Electromagnetic annihilation rates of χ_{c0} and χ_{c2} with both relativistic and QCD radiative corrections

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We estimate the electromagnetic decay rates of $\chi_{c0}\to\gamma\gamma$ and $\chi_{c2}\to\gamma\gamma$ by taking into account both relativistic and QCD radiative corrections. The decay rates are derived in the Bethe-Salpeter formalism and the QCD radiative corrections are included in accordance with the factorization assumption. Using a QCD-inspired interquark potential, we obtain relativistic BS wave functions of χ_{c0} and χ_{c2} by solving the BS equation for the corresponding ${}^{2S+1}L_J$ states. Our numerical result for the ratio $R=\Gamma(\chi_{c0}\to\gamma\gamma)/\Gamma(\chi_{c2}\to\gamma\gamma)$ is about 11–13, which agrees with the update E760 experiment data. Explicit calculations show that in addition to the QCD radiative corrections which may increase the ratio R by about a factor of 2, the relativistic corrections due to spin-dependent interquark forces induced by gluon exchange also enhance the ratio R substantially and its value is insensitive to the choice of parameters that characterize the interquark potential. Our expressions for the decay widths are identical with that obtained in the NRQCD theory to the next-to-leading order in v^2 and α_s. Moreover, we have determined two new coefficients in the nonperturbative matrix elements for these decay widths. [S0556-2821(96)05315-5]

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I. INTRODUCTION

Charmonium physics is in the boundary domain between perturbative and nonperturbative QCD. Charmonium decays may provide useful information on understanding the nature of interquark forces and decay mechanisms. Both QCD radiative corrections and relativistic corrections are important for charmonium decays, because for charmonium the strong coupling constant $\alpha_s(m_c)\approx 0.3$ [defined in the modified minimal subtraction $\overline{\text{MS}}$ scheme] and the velocity squared of the quark in the meson rest frame $v^2\approx 0.3$ both are not small. Decay rates of heavy quarkonium in the nonrelativistic limit with QCD radiative corrections have been studied (see, e.g., Refs. [1–4]). However, the decay rates of many processes are subject to substantial relativistic corrections [4]. With this problem in mind, people have studied relativistic corrections to the decay rates of S-wave charmonium, η_c, J/ψ, and their radial excited states [5–7]. These results show that relativistic effects are significant in the $c\bar{c}$ systems especially for the hadronic decays of J/ψ. In the present paper, we will investigate the relativistic corrections to the electromagnetic decays of P-wave charmonium states $\chi_{c0}\to\gamma\gamma$ and $\chi_{c2}\to\gamma\gamma$.

The P-wave charmonium decays are interesting. Now their experimental results are quite uncertain. The Crystal Ball group (see [8,16] and references therein) gives $\Gamma(\chi_{c0}\to\gamma\gamma)=4.0\pm 2.8$ keV. But for $\Gamma(\chi_{c2}\to\gamma\gamma)$, its central value differs significantly among various experiments [9–11], and the ratio of the photonic width of χ_{c0} to that of χ_{c2} measured by E760 is much larger than that measured by other two groups. Theoretically, in the nonrelativistic limit the ratio is $\frac{15}{4}$ [1], and it will increase to about 7.4 [8] if QCD radiative corrections are considered. On the other hand, in some approaches the relativistic corrections to this ratio were found to reduce its value significantly [19,20]. Recently a rigorous factorization formula which is based on nonrelativistic QCD (NRQCD) has been developed for calculations of inclusive decay rates of heavy quarkonium. In this approach the decay widths factor into a set of long distance matrix elements of NRQCD with each multiplied by a short distance coefficient. To any given order of relative velocity v of heavy quarks and antiquarks, the decay rates are determined by several nonperturbative factors which can be evaluated using QCD lattice calculations or extracted by fitting the data. The study of the photonic decays of χ_{c0} and χ_{c2} can also provide a determination for the nonperturbative factors in the decays of P-wave quarkonium.

In this paper, we will use the Bethe-Salpeter (BS) formalism [12] to derive the decay amplitudes and to calculate the decay widths of $\chi_{c0}\to\gamma\gamma$ and $\chi_{c2}\to\gamma\gamma$. The meson will be treated as a bound state consisting of a pair composed of a constitutent quark and an antiquark (higher Fock states such as $|Q\bar{Q}g\rangle$ and $|Q\bar{Q}gg\rangle$ are neglected because they do not contribute to electromagnetic decays) and described by the BS wave function which satisfies the BS equation. A phenomenological QCD-inspired interquark potential will be used to solve for the wave functions and to calculate the decay widths. Both relativistic and QCD radiative corrections to next-to-leading order will be considered based on the factorization assumption for the long distance and short distance effects. The remainder of this paper is organized as follows. In Sec. II we derive the reduced BS equation for any angular momentum state ${}^{2S+1}L_J$ of heavy mesons. In Sec. III we give the decay amplitudes of $\chi_J\to\gamma\gamma$ $(J=0,2)$ and use the solved relativistic BS wave functions to calculate the numerical results of decay widths. A summary and discussion will be given in the last section.

II. REDUCED BS EQUATIONS FOR ANY ANGULAR MOMENTUM STATE $^{2S+1}L_J$ OF HEAVY MESONS

Define the Bethe-Salpeter wave function, in general, for a $Q_1\overline{Q}_2$ bound state $|P\rangle$ with overall mass M and momentum $P=(\sqrt{\vec{P}^2+M^2},\vec{P})$,

$$\chi(x_1,x_2)=\langle 0|T\psi_1(x_1)\overline{\psi_2}(x_2)|P\rangle, \tag{1}$$

and transform it into momentum space:

$$\chi_P(p)=e^{-iP\cdot X}\int d^4x e^{-ip\cdot x}\chi(x_1,x_2). \tag{2}$$

Here p_1 (m_1) and p_2 (m_2) represent the momenta (masses) of the quark and antiquark, respectively:

$$X=\eta_1x_1+\eta_2x_2,\quad x=x_1-x_2,$$

$$P=p_1+p_2,\quad p=\eta_2p_1-\eta_1p_2,$$

where $\eta_i=m_i/(m_1+m_2)$ $(i=1,2)$.

