A + \mathcal{C}_{1A})$	
Reaction	$\rightarrow \Sigma^+ \eta_8$		$\rightarrow \Lambda \pi^+$ $2$		$ ightarrow \Xi^0 K^+$		$ ightarrow p ec K_0$	$\Xi_c^{0A} { ightarrow} \Sigma^+ \pi^-$			${\rightarrow}\Sigma^{-}\pi^{+}$			$ ightarrow \Sigma^0 \pi^0$	

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TABLE II	

Amplitudes with	Amplitudes with	Amplitudes with SU(3)-symmetry breaking
SU(3) symmetry	SU(3)-symmetry breaking	and final-state interactions
$-\frac{1}{2\sqrt{6}}(2\mathcal{B}_A-\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A)$	$-\frac{1}{2\sqrt{6}}(2\mathcal{B}_A-\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A-\mathcal{E}_A-\mathcal{E}'_A)$	$\left[-\frac{1}{2\sqrt{6}}(2\mathcal{B}_A-\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A-\mathcal{E}_A-\mathcal{E}'_A)+\frac{1}{6\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}'_S)\right.$
		$+\frac{1}{6\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}'_{5}+\mathcal{E}_{5})+\frac{1}{6\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}'_{5}+\mathcal{E}_{5})\bigg]\exp(i\delta_{1}^{\Sigma}\eta_{0})$
$\frac{1}{4\sqrt{3}}(2B_A - 3B'_A - \mathcal{C}_{1A} - \mathcal{C}'_A)$	$\frac{1}{4\sqrt{3}}(2\mathcal{B}_A-3\mathcal{B}_A'-\mathcal{C}_{1A}-\mathcal{C}_A'+\mathcal{E}_A+\mathcal{E}_A')$	$\left[\frac{1}{4\sqrt{3}}(2\mathcal{B}_A - 3\mathcal{B}'_A - \mathcal{C}_{1A} - \mathcal{C}'_A + \mathcal{E}_A + \mathcal{E}'_A) + \frac{1}{12}(5\mathcal{C}'_S + 2\mathcal{C}_{2S} - 5\mathcal{E}_S)\right] \exp(i\delta_1^S \eta_S)$
$+rac{1}{12}(5\mathcal{C}'_{5}+2\mathcal{C}_{2S})$	$+rac{1}{2}(5\mathcal{C}'_{5}+2\mathcal{C}_{2S}-5\mathcal{E}_{S})$	
$\frac{1}{4\sqrt{3}}(2\mathcal{B}_A-\mathcal{B}'_A+\mathcal{C}_{1A}+\mathcal{C}'_A)$	$\frac{1}{4\sqrt{3}}(2\mathcal{B}_A \!-\!\mathcal{B}_A'\!+\!\mathcal{C}_{1A}\!+\!\mathcal{C}_A'\!+\!\mathcal{E}_A'\!+\!\mathcal{E}_A')$	$\left[\frac{1}{4\sqrt{3}}(2B_A - B'_A + \mathcal{C}_{1A} + \mathcal{C}'_A - \mathcal{E}_A + \mathcal{E}'_A) + \frac{1}{4}(2\mathcal{C}_{2S} + \mathcal{C}'_S + \mathcal{E}_S)\right] \exp(i\delta_0^{\Lambda,\mathbb{T}})$
$+\frac{1}{4}(2\mathcal{C}_{2\mathcal{S}}+\mathcal{C}'_{\mathcal{S}})$	$+rac{1}{4}(2\mathcal{C}_{2S}+\mathcal{C}'_{S}+\mathcal{E}_{S})$	
$-\frac{1}{2\sqrt{2}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}'_A)$	$-\frac{1}{2\sqrt{2}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}_A'-\mathcal{E}_A+\mathcal{E}_A')$	$\left[-\frac{1}{2\sqrt{2}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}_A'-\mathcal{E}_A+\mathcal{E}_A')+\frac{1}{2\sqrt{6}}(2\mathcal{C}_{2S}-\mathcal{C}_S'-\mathcal{E}_S)\right]\exp(i\delta_0^{\Lambda\eta_0})$
$+\frac{1}{2\sqrt{6}}(2\mathcal{C}_{2S}-\mathcal{C}'_{3})$	$+rac{1}{2\sqrt{6}}(2\mathcal{C}_{2S}\!-\!\mathcal{C}'_{S}\!-\!\mathcal{E}_{S})$	
$rac{1}{4}(\mathcal{B}'_A+\mathcal{C}'_A+\mathcal{C}_{1_A})$	$rac{1}{4}(\mathcal{B}'_A+\mathcal{C}'_A+\mathcal{C}_{1A})$	$\left[\frac{1}{4}(\mathcal{B}'_A+\mathcal{C}'_A+\mathcal{C}_{1A})+\frac{1}{4\sqrt{3}}(2\mathcal{C}_{2S}-\mathcal{C}'_S)\right]\exp(i\delta_0^{\Lambda\gamma_S})$
$+\frac{1}{4\sqrt{3}}(2\mathcal{C}_{2S}\!-\!\mathcal{C}_S')$	$+\frac{1}{4\sqrt{3}}(2\mathcal{C}_{2S}\!-\!\mathcal{C}_S')$	- -
$\frac{1}{2}(B_A-\mathcal{C}_{1A}-\mathcal{C}_A')$	$rac{1}{2}(\mathcal{B}_A-\mathcal{C}_{1A}-\mathcal{C}'_A+\mathcal{E}_A)$	$\frac{1}{4} \Biggl\{ \left[ \left( \mathcal{A}_A + \mathcal{B}_A - \mathcal{C}_{1A} - \mathcal{C}_A' - 2\mathcal{C}_{2S} + \mathcal{E}_A + \mathcal{E}_A' \right) - \frac{1}{\sqrt{3}} \left( \mathcal{C}_S' + \mathcal{E}_S \right) \Biggr] \exp(i \vartheta_1^{\overline{=}K}) \Biggr\}$
$-rac{1}{2\sqrt{3}}\mathcal{C}_{S}'$	$-rac{1}{2\sqrt{3}}(\mathcal{C}'_{S}+\mathcal{E}_{S})$	$+\left[\left(-\mathcal{A}_{A}+\mathcal{B}_{A}-\mathcal{C}_{1A}-\mathcal{C}'_{A}+2\mathcal{C}_{2S}+\mathcal{E}_{A}-\mathcal{E}'_{A}\right)-\frac{1}{\sqrt{3}}\left(\mathcal{C}'_{S}+\mathcal{E}_{S}\right)\left[\exp(i\delta_{0}^{\Xi K})\right]$
$\frac{1}{2} \left( \mathcal{A}_A - \frac{2}{\sqrt{3}} \mathcal{C}_{2S} \right)$	$rac{1}{2} \Big( \mathcal{A}_A - rac{2}{\sqrt{3}} \mathcal{C}_{2S} + \mathcal{E}_A' \Big)$	$\frac{1}{4}\left\{\left[\left(\mathcal{A}_{A}+\mathcal{B}_{A}-\mathcal{C}_{1A}-\mathcal{C}_{A}^{\prime}-2\mathcal{C}_{2S}+\mathcal{E}_{A}+\mathcal{E}_{A}^{\prime}\right)-\frac{1}{\sqrt{3}}\left(\mathcal{C}_{S}^{\prime}+\mathcal{E}_{S}\right)\right]\exp(i\partial_{1}^{\Xi}K)$
		$-\left[\left(-\mathcal{A}_{A}+\mathcal{B}_{A}-\mathcal{C}_{1A}-\mathcal{C}_{A}'+2\mathcal{C}_{2S}+\mathcal{E}_{A}-\mathcal{E}_{A}'\right)-\frac{1}{\sqrt{3}}\left(\mathcal{C}_{S}'+\mathcal{E}_{S}\right)\left[\exp(i\widehat{\delta}_{0}^{\overline{=}K})\right]$

