

Sum rules for radiative and strong decays of heavy mesons

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We present analogues of the Cabibbo-Radicati and Adler-Weisberger sum rules for heavy mesons. The former expresses the sum of the radiative widths of excited heavy mesons in terms of the isovector charge radius of the ground-state heavy meson, while the latter relates the pionic widths of heavy excited mesons and can be used to set a model-independent upper bound on the pion coupling of the P -wave heavy mesons to the ground state meson. [S0556-2821(96)02515-5]

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We present in this paper two model-independent sum rules for the photon and pion couplings to the light constituents in a heavy meson. They relate properties of the ground state such as charge radius, magnetic moment, and axial vector coupling to matrix elements which govern the radiative and strong transitions between excited heavy meson states and the ground state. The latter parameters can be, in principle, extracted from experiment and in fact, a few of them have already been determined in this way. The sum rules themselves are not new, they are direct analogues of the Cabibbo-Radicati (CR) [1] and Adler-Weisberger (AW) [2,3] sum rules, respectively, familiar from current algebra. However, in this new context they turn out to be considerably more predictive than in their original application, due to additional constraints on the quantum numbers of the available final states. Moreover, when considered in the large- N_c limit, the form of the sum rules simplifies further due to the suppression of the continuum contribution. The CR sum rule reduces in this limit to a constituent-quark sum rule familiar from nonrelativistic quantum mechanics, connecting the charge radius of the ground state to a sum over electric dipole matrix elements between excited states and the ground state [4].

These sum rules are of interest from a phenomenological point of view, as they can be used to place constraints on the photon and pion couplings of low-lying heavy mesons to excited ones. As a sample application, we derive model-independent upper bounds on the pionic decay widths of the charmed P -wave heavy mesons with the quantum numbers of the light degrees of freedom $s^{\pi/2} = 1/2^+$.

The Cabibbo-Radicati sum rule can be derived (we follow here the derivation given in [5–7]) by considering the forward-scattering amplitude of isovector photons of energy ω and helicity λ on a target B :

$$f^{ab}(\omega, \lambda) = \frac{i}{4\pi} \int dx e^{-iq \cdot x} \langle B | T(J^a \cdot e_\lambda)(x) (J^b \cdot e_\lambda^*)(0) | B \rangle, \quad (1)$$

with $q^0 = \omega$, $q^2 = 0$, and $J_\mu^a = e\bar{q}\gamma_\mu t^a q$ ($t^a = \tau^a/2$). The states $|B\rangle$ are normalized noncovariantly to 1. The scattering amplitude f can be written in terms of four invariants $f_\pm^{(\pm)}(\omega)$ as (J_z is the z projection of the target spin)

$$f^{ab}(\omega, \lambda) = \langle B | \frac{1}{2} \{t^a, t^b\} [f_+^{(+)}(\omega) + f_-^{(+)}(\omega)\lambda\omega J_z] | B \rangle + \langle B | \frac{1}{2} [t^a, t^b] [f_+^{(-)}(\omega)\lambda J_z + \omega f_-^{(-)}(\omega)] | B \rangle. \quad (2)$$

The assumption of an unsubtracted dispersion relation [8] for $f_-^{(-)}(\omega)$,

$$f_-^{(-)}(0) = \frac{2}{\pi} \int_0^\infty \frac{d\omega'}{\omega'} \text{Im} f_-^{(-)}(\omega'), \quad (3)$$

in combination with the low-energy theorem [5–7] $\lim_{\omega \rightarrow 0} f_-^{(-)}(\omega) = 1/\pi(-R_V^2/6 + \mu_V^2/2)$ (for a target of isospin $\frac{1}{2}$ and spin $\frac{1}{2}$) leads to the final form of the CR sum rule:

$$\frac{R_V^2}{6} - \frac{\mu_V^2}{2} = \frac{1}{\pi e^2} \int_0^\infty \frac{d\omega'}{\omega'} [2\sigma_{1/2}(\omega') - \sigma_{3/2}(\omega')]. \quad (4)$$

On the left-hand side (LHS) R_V and μ_V are the isovector charge radius and magnetic moment of the target. On the right-hand side (RHS) the optical theorem has been used to express $\text{Im} f_-^{(-)}(\omega)$ in terms of the cross sections for inclusive photoproduction by an isovector photon with $I_z = 0$ of final states with isospin 1/2 and 3/2, respectively.

