Rederivation of the spin-dependent next-to-leading order splitting functions

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We perform a new calculation of the polarized next-to-leading order splitting functions, using the method developed by Curci, Furmanski, and Petronzio. We confirm the results of the recent calculation by Mertig and van Neerven. [S0556-2821(96)05215-0]

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I. INTRODUCTION

The past few years have seen much progress in our knowledge about the nucleon's spin structure due to the experimental study of the spin asymmetries $A_1^N(x,Q^2) \approx g_1^N(x,Q^2)/F_1^N(x,Q^2)$ $(N=p,n,d)$ in deep-inelastic scattering (DIS) with longitudinally polarized lepton beams and nucleon targets. Previous data on A_1^p by the SLAC-Yale Collaboration [1] have been succeeded by more accurate data from [2–4], which also cover a wider range in (x,Q^2), and results on A_1^n and A_1^d have been published in [5] and [6,7], respectively.

On the theoretical side, it has become possible to perform a fully consistent study of longitudinally polarized DIS in next-to-leading order (NLO) of QCD, since recently complete results for the spin-dependent two-loop anomalous dimensions, needed for the NLO evolution of polarized parton distributions, have been presented for the first time [8,9], calculated within the operator product expansion (OPE). A first phenomenological NLO study has been presented in [11], later followed by the analyses in [12].

The calculation of the NLO anomalous dimensions or splitting functions is in general very complicated. This is true in particular for the polarized case, where the Dirac matrix γ_5 and the antisymmetric Levi-Cività tensor $\epsilon_{\mu\nu\rho\sigma}$ enter the calculation as projectors on to definite helicity states of the involved particles. These (genuinely *four*-dimensional) quantities lead to certain complications when dimensional regularization, which probably represents the only viable method of regularization in such a calculation, is used. In fact [8], was recently revised since an error related to the treatment of γ_5 was found. Although the results of [8] now fulfill a relation motivated from supersymmetry, which appears to be an important constraint, it seems necessary to perform an independent calculation of the polarized two-loop splitting functions to check the results of [8]. This is the purpose of this paper.

In the unpolarized case, two different methods have been used to obtain the next-to-leading order splitting functions. The first calculation [13] was performed within the OPE. Afterwards Curci, Furmanski, and Petronzio [14,15] used a technique which is as close as possible to parton model intuition since it is based explicitly on the factorization properties of mass singularities in the lightlike axial gauge [16] and on the generalized ladder expansion [17]. Note that the results of [15] satisfied the above-mentioned supersymmetric relation [18], but were in disagreement with the first calculation [13] for the NLO gluon-to-gluon splitting function. The controversy was resolved by a recalculation [19] of this splitting function in the OPE which confirmed the result of [15]. In this paper we exploit the method and results of [14,15] to rederive the spin-dependent next-to-leading order splitting functions. To deal with γ_5 and the ϵ tensor we use the 't Hooft–Veltman–Breitenlohner–Maison (HVBM) scheme [20], which still seems to be the most consistent prescription [21]. Section II sets the framework for the calculation, some details of which are then given in Sec. III. In Sec. IV we present our results.

II. FRAMEWORK

In this section we outline the framework for our calculation. We mainly focus on the new features in the polarized case; more details on the method itself can be found in the original works [16,14]. We reserve a more detailed description of our calculation to a future publication.

The general strategy consists of a rearrangement of the perturbative expansion which makes explicit the factorization into a part which does not contain any mass singularity and another one which contains all (and only) mass singularities. Figure 1 represents the matrix element squared for polarized virtual (spacelike) photon-quark scattering. The blob ΔM is expanded into two particle irreducible (2PI) kernels C_0 and K_0. In the axial gauge these 2PI kernels have been proven [16] to be finite as long as the external legs are kept unintegrated, such that all collinear singularities originate from the integrations over the momenta flowing in the lines connecting the various kernels. The generalized ladder in Fig. 1 can be written as [14,22]

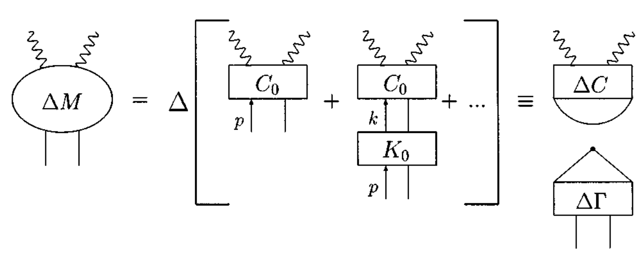

FIG. 1. The matrix element squared for polarized photon-quark interaction, its expansion in terms of 2PI kernels C_0 and K_0, and its final factorized form.