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eaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
$ ightarrow pK^-$	$-\frac{1}{2}(\mathcal{C}'_A+\mathcal{C}_{1A})$	$-rac{1}{2}(\mathcal{C}_A'+\mathcal{C}_{1A}-\mathcal{E}_A)$	$\left[ \left( \mathcal{B}_A - 2\mathcal{C}'_A - 2\mathcal{C}_{1A} + 2\mathcal{E}_A \right) \exp(i \partial_1^{VK}) + \left( -\mathcal{B}_A + \frac{2}{\sqrt{3}} \left( \mathcal{C}'_S + \mathcal{E}_S \right) \right) \exp(i \partial_0^{VK}) \right]$
	$+\frac{1}{2\sqrt{3}}c_s'$	$+rac{1}{2\sqrt{3}}(\mathcal{C}'_{S}+\mathcal{E}_{S})$	
$\rightarrow K_{10}$	$rac{1}{2}(\mathcal{B}_A-\mathcal{C}_A'-\mathcal{C}_{1A})$	$rac{1}{2}(\mathcal{B}_A-\mathcal{C}_A'-\mathcal{C}_{1A}+\mathcal{E}_A)$	$\frac{1}{4} \left[ \left( \mathcal{B}_A - 2\mathcal{C}'_A - 2\mathcal{C}_{1A} + 2\mathcal{E}_A \right) \exp(i \delta_1^{VK}) - \left( -\mathcal{B}_A + \frac{2}{\sqrt{3}} \left( \mathcal{C}_S^* + \mathcal{E}_S \right) \right) \exp(i \delta_0^{VK}) \right]$
	$-rac{1}{2\sqrt{3}C_s}$	$-rac{1}{2\sqrt{3}}\left(\mathcal{C}_{S}'+\mathcal{E}_{S} ight)$	
		(c) Quark-mixing-doubly-suppres	sed modes
$\Lambda_c^+ { ightarrow} pK^0$	$-\frac{1}{2}(\mathcal{B}_A-\mathcal{B}_A')$	$-rac{1}{2}(\mathcal{B}_A-\mathcal{B}_A')$	$\frac{1}{4} \big[ (\mathcal{A}_A + \mathcal{B}_A') \exp(i  \delta_1^{VK}) + (-\mathcal{A}_A - 2 \mathcal{B}_A + \mathcal{B}_A') \exp(i  \delta_0^{VK}) \big]$
$\rightarrow nK^+$	$rac{1}{2}(\mathcal{A}_A+\mathcal{B}_A)$	$\frac{1}{2}(\mathcal{A}_A + \mathcal{B}_A)$	$\frac{1}{4} \big[ (\mathcal{A}_A + \mathcal{B}_A') \exp(i \mathcal{S}_1^{VK}) - (-\mathcal{A}_A - 2\mathcal{B}_A + \mathcal{B}_A') \exp(i \mathcal{S}_0^{VK}) \big]$
$\exists_c^{+A} \longrightarrow \Sigma^0 K^+$	$\frac{1}{2\sqrt{2}}A_A-\frac{1}{\sqrt{6}}\mathcal{C}_{2S}$	$\frac{1}{2\sqrt{2}}A_A-\frac{1}{\sqrt{6}}\mathcal{C}_{2S}$	$\frac{1}{3\sqrt{6}} \left[ (\sqrt{3}\mathcal{A}_A + \sqrt{3}\mathcal{B}'_A - 2\mathcal{C}_{2S} - 2\mathcal{C}'_S) \exp(i\delta_{3/2}^{\Sigma K}) \right]$
			$- \left( - \frac{\sqrt{3}}{2} \mathcal{A}_A + \sqrt{3} \mathcal{B}'_A + \mathcal{C}_{2,S} - 2 \mathcal{C}_S^t \right) \exp(i \delta_{1/2}^{\Sigma K}) \bigg]$
${\rightarrow} \Sigma^+ K^0$	$rac{1}{2}B_A'-rac{1}{\sqrt{3}}C_S'$	$rac{1}{2}B_A'-rac{1}{\sqrt{3}}C_S'$	$\frac{1}{3\sqrt{6}} \bigg[ (\sqrt{3}\mathcal{A}_A + \sqrt{3}\mathcal{B}_A' - 2\mathcal{C}_{2S} - 2\mathcal{C}_S') \exp(i\delta_{3/2}^{\Sigma K}) \bigg]$
			$+2\left(-\frac{\sqrt{3}}{2}\mathcal{A}_{A}+\sqrt{3}\mathcal{B}_{A}^{\prime}+\mathcal{C}_{2S}-2\mathcal{C}_{S}^{\prime}\right)\exp(i\delta_{1/2}^{\Sigma K})\right]$
$ ightarrow \Lambda K^+$	$rac{1}{\sqrt{6}}igg(-rac{1}{2}\mathcal{A}_A+\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A}igg)$	$rac{1}{\sqrt{6}}igg(-rac{1}{2}\mathcal{A}_A+\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A}igg)$	$rac{1}{\sqrt{6}} igg( -rac{1}{2} \mathcal{A}_A + \mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A} igg) \exp(i \delta_{1/2}^{\Lambda K})$
$ ightarrow$ $n \pi^+$	$\frac{1}{2}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$\frac{1}{2}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})-\frac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$\frac{1}{3\sqrt{6}} \bigg[ (-\sqrt{6}\mathcal{C}_{1A} + \sqrt{6}\mathcal{C}_{2A} + \sqrt{2}\mathcal{C}_{S}') \exp(i\partial_{3/2}^{V,\pi}) \bigg]$
			$-\sqrt{2} \left( \frac{\sqrt{3}}{2} \mathcal{C}_{1A} - \frac{\sqrt{3}}{2} \mathcal{C}_{2A} + \mathcal{C}'_{S} + \mathcal{C}_{2S} \right) \exp(i  \delta_{12}^{V\pi}) \bigg]$

TABLE II. (Continued).

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TABLE II.

