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Quark mass corrections to the Z boson decay rates

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The results of a perturbative QCD evaluation of the $\sim m_f^2/M_Z^2$ contributions to $\Gamma_{Z\to b\bar{b}}$ and $\Gamma_{Z\to \mathrm{hadrons}}$ for the quark masses $m_f \ll M_Z$ are presented. The recent results due to the combination of renormalization group constraints and the results of several other calculations are independently confirmed by direct computation. Some existing confusion in the literature is clarified. In addition, the calculated $O(\alpha_s^2)$ correction to the correlation function in the axial channel is a necessary ingredient for the yet uncalculated axial part of the $O(\alpha_s^3)$ mass correction to the Z decay rates. The results can be applied to the τ hadronic width.

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Recent analyses show that the result at the CERN e^+e^- collider LEP for the branching fraction $\Gamma_{Z\to b\bar{b}}/\Gamma_{Z\to \mathrm{hadrons}}=0.2208\pm0.0024$ [1] differs from (is larger than) the standard model prediction [2] by 2σ–2.5σ with the top quark mass at around 174 GeV [3]. Although the search for implications of this fact beyond the standard model has already begun (see, e.g., [4]), a further analysis within the standard model is still an important issue.

Briefly, the current state of the perturbative QCD evaluation of the $\Gamma_{Z\to \mathrm{hadrons}}$ and related quantities is as follows. The QCD contributions are evaluated to $O(\alpha_s^3)$ in the limits $m_f=0$ ($f=u,d,s,c,b$) and $m_t\to\infty$ [5]. The leading correction to the above results due to the $O(\alpha_s^3)$ triangle anomaly-type diagrams with the virtual top quark has also been calculated in the limit $m_t\to\infty$ [6] and this result has been extended to order $(M_Z^2/m_t^2)^3$ in Ref. [7]. The calculations to $O(\alpha_s^2)$ are more complete. Indeed, the electron-positron annihilation R ratio was calculated long ago [8] in the limits $m_f=0$ ($f=u,d,s,c,b$) and $m_t\to\infty$. The corrections due to the large top-bottom mass splitting have been evaluated in [9,10] and thus the axial channel, applicable to $\Gamma_{Z\to \mathrm{hadrons}}$, has also been validated. The $O(\alpha_s^2)$ effects of the virtual heavy quark in the decays of the Z boson have been evaluated in [11,12]. The same effect has been studied previously in [13] in the limit $m_t\to\infty$. The present knowledge of the high order QCD and electroweak corrections to $\Gamma_{Z\to \mathrm{hadrons}}$ is summarized in a review article [2], providing all essential details.

The subject of the present work is the correction due to the nonvanishing ''light'' quark masses. The following discussion will be for the quark of flavor f ($f\neq t$) in general. However, in fact, only $f=b$ is a relevant case and the masses of u, d, s, and c quarks can safely be ignored at the Z mass scale. Note that the corrections of order $\alpha_s^3 m_b^2/M_Z^2$ for the vector part of the $Z\to b\bar{b}$ decay rate and the corrections of order $\alpha_s^2 m_b^2/M_Z^2$ for the axial part were obtained in Ref. [14] and Ref. [15], respectively. Those evaluations are based on an indirect approach, using the renormalization group constraints and the results of earlier calculations of the correlation functions in the vector [16,17] and scalar channels [18]. One of the aims of the present work is to obtain the quark mass corrections to $\Gamma_{Z\to q_f\bar{q}_f}$ in both channels by a direct calculation and to check the method and the results used in [14,15] simultaneously. Moreover, it should be stressed that there is a disagreement between the results of [16] and [17]. It is shown in the present paper that [16] is correct and [17] is incorrect.[1]

The quantity R_Z is defined as the ratio of the hadronic and electronic Z *widths:*

$$R_Z=\sum_{f=u,d,s,c,b}\Gamma_{Z\to q_f\bar{q}_f}/\Gamma_{Z\to e^+e^-}. \tag{1}$$

The partial decay width can be evaluated as the imaginary part

$$\Gamma_{Z\to q_f\bar{q}_f}=-\frac{1}{M_Z}\mathrm{Im}\Pi(m_f,s+i0)\Big|_{s=M_Z^2}, \tag{2}$$

where the function Π is defined through a correlation function of two flavor-diagonal quark currents:

$$i\int d^4x e^{iqx}\langle Tj_\mu^f(x)j_\nu^f(0)\rangle_0=g_{\mu\nu}\Pi(m_f,Q^2)-Q_\mu Q_\nu\Pi'(m_f,Q^2). \tag{3}$$