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$$\Delta M = \Delta[C_0(1+K_0+K_0^2+\cdots)] = \Delta\left[C_0 \frac{1}{1-K_0}\right] \equiv \Delta C \Delta\Gamma. \tag{1}$$

ΔC and $\Delta\Gamma$ have been decoupled by projectors which for the longitudinally polarized case read (A, B being two kernels)

$$\Delta(A\Delta P_F B) = (\Delta A^{ij}(\not{k}\gamma_5)_{ij})[\mathrm{PP}]\left(\frac{(\gamma_5 \not{n})^{kl}}{4kn}\Delta B_{kl}\right) \tag{2}$$

for polarized quarks and

$$\Delta(A\Delta P_G B) = \left(\Delta A_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\frac{k_\rho n_\sigma}{2kn}\right)[\mathrm{PP}]\left(\epsilon^{\alpha\beta\gamma\delta}\frac{k_\gamma n_\delta}{kn}\Delta B_{\alpha\beta}\right) \tag{3}$$

for polarized gluons, setting $k^2=0$ in the part containing the kernel ΔA and taking the pole part [PP] of the projection on kernel ΔB. In Eqs. (2) and (3) i,j,k,l are spinor and the greek letters are Lorentz indices. n is the vector to be introduced in the axial gauge with $n^2=0$ for the lightlike gauge. The last expression in Eq. (1) displays the factorization of mass singularities [14], which in dimensional regularization ($d=4-2\epsilon$) appear as poles in $1/\epsilon$. More explicitly, the contribution to the (partonic) spin-dependent deep-inelastic structure function g_1 reads

$$g_1\left(\frac{Q^2}{\mu^2},x,\alpha_s,\frac{1}{\epsilon}\right) = \Delta C\left(\frac{Q^2}{\mu^2},x,\alpha_s\right)\otimes\Delta\Gamma\left(x,\alpha_s,\frac{1}{\epsilon}\right), \tag{4}$$

where the convolution $\otimes$ is defined as usual by

$$(f\otimes g)(x) \equiv \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right). \tag{5}$$

In Eq. (4) we have introduced the virtuality of the photon Q^2, the unit of mass μ in dimensional regularization, the Bjørken variable $x=Q^2/2pq$, and the strong coupling α_s. Equation (4) has a clear partonic interpretation: $\Delta\Gamma(x,\alpha_s,1/\epsilon)$ describes the density of partons in the parent quark, is independent of the hard process considered, and contains all the collinear singularities (poles in ϵ), whereas $\Delta C(Q^2/\mu^2,x,\alpha_s)$ is the (process-dependent) polarized short-distance cross section. As was shown in [14], $\Delta\Gamma$ does not depend on Q^2, which is a consequence of the finiteness of the kernel K_0 in the axial gauge [16] and allows for the derivation of a ''renormalization group'' equation for ΔC with $\Delta\Gamma$ related to the ''anomalous dimension.'' Thus $\Delta\Gamma$, to be convoluted with bare (''unrenormalized'') parton densities which must cancel its $1/\epsilon$ poles, is equivalent to the respective Altarelli-Parisi [23] kernels: e.g.,

$$\Delta\Gamma\left(x,\alpha_s,\frac{1}{\epsilon}\right) = \delta(1-x) - \frac{1}{\epsilon}\left[\left(\frac{\alpha_s}{2\pi}\right)\Delta P_{qq}^{(0)}(x) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{qq}^{(1)}(x)+\cdots\right] + O\left(\frac{1}{\epsilon^2}\right) \tag{6}$$

for the nonsinglet (NS) case. The final NLO expression for the (physical) spin-dependent nucleon structure function g_1 then reads

$$g_1(x,Q^2) = \tfrac{1}{2}\sum_q^{N_f} e_q^2 \Bigg\{ \Delta q(x,Q^2) + \Delta\overline{q}(x,Q^2) + \frac{\alpha_s(Q^2)}{2\pi}\left[\Delta C_q \otimes (\Delta q + \Delta\overline{q}) + \frac{1}{N_f}\Delta C_g \otimes \Delta g\right](x,Q^2)\Bigg\}, \tag{7}$$

where N_f is the number of active flavors and where in the full singlet case two short-distance cross sections ΔC_q and ΔC_g for scattering off incoming polarized quarks or gluons, respectively, exist. Here, the polarized parton distributions $\Delta p \equiv p^\uparrow - p^\downarrow$ $(p=q,g)$ are to be evolved according to the spin-dependent Altarelli-Parisi [23] evolution equations which to NLO read (see, e.g., [24])