We begin with the bound-state BS equation [12] in momentum space:

$$(\not p_1-m_1)\chi_P(p)(\not p_2+m_2)=\frac{i}{2\pi}\int d^4kG(P,p-k)\chi_P(k), \tag{3}$$

where $G(P,p-k)$ is the interaction kernel which dominates the interquark dynamics. In solving Eq. (3), we will employ the instantaneous approximation since for heavy quarks the interaction is dominated by instantaneous potentials. Meanwhile, we will neglect negative energy projectors in the quark propagators, since they are of even higher orders. Defining the three-dimensional BS wave function

$$\Phi_P(\vec{p})=\int dp^0\chi_P(p),$$

we then get the reduced Salpeter equation for $\Phi_P(\vec{p})$:

$$(M-E_1-E_2)\Phi(\vec{p})=\Lambda^1_+\gamma_0\int d^3kG(P,\vec{p}-\vec{k})\Phi(\vec{k})\gamma_0\Lambda^2_-. \tag{4}$$

Here $G(P,\vec{p}-\vec{k})$ represents the instantaneous potential, Λ_+ (Λ_-) are the positive (negative) energy projector operators for the quark and antiquark, respectively:

$$\Lambda^1_+=\frac{E_1+\gamma_0\vec{\gamma}\cdot\vec{p_1}+m\gamma_0}{2E_1},$$

$$\Lambda^2_-=\frac{E_2-\gamma_0\vec{\gamma}\cdot\vec{p_2}-m\gamma_0}{2E_2},$$

$$E_1=\sqrt{\vec{p_1}^2+m_1^2},\quad E_2=\sqrt{\vec{p_2}^2+m_2^2}.$$

We will follow a phenomenological approach by using the QCD-inspired interquark potentials, which are supported by both lattice QCD and heavy quark phenomenologies, as the kernel in the BS equation. The potentials include a long-range confinement potential (Lorentz scalar) and a short-range one-gluon-exchange potential (Lorentz vector):

$$V(r)=V_S(r)+\gamma_\mu\otimes\gamma^\mu V_V(r),$$

$$V_S(r)=\lambda r\frac{(1-e^{-\alpha r})}{\alpha r},$$

$$V_V(r)=-\frac{4\alpha_s(r)}{3r}e^{-\alpha r}, \tag{5}$$

where the introduction of the factor $e^{-\alpha r}$ is to regulate the infrared divergence and also to incorporate the color-screening effects of dynamical light quark pairs on the $Q\overline{Q}$ linear confinement potential. In momentum space the potentials become

$$G(\vec{p})=G_S(\vec{p})+\gamma_\mu\otimes\gamma^\mu G_V(\vec{p}),$$

$$G_S(\vec{p})=-\frac{\lambda}{\alpha}\delta^3(\vec{p})+\frac{\lambda}{\pi^2}\frac{1}{(\vec{p}^2+\alpha^2)^2},$$

$$G_V(\vec{p})=-\frac{2}{3\pi^2}\frac{\alpha_s(\vec{p})}{\vec{p}^2+\alpha^2}, \tag{6}$$

where $\alpha_s(\vec{p})$ is the quark-gluon running coupling constant and is assumed to become a constant of order 1 as $\vec{p}^2\to 0$:

$$\alpha_s(\vec{p})=\frac{12\pi}{27}\frac{1}{\ln(a+\vec{p}^2/\Lambda^2_{\rm QCD})}.$$

The constants λ, α, a, and $\Lambda_{\rm QCD}$ are the parameters that characterize the potential.

For any given angular momentum state $^{2S+1}L_J$ of mesons, its three-dimensional wave function in the rest frame of mesons takes the following two forms.

(i) $S=0$, then $J=L$:

$$\Phi_{Lm}(\vec{p})=\Lambda^1_+\gamma_0(1+\gamma_0)\gamma_5\gamma_0\Lambda^2_-Y_{Lm}(\hat{p})\phi(p). \tag{7}$$

(ii) $S=1$, then $J=L-1,L,L+1$ for $L\neq 0$ or $J=1$ for $L=0$:

$$\Phi_{JM}(\vec{p})=\sum_{l,m}\langle JM|1lLm\rangle\Lambda^1_+\gamma_0(1+\gamma_0)\gamma_l\gamma_0\Lambda^2_-\times Y_{Lm}(\hat{p})\phi(p). \tag{8}$$

Here $Y_{Lm}(\hat{p})$ is the spherical harmonic function and $\langle JM|1lLm\rangle$ is the Clebsch-Gordan coefficient. Substituting Eqs. (7) and (8) into Eq. (4), one derives the equations for the scalar wave function $\phi(p)$.

(i) $S=0$:

$$[M-E_1(p)-E_2(p)]g_1(p)\phi(p)=-\frac{E_1(p)E_2(p)+m_1m_2+\vec{p}^2}{4E_1(p)E_2(p)}\int d^3k[G_S(\vec{p}-\vec{k})-4G_V(\vec{p}-\vec{k})]g_1(k)P_L(\cos\Theta)\phi(k)$$
$$-\frac{E_1(p)m_2+E_2(p)m_1}{4E_1(p)E_2(p)}\int d^3k[G_s(\vec{p}-\vec{k})+2G_V(\vec{p}-\vec{k})]g_2(k)P_L(\cos\Theta)\phi(k)$$
$$+\frac{E_1(p)+E_2(p)}{4E_1(p)E_2(p)}\int d^3kG_S(\vec{p}-\vec{k})\vec{p}\cdot\vec{k}g_3(k)P_L(\cos\Theta)\phi(k)+\frac{m_1-m_2}{4E_1(p)E_2(p)}$$
$$\times\int d^3k[G_S(\vec{p}-\vec{k})+2G_V(\vec{p}-\vec{k})]\vec{p}\cdot\vec{k}g_4(k)P_L(\cos\Theta)\phi(k), \tag{9}$$