We will take as target a pseudoscalar heavy meson with quark content $\bar{Q}u$ ($I, I_z = \frac{1}{2}, +\frac{1}{2}$), denoted generically as $|B_i\rangle$ ($i = u, d$). There are a number of specific points which have to be addressed in connection with this choice. First, the sum rule (4) has been derived under the assumption that the target is a spin-1/2 particle, whereas the $|B_i\rangle$ meson has spin zero. However, in the heavy mass limit $m_Q \rightarrow \infty$ the dynamics of the heavy quark decouples from that of the light constituents. As a result, the target $|B_i\rangle = 1/\sqrt{2}(|Q^\dagger q_i^\dagger\rangle - |Q^\dagger q_i^\downarrow\rangle)$ can be effectively considered as a coherent superposition of polarized spin-1/2 particles.

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Second, for such a target the isovector and isoscalar electromagnetic parameters are related in the SU(3)-symmetric limit. The reason for this is that heavy mesons containing only one light quark transform according to the $\mathbf{3}$ representation of SU(3). The electromagnetic current transforms as an octet and there is only one way of combining $\mathbf{3}$, $\mathbf{8}$, and $\bar{\mathbf{3}}$ to a singlet [10–12]. Previous applications of the CR sum rule used a proton or a pion as target, which belongs to SU(3) octets. Therefore, their isovector and isoscalar electromagnetic form factors remain unrelated even in the SU(3) limit. The assumption of SU(3) symmetry simplifies very much the sum rule, and the main part of the discussion below will be restricted to this case [except in the numerical evaluation [see Eq. (12)], where we make use of all information available]. However, the sum rule can be modified to include SU(3)-breaking effects and a detailed discussion will be given elsewhere [9].

From the above observation it follows that the elastic e.m. form factor of a B meson can be written in terms of just one function $F(q^2)$ as $\langle B_i | J_\mu | B_j \rangle = e F(q^2) v_\mu Q_{ij}$ with $J_\mu = e \bar{q} \gamma_\mu Q q$ and $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3})$ is the light quark charge matrix. Current conservation gives $F(0) = 1$. The charge radius appearing on the LHS of Eq. (4) is defined as $R_V^2/6 = dF(q^2)/dq^2|_{q^2=0}$.

The isovector magnetic moment μ_V of the light constituents in Eq. (4) is related to the parameter β introduced in [11]¹ which describes the radiative decay $B^* \rightarrow B \gamma$ $\langle B(v) | \bar{q} \gamma_\mu q | B^*(v, \epsilon) \rangle = -i \beta \epsilon_{\mu\nu\rho\sigma} v^\nu k^\rho \epsilon^\sigma$ (k is the photon momentum) as $\mu_V = \beta/2$. The corresponding decay rate is equal to $\Gamma = \frac{1}{3} \alpha Q^2 \beta^2 |\vec{k}|^3$, with Q the light quark charge in units of e .

Finally, another distinctive feature of our problem is the absence of resonant final states with isospin 3/2. Therefore, $\sigma_{3/2}(\omega)$ in Eq. (4) only receives contributions from continuum states. There is one limit in which these contributions are completely suppressed, and this is the large- N_c limit. In this case the Cabibbo-Radicati sum rule is saturated with the resonances alone, and the corresponding photoproduction cross sections can be expressed in terms of the decay widths for the inverse process $B^{\text{exc}} \rightarrow B \gamma$. One obtains, in this way, the particularly simple result

$$\frac{R_V^2}{6} - \frac{\beta^2}{8} = \frac{1}{8\alpha Q^2} \sum_{\text{exc}} (2J+1) \frac{\Gamma(B^{\text{exc}} \rightarrow B \gamma)}{|\vec{k}|^3}. \quad (5)$$

The summation runs over all excited states of the $|B\rangle$ meson and J is the spin of each state. The form of the sum rule can be simplified by deleting the second term on the left-hand-side (LHS) and extending the summation over the B^* state as well.