Here, Q^2 is a large ($\sim -M_Z^2$) Euclidean momentum. According to the standard model, the neutral weak current of quarks coupled to Z bosons is

$$j_\mu^f=(G_F M_Z^2/2\sqrt{2})^{1/2}(g_f^V\overline{q_f}\gamma_\mu q_f+g_f^A\overline{q_f}\gamma_\mu\gamma_5 q_f), \tag{4}$$

where the electroweak vector and axial vector couplings are defined in the standard way:

$$g_f^V=2I_f^{(3)}-4e_f\sin^2\Theta_W,\ \ g_f^A=2I_f^{(3)}.$$

The Π function may be decomposed into vector and axial parts:

$$\Pi(m_f,Q^2)=\Pi^V(m_f,Q^2)+\Pi^A(m_f,Q^2). \tag{5}$$

[1]Numerically the disagreement is not large. The error in [17] was repeated in [19]. The second erratum to [19] remedies the situation, citing a previous version of the present work and other confirmation in Ref. [25]. Unfortunately, in [14], the result of [17] was used.

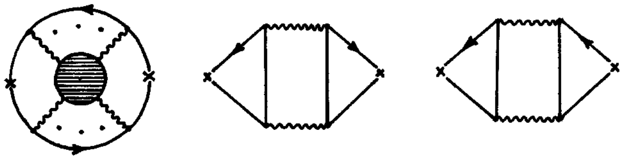

FIG. 1. Feynman diagrams contributing to the correlation function to $O(\alpha_s^2)$. The cut diagrams contribute to $\Gamma_{Z\to q_f\bar{q}_f}$ at the same order.

The Feynman diagrams that contribute to Π to $O(\alpha_s^2)$ are shown in Fig. 1. The effects of the last two diagrams in Fig. 1 (so called triangle anomaly-type diagrams) have been studied in [9,10] and will not be considered here. In the first diagram, the shaded bulb includes any interactions of quarks and gluons (or ghosts) allowed in QCD and the dots cover any number of gluon propagators that gives one-, two-, and three-loop topologies. The crosses denote current vertices corresponding to the vector or the axial part of the current (4).

Because the problem scale ($\sim M_Z$) is much larger than the quark masses, the following expansion is legitimate:

$$\Pi^{V/A}(m_f,m_v,Q^2)=\Pi_1^{V/A}(Q^2)+\frac{m_f^2}{Q^2}\Pi_{m_f^2}^{V/A}(Q^2)+\sum_{v=u,d,s,c,b}\frac{m_v^2}{Q^2}\Pi_{m_v^2}^{V/A}(Q^2)+\cdots. \tag{6}$$

The last term in the above expansion is due to the particular topological types of three-loop diagrams containing virtual fermionic loops. The effects of the virtual top quark in the decays of the Z boson have been studied in [11–13] and will not be discussed here. The ellipsis in Eq. (6) covers the terms $\sim m_f^4/Q^4$ and higher orders. Those terms at $-Q^2=M_Z^2$ for the Z decay are heavily suppressed and can safely be ignored.

The expansion coefficients Π_i can be calculated in a way similar to the one used in [20] for the evaluation of the fermionic decay rates of the Higgs boson. In fact, the whole calculational procedure can be combined in one equation (in the limit $m_t\to\infty$)

$$\Gamma_{Z\to q_f\bar{q}_f}=-\sum_{i=V,A}\sum_{\substack{n,k=0,1\\ n+k\leq 1}}\frac{1}{(2n)!(2k)!}\frac{1}{M_Z}\,\mathrm{Im}\left\{Z_m^{2(n+k)}m_f^{2n}m_v^{2k}\left[\left(\frac{d}{dm_f^B}\right)^{2n}\left(\frac{d}{dm_v^B}\right)^{2k}\Pi^i(\alpha_s^B,m_f^B,m_v^B,s+i0)\right]_{\substack{m_f^B=m_v^B=0\\ \alpha_s^B\to Z_\alpha\alpha_s}}\right\}_{s=M_Z^2} \tag{7}$$

where B labels the unrenormalized quantities. Z_m and Z_α are the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$) renormalization constants of the quark mass and the strong coupling, respectively, and can be found, for instance, in [20]. The summation over the virtual quark flavors $v=u,d,s,c,b$ is assumed in Π^i. Note that the introduction of the so called D function (see, e.g., [14,15]), which is the Π function differentiated with respect to Q^2, is not necessary in this calculation. Note also that Eq. (7) does not include important effects from the last two diagrams in Fig. 1 (evaluated in [9,10]) and the virtual top quark effects (evaluated in [11–13]).