where

$$g_1(p)=\frac{[E_1(p)+m_1][E_2(p)+m_2]+\vec{p}^2}{4E_1(p)E_2(p)},$$

$$g_2(p)=\frac{[E_1(p)+m_1][E_2(p)+m_2]-\vec{p}^2}{4E_1(p)E_2(p)},$$

$$g_3(p)=\frac{E_1(p)+m_1+E_2(p)+m_2}{4E_1(p)E_2(p)},$$

$$g_4(p)=\frac{E_1(p)+m_1-E_2(p)-m_2}{4E_1(p)E_2(p)},$$

$$E_1(p)=\sqrt{\vec{p}^2+m_1^2},$$

$$E_2(p)=\sqrt{\vec{p}^2+m_2^2}.$$

(ii) $S=1$:

$$[M-E_1(p)-E_2(p)]f_8(p)\phi(p)=\frac{1}{4E_1(p)E_2(p)}\Bigg\{\int d^3k\{[2G_V(\vec{p}-\vec{k})-G_S(\vec{p}-\vec{k})]f_1(k)(m_1+m_2)-G_s(\vec{p}-\vec{k})f_2(k)$$
$$\times[E_1(p)+E_2(p)]\}P_L(\cos\Theta)\phi(k)+\Bigg[\int d^3k[4G_V(\vec{p}-\vec{k})+G_S(\vec{p}-\vec{k})]f_8(k)$$
$$\times[E_1(p)E_2(p)-m_1m_2+\vec{p}^2]+[G_S(\vec{p}-\vec{k})-2G_V(\vec{p}-\vec{k})]f_7(k)(m_1E_2(p)-m_2E_1(p))\Bigg]$$
$$\times P_J(\cos\Theta)\frac{k}{p}\phi(k)+\int d^3k\{[2G_V(\vec{p}-\vec{k})-G_S(\vec{p}-\vec{k})]f_5(k)(m_1+m_2)$$
$$-G_S(\vec{p}-\vec{k})f_6(k)(E_1(p)+E_2(p))\}\vec{p}\cdot\vec{k}P_J(\cos\Theta)\frac{k}{p}\phi(k)\Bigg\}, \tag{10}$$

where

$$f_1(p)=\frac{1}{4E_1(p)E_2(p)}\{[E_1(p)+m_1][E_2(p)+m_2]+\vec{p}^2\},$$

$$f_2(p)=\frac{1}{4E_1(p)E_2(p)}\{[E_1(p)+m_1][E_2(p)+m_2]-\vec{p}^2\},$$

$$f_3(p)=f_4(p)=\frac{2[E_1(p)+m_1]}{4E_1(p)E_2(p)},$$

$$f_5(p)=-f_6(p)=-\frac{2}{4E_1(p)E_2(p)},$$

$$f_7(p)=\frac{1}{4E_1(p)E_2(p)}[E_1(p)+m_1-E_2(p)-m_2],$$

$$f_8(p)=\frac{1}{4E_1(p)E_2(p)}[E_1(p)+m_1+E_2(p)+m_2].$$

The normalization condition $\int d^3p \mathrm{Tr}\{\Phi^+(\vec{p})\Phi(\vec{p})\} = 1/(2\pi)^3$ for the BS wave function $\phi(p)$ leads to

$$\int dpp^3 \frac{[E_1(p)+m_1][E_2(p)+m_2]}{4E_1E_2}\phi^2(p) = \frac{1}{(4\pi)^3}. \tag{11}$$

To the leading order in the nonrelativistic limit, Eqs. (9) and (10) can be reduced to the ordinary nonrelativistic Schrödinger equation for orbital angular momentum L with simply a spin-independent linear plus Coulomb potential. Solving the full equation (9) or (10), we can get the spectra and wave functions with relativistic corrections for any given angular momentum state ${}^{2S+1}L_J$ of heavy mesons. With these wave functions we can calculate hadronic matrix elements in processes involving the corresponding states, and the relativistic corrections due to interquark dynamics are included automatically in them. This approach is different from conventional ones which start from the Schrödinger equation with all relativistic effects considered perturbatively.

III. DECAY RATES OF $\Gamma(\chi_{c0}\to\gamma\gamma)$ AND $\Gamma(\chi_{c2}\to\gamma\gamma)$

Electromagnetic decays of χ_{c0} and χ_{c2} proceed via the annihilation of $c\bar{c}$ to two photons. Here only electromagnetic interactions are considered, and color-octet components which contribute significantly in hadronic decays of P-wave quarkonium do not contribute to electromagnetic decay widths, because final states are the photons which cannot be produced via the annihilation of color-octet $Q\bar{Q}$ pair. So photonic decays of χ_{cJ} for $J=0,2$ can be well expressed in the BS formalism and relativistic corrections are incorporated systematically in the decay rates. In the BS formalism the annihilation matrix elements can be written as

$$\langle 0|\overline{Q}IQ|P\rangle = \int d^4p \mathrm{Tr}[I(p,P)\chi_P(p)], \tag{12}$$

where $I(p,P)$ is the interaction vertex of the $Q\bar{Q}$ with other fields (e.g., the photons or gluons) which, in general may also depend on the variable q^0 (the time component of the relative momentum). If $I(p,P)$ is independent of p^0 (see [1,5]), the equation can be written as

$$\langle 0|\overline{Q}IQ|P\rangle = \int d^3p \mathrm{Tr}[I(\vec{p},P)\Phi_P(\vec{p})], \tag{13}$$