Amplitudes with SU(3)-symmetry breaking and final-state interactions	$\frac{1}{3\sqrt{6}} \bigg[ \sqrt{2} (-\sqrt{6}\mathcal{C}_{1A} + \sqrt{6}\mathcal{C}_{2A} + \sqrt{2}\mathcal{C}_{S}') \exp(i\delta_{3/2}^{N,\pi}) \\$	$+ \left(\frac{\sqrt{3}}{2}\mathcal{C}_{1A} - \frac{\sqrt{3}}{2}\mathcal{C}_{2A} + \mathcal{C}'_{S} + \mathcal{C}_{2S}\right) \exp(i\delta^{V\pi}_{1/2}) \bigg]$	$\left[\frac{1}{2\sqrt{3}}(-\mathcal{B}_A-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{6}\mathcal{C}_{2S}-\frac{1}{3}\mathcal{C}'_S\right]\exp(i\delta_{1/2}^{V\eta_0})$	$\left[\frac{1}{2\sqrt{6}}(2\mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6\sqrt{2}}\mathcal{C}_{2S} - \frac{1}{3}\mathcal{C}_S'\right] \exp(i\delta_{1/2}^{N\eta_8})$	$rac{1}{\sqrt{6}}(\mathcal{B}_A-rac{1}{2}\mathcal{B}_A'-\mathcal{C}_{1A}-\mathcal{C}_A')\mathrm{exp}(i\delta_{1/2}^{\Lambda K})$	$\frac{1}{3\sqrt{6}} \bigg[ (\sqrt{3}\mathcal{A}_A + \sqrt{3}\mathcal{B}_A' - 2\mathcal{C}_{2S} - 2\mathcal{C}_S') \exp(i\delta_{3/2}^{\Sigma K}) \bigg]$	$+\left(-\sqrt{3}\mathcal{A}_{A}+\frac{\sqrt{3}}{2}B_{A}^{\prime}+2\mathcal{C}_{2S}-\mathcal{C}_{S}^{\prime}\right)\exp(i\partial_{1/2}^{\Sigma})\right]$	$\frac{1}{6\sqrt{3}} \bigg[ (\sqrt{3}\mathcal{A}_A + \sqrt{3}\mathcal{B}_A - 2\mathcal{C}_{2S} - 2\mathcal{C}_S) \exp(i\delta_{3/2}^{\Sigma K}) \bigg]$	$-2 \Big( -\sqrt{3}\mathcal{A}_A + \frac{\sqrt{3}}{2}\mathcal{B}_A + 2\mathcal{C}_{2S} - \mathcal{C}_S^* \Big) \!\exp(i\delta_{1/2}^{\mathcal{S}K}) \Big]$	$\frac{1}{3\sqrt{3}}\left[\left(\mathcal{C}_{S}^{\prime}+\mathcal{C}_{2S}\right)\exp(i\partial_{3/2}^{\prime/\pi})-\left(\frac{3\sqrt{3}}{2}(\mathcal{C}_{1A}+\mathcal{C}_{A}^{\prime})+\mathcal{C}_{2S}-\frac{1}{2}\mathcal{C}_{S}^{\prime}\right)\exp(i\partial_{1/2}^{\prime/\pi})\right]$	$\frac{1}{3\sqrt{6}} \left[ 2(\mathcal{C}_{S}' + \mathcal{C}_{2S}) \exp(i\beta_{3/2}^{N}) + \left(\frac{3\sqrt{3}}{2}(\mathcal{C}_{1A} + \mathcal{C}_{A}') + \mathcal{C}_{2S} - \frac{1}{2}\mathcal{C}_{S}'\right) \exp(i\beta_{1/2}^{N}) \right]$	$\left[-\frac{1}{2\sqrt{3}}(B_A+\mathcal{C}_{1A}+\mathcal{C}'_A)+\frac{1}{3\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}'_S)\right]\exp(i\delta_{1/2}^{V\eta_0})$	$\left[\frac{1}{2\sqrt{3}}(2B_A - \mathcal{C}_{1A} - \mathcal{C}'_A) + \frac{1}{6}(2\mathcal{C}_{2S} - \mathcal{C}'_S)\right] \exp(i\delta_{1/2}^{V_T g_S})$
Amplitudes with SU(3)-symmetry breaking	$\frac{1}{2\sqrt{2}}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{2\sqrt{6}}\mathcal{C}_{2S}+\frac{1}{\sqrt{6}}\mathcal{C}'_{S}$		$\frac{1}{2\sqrt{3}}(-\mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6}\mathcal{C}_{2S} - \frac{1}{3}\mathcal{C}_{S}'$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6\sqrt{2}}\mathcal{C}_{2S} - \frac{1}{3\sqrt{2}}\mathcal{C}_{S}'$	$rac{1}{\sqrt{6}} ig( \mathcal{B}_A - rac{1}{2} \mathcal{B}'_A - \mathcal{C}_{1A} - \mathcal{C}'_A ig)$	$rac{1}{2\sqrt{2}}B_A'-rac{1}{\sqrt{6}}\mathcal{C}_S'$		$rac{1}{2}\mathcal{A}_A - rac{1}{\sqrt{3}}\mathcal{C}_{2S}$		$-\frac{1}{2}(\mathcal{C}_{1A}+\mathcal{C}'_A)+\frac{1}{2\sqrt{3}}\mathcal{C}'_S$	$\frac{1}{2\sqrt{2}}(\mathcal{C}_{1A}+\mathcal{C}'_A)+\frac{1}{2\sqrt{6}}(\sqrt{2}\mathcal{C}_{2S}+\mathcal{C}'_S)$	$-\frac{1}{2\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}'_A)+\frac{1}{6}(2\mathcal{C}_{2S}-\mathcal{C}'_S)$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_A-\mathcal{C}_{1A}-\mathcal{C}_A')+\frac{1}{6\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}_S')$
Amplitudes with SU(3) symmetry	$\frac{1}{2\sqrt{2}}(-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{2\sqrt{6}}\mathcal{C}_{2S}+\frac{1}{\sqrt{6}}\mathcal{C}'_{S}$		$\frac{1}{2\sqrt{3}}(-\mathcal{B}_{A}-\mathcal{C}_{1A}+\mathcal{C}_{2A})+\frac{1}{6}\mathcal{C}_{2S}-\frac{1}{3}\mathcal{C}_{S}'$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_{A} - \mathcal{C}_{1A} + \mathcal{C}_{2A}) + \frac{1}{6\sqrt{2}}\mathcal{C}_{2S} - \frac{1}{3\sqrt{2}}\mathcal{C}_{S}'$	$rac{1}{\sqrt{6}} ig( \mathcal{B}_A - rac{1}{2} \mathcal{B}'_A - \mathcal{C}_{1A} - \mathcal{C}'_A ig)$	$\frac{1}{2\sqrt{2}}B_A'-\frac{1}{\sqrt{6}}\mathcal{C}_S'$		$\frac{1}{2}\mathcal{A}_A - \frac{1}{\sqrt{3}}\mathcal{C}_{2S}$		$-\frac{1}{2}(\mathcal{C}_{1A}+\mathcal{C}'_A)+\frac{1}{2\sqrt{3}}\mathcal{C}'_S$	$rac{1}{2\sqrt{2}}(\mathcal{C}_{1A}+\mathcal{C}'_A)+rac{1}{2\sqrt{6}}(2\mathcal{C}_{2S}+\mathcal{C}'_S)$	$-\frac{1}{2\sqrt{3}}(\mathcal{B}_A+\mathcal{C}_{1A}+\mathcal{C}_A')+\frac{1}{6}(2\mathcal{C}_{2S}-\mathcal{C}_S')$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_A-\mathcal{C}_{1A}-\mathcal{C}_A')+\frac{1}{6\sqrt{2}}(2\mathcal{C}_{2S}-\mathcal{C}_S')$
Reaction	$\rightarrow p\pi^0$		$ ightarrow p$ $\eta_0$	$ o p  \eta_8$	$\Xi_c^{0A} { ightarrow} \Lambda^0 K^0$	$ ightarrow \Sigma^0 K^0$		$ ightarrow \Sigma^- K^+$		$\rightarrow p  \pi^-$	$\rightarrow n  \pi^0$	$ ightarrow n \eta_0$	$ ightarrow n\eta_8$

for quark-mixing-suppressed modes,

$$|A(\Xi_{c}^{+A} \to \Sigma^{+} K^{0})|^{2} = 2|A(\Xi_{c}^{0A} \to \Sigma^{0} K^{0})|^{2},$$
$$|A(\Xi_{c}^{0A} \to \Sigma^{-} K^{+})|^{2} = 2|A(\Xi_{c}^{+A} \to \Sigma^{0} K^{+})|^{2}, \quad (59)$$

for quark-mixing-doubly-suppressed modes, and relations between the squares of quark-mixing-allowed, -suppressed, and -doubly-suppressed amplitudes:

$$\begin{split} &|A(\Lambda_{c}^{+} \to p \eta_{0})|^{2} = s_{1}^{2} |A(\Lambda_{c}^{+} \to \Sigma^{+} \eta_{0})|^{2}, \\ &|A(\Xi_{c}^{0A} \to \Xi^{-}K^{+})|^{2} = s_{1}^{2} |A(\Xi_{c}^{0A} \to \Xi^{-}\pi^{+})|^{2}, \\ &|A(\Xi_{c}^{0A} \to \Sigma^{+}\pi^{-})|^{2} = s_{1}^{2} |A(\Xi_{c}^{0A} \to \Sigma^{+}K^{-})|^{2}, \\ &|A(\Xi_{c}^{+A} \to \Sigma^{+}K^{0})|^{2} = s_{1}^{4} |A(\Lambda_{c}^{+} \to p\overline{K}^{0})|^{2}, \\ &|A(\Xi_{c}^{+A} \to \pi^{+})|^{2} = s_{1}^{4} |A(\Lambda_{c}^{+} \to \Xi^{0}K^{+})|^{2}, \\ &|A(\Xi_{c}^{0A} \to \Sigma^{-}K^{+})|^{2} = s_{1}^{4} |A(\Xi_{c}^{0A} \to \Xi^{-}\pi^{+})|^{2}, \\ &|A(\Xi_{c}^{0A} \to p \pi^{-})|^{2} = s_{1}^{4} |A(\Xi_{c}^{0A} \to \Sigma^{+}K^{-})|^{2}, \\ &|A(\Lambda_{c}^{+} \to p K^{0})|^{2} = s_{1}^{4} |A(\Xi_{c}^{+A} \to \Xi^{0}\pi^{+})|^{2}, \\ &|A(\Lambda_{c}^{+} \to p K^{0})|^{2} = s_{1}^{4} |A(\Xi_{c}^{+A} \to \Sigma^{+}\overline{K}^{0})|^{2}, \\ &|A(\Xi_{c}^{+} \to p \eta_{0})|^{2} = s_{1}^{4} |A(\Lambda_{c}^{+} \to \Sigma^{+}\eta_{0})|^{2}, \end{split}$$

where  $s_1 = \sin \theta_1$  and  $\theta_1$  is the usual quark-mixing angle.