In the $\overline{\mathrm{MS}}$ [21] analytical calculations of the one-, two-, and three-loop dimensionally regularized [22] Feynman diagrams, the FORM [23] program HEPLOOPS [24] is used.

For the massless limit coefficients $\Pi_1^{V/A}$ in the expansion (6), the known results are obtained. The perturbative expansion of the $\sim m_f^2$ part in the right-hand side (RHS) of Eq. (6) has the form

$$\frac{m_f^2(\mu)}{Q^2}\Pi_{m_f^2}^{V/A}[\alpha_s(\mu),Q^2]+\sum_{v=u,d,s,c,b}\frac{m_v^2(\mu)}{Q^2}\Pi_{m_v^2}^{V/A}[\alpha_s(\mu),Q^2]$$
$$=\frac{G_F M_Z^2}{8\sqrt{2}\pi^2}(g_f^{V/A})^2\sum_{i=0}^{3}\sum_{j=0}^{i+1}\left(\frac{\alpha_s(\mu)}{\pi}\right)^i\ln^j\frac{\mu^2}{Q^2}\left(m_f^2(\mu)d_{ij}^{V/A}+e_{ij}\sum_{v=u,d,s,c,b}m_v^2(\mu)\right). \tag{8}$$

The coefficients e_{ij} are the same in both channels for obvious reasons. Moreover, they get nonzero values starting at the three-loop level ($i\geq 2$). The summation index j runs from zero to $i+1$ since, in general, the maximum power of the pole that can be produced by a multiloop Feynman diagram is equal to the number of loops.

The direct computation of all relevant one-, two-, and three-loop Feynman diagrams for the standard QCD with the $\mathrm{SU}_c(3)$ gauge group gives in the vector channel

$$d_{00}^V=-6;$$
$$d_{10}^V=-16,\quad d_{11}^V=-12; \tag{9}$$
$$d_{20}^V=-\tfrac{19691}{72}-\tfrac{124}{9}\zeta(3)+\tfrac{1045}{9}\zeta(5)+N_f\tfrac{95}{12},$$
$$d_{21}^V=-\tfrac{253}{2}+N_f\tfrac{13}{3},\quad d_{22}^V=-\tfrac{57}{2}+N_f;$$
$$e_{00}=e_{1j}=0,\quad e_{20}=32/3-8\zeta(3),\quad e_{21}=e_{22}=0.$$

Note that $d^V_{i,i+1}=e_{i,i+1}=0$, because the highest poles cancel at each order after the summation of Feynman graphs within each gauge-invariant set. This is the consequence of the conservation of current. The above results fully confirm the findings of [16] (see [25]). On the other hand, the $\zeta(3)$ coefficient in d_{20} disagrees[2] with the incorrect one presented in [17]. Fortunately, the numerical difference is small.

It can be shown that the vector part of the LHS of Eq. (8) is invariant under the renormalization group transformations and obeys the renormalization group equation

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha_s)\alpha_s\frac{\partial}{\partial\alpha_s}-\gamma_m(\alpha_s)\sum_{l=f,v} m_l\frac{\partial}{\partial m_l}\right)\times[\text{vector part of the LHS of Eq. (8)}]=0, \tag{10}$$

where $\beta(\alpha_s)$ and $\gamma_m(\alpha_s)$ are the QCD β function and the quark mass anomalous dimension which have been evaluated to three-loop level in Refs. [26] and [27], respectively. From Eqs. (8) and (10), it is straightforward (similar to [20]) to get the $O(\alpha_s^3)$ logarithmic coefficients in the vector channel:

$$d^V_{31}=2(\beta_0+\gamma_0)d^V_{20}+(\beta_1+2\gamma_1)d^V_{10}+2\gamma_2 d^V_{00},$$