It is interesting that a very similar sum rule can be obtained in the nonrelativistic constituent quark model (NRCQM) [13] (and also in atomic physics [4]) by writing

$$\langle B | \vec{x} \cdot \vec{x} | B \rangle = \sum_n^{(\ell=1)} |\langle B | \vec{x} | n \rangle|^2. \quad (6)$$

The radiative decay rate in the electric dipole approximation is given by $\Gamma_{E1} = \frac{4}{3} \alpha Q^2 |\vec{k}|^3 |\langle B | \vec{x} | n \rangle|^2$, which yields upon insertion into Eq. (6) a sum rule for the heavy meson charge radius:

$$\frac{R^2}{6} = \frac{1}{8\alpha Q^2} \sum_{\text{exc}}^{(\ell=1)} (2J+1) \frac{\Gamma_{E1}(B^{\text{exc}} \rightarrow B \gamma)}{|\vec{k}|^3}. \quad (7)$$

Although very similar to Eq. (5), the summation over excited states extends in Eq. (7) only over P -wave states, which are connected to the ground state by a $E1$ transition, whereas in the exact sum rule (5) *all* excited states contribute.

When the large- N_c limit is relaxed, the integral on the right-hand side of the sum rule (4) will receive, in addition to the contributions from the excited heavy mesons (5), also contributions from continuum states such as $(B\pi)$, $(B^*\pi)$, etc. These can be calculated reliably in heavy hadron chiral perturbation theory [14–16], as long as the pion momentum is smaller than the chiral symmetry-breaking scale $\Lambda_\chi \simeq 1$ GeV. By keeping only the B and B^* mesons in intermediate states, we obtain

$$\sigma_{1/2}^{(\pi B)}(\omega) = 2\sigma_{3/2}^{(\pi B)}(\omega) = \frac{\pi}{18} \left(\frac{eg\beta}{2\pi f_\pi} \right)^2 \frac{|\vec{p}|}{\omega} (\omega^2 - m_\pi^2), \quad (8)$$

$$\begin{aligned} \sigma_{1/2}^{(\pi B^*)}(\omega) = & \frac{2\pi}{3} \left(\frac{eg}{2\pi f_\pi} \right)^2 \frac{|\vec{p}|}{\omega} \left\{ 1 + \frac{m_\pi^2}{\omega^2} + \frac{m_\pi^2}{2\omega|\vec{p}|} \ln \frac{\omega - |\vec{p}|}{\omega + |\vec{p}|} \right. \\ & - \frac{1}{4} \beta \omega \left(1 + \frac{m_\pi^2}{2\omega|\vec{p}|} \ln \frac{\omega - |\vec{p}|}{\omega + |\vec{p}|} \right) \\ & \left. + \frac{1}{24} \beta^2 (\omega^2 - m_\pi^2) \right\}, \quad (9) \end{aligned}$$

$$\begin{aligned} \sigma_{3/2}^{(\pi B^*)}(\omega) = & \frac{\pi}{3} \left(\frac{eg}{2\pi f_\pi} \right)^2 \frac{|\vec{p}|}{\omega} \left\{ 1 + \frac{m_\pi^2}{\omega^2} + \frac{m_\pi^2}{2\omega|\vec{p}|} \ln \frac{\omega - |\vec{p}|}{\omega + |\vec{p}|} \right. \\ & + \frac{1}{2} \beta \omega \left(1 + \frac{m_\pi^2}{2\omega|\vec{p}|} \ln \frac{\omega - |\vec{p}|}{\omega + |\vec{p}|} \right) \\ & \left. + \frac{1}{6} \beta^2 (\omega^2 - m_\pi^2) \right\}, \quad (10) \end{aligned}$$

where $|\vec{p}| = \sqrt{\omega^2 - m_\pi^2}$ is the pion momentum. The static approximation for the heavy meson has been used in deriving these expressions $\omega/m_B \simeq 0$.