$$\frac{d}{d\ln Q^2}(\Delta q + \Delta\overline{q} - \Delta q' - \Delta\overline{q}') = \frac{\alpha_s}{2\pi}(\Delta P_{qq}^V + \Delta P_{q\overline{q}}^V)\otimes(\Delta q + \Delta\overline{q} - \Delta q' - \Delta\overline{q}'), \tag{8}$$

$$\frac{d}{d\ln Q^2}(\Delta q - \Delta\overline{q}) = \frac{\alpha_s}{2\pi}(\Delta P_{qq}^V - \Delta P_{q\overline{q}}^V)\otimes(\Delta q - \Delta\overline{q}) \tag{9}$$

for the NS quark densities and

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}\Delta P_{qq}^V + \Delta P_{q\overline{q}}^V + \Delta P_{qq}^S & \Delta P_{qg}\\ \Delta P_{gq} & \Delta P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma\\ \Delta g\end{pmatrix} \tag{10}$$

in the singlet sector, where $\Delta\Sigma \equiv \Sigma_q(\Delta q + \Delta\overline{q})$ and the argument (x,Q^2) has been omitted from all parton densities. To NLO, all splitting functions in Eqs. (8)–(10) have the perturbative expansion

$$\Delta P_{ij} = \Delta P_{ij}^{(0)} + \frac{\alpha_s}{2\pi}\Delta P_{ij}^{(1)}. \tag{11}$$

The entries $\Delta P_{q\overline{q}}^V$ and ΔP_{qq}^S start to be nonzero only beyond leading order. For future reference it is convenient to introduce the NLO combinations

$$\Delta P_{qq}^{\pm,(1)} = \Delta P_{qq}^{V,(1)} \pm \Delta P_{q\overline{q}}^{V,(1)}, \tag{12}$$

which according to Eqs. (8) and (9) govern the NLO part of the evolution in the NS sector. ΔP_{qq}^S is called the ''pure singlet'' splitting function since it only appears in the singlet case.

III. THE CALCULATION

Before giving some details of the calculation, we note that the use of the lightlike $(n^2=0)$ axial gauge in practical calculations has been a matter of debate for a long time now [25,26]. The great computational advantage and success it has brought for, e.g., perturbative NLO QCD calculations in DIS [14,15,27] or jet calculus [28] has not always been matched by the theoretical understanding of why it worked so well [14,29]. The problems connected with the lightlike axial gauge are due in the first place to the presence of spurious singularities in loop and phase space integrals coming from the vector propagator in this gauge, which are neither of ultraviolet nor of infrared origin. We will follow [14,15] to use the Cauchy principle value (PV) prescription to deal with such $1/(n\cdot l)$ terms (where l is some momentum):

$$\frac{1}{ln}\rightarrow\frac{ln}{(ln)^2+\delta^2(pn)^2}. \tag{13}$$

All the resulting divergencies of this type can then be transformed into the basic integrals

$$I_i\equiv\int_0^1 dy\,\frac{y\ln^i y}{y^2+\delta^2}\quad (i=0,1). \tag{14}$$

I_0 and I_1 have to cancel out in the final answer. As was discussed in [14], use of the PV prescription entails renormalization ''constants'' which depend on I_0 and the infinite-momentum-frame (IMF) variable x [30].

Some representatives of the graphs to be evaluated in the calculation of the $\Delta P_{ij}^{(1)}$ are shown in Fig. 2. As indicated by the dashed lines in Fig. 2, some graphs possess real and virtual cuts. We do not need to calculate the contributions from genuine two-loop graphs (not shown in Fig. 2) to the diagonal splitting functions $\Delta P_{qq}^{V,(1)}$ and $\Delta P_{gg}^{(1)}$, which are $\sim\delta(1-x)$. These are the same as for the unpolarized case, where they were determined via constraints from momentum conservation in [14,15,31] and also explicitly calculated in [13,8].

The calculation of the real emission graphs is rather involved. This is true in particular for the polarized case when using the HVBM scheme since in this method the $(d=4-2\epsilon)$-dimensional space-time is explicitly decomposed into the usual four dimensions in which γ_5 anticommutes with the other Dirac matrices and the (-2ϵ)-dimensional part, where it commutes. Thus the squared matrix elements of the graphs will depend on the usual ''d-dimensional'' scalar products such as $l_1\cdot l_2$ etc. [see Fig. 2(a) for notation of the momenta], but also on ''$(d-4)$-dimensional'' ones, denoted by $\hat{l}_1\cdot\hat{l}_2$, $\hat{k}^2$, etc. [32]. It is most convenient to work in the IMF parametrization of the momenta [14] which in our case takes the form