$$\begin{aligned}d^V_{32}=&(\beta_0+\gamma_0)[2\gamma_1 d^V_{00}+(\beta_0+2\gamma_0)d^V_{10}]\\&+(\beta_1+2\gamma_1)\gamma_0 d^V_{00},\end{aligned} \tag{11}$$

$$d^V_{33}=\tfrac{2}{3}\gamma_0(\beta_0+\gamma_0)(\beta_0+2\gamma_0)d^V_{00},\quad d_{34}=0,$$

$$e_{31}=2(\beta_0+\gamma_0)e_{20},\quad e_{32}=e_{33}=e_{34}=0.$$

The known perturbative coefficients β_n and γ_n of the QCD β function and the quark mass anomalous dimension γ_m with the proper normalization factors may be found, e.g., in [20]. Similar equations were obtained for the Adler D function perturbative coefficients in Ref. [14]. The only missing coefficients at $O(\alpha_s^3)$ in Eq. (8) for the vector channel are the nonlogarithimc terms d^V_{30} and e_{30}. However, these terms have a zero imaginary part and do not contribute to the decay rate to $O(\alpha_s^3)$.

In the axial channel, the direct computation of the relevant one-, two-, and three-loop Feynman graphs yields

$$d^A_{00}=6/\varepsilon+6,\quad d^A_{01}=6;$$

$$d^A_{10}=-\frac{6}{\varepsilon^2}+\frac{5}{\varepsilon}+\frac{107}{2}-24\zeta(3),\quad d^A_{11}=22,\quad d^A_{12}=6;$$

$$\begin{aligned}d^A_{20}=&\frac{19}{2\varepsilon^3}-\frac{99}{4\varepsilon^2}+\left(\frac{455}{36}-\zeta(3)\right)\frac{1}{\varepsilon}+\frac{3241}{6}-387\zeta(3)\\&-\frac{3}{2}\zeta(4)+165\zeta(5)-N_f\left(\frac{1}{3\varepsilon^3}-\frac{5}{6\varepsilon^2}+\frac{2}{3\varepsilon}+\frac{857}{36}\right.\\&\left.-\frac{32}{3}\zeta(3)\right),\end{aligned} \tag{12}$$

$$d^A_{21}=\tfrac{8221}{24}-117\zeta(3)-N_f[\tfrac{151}{12}-4\zeta(3)],$$

$$d^A_{22}=\tfrac{155}{2}-\tfrac{8}{3}N_f,\quad d^A_{23}=\tfrac{19}{2}-\tfrac{1}{3}N_f,$$

where $\varepsilon=(4-D)/2$ is the deviation of the dimension of spacetime from its physical value 4 within the dimensional regularization [22]. Note that the nonlogarithmic terms in d^A_{i0} contain poles, which cannot be removed by the renormalization of the quark mass and the coupling and have to be subtracted independently. However, one need not worry about those poles, since the imaginary parts of nonlogarithmic terms vanish anyway and do not contribute in the decay rate.

The above mass corrections to the three-loop correlation function of the axial vector quark currents are the new results of the present paper.

One may try to use the renormalization group arguments to obtain the $O(\alpha_s^3)$ logarithmic terms similarly to the vector channel [Eq. (11)]. However, to do so, the knowledge of the $O(\alpha_s^3)$ anomalous dimension is necessary along with the calculated d^A_{20} coefficient. In fact, as was discovered in [15], using the axial Ward identity, this anomalous dimension can be connected to the correlation function of the quark scalar currents which, however, is also known only to $O(\alpha_s^2)$ [18,20].

From Eqs. (6)–(9), (11), and (12), one obtains, for the decay rate,

$$\begin{aligned}\Gamma_{Z\to q_f\bar{q}_f}=&\frac{G_F M_Z^3}{8\sqrt{2}\pi}\sum_{k=V,A}(g_f^k)^2\sum_{i=0}^{3}\sum_{j=0}^{i}\left(\frac{\alpha_s(\mu)}{\pi}\right)^i\\&\times\ln^j\frac{\mu^2}{M_Z^2}\left(a^k_{ij}+\frac{m_f^2(\mu)}{M_Z^2}b^k_{ij}\right.\\&\left.+\sum_{v=u,d,s,c,b}\frac{m_v^2(\mu)}{M_Z^2}c^k_{ij}\right).\end{aligned} \tag{13}$$