Note that the above quark-diagram relations can also be reproduced in the SU(3) Hamiltonian approach of Savage and Springer<sup>1</sup> (SS) [6] except for Eq. (59) and the first and last relations in Eq. (60). We believe that when the use of the SU(3) Hamiltonian in which the symmetry amplitudes are tensor decomposed is done correctly to incorporate the symmetry properties of the baryon wave function, the reduced matrix element *a* defined in Ref. [6] should not contribute and all aforementioned SU(3) quark-diagram results will be reproduced.

The relations between the SU(3) reduced matrix elements of Ref. [6] and the quark-diagram amplitudes  $are^2$ 

$$a = 0, \quad b = -\frac{1}{4}(\mathcal{C}'_{A} + \mathcal{C}_{2A}) - \frac{1}{4\sqrt{3}}(\mathcal{C}'_{S} + \mathcal{C}_{2S}),$$
$$c = \frac{1}{4}(\mathcal{A}_{A} + \mathcal{B}'_{A}) - \frac{1}{2\sqrt{3}}(\mathcal{C}'_{S} + \mathcal{C}_{2S}), \quad d = \frac{1}{2\sqrt{3}}(\mathcal{C}'_{S} + \mathcal{C}_{2S}),$$

$$e = \frac{1}{8} (\mathcal{A}_A - \mathcal{B}'_A) + \frac{1}{4\sqrt{3}} (\mathcal{C}'_S - \mathcal{C}_{2S}), \tag{61}$$

$$f = -\frac{1}{8}(2C_{1A} + C'_A - C_{2A}) + \frac{1}{8\sqrt{3}}(C'_S - C_{2S}),$$
$$g = \frac{1}{8}(A_A + 2B_A - B'_A).$$

At first sight, it appears that there are six independent SU(3) parameters, but eight different quark amplitudes. However, one may make the redefinition (this redefinition is not unique)

$$\widetilde{\mathcal{A}} = \mathcal{A}_{A} - \frac{2}{\sqrt{3}} \mathcal{C}'_{S}, \quad \widetilde{\mathcal{B}}' = \mathcal{B}'_{A} - \frac{2}{\sqrt{3}} \mathcal{C}_{2S}, \quad \widetilde{\mathcal{C}}_{S} = \mathcal{C}'_{S} + \mathcal{C}_{2S},$$
$$\widetilde{\mathcal{C}}' = \mathcal{C}'_{A} - \frac{1}{\sqrt{3}} \mathcal{C}'_{S} + \mathcal{C}_{1A}, \quad \widetilde{\mathcal{C}}_{2} = \mathcal{C}_{2A} - \frac{1}{\sqrt{3}} \mathcal{C}_{2S} - \mathcal{C}_{1A}, \quad (62)$$

so that the amplitudes for the decay modes in Table I can be expressed in terms of the six quark-diagram terms  $\widetilde{\mathcal{A}}$ ,  $\widetilde{\mathcal{B}}'$ ,  $\mathcal{B}_A$ ,  $\widetilde{\mathcal{C}}'$ ,  $\widetilde{\mathcal{C}}_2$ , and  $\widetilde{\mathcal{C}}_S$ .

Table Ia for quark-mixing-allowed decays  $B_c(\overline{3}) \rightarrow B(8) + M(8)$  was also previously considered by Kohara [10]. In Kohara's results there are eight quark diagrams  $a_K, b_K, c_K, d_{1K}, d_{2K}, d_{3K}, d_{4K}$ , and  $e_K$ .<sup>3</sup> The relations between our quark-diagram amplitudes and those in [10] are

$$a_{K} = \frac{\sqrt{6}}{8} \mathcal{A}_{A}, \quad b_{K} = \frac{\sqrt{6}}{8} \mathcal{B}_{A}', \quad c_{K} = \frac{\sqrt{6}}{4} \mathcal{B}_{A},$$

$$d_{1K} = \frac{\sqrt{6}}{4} \mathcal{C}_{A}' - \frac{1}{2\sqrt{2}} \mathcal{C}_{S}', \quad d_{2K} = \frac{1}{\sqrt{2}} \mathcal{C}_{S}', \quad (63)$$

$$d_{3K} = \frac{\sqrt{6}}{4} \mathcal{C}_{2A} - \frac{1}{2\sqrt{2}} \mathcal{C}_{2S}, \quad d_{4K} = \frac{1}{\sqrt{2}} \mathcal{C}_{2S}, \quad e_{K} = \frac{\sqrt{6}}{4} \mathcal{C}_{1A},$$

which are obtained by comparing Table I of [10] with our Table IIa. As emphasized before, we consider it conceptually clearer and in practice simpler to work with the orthonormal basis of quark states.

## V. SEXTET CHARMED BARYON DECAYS

#### A. Quark-diagram scheme for $B_c(6) \rightarrow B(10) + M(8)$

There are six independent quark-diagram amplitudes for  $B_c(6) \rightarrow B(10) + M(8)$ . The amplitudes  $\mathcal{B}$  and  $\mathcal{C}_1$  are forbidden owing to the Pati-Woo theorem. The relevant diagrams and amplitudes are exhibited in Fig. 3 and Table III, respectively.

<sup>&</sup>lt;sup>1</sup>Note that the reduced matrix elements *a*, *b*, *c*, and *d* introduced in Ref. [6] are associated with the operator  $O_{15}$ , which transforms as a  $\overline{15}$  under flavor SU(3) and is symmetric in color indices and hence cannot induce a baryon-baryon transition. In other words, *baryon-pole diagrams are prohibited by the operator*  $O_{\overline{15}}$ .

<sup>&</sup>lt;sup>2</sup>Using Table II and the relations (61), one can perform a crosscheck on the SU(3) amplitudes given in Tables I–III of Ref. [6]. For example, we find a sign error in Table III [6], namely, the squared matrix elements for  $\Xi_c^0 \rightarrow \Lambda^0 K^0$  should read  $(1/6)|a-2b+c+2e-4f-4g|^2$ .

<sup>&</sup>lt;sup>3</sup>In order to avoid notation confusion with the SU(3) parameters of SS [6], we add a subscript *K* to Kohara's quark-diagram amplitudes [10].



FIG. 3. Quark diagrams for the decay  $B_c(6) \rightarrow B(10) + M(8)$ .

From Table III we obtain the following relations, in the absence of both SU(3) violations and final-state interactions:

$$|A(\Omega_{c}^{0} \rightarrow \Sigma^{*+} K^{-})|^{2} = 2|A(\Omega_{c}^{0} \rightarrow \Sigma^{*0} \overline{K}^{0})|^{2},$$
  

$$|A(\Omega_{c}^{0} \rightarrow \Sigma^{*0} \eta_{0})|^{2} = 2|A(\Omega_{c}^{0} \rightarrow \Sigma^{*0} \eta_{8})|^{2},$$
  

$$|A(\Omega_{c}^{0} \rightarrow \Sigma^{*0} \eta_{0})|^{2} = \frac{1}{2}s_{1}^{2}|A(\Omega_{c}^{0} \rightarrow \Xi^{*0} \eta_{0})|^{2},$$
 (64)  

$$|A(\Omega_{c}^{0} \rightarrow \Sigma^{*+} \pi^{-})|^{2} = s_{1}^{2}|A(\Omega_{c}^{0} \rightarrow \Sigma^{*+} K^{-})|^{2},$$
  

$$|A(\Omega_{c}^{0} \rightarrow \Xi^{*-} K^{+})|^{2} = \frac{1}{3}s_{1}^{2}|A(\Omega_{c}^{0} \rightarrow \Omega^{-} K^{+})|^{2}.$$

It is interesting to note that the  $\Omega_c^0$  decays into  $\Delta^0 \overline{K}^0$  and  $\Delta^+ K^-$  are prohibited in the quark-diagram scheme,

$$|A(\Omega_{c}^{0} \rightarrow \Delta^{0} \overline{K}^{0})|^{2} = 0, |A(\Omega_{c}^{0} \rightarrow \Delta^{+} K^{-})|^{2} = 0, (65)$$

as the quark diagram  $C_1$  is not allowed by the Pati-Woo theorem. Consequently, the corresponding reduced matrix element  $\alpha$  makes no contribution.