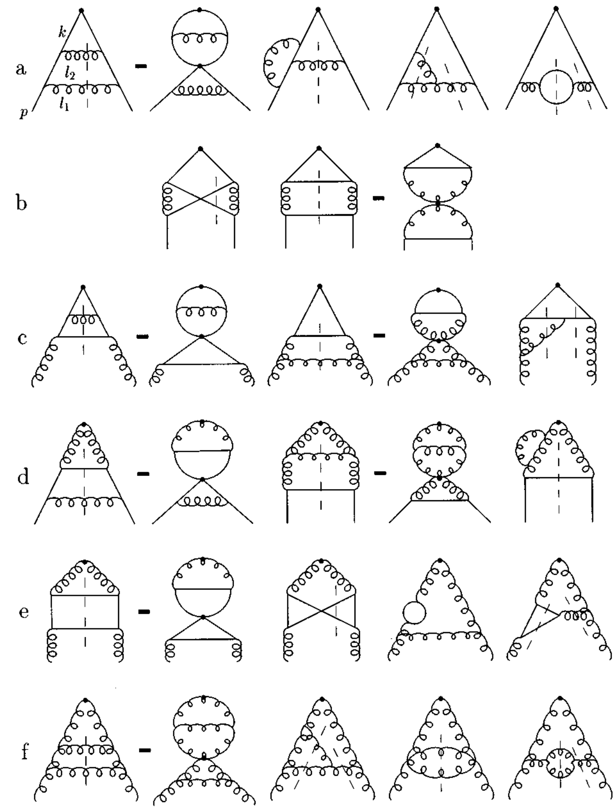

FIG. 2. Some representative Feynman graphs to be evaluated in the calculation of (a) $(\Delta)P_{qq}^{V,(1)}$, (b) $(\Delta)P_{qq}^{V,(1)}$, $(\Delta)P_{qq}^{S,(1)}$ (c) $(\Delta)P_{qg}^{(1)}$, (d) $(\Delta)P_{gq}^{(1)}$, and (e), (f) $(\Delta)P_{gg}^{(1)}$. Subtraction of ''doubly collinear'' graphs is indicated.

$$p=(P,\vec{0}_{xy},P,\vec{0}_{d-4}),$$

$$n=\left(\frac{pn}{2P},\vec{0}_{xy},-\frac{pn}{2P},\vec{0}_{d-4}\right),$$

$$k=\left(xP+\frac{k^2+\widetilde{k}^{\,2}}{4xP},\vec{k}_T,xP-\frac{k^2+\widetilde{k}^{\,2}}{4xP},\hat{\vec{k}}\right),$$

$$l_1=(l_1^0,\vec{l}_1^{\,xy},l_1^z,\hat{\vec{l}}_1), \tag{15}$$

where $x=kn/pn$ is interpreted as the IMF momentum fraction of the incoming momentum p carried by k, and $\widetilde{k}^2=k_x^2+k_y^2+\hat{k}^2\equiv k_T^2+\hat{k}^2$ is the total transverse momentum squared of k relative to the axis defined by p,n. We split the $(d-4)$-dimensional components of l_1 into a part $\hat{l}_1^{\parallel}$ parallel to those of k and a transverse part $\hat{l}_1^{\perp}$. According to our definitions, only k, l_1, and $l_2=p-k-l_1$ possess such components. When performing the phase space integrations one has to carefully take into account the $(d-4)$-dimensional terms. The contribution of each real graph to $\Delta\Gamma(x,\alpha_s,1/\epsilon)$ is given by the integration of the squared matrix elements [with the projectors in Eqs. (2) and (3) being acted upon] over the phase space

$$R\equiv\int\frac{d^dk}{(2\pi)^d}x\,\delta\left(x-\frac{kn}{pn}\right)\int\frac{d^dl_1}{(2\pi)^{d-1}}\int\frac{d^dl_2}{(2\pi)^{d-1}}(2\pi)^d\delta^{(d)}(p-k-l_1-l_2)\,\delta(l_1^2)\,\delta(l_2^2), \tag{16}$$

which is conveniently written as

$$x^{\epsilon}(1-x)^{1-2\epsilon}\int_{-Q^2}^{0}dk^2(-k^2)^{1-2\epsilon}\int_0^1 dw[w(1-w)]^{-\epsilon}\int_0^1 d\tilde{\kappa}[\tilde{\kappa}(1-\tilde{\kappa})]^{-\epsilon}\int_0^1 dv[v(1-v)]^{-1/2-\epsilon}\left((-\epsilon)\int_0^1 d\hat{\kappa}\,\hat{\kappa}^{-1-\epsilon}\right)$$
$$\times\left((-\tfrac{1}{2}-\epsilon)\int_0^1 d\lambda^{\perp}(\lambda^{\perp})^{-3/2-\epsilon}\right)\left(\frac{1}{\pi}\int_0^1 d\lambda^{\parallel}[\lambda^{\parallel}(1-\lambda^{\parallel})]^{-1/2}\right), \tag{17}$$