The massless limit coefficients $a^{V/A}_{ij}$ are calculated up to $O(\alpha_s^3)$ [5]. The coefficients b_{ij} read

$$b^V_{00}=0,\quad b^A_{00}=-6;$$

$$b^V_{10}=12,\quad b^V_{11}=0,\quad b^A_{10}=-22,\quad b^A_{11}=-12;$$

$$b^V_{20}=\tfrac{253}{2}-\tfrac{13}{3}N_f,\quad b^V_{21}=57-2N_f,\quad b^V_{22}=0, \tag{14}$$

$$\begin{aligned}b^A_{20}=&-\tfrac{8221}{24}+57\zeta(2)+117\zeta(3)\\&+N_f[\tfrac{151}{12}-2\zeta(2)-4\zeta(3)],\end{aligned}$$

$$b^A_{21}=-155+\tfrac{16}{3}N_f,\quad b^A_{22}=-\tfrac{57}{2}+N_f.$$

The $O(\alpha_s^3)$ coefficients for the vector part read

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[2] Contrary to the previous belief (see, e.g., comments in [14,19]) that the result of [16] has been corrected in [17].

$$b_{30}^V = 2522 - \tfrac{855}{2}\zeta(2) + \tfrac{310}{3}\zeta(3) - \tfrac{5225}{6}\zeta(5) - N_f[\tfrac{4942}{27} - 34\zeta(2) + \tfrac{394}{27}\zeta(3) - \tfrac{1045}{27}\zeta(5)] + N_f^2[\tfrac{125}{54} - \tfrac{2}{3}\zeta(2)], \quad (15)$$

$$b_{31} = \tfrac{4505}{4} - \tfrac{175}{2}N_f + \tfrac{13}{9}N_f^2, \quad b_{32} = \tfrac{855}{4} - 17N_f + \tfrac{1}{3}N_f^2,$$

$$b_{33} = 0.$$

The coefficients c_{ij} in both channels are

$$c_{1j} = c_{2j} = 0, \quad c_{30} = -80 + 60\zeta(3) + N_f[\tfrac{32}{9} - \tfrac{8}{3}\zeta(3)],$$

$$c_{31} = c_{32} = c_{33} = 0. \quad (16)$$

The evaluation of $O(\alpha_s^3)$ coefficients b_{3j}^A for the axial part requires the corresponding four-loop calculations. The $\zeta(2)$ terms in the above coefficients are due to the imaginary part of the term $\sim \ln^3(\mu^2/s)$, which appears in the $O(\alpha_s^3)$ coefficients of the correlation function.

Taking $\mu = M_Z$ and $N_f = 5$ and recalling the known massless limit coefficients [5], one obtains, numerically,

$$\begin{aligned}\Gamma_{Z\to q_f\bar{q}_f} = \frac{G_F M_Z^3}{8\sqrt{2}\pi}\Bigg\{&(2I_f^{(3)} - 4e_f\sin^2\Theta_W)^2\Bigg[\left(1 + \frac{2m_f^2(M_Z)}{M_Z^2}\right)\sqrt{1 - \frac{4m_f^2(M_Z)}{M_Z^2}} + \frac{\alpha_s(M_Z)}{\pi}\left(1 + 12\frac{m_f^2(M_Z)}{M_Z^2}\right) + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^2\\
&\times\left(1.4092 + 104.833\frac{m_f^2(M_Z)}{M_Z^2}\right) + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^3\left(-12.805 + 547.879\frac{m_f^2(M_Z)}{M_Z^2} - 6.126\,23\sum_{v=u,d,s,c,b}\frac{m_v^2(M_Z)}{M_Z^2}\right)\Bigg]\\
&+(2I_f^{(3)})^2\Bigg[\left(1 - \frac{4m_f^2(M_Z)}{M_Z^2}\right)^{3/2} + \frac{\alpha_s(M_Z)}{\pi}\left(1 - 22\frac{m_f^2(M_Z)}{M_Z^2}\right) + \left(\frac{\alpha_s(M_Z)}{\pi}\right)^2\left(1.4092 - 85.7136\frac{m_f^2(M_Z)}{M_Z^2}\right)\\
&+\left(\frac{\alpha_s(M_Z)}{\pi}\right)^3\left(-12.767 + (\text{unknown})\frac{m_f^2(M_Z)}{M_Z^2} - 6.126\,23\sum_{v=u,d,s,c,b}\frac{m_v^2(M_Z)}{M_Z^2}\right)\Bigg]\Bigg\}, \quad (17)\end{aligned}$$