For process $\chi_{c0}\to\gamma\gamma$ or $\chi_{c2}\to\gamma\gamma$ with the momenta and polarizations of photons k_1,ϵ_1 and k_2,ϵ_2, the decay amplitude can be written as

$$T = \langle 0|\bar{c}\Gamma_{\mu\nu}c|\chi_{cJ}\rangle \epsilon_1^\mu \epsilon_2^\nu \tag{14}$$

for $J=0,2$, where

$$\Gamma_{\mu\nu} = e^2\left[\gamma_\nu \frac{1}{\not{p}_1-\not{k}_1-m}\gamma_\mu + \gamma_\mu \frac{1}{\not{k}_1-\not{p}_2-m}\gamma_\nu\right].$$

Here p_1 (p_2) is the charm quark (antiquark) momentum, and their time components satify $p_1^0+p_2^0=M$, as [1,5] we take

$$p_1^0 = p_2^0 = \frac{M}{2}. \tag{15}$$

Therefore, the amplitude T becomes independent of p^0. In terms of T the decay rates can be written as

$$\Gamma(\chi_{cJ}\to\gamma\gamma) = \frac{1}{2!}\frac{1}{(2J+1)}\sum_{J_Z}\sum_{\text{polar}}\int |T|^2 d\Omega \tag{16}$$

for $J=0,2$, where the spin is averaged over the initial state, and the photon polarization is summed over the final state according to

$$\sum_{\text{polar}} \varepsilon^\mu(k_1)\varepsilon^{\nu *}(k_1) = -g^{\mu\nu}.$$

Substituting the BS wave function (8) into Eq. (16), we get

$$\Gamma(\chi_{c0}\to\gamma\gamma) = 24 e_Q^4\alpha^2(c_1+3c_2+2c_3)^2, \tag{17}$$

$$\Gamma(\chi_{c2}\to\gamma\gamma) = \frac{48 e_Q^4\alpha^2}{5}(c_1^2-2c_1c_3+7c_3^2), \tag{18}$$

where

$$c_1 = \int d^3p \frac{1}{(\vec{p}-\vec{k})^2+m^2}\left\{\left[-E^2-mE-\frac{\vec{p}^2}{2}+\frac{3(\vec{p}\cdot\hat{k})^2}{2}\right]\vec{p}\cdot\vec{k} + \left[-\frac{\vec{p}^4}{4}+\frac{3\vec{p}^2(\vec{p}\cdot\hat{k})^2}{2}-\frac{5(\vec{p}\cdot\hat{k})^4}{4}\right]\right\}\frac{\phi(p)}{p},$$

$$c_2 = \int d^3p \frac{1}{(\vec{p}-\vec{k})^2+m^2}\left\{\frac{\vec{p}\cdot\vec{k}}{2}[\vec{p}^2-(\vec{p}\cdot\hat{k})^2] + \frac{1}{4}[\vec{p}^2-(\vec{p}\cdot\hat{k})^2]^2\right\}\frac{\phi(p)}{p},$$

$$c_3 = \int d^3p \frac{1}{(\vec{p}-\vec{k})^2+m^2}\left\{\frac{-E^2-mE}{2}[\vec{p}^2-(\vec{p}\cdot\hat{k})^2] + \frac{1}{4}[\vec{p}^2-(\vec{p}\cdot\hat{k})^2]^2\right\}\frac{\phi(p)}{p}.$$

In the nonrelativistic limit, Eqs. (17) and (18) reduce to

$$\Gamma(\chi_{c0}\to\gamma\gamma) = \frac{24 e_Q^4\alpha^2}{m^4}\left|\int d^3pp\phi_{\chi_{c0}}(p)\right|^2,$$

$$\Gamma(\chi_{c2}\to\gamma\gamma) = \frac{32 e_Q^4\alpha^2}{5m^4}\left|\int d^3pp\phi_{\chi_{c2}}(p)\right|^2.$$

Using the Fourier transformation of wave functions,

$$\int d^3pp\phi_{\chi_{cJ}}(p) = \frac{3}{\sqrt{8}}R'_{\chi_{cJ}}(0),$$

we derive the well-known result in coordinate space, which is consistent with that given in [1]

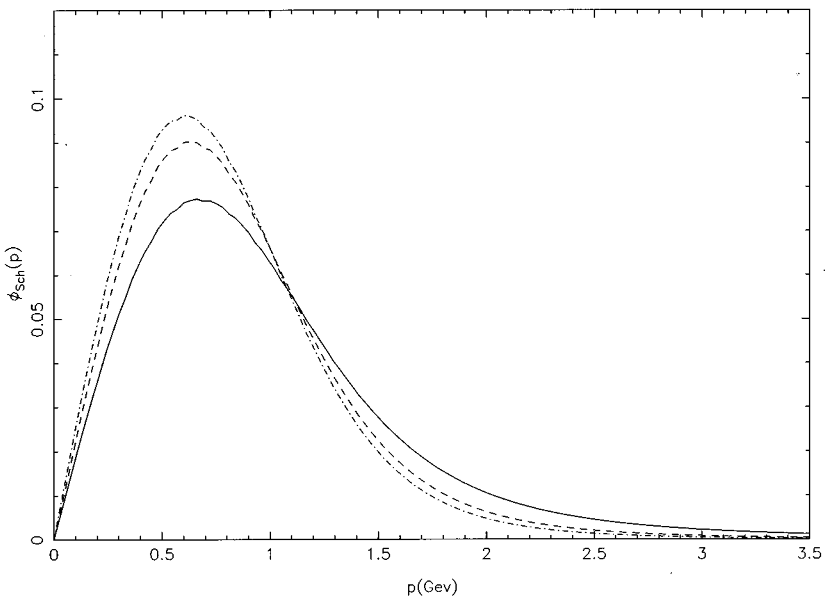

FIG. 1. Wave functions $\phi_{\rm Sch}(p)$ (normalized in momentum space) of χ_{c0} (solid line), χ_{c1} (dashed line), and χ_{c2} (dot-dashed line) by solving BS equations with $m_c=1.5$ GeV.

$$\Gamma(\chi_{c0}\rightarrow\gamma\gamma)=\frac{27e_Q^4\alpha^2}{m^4}|R'_{\chi_{c0}}(0)|^2, \tag{19}$$

$$\Gamma(\chi_{c2}\rightarrow\gamma\gamma)=\frac{36e_Q^4\alpha^2}{5m^4}|R'_{\chi_{c2}}(0)|^2, \tag{20}$$

where $R'_{\chi_{cJ}}(0)$ is the derivative of the radial wave function at the origin, and in the nonrelativistic limit, $R'_{\chi_{c0}}(0)=R'_{\chi_{c2}}(0)$, due to heavy quark spin symmetry.