where we have omitted trivial prefactors and defined

$$\hat{k}^2=-k^2(1-x)\hat{\kappa}\tilde{\kappa},$$
$$\tilde{k}^2=-k^2(1-x)\tilde{\kappa},$$
$$l_1^0+l_1^z=2P(1-x)w=2P\frac{l_1n}{pn},$$
$$(l_1^0)^2-(l_1^z)^2=c_1^2+v(c_2^2-c_1^2)=\frac{1}{P}(l_1^0+l_1^z)(l_1p),$$
$$\hat{l}_1^{\parallel}=\lambda_1+\lambda^{\parallel}(\lambda_2-\lambda_1),$$
$$(\hat{l}_1^{\perp})^2=v(1-v)(c_1+c_2)^2\lambda^{\perp}, \tag{18}$$

with

$$c_{1,2}\equiv\sqrt{\frac{-k^2(1-x)w}{x}}\left[\sqrt{(1-w)(1-\tilde{\kappa})}\mp\sqrt{xw\tilde{\kappa}}\right],$$

$$\lambda_{1,2}=-\frac{1}{2}\frac{\hat{\kappa}}{\hat{k}w}[(l_1^0)^2-(l_1^z)^2-c_1c_2]$$
$$\mp(c_1+c_2)\sqrt{(1-\hat{\kappa})(1-\lambda^{\perp})v(1-v)}.$$

Note that the last three integrals in Eq. (17) are all unity if no dependence on $(d-4)$-dimensional scalar products occurs, which of course is always the case in the unpolarized situation. If present, such $(d-4)$-dimensional terms only give contributions proportional to ϵ after the last three integrals in Eq. (17) have been performed. One also sees from the definition of $l_1^0+l_1^z$ in Eqs. (18) that the divergences from the gauge propagator $1/(l_1n)$ appear at $w\to 0$ [and at $w\to 1$ for $1/(l_2n)$].

If we are only interested in the final answer, it is satisfactory that it turns out to be possible to quite easily infer the effective contributions of the nontrivial [33] virtual, in particular the vertex correction, graphs to the polarized splitting functions from the known results [14,15] for the *un*polarized $P_{ij}^{(1)}$ such that these contributions need not be calculated all over again. Considering, for instance, the last graph in Fig. 2(c) (which contributes to $\Delta P_{qg}^{(1)}$), the strategy for this goes as follows. The final result for the graph is the sum of the real-cut and the virtual-cut contributions, as indicated by the dashed lines. Since the graph is 2PI, the sum is finite in the axial gauge before the integration over k^2 is performed. After integrating the matrix element for the *real* part of the graph over all variables in Eq. (17) except for w (and k^2), it contains terms $\sim 1/w$ from the $1/(l_1n)$ gauge propagator, but also, as it turns out, from quantities $\sim 1/[(l_1p)(l_1l_2)]$ in the original matrix element. In principle, the $1/w$ terms from the latter should be treated in dimensional regularization, where they give rise to (infrared) $1/\epsilon$ [see Eq. (17)] and eventually (together with $1/\epsilon$ poles from the $\tilde{\kappa}$ integration) to $1/\epsilon^2$ poles, to be canceled from similar terms in the virtual contributions. In contrast to this, the $1/w$ terms from the gauge propagator are subject to the PV prescription (13). If one now treats *all* terms $\sim 1/w$ in the same way according to Eq. (13), the result is that the real part of the graph becomes entirely *finite* in itself. This means that the virtual part (vertex correction) of the graph is finite as well in this regularization [34], apart of course from the ultraviolet poles which are removed by renormalization. One can then make an ansatz for the unrenormalized vertex correction

$$V^{\mu}\equiv\left(\frac{A_0}{\epsilon}+A_1\right)\gamma^{\mu}+\sum_i B_i\mathcal{D}_i^{\mu},$$