In these formulas g is the $BB^*\pi$ coupling in the heavy mass limit [14–16]. Experimental data on branching ratios of D^* decays [10–12] give $0.09 \leq g^2 \leq 0.5$ (with 90% confidence limits). We will adopt for our estimates the upper limit $g^2 = 0.5$ (note though that QCD sum rule computations suggest significantly lower values $g^2 \simeq 0.1$ [17–20]). As already mentioned, β describes the $B^* \rightarrow B \gamma$ decay in the heavy mass limit. It has been determined simultaneously with g in [10–12] such that their values are correlated: larger values for g favor larger values for β . The limits quoted are $2 \leq \beta \leq 6 \text{ GeV}^{-1}$ [10–12]. Recent QCD sum rule and model

¹The extra (–) sign is due to the fact that we are considering heavy mesons with one heavy antiquark.

calculations [21,17,20] give, on the other hand, smaller values, around $\beta=2-3 \text{ GeV}^{-1}$ ([20] finds $\beta \approx 1 \text{ GeV}^{-1}$). We will use in our estimates below $\beta=3-4 \text{ GeV}^{-1}$.

Previous experience with the Cabibbo-Radicati sum rule [22–24] suggests that the saturation region includes states with an excitation energy of the order of a few GeV. Inserting Eqs. (8)–(10) into Eq. (4) with an upper cutoff of 1 GeV gives, for the continuum contribution on the RHS of the sum rule (4),

$$\begin{aligned} I_{\text{cont}}^{(\pi B)} + I_{\text{cont}}^{(\pi B^*)} + \dots &= \left(\frac{g}{2\pi f_\pi} \right)^2 0.036\beta^2 + \left(\frac{g}{2\pi f_\pi} \right)^2 \\ &\times (1.519 - 0.321\beta) + \dots \\ &= 0.639(0.589)(\text{GeV}^{-2}) + \dots \end{aligned} \quad (11)$$

The ellipsis stands for other continuum contributions, which are expected to be less important as they have less phase space available. The two numbers correspond to $\beta=3(4) \text{ GeV}^{-1}$. The pion decay constant is $f_\pi=0.132 \text{ GeV}$.

We are now in a position to discuss the numerical values of the two sides of the sum rule (4). It can be written as

$$\begin{aligned} 2.161 &= 0.321(0.846) + (0.405 \pm 0.067) + (0.811 \pm 0.135) \\ &+ 0.639(0.589) + \dots (\text{GeV}^{-2}) \end{aligned}$$

$$\frac{R_V^2}{6} = (B^*) + (P_{1/2}) + (P_{3/2}) + (\text{continuum}). \quad (12)$$

On the LHS, the isovector charge radius R_V has been obtained from a version of vector-meson dominance where the contributions of the two lowest $I=1$ ($J^{PC}=1^{-}$) vector mesons are kept $R_V^2/6=(1/m_\rho^2+1/m_{\rho'}^2)$. This value follows from a minimal ansatz for the elastic form factor chosen to conform with the QCD counting rules at large q^2 [25]. On the RHS, the B^* contribution has been computed by including also the nonanalytic SU(3)-violating contributions obtained in [11] $\mu_V^2=\frac{1}{4}(\beta-g^2 m_K/4\pi f_K^2-g^2 m_\pi/2\pi f_\pi^2)$. The contributions of the two lowest-lying P -wave states with $s_{\not{L}}^{\pi}=\frac{1}{2}^+, \frac{3}{2}^+$ were computed in the dipole approximation using the wave functions of the Isgur-Scora-Grinstein-Wise (ISGW) ISGW model [26] with the updated parameters of the ISGW2 model [27]. In the absence of the spin-orbit interaction, responsible for the splitting of these two multiplets, their contributions to the sum rule are in the ratio $P_{1/2}:P_{3/2}=1:2$. The errors shown correspond to the 30% accuracy expected from the model when predicting radiative decay matrix elements [27].

One obtains in this way for the RHS of the sum rule 2.176 ± 0.202 (2.651 ± 0.202) GeV^{-2} . The agreement with the LHS is certainly better than one could have expected from our qualitative estimates. This strongly suggests that the CR sum rule is very close to saturation with the first few excited states and the continuum up to excitation energies of about 1 GeV.

We would like to make a few additional comments about the different contributions on the RHS of Eq. (12).