Recently, in the framework of NRQCD the factorization formulas for the long distance and short distance effects were found to involve a double expansion in the quark relative velocity v and in the QCD coupling constant α_s [13,14]. To next-to-leading order in both v^2 and α_s, as an approximation, we may write

$$\Gamma(\chi_{c0}\rightarrow\gamma\gamma)=24e_Q^4\alpha^2(c_1+3c_2+2c_3)^2 \times\left[1+\frac{\alpha_s}{\pi}\left(\frac{\pi^2}{3}-\frac{28}{9}\right)\right], \tag{21}$$

$$\Gamma(\chi_{c2}\rightarrow\gamma\gamma)=\frac{48e_Q^4\alpha^2}{5}(c_1^2-2c_1c_3+7c_3^2)\left(1-\frac{\alpha_s}{\pi}\frac{16}{3}\right), \tag{22}$$

where we have used QCD radiative corrections given in [15]. We must emphasize that above factorization formulas are correct only to next-to-leading order in v^2 and α_s. If higher order effects are involved, the decay widths cannot be factored into a integral of wave functions and a coefficient that can be written as a series of α_s. NRQCD has applied a more general factorization formula for quarkonium decay rates, which will be discussed in detail later.

For the heavy quarkonium $c\bar{c}$ systems, $m_1=m_2=m_c$, Eqs. (9) and (10) become much simpler. We take the following parameters which appear in the potential (5):

$$m_c=1.5\ \text{GeV},\quad \lambda=0.23\ \text{GeV}^2,\quad \Lambda_{\rm QCD}=0.18\ \text{GeV},$$

$$\alpha=0.06\ \text{GeV},\quad a=e=2.7183. \tag{23}$$

With these values the mass spectrum of charmonium are found to fit the data well. In Figs. 1 and 2 the solved scalar wave functions both in momentum and coordinate space for P-wave triplet χ_{cJ} states are shown and we can see explicitly the differences between wave functions for $J=0,2$ while they are the same in the nonrelativistic limit. Substituting $\phi_{\chi_{c0}}(p)$ and $\phi_{\chi_{c2}}(p)$ into Eqs. (21) and (22), we get

$$\Gamma(\chi_0\rightarrow\gamma\gamma)=5.32\ \text{keV},$$

$$\Gamma(\chi_2\rightarrow\gamma\gamma)=0.44\ \text{keV},$$

where the meson mass M in Eq. (15) is taken to be the observed physical value. The ratio of the widths is

$$R=\frac{\Gamma(\chi_0\rightarrow\gamma\gamma)}{\Gamma(\chi_2\rightarrow\gamma\gamma)}=12.1. \tag{24}$$

Our results are satisfactory, as compared with the Particle Data Group experimental values [16] $\Gamma(\chi_{c0}\rightarrow\gamma\gamma)=5.6\pm3.2$ keV and $\Gamma(\chi_{c2}\rightarrow\gamma\gamma)=0.32\pm0.1$ keV. Here in the above calculations the value of $\alpha_s(m_c)$ in the QCD radiative correction factor in Eqs. (21) and (22) is chosen to be 0.29 [4], which is also consistent with our determination from the ratio of $B(J/\psi\rightarrow3g)$ to $B(J/\psi\rightarrow e^+e^-)$ [5].

Moreover, in order to see the sensitivity of the decay widths to the parameters, especially the charm quark mass, we use another two sets of parameters:

$$m_c=1.4\ \text{GeV},\quad \lambda=0.24\ \text{GeV}^2,$$

$$m_c=1.6\ \text{GeV},\quad \lambda=0.22\ \text{GeV}^2,$$

with the other parameters remaining unchanged (the heavy quarkonia mass spectra are not sensitive to a and α for $\alpha\leqslant0.06$ GeV). By the same procedure, we obtain

$$\Gamma(\chi_0\rightarrow\gamma\gamma)=5.82(4.85)\ \text{keV},$$

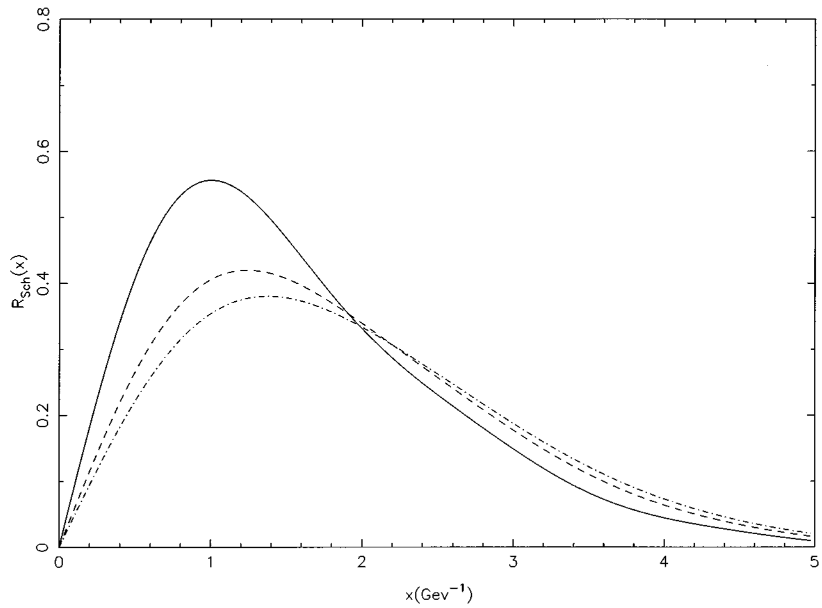

FIG. 2. Wave functions $R_{\rm Sch}(x)$ (normalized in coordinate space) of χ_{c0} (solid line), χ_{c1} (dashed line), and χ_{c2} (dot-dashed line) by solving BS equations with $m_c = 1.5$ GeV.