where the sum runs over the possible Lorentz and Dirac structure $\mathcal{D}_i^{\mu}$ of the vertex which is not proportional to the tree vertex γ^{μ}. The coefficient A_0 is known from the renormalization constant for the qqg vertex in the lightlike axial gauge as given in [14]. Inserting the vertex V^{μ} into the underlying LO graph one finds after renormalization that its contribution to the NLO splitting function depends on A_0, A_1, and a certain combination $f(B_i)$ of the B_i, which, crucially, is the *same* in the unpolarized and the polarized cases. Thus, subtracting the sum of all real-emission graphs in the unpolarized case from the corresponding final results listed in [14,15], one can straightforwardly read off A_1 and the combination $f(B_i)$, which makes the transfer to the polarized case directly possible. Note that when inserting V^{μ} into the LO graph in the polarized case, the HVBM scheme introduces dependence of the result on the $(d-4)$-dimensional scalar product $\hat{k}^2$. One finds that after integration, effectively,

$$\hat{k}^2\to\frac{\epsilon}{1-\epsilon}k^2(1-x). \tag{19}$$

We found that our strategy for determining the contributions from virtual graphs, which relies on the success of the unpolarized calculation [14,15], worked in all cases. It makes the whole calculation considerably simpler and is sufficient if we are just interested in providing a check on the results of [8]. We reserve a presentation of a full-fledged calculation of all (real *and* virtual) contributions to a future publication.

We finally note that whenever considering a genuine ladder graph with two parallel rungs, subtraction of the ''doubly collinear'' graph (see Fig. 2) is required within the method of [14]. The result for this is given by convoluting the d-dimensional leading order splitting function standing for the upper part with the four-dimensional one representing the lower part of the diagram and including a factor $(1-x)^{-\epsilon}$ from phase space in the convolution. In $4-2\epsilon$ dimensions the polarized LO splitting functions read, for $x\neq 1$ in the HVBM scheme [35],

$$\Delta P_{qq}^{(0)}(x,\epsilon)=C_F\left(\frac{1+x^2}{1-x}+3\epsilon(1-x)\right),$$

$$\Delta P_{qg}^{(0)}(x,\epsilon)=2T_R N_f[2x-1-2\epsilon(1-x)],$$

$$\Delta P_{gq}^{(0)}(x,\epsilon)=C_F[2-x+2\epsilon(1-x)],$$

$$\Delta P_{gg}^{(0)}(x,\epsilon)=2C_A\left(\frac{1}{1-x}-2x+1+2\epsilon(1-x)\right), \quad (20)$$

where $C_F=4/3$, $C_A=3$, $T_R=1/2$, and N_f is the number of active flavors. Equation (19) is needed to derive these results.

IV. RESULTS

In the normalization of [14,15] our modified minimal subtraction scheme ($\overline{\mathrm{MS}}$) results read

$$\Delta P_{qq}^{\pm,(1)}(x)=P_{qq}^{\mp,(1)}(x)-2\beta_0 C_F(1-x), \quad (21)$$

$$\Delta P_{qq}^{S,(1)}(x)=\Delta\widetilde{P}_{qq}^{S,(1)}(x),$$

$$\Delta P_{qg}^{(1)}(x)=\Delta\widetilde{P}_{qg}^{(1)}(x)+4C_F(1-x)\otimes\Delta P_{qg}^{(0)}(x),$$

$$\Delta P_{gq}^{(1)}(x)=\Delta\widetilde{P}_{gq}^{(1)}(x)-4C_F(1-x)\otimes\Delta P_{gq}^{(0)}(x),$$

$$\Delta P_{gg}^{(1)}(x)=\Delta\widetilde{P}_{gg}^{(1)}(x), \quad (22)$$

where $\beta_0=11C_A/3-4T_R N_f/3$ and $\Delta P_{ij}^{(0)}(x)\equiv\Delta P_{ij}^{(0)}(x,0)$ [see Eq. (20)]. The $\Delta\widetilde{P}_{ij}^{(1)}$ [36] are the results of [8], and the unpolarized $P_{qq}^{\pm,(1)}$ can be found in [14]. As was already discussed in [11,37] and indicated in Eq. (21), the ''+'' and ''−'' combinations of the NS splitting functions as defined in Eq. (12) interchange their role in the polarized case, such that, according to Eqs. (8), (12), and (21), the combination $\Delta P_{qq}^{+,(1)}=P_{qq}^{-,(1)}-2\beta_0 C_F(1-x)$ would govern the Q^2 evolution of, e.g., the polarized NS quark combination

$$\Delta A_3(x,Q^2)=(\Delta u+\Delta\overline{u}-\Delta d-\Delta\overline{d})(x,Q^2).$$