where for the Born terms their well known exact expressions [2] are used. (These terms have once again been reevaluated here.) It should be stressed that, in order to obtain a complete (up to date) standard model expression for the decay rate, the following known QCD contributions should also be included: (i) the $O(\alpha_s^2)$ corrections due to the large mass splitting within the t-b doublet [9,10]; (ii) the $O(\alpha_s^2)$ effects due to the virtual heavy quark [11–13]; (iii) the $O(\alpha_s^3)$ corrections coming from the triangle anomaly-type graphs in the limit $m_t \to \infty$ [6]. One also needs to include the electroweak corrections. All those corrections can be found in [2].

The calculated quark mass corrections to $O(\alpha_s^2)$ and $O(\alpha_s^3)$ gave about 10–20 % corrections to the corresponding massless results and are of marginal importance for the high precision analysis at LEP. It is reasonable to expect that the missing $O(\alpha_s^3)$ correction in the axial part will be of order similar to the corresponding vector part result. However, the calculated mass corrections are important in low energy analysis, e.g., at SLAC e^+e^- storage ring PEP and the DESY e^+e^- collider PETRA (or B factory), where the vector part of Eq. (17) is relevant. At low energies, the corrections $\sim \alpha_s^2 m_b^4/s^2$ [28] may also have a phenomenological relevance [29]. However, at the Z mass scale these corrections, of course, are negligible.

For the $Z \to bb$ decay mode, the $O(\alpha_s^2)$ mass corrections agree with the ones obtained in [14,15] using an indirect approach, based on renormalization group arguments and the results of [17,18]. However, at the $O(\alpha_s^3)$, there is a small disagreement. This, in fact, is due to the incorrect numerical coefficient for the $\zeta(3)$ term in [17], which was used in[3] [14]. In the previous equations, the strong coupling $\alpha_s(M_Z)$ and the quark mass $m_f(M_Z)$ are understood as the $\overline{\text{MS}}$ quantities renormalized at the Z mass. The relation between the $\overline{\text{MS}}$ running quark mass and the pole mass is derived from the on shell results of [30] (see [20]):

$$m_f^{(N)}(\mu) = m_f\Bigg\{1 - \frac{\alpha_s^{(N)}(\mu)}{\pi}\left(\frac{4}{3} + \ln\frac{\mu^2}{m_f^2}\right) - \left(\frac{\alpha_s^{(N)}(\mu)}{\pi}\right)^2\Bigg[K_f - \frac{16}{9} + \sum_{m_f < m_{f'} < \mu}\delta(m_f, m_{f'}) + (\tfrac{157}{24} - \tfrac{13}{36}N)\ln\frac{\mu^2}{m_f^2} + (\tfrac{7}{8} - \tfrac{1}{12}N)\ln^2\frac{\mu^2}{m_f^2}\Bigg]\Bigg\}, \quad (18)$$

[3] There is also a misprint in Eq. (23) for the general expression for the $O(\alpha_s^3)$ term in [14]: the division factor 92 should be replaced by 96.

where m_f is the pole mass of the quark, the superscript N indicates that the corresponding quantity is evaluated for the N active flavors of quarks, μ is an arbitrary scale (in the case of Z decay, $\mu = M_Z$ and $N=5$),

$$K_f = 16.006\,50 - 1.041\,37N + \frac{4}{3}\sum_{m_l \leq m_f} \Delta\left(\frac{m_l}{m_f}\right),$$

$$\delta(m_f, m_{f'}) = -1.041\,37 + \frac{4}{3}\Delta\left(\frac{m_{f'}}{m_f}\right), \tag{19}$$

and the numerical values for the Δ at the relevant quark mass ratios are given in [20]. Numerically, in the case of the $Z \to b\bar{b}$ decay mode,[4] $K_b \approx 12.5$ and the sum over $m_{f'}$ drops out in Eq. (18).

The calculated mass corrections to the correlation functions are relevant for the hadronic decay rates of the τ lepton.

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[4] Slightly higher than the one given in [15].

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