$$\Gamma(\chi_2 \to \gamma\gamma) = 0.50(0.39) \ \ \text{keV},$$

and the ratio

$$R = \frac{\Gamma(\chi_0 \to \gamma\gamma)}{\Gamma(\chi_2 \to \gamma\gamma)} = 11.8(12.5) \tag{25}$$

for $m_c = 1.4(1.6)$ GeV, respectively.

We find that the widths are decreased with the decreasing of λ. This is obvious since the wave function in coordinate space will become broader when the slope of the linear potential is decreased and the corresponding wave function in momentum space will become narrower; so the effective decay couplings become smaller. It is interesting to note that the ratio of two photonic decay widths of χ_{c0} and χ_{c2} is almost unchanged and is insensitive to the choice of parameters.

In order to see further the sensitivity of the value of R to the parameters in the potentials, we have also solved the BS equation by using two extremely different values:

$$\lambda = 0.25 \ \ \text{GeV}^2, \quad \Lambda_{\rm QCD} = 0.25 \ \ \text{GeV}$$

and

$$\lambda = 0.15 \ \ \text{Gev}^2, \quad \Lambda_{\rm QCD} = 0.15 \ \ \text{GeV},$$

with other parameters being the same as that in Eqs. (23). We obtain the decay widths

$$\Gamma(\chi_0 \to \gamma\gamma) = 3.36(6.53) \ \ \text{keV},$$

$$\Gamma(\chi_2 \to \gamma\gamma) = 0.24(0.54) \ \ \text{keV}, \tag{26}$$

and the ratio

$$R = \frac{\Gamma(\chi_0 \to \gamma\gamma)}{\Gamma(\chi_2 \to \gamma\gamma)} = 14.1(12.2) \tag{27}$$

for $\lambda = 0.15(0.25)$ GeV^2 and $\Lambda_{\rm QCD} = 0.15(0.25)$ GeV, respectively. This indicates that we can obtain a rather stable range for the value of R, despite the uncertainty in the estimate of the decays widths. Indeed, it is difficult to control the systematic accuracy within the potential model. In particular, the spin-independent relativistic correction to the confinement potential and the retardation correction connected with confinement are far from being thoroughly understood. This also causes an uncertainty in the estimate of decay widths. Nevertheless, from Eqs. (26) and (27) it can be seen that two group of very different parameters lead to different decay widths but give very close values for R.

Finally we discuss the relation between our approach and the NRQCD theory. Recently, a general factorization formula which is based on nonrelativistic QCD (NRQCD) has been developed for studying the inclusive cross sections of production and decay of heavy quarkonium. In this formalism the quarkonium decay rates can be written as a sum of a set of matrix elements to any given order in v^2, with each matrix element multiplied by a coefficient which can be calculated in perturbative QCD. This approach has been proved successful in the application of some processes involving heavy quarkonium [17,18]. In NRQCD, the electromagnetic decay rates of χ_{c0} and χ_{02} to next-to-leading order in v^2 can be written as

$$\Gamma(\chi_0 \to \gamma\gamma) = \frac{2\,{\rm Im} f_{\rm EM}(^3p_0)}{m^4} \langle \chi_{c0} | \mathcal{O}_{\rm EM}(^3P_0) | \chi_{c0} \rangle + \frac{2\,{\rm Im} g_{\rm EM}(^3p_0)}{m^6} \langle \chi_{c0} | \mathcal{G}_{\rm EM}(^3P_0) | \chi_{c0} \rangle, \tag{28}$$

$$\Gamma(\chi_2 \to \gamma\gamma) = \frac{2\,{\rm Im} f_{\rm EM}(^3p_2)}{m^4} \langle \chi_{c2} | \mathcal{O}_{\rm EM}(^3P_2) | \chi_{c2} \rangle + \frac{2\,{\rm Im} g_{\rm EM}(^3p_2)}{m^6} \langle \chi_{c2} | \mathcal{G}_{\rm EM}(^3P_2) | \chi_{c2} \rangle, \tag{29}$$

where

$$\mathcal{O}_{\rm EM}(^3P_0)=\frac{1}{3}\psi^+\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)\cdot\vec{\sigma}\chi|0\rangle\langle 0|\chi^+\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)\cdot\vec{\sigma}\psi,$$

$$\mathcal{O}_{\rm EM}(^3P_2)=\psi^+\left(-\frac{i}{2}\overleftrightarrow{D}\right)^{(i}\sigma^{j)}\chi|0\langle 0|\chi^+\left(-\frac{i}{2}\overleftrightarrow{D}\right)^{(i}\sigma^{j)}\psi,$$

$$\mathcal{G}_{\rm EM}(^3P_0)=\frac{1}{2}\left[\frac{1}{3}\psi^+\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)\cdot\vec{\sigma}\chi|0\rangle \times\langle 0|\chi^+\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)\cdot\vec{\sigma}\psi+{\rm H.c}\right],$$

$$\mathcal{G}_{\rm EM}(^3P_0)=\frac{1}{2}\left[\psi^+\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(-\frac{i}{2}\overleftrightarrow{D}\right)^{(i}\sigma^{j)}\chi|0\rangle \times\langle 0|\chi^+\left(-\frac{i}{2}\overleftrightarrow{D}\right)^{(i}\sigma^{j)}\psi+{\rm H.c}\right], \qquad (30)$$