Since the first moment (x integral) of the latter corresponds to the nucleon matrix element of the NS axial vector current $\overline{q}\gamma^{\mu}\gamma_5\lambda_3 q$ which is conserved, it has to be Q^2 independent [38]. Keeping in mind that the first moment of the unpolarized $P_{qq}^{-,(1)}$ vanishes already due to fermion number conservation [14], it becomes obvious that the additional term $-2\beta_0 C_F(1-x)$ in Eq. (21) spoils the Q^2 independence of the first moment of $\Delta A_3(x,Q^2)$. It is therefore necessary to perform a factorization scheme transformation to the results in Eqs. (21) and (22) in order to remove this additional term which, as pointed out in [39,35,37], is typical of the HVBM scheme with its not fully anticommuting γ_5 and trivially would not be present in a scheme with a fully anticommuting γ_5 since then the two γ_5 matrices appearing in the relevant graphs could be removed by anticommuting them towards each other and using $\gamma_5^2=1$ [cf. Fig. 2(a)]. We note that all these observations were already made in the original calculation of [8] in the OPE where, however, the removal of the additional term $\sim(1-x)$ corresponds to a finite renormalization rather than a factorization scheme transformation. It is nice to recover this analogy between our results and those of [8,40]. The factorization scheme transformation for $\Delta P_{qq}^{\pm,(1)}$ also affects the singlet sector since, according to Eq. (10), $\Delta P_{qq}^{+,(1)}+\Delta P_{qq}^{S,(1)}$ occurs in the evolution of the NLO quark singlet $\Delta\Sigma$. The transformation reads, in general (see, e.g., [8,41]),

$$\Delta P_{qq}^{\pm,(1)}=\Delta\hat{P}_{qq}^{\pm,(1)}-2\beta_0 z_{qq},$$

$$\Delta P_{qq}^{S,(1)}=\Delta\hat{P}_{qq}^{S,(1)},$$

$$\Delta P_{qg}^{(1)}=\Delta\hat{P}_{qg}^{(1)}+4z_{qq}\otimes\Delta P_{qg}^{(0)},$$

$$\Delta P_{gq}^{(1)}=\Delta\hat{P}_{gq}^{(1)}-4z_{qq}\otimes\Delta P_{gq}^{(0)},$$

$$\Delta P_{gg}^{(1)}=\Delta\hat{P}_{gg}^{(1)}, \quad (23)$$

where the $\Delta\hat{P}_{ij}^{(1)}$ now are the NLO splitting functions on the left-hand sides of Eqs. (21) and (22) and the $\Delta P_{ij}^{(1)}$ are the *new* splitting functions after the scheme transformation. One immediately sees that the choice

$$z_{qq}=-C_F(1-x) \quad (24)$$

leads to $\Delta P_{qq}^{\pm,(1)}=P_{qq}^{\mp,(1)}$ and thus now yields the required vanishing of the first moment of $\Delta P_{qq}^{+,(1)}$. Even more, the transformation (23) and (24) removes *all* additional terms on the right-hand sides of Eqs. (21) and (22) simultaneously, bringing our final result into complete agreement with the revised one of [8]. We finally note that the above factorization scheme transformation also changes the quark short-distance cross section (coefficient function) ΔC_q in Eq. (7), since the combination

$$\Delta C_q-\frac{2\Delta P_{qq}^{\pm,(1)}}{\beta_0}$$

must be independent of the choice of the factorization scheme convention [41]. As was shown in [39,42,37], only *after* the transformation (23) and (24) does ΔC_q in the $\overline{\rm MS}$ scheme take the form

$$\Delta C_q(x)=C_F\left[(1+x^2)\left(\frac{\ln(1-x)}{1-x}\right)_+-\frac{3}{2}\frac{1}{(1-x)_+}-\frac{1+x^2}{1-x}\ln x+2+x-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right], \qquad (25)$$

i.e., does it become the previously calculated [43] $O(\alpha_s)$ quark correction to g_1 giving rise to, e.g., the correct first order correction $(1-\alpha_s/\pi)$ to the Bjørken sum rule. In Eq. (25),

$$\int_0^1 dz f(z)[g(z)]_+\equiv\int_0^1 dz[f(z)-f(1)]g(z).$$

For completeness we note that the NLO $\overline{\rm MS}$ gluonic short-distance cross section ΔC_g in Eq. (7) remains unaffected by the transformation (23) and (24) and reads (see, e.g., [8])

$$\Delta C_g(x)=2T_RN_f\left[(2x-1)\left(\ln\frac{1-x}{x}-1\right)+2(1-x)\right]. \qquad (26)$$