We note that the above quark-diagram relations except for the last one listed in (65) cannot be reproduced in the SU(3)IR approach of SS unless the reduced matrix elements  $\alpha$  and  $\delta$  do not contribute. Therefore in the SU(3) limit there are only four independent quark-diagram amplitudes or reduced matrix elements. Relations between the quark-diagram amplitudes and the symmetry parameters [see Eq. (25) of Ref. [6]] are given by

$$\mathcal{A}_{S} = \beta - 2 \eta, \quad \mathcal{B}'_{S} = \beta + 2 \eta,$$
$$\mathcal{C}'_{S} = \gamma + 2\lambda, \quad \mathcal{C}_{2S} = \gamma - 2\lambda. \tag{66}$$

## B. Quark-diagram scheme for $B_c(6) \rightarrow B(8) + M(8)$

We discuss in this section the decays of sextet charmed baryons into an octet baryon and a pseudoscalar meson. The relevant quark diagrams and amplitudes are shown in Fig. 4 and Table IV, respectively.

In the SU(3)-symmetry approach [6], there exist no relations between the decays of  $\Omega_c^0 \rightarrow B(8) + M(8)$ . However, from Table IV we obtain

$$|A(\Omega_{c}^{0} \to n\bar{K}^{0})|^{2} = |A(\Omega_{c}^{0} \to pK^{-})|^{2},$$
  
$$|A(\Omega_{c}^{0} \to \Sigma^{+}K^{-})|^{2} = 2|A(\Omega_{c}^{0} \to \Sigma^{0}\bar{K}^{0})|^{2}.$$
 (67)

These relations cannot be reproduced in the SU(3) approach [6] unless the contributions due to the SU(3) parameters *a* 

		- -	
Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
		(a) Quark-mixing-allov	wed modes
$\Omega_c^0\!\!\rightarrow\!\Xi\!*^0ec{K}^0$	$rac{1}{\sqrt{3}}\mathcal{B}_{S}'$	$rac{1}{\sqrt{3}}\mathcal{B}_{S}'$	$\frac{1}{\sqrt{3}}\mathcal{B}_{s}^{\prime}\mathrm{exp}(i\delta_{1}^{\Xi^{*}\overline{K}})$
${ ightarrow} \Omega^- \pi^+$	$\mathcal{A}_S$	$\mathcal{A}_{S}$	$\mathcal{A}_{ m S}{ m exp}(i\delta_{1}^{ m I\pi})$
		(b) Quark-mixing-suppre	essed modes
$\Omega_c^0\!\!\rightarrow\!\Xi^{*-}\pi^+$	$\frac{1}{\sqrt{3}}\left(-\mathcal{A}_{S}+\mathcal{C}_{2S}\right)$	$rac{1}{\sqrt{3}}(-\mathcal{A}_{S}+\mathcal{C}_{2S}+\mathcal{E}_{S}')$	$\frac{1}{3\sqrt{3}}\left[\left(-\mathcal{A}_{S}+\mathcal{B}_{S}'+2\mathcal{C}_{2S}+2\mathcal{E}_{S}'\right)\exp(i\delta_{22}^{\Xi^{*}\pi})+\left(-2\mathcal{A}_{S}-\mathcal{B}_{S}'+\mathcal{C}_{2S}+\mathcal{E}_{S}'\right)\exp(i\delta_{12}^{\Xi^{*}\pi})\right]$
$\rightarrow \Pi^{*0} \boldsymbol{\pi}^0$	$\frac{1}{\sqrt{6}}(B_S'+\mathcal{C}_{2,\zeta})$	$rac{1}{\sqrt{6}}(\mathcal{B}_{S}^{\prime}+\mathcal{C}_{2S}+\mathcal{E}_{S}^{\prime})$	$\frac{1}{3\sqrt{6}} \left[ 2\left( -\mathcal{A}_{S} + \mathcal{B}_{S}' + 2\mathcal{C}_{2S} + 2\mathcal{E}_{S}'\right) \exp(i\delta_{32}^{\Xi^{*}\pi}) - \left( -2\mathcal{A}_{S} - \mathcal{B}_{S}' + \mathcal{C}_{2S} + \mathcal{E}_{S}'\right) \exp(i\delta_{12}^{\Xi^{*}\pi}) \right]$
$\to \Xi^{*0} \boldsymbol{\eta}_0$	$\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_{S})$	$\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_{S}+\mathcal{E}_{S}+\mathcal{E}'_{S})$	$\left[\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{C}'_S+\mathcal{E}_S+\mathcal{E}'_S)\right]\exp(i\delta^{\Xi*}_{1/2}\eta)$
$\to \Xi^{*0} \boldsymbol{\eta}_8$	$-rac{1}{3\sqrt{2}}(3B_s'-\mathcal{C}_{2S}+2\mathcal{C}_s')$	$-\frac{1}{3\sqrt{2}}(3\mathcal{B}_{S}^{\prime}-\mathcal{C}_{2S}+2\mathcal{C}_{S}^{\prime}+\mathcal{E}_{S}^{\prime}-2\mathcal{E}_{S})$	$\left[-\frac{1}{3\sqrt{2}}(3B_{S}^{\prime}-\mathcal{C}_{2S}+2C_{S}^{\prime}+\mathcal{E}_{S}^{\prime}-2\mathcal{E}_{S})\right]\exp(i\delta_{1/2}^{\Xi^{*}}\eta)$
$\rightarrow \Sigma^{*0} \overline{K}^{0}$	$\frac{1}{\sqrt{6}C_s'}$	$rac{1}{\sqrt{6}}(\mathcal{C}_{S}^{\prime}+\mathcal{E}_{S})$	$\frac{1}{3\sqrt{6}}(3\mathcal{C}'_{S}+\mathcal{E}_{S})[4\exp(i\delta^{\Sigma*\bar{K}}_{3/2})-\exp(i\delta^{\Sigma*\bar{K}}_{1/2})]$
${\rightarrow} \Sigma^{*+} K^{-}$	$\frac{1}{\sqrt{3}C_{S}}$	$rac{1}{\sqrt{3}}(\mathcal{C}'_{S}+\mathcal{E}_{S})$	$\frac{1}{3\sqrt{3}}(\mathcal{C}'_{S}+\mathcal{E}_{S})[2\exp(i\delta^{\Sigma*\overline{K}}_{3/2})+\exp(i\delta^{\Sigma*\overline{K}}_{1/2})]$
$ ightarrow \Omega^- K^+$	$\mathcal{A}_{S} + \mathcal{C}_{2S}$	$\mathcal{A}_{S} + \mathcal{C}_{2S} + \mathcal{E}_{S}'$	$(\mathcal{A}_{S} + \mathcal{C}_{2S} + \mathcal{E}'_{S}) \exp(i\delta^{\Omega K}_{1/2})$

TABLE III. Quark-diagram amplitudes for  $B_c(6) \rightarrow B(10+M(8))$ .

Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
		Quark-mixing-doubly-suppre	ssed modes
$\Omega_c^0 { o} \Sigma^{*+} \pi^-$	$\frac{1}{\sqrt{3}}c'_s$	$rac{1}{\sqrt{3}}\mathcal{C}'_S$	$\frac{1}{2\sqrt{3}}\left[\frac{2}{3}\mathcal{C}_{2S}\exp(i\delta_{2}^{\mathbb{Z}}\pi) + (\mathcal{C}_{S}' - \mathcal{C}_{2S})\exp(i\delta_{1}^{\mathbb{Z}}\pi) + \left(\mathcal{C}_{S}' + \frac{1}{3}\mathcal{C}_{2S}\right)\exp(i\delta_{0}^{\mathbb{Z}}\pi)\right]$
$\rightarrow \Sigma^{*-}\pi^{+}$	$\frac{1}{\sqrt{3}}c_{2S}$	$\frac{1}{\sqrt{3}}C_{2S}$	$\frac{1}{2\sqrt{3}} \left[ \frac{2}{3} \mathcal{C}_{2S} \exp(i\partial_{2}^{\mathbb{Z}}\pi) - (\mathcal{C}_{S}' - \mathcal{C}_{2S}) \exp(i\partial_{1}^{\mathbb{Z}}\pi) + \left(\mathcal{C}_{S}' + \frac{1}{3}\mathcal{C}_{2S}\right) \exp(i\partial_{0}^{\mathbb{Z}}\pi) \right]$
$ ightarrow \Sigma^{*0} \pi^0$	$rac{1}{2\sqrt{3}}(\mathcal{C}_{2S}-\mathcal{C}'_S)$	$rac{1}{2\sqrt{3}}(\mathcal{C}_{2S}\!-\!\mathcal{C}'_{S})$	$\frac{1}{2\sqrt{3}} \left[ \frac{4}{3} \mathcal{C}_{2s} \exp(i\delta_2^{\mathbb{X}}\pi) - \left( \mathcal{C}_s' + \frac{1}{3} \mathcal{C}_{2s} \right) \exp(i\delta_0^{\mathbb{X}}\pi) \right]$
$ ightarrow \Sigma^{*0} \eta_0$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S} + \mathcal{C}'_S)$	$\frac{1}{3\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{C}'_{S})$	$rac{1}{3\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{C}'_S) \mathrm{exp}(i \delta_1^{\Sigma*} \eta_0)$
${ ightarrow} \Sigma^{*0} \eta_8$	$rac{1}{6}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$rac{1}{6}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$\frac{1}{6}(\mathcal{C}_{2S}+\mathcal{C}_{S}')\mathrm{exp}(i\delta_{1}^{S^{*}\eta_{8}})$
$\to \Xi^{*0} K^0$	$rac{1}{\sqrt{3}}(B'_S+\mathcal{C}'_S)$	$rac{1}{\sqrt{3}}(\mathcal{B}_{S}^{\prime}+\mathcal{C}_{S}^{\prime})$	$\frac{1}{2\sqrt{3}}\left[\left(\mathcal{A}_{S}+\mathcal{B}_{S}'+\mathcal{C}_{2S}+\mathcal{C}_{S}'\right)\exp(i\vartheta_{1}^{\Xi^{*}K})+\left(-\mathcal{A}_{S}+\mathcal{B}_{S}'-\mathcal{C}_{2S}+\mathcal{C}_{S}'\right)\exp(i\vartheta_{0}^{\Xi^{*}K})\right]$
${}^+ M^+ K^+$	$\frac{1}{\sqrt{3}}(\mathcal{A}_S + \mathcal{C}_{2S})$	$\frac{1}{\sqrt{3}}(\mathcal{A}_S + \mathcal{C}_{2S})$	$\frac{1}{2\sqrt{3}}\left[\left[\left(\mathcal{A}_{S}+\mathcal{B}_{S}'+\mathcal{C}_{2S}+\mathcal{C}_{S}'\right)\exp(i\delta_{1}^{\Xi^{*}K}\right)-\left(-\mathcal{A}_{S}+\mathcal{B}_{S}'-\sqrt{2}\mathcal{C}_{2S}+\sqrt{2}\mathcal{C}_{S}'\right)\exp(i\delta_{0}^{\Xi^{*}K})\right]$
$ ightarrow \Delta^0 ec K^0$	0	0	0
$\rightarrow \Delta^+ K^-$	0	0	0





FIG. 4. Quark diagrams for the decay  $B_c(6) \rightarrow B(8) + M(8)$ .

and d vanish. Therefore Eq. (67) will provide a good test of the quark-diagram scheme. Unfortunately, these processes are either singly or quark-mixing-doubly suppressed. We do not expect that an encouraging experimental verification will come out soon.

The relations between quark-diagram amplitudes and SU(3) reduced matrix elements are found to be

$$a = d = 0, \quad b = -\frac{1}{2\sqrt{3}}(\mathcal{A}_{S} + \mathcal{B}'_{S}),$$
$$c + l = \frac{1}{4}(\mathcal{C}'_{A} + \mathcal{C}_{2A}) - \frac{1}{4\sqrt{3}}(\mathcal{C}'_{S} + \mathcal{C}_{2S}),$$

$$e - l = \frac{1}{2\sqrt{3}}(\mathcal{C}'_{S} + \mathcal{C}_{2S}), \quad f = \frac{1}{8}(\mathcal{C}'_{A} - \mathcal{C}_{2A}) - \frac{1}{8\sqrt{3}}(\mathcal{C}'_{S} - \mathcal{C}_{2S}),$$
(68)

$$g = \frac{1}{4\sqrt{3}}(\mathcal{A}_{S} - \mathcal{B}'_{S}) + \frac{1}{4\sqrt{3}}(\mathcal{C}'_{S} - \mathcal{C}_{2S}),$$
  
$$h = -\frac{1}{8}(\mathcal{C}'_{A} - \mathcal{C}_{2A}) + \frac{1}{8\sqrt{3}}(\mathcal{C}'_{S} - \mathcal{C}_{2S}) + \frac{1}{4}\mathcal{C}_{1A},$$
  
$$k = -\frac{1}{4\sqrt{3}}(\mathcal{A}_{S} - \mathcal{B}'_{S}) - \frac{1}{4}\mathcal{B}_{A}.$$