where $\vec{D}$ is the space component of covariant derivate D^μ, and ψ and χ are two-component operators of quarks and antiquarks, respectively. If identifying the quark operator expectation values with the deriatives of wave functions at the origin, the decay widths can be written as

$$\Gamma(\chi_0\to\gamma\gamma)=\frac{9{\rm Im}f_{\rm EM}(^3p_0)}{\pi m^4}|R'_{\chi_0}(0)|^2+\frac{15{\rm Im}g_{\rm EM}(^3p_0)}{\pi m^6}{\rm Re}[R^{(3)}_{\chi_0}(0)R'_{\chi_0}(0)], \qquad (31)$$

$$\Gamma(\chi_2\to\gamma\gamma)=\frac{9{\rm Im}f_{\rm EM}(^3p_2)}{\pi m^4}|R'_{\chi_2}(0)|^2+\frac{15{\rm Im}g_{EM}(^3p_2)}{\pi m^6}{\rm Re}[R^{(3)}_{\chi_2}(0)R'_{\chi_2}(0)]. \qquad (32)$$

In comparision with NRQCD, we take the on-shell condition, which assumes the quark and antiquark to be on the mass shell $p_1^0=p_2^0=E=\sqrt{m^2+\vec{p}^2}$, instead of Eq. (15). The advantage of this assumption is that gauge invariance is maintained for the on-shell quarks but at the price of treating the quark and antiquark just as free particles in a bound state. An apparent problem in this scheme is that with a fixed value of the meson mass M (e.g., its observed value), if the quark mass takes a fixed value, then $\vec{p}^2$ will be fixed but not weighted by the wave function as in the usual bound-state description. In order to connect the decay process, which occurs at short distances, where quarks are approximately on shell, with bound state wave function, which is mainly determined by the long distance confinement force, we have to make a compromise between the on-shell condition and the bound-state description. We will expand the annihilation amplitudes (21) and (22) in terms of $\vec{p}^2/m^2$ and allow $\vec{p}^2$ (and so the meson mass accordingly) to vary in accordance with the bound-state wave function $\phi(p)$ which is determined by the long distance dynamics or, phenomenologically, by some dynamical models. With this treatment, we get the decay widths to next-to-leading order of $\vec{p}^2/m^2$,

$$\Gamma(\chi_0\to\gamma\gamma)=\frac{3e_Q^4\alpha^2}{m^4}\left|\int d^3pp\left(1-\frac{\vec{p}^2}{6m^2}\right)\phi_{\rm Sch}(p)\right|^2\times\left[1+\frac{\alpha_s}{\pi}\left(\frac{\pi^2}{3}-\frac{28}{9}\right)\right], \qquad (33)$$

$$\Gamma(\chi_2\to\gamma\gamma)=\frac{4e_Q^4\alpha^2}{5m^4}\left|\int d^3pp\,\phi_{\rm Sch}(p)\right|^2\left(1-\frac{\alpha_s}{\pi}\frac{16}{3}\right), \qquad (34)$$

where the standard Schrödinger wave function (with relativistic corrections) $\phi_{\rm Sch}(p)$ is related to $\phi(p)$ through the normalization condition (11):

$$\phi_{\rm Sch}(p)=\sqrt{2}\left(\frac{m+E}{E}\right)\phi(p),$$

$$(2\pi)^3\int dpp^2|\phi_{\rm Sch}(p)|^2=1.$$

Using the formulas

$$\int d^3pp\,\phi_{\rm Sch}(p)=3R'_{\rm Sch}(0),$$

$$\int d^3pp^3\phi_{\rm Sch}(p)=5R^{(3)}_{\rm Sch}(0),$$

the expressions of Eqs. (33) and (34) are transferred into coordinate space and comparing with that derived from NRQCD, Eqs. (31) and (32), we can easily determine the coefficients

$${\rm Im}f_{\rm EM}(^3P_0)=3\pi e_Q^4\alpha^2\left[1+\frac{\alpha_s}{\pi}\left(\frac{\pi^2}{3}-\frac{28}{9}\right)\right],$$

$${\rm Im}g_{\rm EM}(^3P_0)=-\pi e_Q^4\alpha^2,$$

$${\rm Im}f_{\rm EM}(^3P_2)=\frac{4\pi e_Q^4\alpha^2}{5}\left(1-\frac{\alpha_s}{\pi}\frac{16}{3}\right),$$

$${\rm Im}g_{\rm EM}(^3P_2)=0. \qquad (35)$$

Here we only consider the QCD radiative corrections to leading order coefficients ${\rm Im}f_{\rm EM}(^3P_0)$ and ${\rm Im}f_{\rm EM}(^3P_2)$ which are equal to the results derived in [13]. Moreover, we have determined two new coefficents, i.e., the second one and fourth one in Eqs. (35). These two matrix elements have a suppression factor of v^2, and so we need not to take into account higher order corrections to their coefficients any more.

IV. SUMMARY AND DISCUSSION

In this paper we provide an estimate for the photonic decays of P-wave charmonium with both QCD radiative corrections and relativistic corrections. In the nonrelativistic limit but with first order QCD radiative corrections, the ratio R of the photonic widths is expected to be [for $\alpha_s(m_c)=0.29$]

$$R=\frac{\Gamma(\chi_{c0}\to\gamma\gamma)}{\Gamma(\chi_{c2}\to\gamma\gamma)}=\frac{15}{4}\frac{[1+(\alpha_s/\pi)(\pi^2/3-\frac{28}{9})]}{(1-\frac{16}{3}\alpha_s/\pi)}=7.5,$$

which is enhanced by a factor of 2, as compared with the well-known value $\frac{15}{4}$ obtained without QCD radiative corrections.