Our *final* results for the polarized $\overline{\rm MS}$ NLO splitting functions are given by

$$\Delta P_{qq}^{\pm,(1)}(x)=P_{qq}^{\mp,(1)}(x), \qquad (27)$$

$$\Delta P_{qq}^{S,(1)}(x)=2C_FT_RN_f[(1-x)-(1-3x)\ln x-(1+x)\ln^2 x], \qquad (28)$$

$$\begin{aligned}\Delta P_{qg}^{(1)}(x)=&C_FT_RN_f\{-22+27x-9\ln x+8(1-x)\ln(1-x)+\delta p_{qg}(x)[2\ln^2(1-x)-4\ln(1-x)\ln x+\ln^2x-4\zeta(2)]\}\\&+C_AT_RN_f\{2(12-11x)-8(1-x)\ln(1-x)+2(1+8x)\ln x-2[\ln^2(1-x)-\zeta(2)]\delta p_{qg}(x)\\&-[2S_2(x)-3\ln^2x]\delta p_{qg}(-x)\},\end{aligned} \qquad (29)$$

$$\begin{aligned}\Delta P_{gq}^{(1)}(x)=&C_FT_RN_f[-\tfrac{4}{9}(x+4)-\tfrac{4}{3}\delta p_{gq}(x)\ln(1-x)]+C_F^2\{-\tfrac{1}{2}-\tfrac{1}{2}(4-x)\ln x-\delta p_{gq}(-x)\ln(1-x)+[-4-\ln^2(1-x)\\&+\tfrac{1}{2}\ln^2x]\delta p_{gq}(x)\}+C_AC_F\{(4-13x)\ln x+\tfrac{1}{3}(10+x)\ln(1-x)+\tfrac{1}{9}(41+35x)+\tfrac{1}{2}[-2S_2(x)+3\ln^2x]\delta p_{gq}(-x)\\&+[\ln^2(1-x)-2\ln(1-x)\ln x-\zeta(2)]\delta p_{gq}(x)\},\end{aligned} \qquad (30)$$

$$\begin{aligned}\Delta P_{gg}^{(1)}(x)=&-C_AT_RN_f[4(1-x)+\tfrac{4}{3}(1+x)\ln x+\tfrac{20}{9}\delta p_{gg}(x)+\tfrac{4}{3}\delta(1-x)]-C_FT_RN_f[10(1-x)+2(5-x)\ln x+2(1+x)\ln^2x\\&+\delta(1-x)]+C_A^2\{\tfrac{1}{3}(29-67x)\ln x-\tfrac{19}{2}(1-x)+4(1+x)\ln^2x-2S_2(x)\delta p_{gg}(-x)+[\tfrac{67}{9}-4\ln(1-x)\ln x+\ln^2x\\&-2\zeta(2)]\delta p_{gg}(x)+[3\zeta(3)+\tfrac{8}{3}]\delta(1-x)\},\end{aligned} \qquad (31)$$

where, as mentioned above, the unpolarized NS pieces $P_{qq}^{\pm,(1)}$ can be found in [14]. We have defined [44]:

$$\delta p_{qg}(x)\equiv 2x-1,$$

$$\delta p_{gq}(x)\equiv 2-x,$$

$$\delta p_{gg}(x)\equiv\frac{1}{(1-x)_+}-2x+1. \qquad (32)$$

Furthermore, we have, in Eqs. (27)–(31), $\zeta(2)=\pi^2/6$, $\zeta(3)\approx 1.202\,057$, and

$$S_2(x)\equiv\int_{x/(1+x)}^{1/(1+x)}\frac{dz}{z}\ln\left(\frac{1-z}{z}\right).$$

For relating our results to those of [8] the relation

$$S_2(x)=-2\mathrm{Li}_2(-x)-2\ln x\ln(1+x)+\tfrac{1}{2}\ln^2x-\zeta(2)$$

is needed, where $\mathrm{Li}_2(x)$ is the dilogarithm [45]. In Eq. (31), the contributions $\sim\delta(1-x)$ to $\Delta P_{gg}^{(1)}$ are the same as those for the unpolarized $P_{gg}^{(1)}$ [31]; they lead to satisfaction of the constraint [46]

$$\int_0^1 dx\Delta P_{gg}^{(1)}(x)=\frac{\beta_1}{4}\equiv\tfrac{17}{6}C_A^2-C_FT_RN_f-\tfrac{5}{3}C_AT_RN_f,$$

valid in the $\overline{\rm MS}$ scheme.

In conclusion, our calculation, which was based on the approach of [16,14] and on using the HVBM [20] prescription for γ_5, has confirmed the recent results of [8] for the spin-dependent two-loop splitting functions $\Delta P_{ij}^{(1)}(x)$. Our results also once more demonstrate the usefulness and applicability of the method of [14] and the lightlike axial gauge in perturbative QCD calculations.

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