		commutin mighin wing	$(\alpha)$ $(\alpha)$ $(\alpha)$ $(\alpha)$
Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitudes with SU(3)-symmetry breaking and final-state interactions
$\Omega^0_c{ o}\Xi^0ec K^0$	$-rac{1}{2}B_A+rac{1}{\sqrt{3}}B_S'$	(a) Quark-mixing-allowe $-\frac{1}{2}B_A + \frac{1}{\sqrt{3}}B'_5$	d mode $\left(-\frac{1}{2}\mathcal{B}_A+\frac{1}{\sqrt{3}}\mathcal{B}_S'\right)\exp(i\partial_1^{\Xi K})$
		(b) Quark-mixing-suppress	ed modes
$\Omega^0_c {\rightarrow} \Sigma^0 \overline{K}^0$	$\frac{1}{2\sqrt{2}}(\mathcal{C}_{1A}-\mathcal{C}_A')+\frac{1}{2\sqrt{6}}\mathcal{C}_S'$	$\frac{1}{2\sqrt{2}}(\mathcal{C}_{1A}-\mathcal{C}'_A+\mathcal{E}_A)+\frac{1}{2\sqrt{6}}(\mathcal{C}'_S+\mathcal{E}_S)$	$\frac{1}{6} \left[ \frac{1}{\sqrt{2}} (\mathcal{C}_{1A} - \mathcal{C}'_A + \mathcal{E}_A) + \frac{1}{\sqrt{6}} (\mathcal{C}'_S + \mathcal{E}_S) \right] \left[ 4 \exp(i \delta_{3Z}^{\mathbf{X}} \overline{k}) - \exp(i \delta_{1Z}^{\mathbf{X}} \overline{k}) \right]$
$ ightarrow \Sigma^+ K^-$	$\frac{1}{2}\left(\mathcal{C}_{1A}-\mathcal{C}_{A}'\right)+\frac{1}{2\sqrt{3}}\mathcal{C}_{S}'$	$rac{1}{2}(\mathcal{C}_{1A}-\mathcal{C}'_A+\mathcal{E}_A)+rac{1}{2\sqrt{3}}(\mathcal{C}'_S+\mathcal{E}_S)$	$\frac{1}{6} \left[ (\mathcal{C}_{1A} - \mathcal{C}'_A + \mathcal{E}_A) + \frac{1}{\sqrt{3}} (\mathcal{C}'_S + \mathcal{E}_S) \right] \left[ 2 \exp(i \delta_{3,2}^{\mathbb{Z}} \widetilde{K}) + \exp(i \delta_{1/2}^{\mathbb{Z}} \widetilde{K}) \right]$
$\stackrel{\mathbf{u}^{+}}{\rightarrow} \Pi^{-}$	$\frac{1}{\sqrt{3}}(-\mathcal{A}_{S}+\mathcal{C}_{2S})$	$rac{1}{\sqrt{3}}(-\mathcal{A}_{S}\!+\!\mathcal{C}_{2S}\!+\!\mathcal{E}_{S}')$	$\frac{1}{3\sqrt{3}}\left[(-\mathcal{A}_{S}+\mathcal{B}_{S}'+2\mathcal{C}_{2S}+2\mathcal{E}_{S}')\exp(i\delta\overline{\mathbb{S}}_{32}^{*\pi})+(-2\mathcal{A}_{S}-\mathcal{B}_{S}'+\mathcal{C}_{2S}+\mathcal{E}_{S}')\exp(i\delta\overline{\mathbb{E}}_{12}^{*\pi})\right]$
$\rightarrow \Pi^0 \boldsymbol{\pi}^0$	$rac{1}{\sqrt{6}}(B_{ m s}'+{\cal C}_{2S})$	$rac{1}{\sqrt{6}}(\mathcal{B}_{S}'+\mathcal{C}_{2S}+\mathcal{E}_{S}')$	$\frac{1}{3\sqrt{6}} \left[ 2(-\mathcal{A}_{S} + \mathcal{B}'_{S} + 2\mathcal{C}_{2S} + 2\mathcal{E}'_{S}) \exp(i\delta_{32}^{\Xi*}\pi) - (-2\mathcal{A}_{S} - \mathcal{B}'_{S} + \mathcal{C}_{2S} + \mathcal{E}'_{S}) \exp(i\delta_{12}^{\Xi*}\pi) \right]$
$\to \Xi^0  \eta_0$	$\frac{1}{2\sqrt{3}}(-\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_A')$	$\frac{1}{2\sqrt{3}}(-\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_A'+\mathcal{E}_A)$	$\left[\frac{1}{2\sqrt{3}}(-\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}'_A+\mathcal{E}_A)-\frac{1}{6}(\mathcal{C}'_S+\mathcal{E}_S)+\frac{1}{3}(\mathcal{C}_{2S}+\mathcal{E}'_S)\right]\exp(i\delta_{1/2}^{\Xi})$
	$-\frac{1}{6}c_{S}'+\frac{1}{3}c_{2S}$	$-\frac{1}{6}(\mathcal{C}'_{S}+\mathcal{E}_{S})+\frac{1}{3}(C_{2S}+\mathcal{E}'_{S})$	
$\to \Pi^0 \eta_8$	$\frac{1}{\sqrt{6}}(\mathcal{B}_A - \mathcal{C}_{1A} + \mathcal{C}_A')$	$rac{1}{\sqrt{6}}(\mathcal{B}_A-\mathcal{C}_{1A}-\mathcal{E}_A+\mathcal{C}_A')$	$\left[\frac{1}{\sqrt{6}}(\mathcal{B}_{A}-\mathcal{C}_{1A}-\mathcal{E}_{A}+\mathcal{C}'_{A})+\frac{1}{3\sqrt{2}}(-3\mathcal{B}'_{S}+\mathcal{C}_{2S}+\mathcal{C}'_{S}+\mathcal{E}'_{S}+\mathcal{E}_{S})\right]\exp(i\delta_{1/2}^{\Xi\eta_{S}})$
	$+\frac{1}{3\sqrt{2}}(-3\mathcal{B}'_{S}+\mathcal{C}_{2S}+\mathcal{C}'_{S})$	$+\frac{1}{3\sqrt{2}}(-3\mathcal{B}_{S}^{\prime}+\mathcal{C}_{2S}+\mathcal{C}_{S}^{\prime}+\mathcal{E}_{S}^{\prime}+\mathcal{E}_{S})$	
$ ightarrow \Lambda \overline{K}^0$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_A')-\frac{1}{2\sqrt{2}}\mathcal{C}_S'$	$\frac{1}{2\sqrt{6}}(2\mathcal{B}_A+\mathcal{C}_{1A}-\mathcal{C}_A'-\mathcal{E}_A)-\frac{1}{2\sqrt{2}}(\mathcal{C}_S'+\mathcal{E}_S)$	$\left[\frac{1}{2\sqrt{6}}(2\mathcal{B}_A + \mathcal{C}_{1A} - \mathcal{C}'_A - \mathcal{E}_A) - \frac{1}{2\sqrt{2}}(\mathcal{C}'_S + \mathcal{E}_S)\right] \exp(i\mathcal{B}_{12}^{\Lambda\bar{K}})$
		(c) Quark-mixing-doubly-supp	ressed modes
$\Omega^0_c \!  ightarrow \! \Lambda  \pi^0$	$rac{1}{4}(\mathcal{C}_{2S}+\mathcal{C}'_S)+rac{1}{4\sqrt{3}}(\mathcal{C}_{2A}+\mathcal{C}'_A)$	$rac{1}{4}(\mathcal{C}_{2S}+\mathcal{C}'_{S})+rac{1}{4\sqrt{3}}(\mathcal{C}_{2A}+\mathcal{C}'_{A})$	$\left[\frac{1}{4}(\mathcal{C}_{2S}+\mathcal{C}'_S)+\frac{1}{4\sqrt{3}}(\mathcal{C}_{2A}+\mathcal{C}'_A)\right]\exp(i\delta_1^\Lambda\pi)$
$ ightarrow \Lambda \eta_0$	$\frac{1}{6\sqrt{2}}(-2\mathcal{B}_{A}+2\mathcal{C}_{1A}+\mathcal{C}_{2A}-\mathcal{C}_{A}')$	$\frac{1}{6\sqrt{2}}(-2\mathcal{B}_A+2\mathcal{C}_{1A}+\mathcal{C}_{2A}-\mathcal{C}_A')$	$\left[\frac{1}{6\sqrt{2}}(-2\mathcal{B}_A+2\mathcal{C}_{1A}+\mathcal{C}_{2A}-\mathcal{C}'_A)+\frac{1}{2\sqrt{6}}(\mathcal{C}_{2S}-\mathcal{C}'_S)\right]\exp(i\delta_0^{\Lambda\eta_0})$
	$+\frac{1}{2\sqrt{6}}(\mathcal{C}_{2S}-\mathcal{C}_{S}')$	$+rac{1}{2\sqrt{6}}(\mathcal{C}_{2S}\!-\!\mathcal{C}'_{S})$	

TABLE IV. Quark-diagram amplitudes for  $B_c(6) \rightarrow B(8) + M(8)$ .

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(Continued).	
TABLE IV.	