Our calculations show that comparing with the nonrelativistic value given above, the relativistic effects will further enhance the ratio R substantially and make

$$R=11-13.$$

This result differs significantly from other theoretical predictions [19,20]. In fact, we know that there are two sources of relativistic corrections: (1) the correction due to relativistic kinematics which appears explicitly in the decay amplitudes and (2) the correction due to interquark dynamical effects (e.g., the well-known Breit-Fermi interactions), which mainly causes the correction to the bound-state wave functions. From the expressions (33) and (34) of the decay rates which have been expanded to the first order of $\vec{p}^2/m^2$, one might expect that the ratio R would become smaller after taking relativistic corrections into account, because the coefficient of the term $\vec{p}^2/m^2$ in Eq. (33) is smaller than that in Eq. (34). This would be true if the dynamical relativistic corrections to the wave functions were completely neglected. Indeed, if we were using the same scalar wave functions $\phi_{\rm Sch}(p)$ for χ_{c0} and χ_{c2} we would find

$$R=5.3-5.9,$$

which is smaller than the nonrelativistic value $R=7.5$. However, the dynamical relativistic effects are very important. The spin-dependent forces (induced mainly by one-gluon exchange) not only cause the fine splittings of masses of χ_{c0}, χ_{c1}, and χ_{c2}, but also make the wave functions of χ_{c0}, χ_{c1}, and χ_{c2} different from each other. Mainly due to the attractive spin-orbital force induced by one-gluon exchange for the 0^{++} meson, the χ_0 wave function in coordinate space becomes narrower than the χ_2 wave function in which the spin-orbital force is repulsive, and therefore the derivative of the wave function at the origin becomes larger for χ_0 than that for χ_2. As a result, the dynamic relativistic effect on R is in the opposite direction to the kinamatic correction and can be even larger. The overall relativistic correction to R is found to be positive. Our result is in agreement with the E760 data and disagrees with the values measured by CLEO and TPC2.

Our expressions for the decay widths are identical with that derived from the rigorous factorization formula to next-to-leading order in v^2 and in α_s. Moreover, we have determined two new coefficients in the nonperturbative matrix elements for these decay widths. For a more accurate estimate, higher order corrections both in v^2 and in α_s should be taken into account. For electromagnetic decays, in general we can estimate them within the $|Q\bar{Q}\rangle$ sector and avoid the difficult problem due to the effects of high Fock states such as $|Q\bar{Q}g\rangle$ and $|Q\bar{Q}gg\rangle$. But we must notice that if higher order matrix elements are included, the decay widths cannot be factored in the way like Eqs. (31) and (32) because the higher order coefficients are different for each nonperturbative factor.

We have solved the BS equation for the bound-state wave functions with QCD-inspired interquark potentials (linear confinement potential plus one-gluon-exchange potential) as the BS kernel. With some popular parameters for the potentials we obtained the wave functions and used them to calculate the decay widths. From Eqs. (24) and (25) it can be seen that different parameters lead to somewhat different photonic decay widths but give very close values for R. This might indicate that our estimate of R is insensitive to the quark mass and potential parameters, and therefore could be a rather reliable result, despite the uncertainty in the estimate of the dynamical relativistic effects. We hope the lattice simulations will give more reliable estimates for these decays within the framework of NRQCD, and can be compared with our results.

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[1] R. Barbieri, R. Gatto, and R. Kögerler, Phys. Lett. **60B**, 183 (1976); R. Barbieri, M. Caffo, and E. Remiddi, *ibid.* **61B**, 465 (1976); R. Barbieri *et al.*, Nucl. Phys. **B162**, 220 (1980).

[2] K. Hagiwara, C. B. Kim, and T. Yoshino, Nucl. Phys. **B177**, 461 (1981).

[3] P. B. Mackenzie and G. P. Lepage, Phys. Rev. Lett. **47**, 1244 (1981).

[4] W. Kwong, P. B. Mackenzie, R. Rosenfeld, and J. L. Rosner, Phys. Rev. D **37**, 3210 (1988).

[5] K. T. Chao, H. W. Huang, J. H. Liu, Y. Q. Liu, and J. Tang, in *Proceedings of the International Conference on Quark Confinement and the Hadron Spectrum*, Come, Italy, 1994, edited by N. Brambilla and G. M. Prosperi (World Scientific, Singapore, 1995), p. 306; K. T. Chao, H. W. Huang, and Y. Q. Liu, Phys. Rev. D **53**, 221 (1996)

[6] W. Y. Keung and I. J. Muzinich, Phys. Rev. D **27**, 1518 (1983).

[7] H. C. Chiang, J. Hufner, and H. J. Pirner, Phys. Lett. B **324**, 482 (1994).

[8] M. L. Mangano and A. Petrelli, Phys. Lett. B **352**, 445 (1995).

[9] E760 Collaboration, T. A. Armstrong *et al.*, Nucl. Phys. **B373**, 35 (1992); Phys. Rev. Lett. **70**, 2988 (1993).

[10] CLEO Collaboration, J. Dominick *et al.*, Phys. Rev. D **50**, 4265 (1994).

[11] TPC/Two-Gamma Collaboration, Phys. Lett. B **302**, 345 (1993).

[12] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951); E. E. Salpeter, *ibid.* **87**, 328 (1952).

[13] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).
[14] E. Braaten, Report No. NUHEP-TH-94-22, hep-ph/9409286 (unpublished).
[15] R. Barbieri, R. Gatto, M. Caffo, and E. Remiddi, Nucl. Phys. **B192**, 61 (1981).
[16] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1171 (1994).
[17] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **46**, 1914 (1992).
[18] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993); E. Braaten and S. Fleming, *ibid.* **74**, 3327 (1995).
[19] L. Bergström, G. Hulth, and H. Snellman, Z. Phys. C **16**, 263 (1983).
[20] Z. P. Li, F. E. Close, and T. Barnes, Phys. Rev. D **43**, 2161 (1991); E. S. Ackleh and T. Barnes, *ibid.* **45**, 232 (1992).