The nonresonant contributions have all the same sign (positive). The dominant contribution to the integral over ω

comes from the (πB^*) two-body state, which can be formed in a $l=0$ partial wave. Therefore, this cross section dominates in the low energy region, to which the sum rule is the most sensitive. The threshold region gives rise to a large $\ln[g/(2\pi f_\pi)]^2 \ln(\Lambda_\chi/m_\pi)$, which is consistent with the expected divergence of the isovector charge radius in the chiral limit $m_\pi \rightarrow 0$ [28].

An important issue which needs to be addressed is that of the contributions of higher excited states. We have estimated these contributions with the help of the simpler NRCQM sum rule (7). In general, the approach to saturation depends on the specific potential adopted. For a harmonic oscillator the sum rule is saturated with the first P -wave states alone and for a particle in a spherical potential well these states contribute over 99% of the total. A similar behavior is expected to be true of any other confining potential. On the other hand, in the hydrogen atom the lowest-lying P -wave states contribute 55% and the continuum states about 28% of the total [4].

The RHS of Eq. (12) is an increasing function of β , with a minimum at $\beta=1.93$ (for $g^2=0.5$). Therefore, requiring equality of the two sides seems to favor values for β of the order of 3 GeV^{-1} . However, the estimate of the P -wave contributions is still too crude to allow setting an useful constraint on β . In case that these contributions turn out to have been overestimated, the sum rule will be well satisfied with larger values of β ($4-6 \text{ GeV}^{-1}$).

Finally, we note that the methods of this paper can be used with little modification to obtain a sum rule for the pion couplings between heavy excited mesons and the ground-state ones B, B^* . To derive it, consider the amplitude for forward scattering of pions on a B meson. The assumption of an unsubtracted dispersion relation [8] for the isospin-odd part of this amplitude plus knowledge of its low-energy limit gives an analogue of the well-known Adler-Weisberger sum rule [2,3]

$$\begin{aligned} 1 - g^2 &= \frac{f_\pi^2}{\pi} \int_{m_\pi}^{\infty} \frac{d\nu}{\nu^2} \sqrt{\nu^2 - m_\pi^2} [\sigma(\pi^- B_u \rightarrow X) \\ &- \sigma(\pi^+ B_u \rightarrow X)]. \end{aligned} \quad (13)$$

On the LHS g is the $BB^*\pi$ coupling defined as above; the integral on the RHS runs over the inclusive cross sections for $B_u \pi^\pm$ scattering with energy ν . Just as in the case of the CR sum rule, the heavy resonances will contribute only to $\sigma(\pi^- B_u \rightarrow X)$, since the $\pi^+ B_u$ state has isospin $3/2$. Separating explicitly the contribution of the resonances from the continuum (I_{cont}), the sum rule (13) can be written as

$$1 = 2\pi f_\pi^2 \sum_{\text{res}} (2J+1) \frac{\Gamma(B_d^{\text{res}} \rightarrow \pi^- B_u)}{\nu^3} + I_{\text{cont}}. \quad (14)$$

Even without a detailed calculation, one can see that I_{cont} is positive, because $B_u \pi^-$ has simply more available channels than $B_u \pi^+$. This observation can be used to produce a model-independent constraint on the couplings of the higher resonances

$$g^2 + h^2 + (0.07 \pm 0.01) + \dots < 1 \quad (15)$$

$$(B^*)P_{1/2}P_{3/2}.$$

The contribution of the $s_{\not{c}}^{\pi}=3/2^+$ heavy mesons has been obtained from existing data on their decay widths [29,30]. On the other hand, the members of the $s_{\not{c}}^{\pi}=1/2^+$ multiplet, whose strong couplings are parametrized by h (defined as in [31]), have not been observed experimentally due to their large width. The relation (15) gives an useful upper bound on $|h|$: $h^2 < 0.9$, where we neglected the small contribution from the $P_{3/2}$ excited states and used the experimental lower limit on g^2 [10–12]. This agrees with the recent QCD sum rule calculation of [32], who find $h^2 = 0.39 \pm 0.31$. For the total pionic widths of the 0^+ and 1^+ charmed states our result

implies the upper bounds $\Gamma(0^+) \leq 590\text{--}1010$ MeV and $\Gamma(1^+) \leq 200\text{--}450$ MeV, corresponding to the mass values $m_{0^+} = m_{1^+} = 2.3\text{--}2.4$ GeV [33].