		TABLE IV. (Cont.	inued).
Reaction	Amplitudes with SU(3) symmetry	Amplitudes with SU(3)-symmetry breaking	Amplitude with SU(3)-symmetry breaking and final-state interactions
$ ightarrow \Lambda \eta_8$	$\frac{1}{12}(4\mathcal{B}_A-4\mathcal{C}_{1A}+\mathcal{C}_{2A}-\mathcal{C}_A')$	$\frac{1}{12}(4\mathcal{B}_A-4\mathcal{C}_{1A}+\mathcal{C}_{2A}-\mathcal{C}_A')$	$\frac{1}{12} \left[ \left( 4\mathcal{B}_A - 4\mathcal{C}_{1A} + \mathcal{C}_{2A} - \mathcal{C}'_A \right) \frac{1}{4\sqrt{3}} (\mathcal{C}_{2S} - \mathcal{C}'_S) \right] \exp(i\beta_0^{\Lambda \eta_S})$
	$+rac{1}{4\sqrt{3}}(\mathcal{C}_{2S}-\mathcal{C}_{S}')$	$+rac{1}{4\sqrt{3}}(\mathcal{C}_{2S}\!-\!\mathcal{C}'_S)$	
$\rightarrow \Sigma^{+} \pi^{-}$	$-\frac{1}{2}\mathcal{C}'_A+\frac{1}{2\sqrt{3}}\mathcal{C}'_S$	$-\frac{1}{2}\mathcal{C}'_A+\frac{1}{2\sqrt{3}}\mathcal{C}'_S$	$\frac{1}{2\sqrt{3}}\left\{\left(-\frac{1}{\sqrt{3}}\mathcal{C}_{2A}+\frac{1}{3}\mathcal{C}_{2S}\right)\exp(i\partial_{2}^{\mathbb{X}}\pi)+\frac{1}{2}\left[\sqrt{3}(-\mathcal{C}_{A}'+\mathcal{C}_{2A})+\mathcal{C}_{S}'-\mathcal{C}_{2S}\right]\exp(i\partial_{1}^{\mathbb{X}}\pi)\right.$
			$+\frac{1}{2}\left(-\sqrt{3}\mathcal{C}_A'-\frac{1}{\sqrt{3}}\mathcal{C}_{2A}+\mathcal{C}_S'+\frac{1}{3}\mathcal{C}_{2S}\right)\exp(i\delta_0^{\mathbb{Z}}\pi)\bigg\}$
$\rightarrow \Sigma^{-} \pi^{+}$	$-rac{1}{2}\mathcal{C}_{2A}+rac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$-\frac{1}{2}\mathcal{C}_{2A}+\frac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$\frac{1}{2\sqrt{3}}\left\{\left(-\frac{1}{\sqrt{3}}\mathcal{C}_{2A}+\frac{1}{3}\mathcal{C}_{2S}\right)\exp(i\delta_{2}^{\mathbb{X}}\pi)-\frac{1}{2}\left[\sqrt{3}(-\mathcal{C}_{A}'+\mathcal{C}_{2A})+\mathcal{C}_{S}'-\mathcal{C}_{2S}\right]\exp(i\delta_{1}^{\mathbb{X}}\pi)\right.$
			$+ \frac{1}{2} \left( -\sqrt{3}\mathcal{C}_A' - \frac{1}{\sqrt{3}}\mathcal{C}_{2A} + \mathcal{C}_S' + \frac{1}{3}\mathcal{C}_{2S} \right) \exp(i\delta_0^{\mathbb{Z}}\pi) \bigg\}$
$ ightarrow \Sigma^0 \pi^0$	$\frac{1}{4}(\mathcal{C}'_{A} - \mathcal{C}_{2A}) + \frac{1}{4\sqrt{3}}(\mathcal{C}_{2S} - \mathcal{C}'_{S})$	$\frac{1}{4}(\mathcal{C}'_{A} - \mathcal{C}_{2A}) + \frac{1}{4\sqrt{3}}(\mathcal{C}_{2S} - \mathcal{C}'_{S})$	$\frac{1}{2\sqrt{3}} \left[ 2 \left( -\frac{1}{\sqrt{3}} \mathcal{C}_{2A} + \frac{1}{3} \mathcal{C}_{2S} \right) \exp(i\delta_2^{\mathbb{X}}\pi) - \frac{1}{2} \left( -\sqrt{3} \mathcal{C}'_A - \frac{1}{\sqrt{3}} \mathcal{C}_{2A} + \mathcal{C}'_S + \frac{1}{3} \mathcal{C}_{2S} \right) \exp(i\delta_0^{\mathbb{X}}\pi) \right]$
${\rightarrow} \Sigma^0 \eta_0$	$-rac{1}{2\sqrt{6}}(\mathcal{C}_{2A}+\mathcal{C}'_A)+rac{1}{6\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$-rac{1}{2\sqrt{6}}(\mathcal{C}_{2A}+\mathcal{C}'_A)+rac{1}{6\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{C}'_S)$	$\left[-\frac{1}{2\sqrt{6}}(\mathcal{C}_{2A}+\mathcal{C}'_A)+\frac{1}{6\sqrt{2}}(\mathcal{C}_{2S}+\mathcal{C}'_S)\right]\exp(i\partial_1^S\eta_0)$
$ ightarrow \Sigma^0 \eta_8$	$-\frac{1}{4\sqrt{3}}(\mathcal{C}'_A+\mathcal{C}_{2A})+\frac{1}{12}(\mathcal{C}'_S+\mathcal{C}_{2S})$	$-\frac{1}{4\sqrt{3}}\left(\mathcal{C}'_A+\mathcal{C}_{2A}\right)+\frac{1}{12}\left(\mathcal{C}'_S+\mathcal{C}_{2S}\right)$	$\left[-\frac{1}{4\sqrt{3}}(\mathcal{C}'_A+\mathcal{C}_{2,A})+\frac{1}{12}(\mathcal{C}'_S+\mathcal{C}_{2,S})\right]\exp(i\delta_1^\Sigma\eta_s)$
$\Omega_c^0\!\!\rightarrow\!\Xi^0K^0$	$rac{1}{\sqrt{3}}\mathcal{B}_{S}^{\prime}-rac{1}{2}\mathcal{C}_{A}^{\prime}-rac{1}{2\sqrt{3}}\mathcal{C}_{S}^{\prime}$	$rac{1}{\sqrt{3}}\mathcal{B}_S'-rac{1}{2}\mathcal{C}_A'-rac{1}{2\sqrt{3}}\mathcal{C}_S'$	$\frac{1}{4\sqrt{3}}\{[2(\mathcal{A}_{S}+\mathcal{B}'_{S})-\sqrt{3}(\mathcal{C}'_{A}+\mathcal{C}_{2A})-(\mathcal{C}'_{S}+\mathcal{C}_{2S})]\exp(i\partial_{1}^{\overline{=}K})$
			$+[2(-\mathcal{A}_{S}+\mathcal{B}_{S}')-\sqrt{3}(\mathcal{C}_{A}'-\mathcal{C}_{2A})-(\mathcal{C}_{S}'-\mathcal{C}_{2S})]\exp(i\delta_{0}^{\overline{=}K})\}$
${}^+ \Pi^- K^+$	$rac{1}{\sqrt{3}}A_{S}-rac{1}{2}\mathcal{C}_{2A}-rac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$rac{1}{\sqrt{3}}A_{S}-rac{1}{2}\mathcal{C}_{2A}-rac{1}{2\sqrt{3}}\mathcal{C}_{2S}$	$\frac{1}{4\sqrt{3}}\{[2(\mathcal{A}_S+\mathcal{B}'_S)-\sqrt{3}(\mathcal{C}'_A+\mathcal{C}_{2A})-(\mathcal{C}'_S+\mathcal{C}_{2S})]\exp(i\delta^{\mathbb{Z}K}_T)-[2(-\mathcal{A}_S+\mathcal{B}'_S)$
			$-\sqrt{3}\left(\mathcal{C}'_{A}-\mathcal{C}_{2A}\right)-\left(\mathcal{C}'_{S}-\mathcal{C}_{2S}\right)\left[\exp(i\delta_{0}^{\Xi K})\right]$
$ ightarrow n K^0$	$\frac{1}{2}c_{1A}$	$\frac{1}{2}c_{1A}$	$rac{1}{2}\mathcal{C}_{1A}\mathrm{exp}(i\delta_1^{VK})$
$ ightarrow p K^-$	$\frac{1}{2}c_{1A}$	$rac{1}{2}\mathcal{C}_{1A}$	$rac{1}{2} \mathcal{C}_{1A} \exp(i \delta_1^{NK})$

Therefore there are seven independent SU(3) parameters and quark-diagram amplitudes.

# VI. CONCLUSIONS

In this paper we have given a general and unified formulation useful for the quark-diagram scheme for baryons. Here we apply it to the two-body nonleptonic weak decays of charmed baryons and express their decay amplitudes in terms of the quark-diagram amplitudes. The effects of finalstate interactions and SU(3) violation arising in the horizontal W-loop quark diagrams are included in the tables. In the absence of SU(3) violation and final-state interactions we have obtained many relations among various decay modes. These relations provide a framework to study these effects. Some of the relations are valid even in the presence of finalstate interactions when each decay amplitude in the relation contains only a single phase shift. It will be interesting to

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compare all these relations with future experimental data.

Our results are consistent with those from the SU(3)IR scheme. In addition, in the quark-diagram scheme we are able to impose the Pati-Woo theorem for weak decays and obtain more specific results than those from the SU(3)IR scheme.

We also note that the quark-mixing-allowed decays of the antitriplet charmed baryon into a decuplet baryon and a pseudoscalar meson can only proceed through the *W*-exchange diagram. Hence the experimental measurement of  $\Lambda_c^+ \rightarrow \Delta^{++} K^-$  implies that the *W*-exchange mechanism plays a significant role in charmed baryon decays.

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