After completing this work the paper [34] appeared, where the Adler-Weisberger sum rule is applied to strong decays of heavy mesons.

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- [1] N. Cabibbo and L.A. Radicati, Phys. Lett. **19**, 697 (1966)
- [2] S. Adler, Phys. Rev. Lett. **14**, 1051 (1965); Phys. Rev. **140**, B736 (1965).
- [3] W. Weisberger, Phys. Rev. Lett. **14**, 1047 (1965); Phys. Rev. **143**, 1302 (1965).
- [4] H. Bethe and E. Salpeter, *Quantum Mechanics of One- and Two-electron Atoms* (Springer-Verlag, Berlin, 1957).
- [5] M.A. Beg, Phys. Rev. **150**, 1276 (1966).
- [6] K. Kawarabayashi and W.W. Wada, Phys. Rev. **152**, 1286 (1966).
- [7] S. Weinberg, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis University Summer Institute on Theoretical Physics (MIT, Cambridge, MA, 1970), pp. 285–393.
- [8] The no-subtraction option follows, as usual, from assuming Regge asymptotics dominated by the $I=1$ ρ trajectory. In addition to this, we request that the resulting sum rules are close to saturation with states of excitation energy much smaller than the heavy quark mass. This assumption is necessary to allow the use of the simplifications appearing in the heavy mass limit and has to be ultimately tested by experiment.
- [9] C.K. Chow and D. Pirjol (unpublished).
- [10] P. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992); **300**, 410(E) (1993).
- [11] J. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).
- [12] H.Y. Cheng *et al.*, Phys. Rev. D **47**, 1030 (1993).
- [13] A. de Rujula, H. Georgi, and S.L. Glashow, Phys. Rev. D **12**, 3589 (1975).
- [14] M. Wise, Phys. Rev. D **45**, 2188 (1992).
- [15] G. Burdman and J.F. Donoghue, Phys. Lett. B **280**, 287 (1992).
- [16] T.M. Yan *et al.*, Phys. Rev. D **46**, 1148 (1992).
- [17] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. B **334**, 175 (1994); **339**, 151 (1994).
- [18] A.G. Grozin and O.I. Yakovlev, Report No. BUDKERINP-94-3, hep-ph/9401267 (unpublished).
- [19] V.M. Belyaev *et al.*, Phys. Rev. D **51**, 6177 (1995).
- [20] H.G. Dosch and S. Narison, Phys. Lett. B **368**, 163 (1996).
- [21] T.M. Aliev *et al.*, Phys. Lett. B **334**, 169 (1994); Phys. Rev. D (to be published).
- [22] F.J. Gilman and H.J. Schnitzer, Phys. Rev. **150**, 1362 (1966).
- [23] S.L. Adler and F.J. Gilman, Phys. Rev. **156**, 1598 (1967).
- [24] C.A. Dominguez and H. Moreno, Phys. Rev. D **13**, 616 (1976).
- [25] M.A. Luty and R. Sundrum, Phys. Rev. D **52**, 1627 (1995); **52**, 5202 (1995).
- [26] N. Isgur, D. Scora, B. Grinstein, and M. Wise, Phys. Rev. D **39**, 799 (1989).
- [27] N. Isgur and D. Scora, Phys. Rev. D **52**, 2783 (1995).
- [28] M.A. Beg and A. Zepeda, Phys. Rev. D **6**, 2912 (1972).
- [29] U. Kilian, J.G. Körner, and D. Pirjol, Phys. Lett. B **288**, 360 (1992).
- [30] A. Falk and M. Luke, Phys. Lett. B **292**, 119 (1992).
- [31] A. Falk, Phys. Lett. B **305**, 268 (1983).
- [32] P. Colangelo *et al.*, Phys. Rev. D **52**, 6422 (1995).
- [33] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); S. Godfrey and R. Kokoski, *ibid.* **43**, 1679 (1991).
- [34] S.R. Beane, Report No. DUKE-TH-95-98, hep-ph/9512228 (unpublished).