

## Polarized structure function $g_2$ in the c.m. bag model

X. Song\*

*Institute of Nuclear and Particle Physics, Department of Physics, University of Virginia, Charlottesville, Virginia 22901*

(Received 1 November 1995)

The spin-dependent structure functions  $g_1(x)$ ,  $g_2(x)$ ,  $g_2^{WW}(x)$ , and  $\bar{g}_2(x)$  and their moments are studied in the c.m. bag model. The results show the following: (i)  $\int_0^1 g_2(x) dx = 0$ , i.e., the Burkhardt-Cottingham sum rule holds; hence,  $g_2(x)$  must have at least one nontrivial zero in addition to  $x=0$  and  $x=1$ . (ii)  $\int_0^1 x^2 g_2(x) dx$  is negative for the proton, neutron, and deuteron. (iii)  $\int_0^1 x^2 g_2(x) dx$  is about one order of magnitude smaller than  $\int_0^1 x^2 g_1(x) dx$ ; hence, the twist-3 matrix element is approximately equal to the twist-2 matrix element. The results are compared with most recent data and predictions from the MIT bag model, lattice QCD, and QCD sum rules. [S0556-2821(96)03113-X]

PACS number(s): 13.60.Hb; 12.39.Ba

### I. INTRODUCTION

In recent years, the investigation of the spin structure of the nucleon by using deep inelastic lepton-hadron scattering (DIS) experiments has been an exciting and controversial field in hadron physics. The experiments performed by the European Muon Collaboration (EMC), and Spin Muon Collaboration (SMC) at CERN [1,2], and E142, E143 at SLAC [3–5] provide direct information on the matrix elements of spin-dependent operators in the nucleon. The spin-dependent DIS cross section is determined by the antisymmetric part of the hadronic tensor:

$$W_{\mu\nu}^A = i \epsilon_{\mu\nu\sigma\rho} \frac{q^\rho}{\nu} \left\{ S^\sigma g_1(x, Q^2) + \left[ S^\sigma - P^\sigma \frac{S \cdot q}{P \cdot q} \right] g_2(x, Q^2) \right\} \quad (1)$$

where  $P$  and  $q$  are the four-vectors of the nucleon and virtual photon momenta.  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  are two spin-dependent structure functions for the Bjorken variable  $x = Q^2/2P \cdot q \equiv Q^2/2M\nu$  with  $Q^2 = -q^2$  is the transferred four-momentum squared and  $S^\sigma = \bar{U}(P, S) \gamma^\sigma \gamma_5 U(P, S)$  is the covariant spin vector of the nucleon.

According to operator product expansion (OPE) analysis, to order  $M^2/Q^2$ , the lowest two moments of  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  can be written (QCD radiative corrections are not included) [6–10]

$$\int_0^1 g_1(x, Q^2) dx = \frac{1}{2} a^{(0)}(Q^2) + \frac{M^2}{9Q^2} [a^{(2)}(Q^2) + 4d^{(2)}(Q^2) + 4f^{(2)}(Q^2)], \quad (2a)$$

$$\int_0^1 g_2(x, Q^2) dx = 0, \quad (2b)$$

$$\int_0^1 x^2 g_1(x, Q^2) dx = \frac{1}{2} a^{(2)}(Q^2) + O\left(\frac{M^2}{Q^2}\right), \quad (2c)$$

$$\int_0^1 x^2 g_2(x, Q^2) dx = -\frac{1}{3} a^{(2)}(Q^2) + \frac{1}{3} d^{(2)}(Q^2) + O\left(\frac{M^2}{Q^2}\right), \quad (2d)$$

where  $a^{(0,2)}(Q^2)$ ,  $d^{(0,2)}(Q^2)$  and  $f^{(2)}(Q^2)$  depend on the nucleon forward matrix elements of twist-2, twist-3, and twist-4 operators, respectively. For example,

$$a^{(0)} = \sum_{q_f=u,d,s,\dots} e_f^2 \Delta q_f, \quad \langle P, S | \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f | P, S \rangle \equiv 2 \Delta q_f S_\mu. \quad (3)$$

$\Delta q_f$  ( $q_f = u, d, s, \dots$ ) are axial charges defined by the above axial-vectorial matrix elements. The singlet axial charge is proportional to the total quark helicity  $\Delta \Sigma = \Delta u + \Delta d + \Delta s$  in the nucleon. Using the semileptonic weak decay data, which are related to nonsinglet axial charges, the 1988 EMC result seemed to indicate that  $\Delta \Sigma$  is surprisingly small and led to the so-called ‘‘spin crisis.’’ Since then, an intensive study of  $g_1$  has been conducted. The efforts both from experimental and theoretical works led to a deeper understanding of internal spin structure of the nucleon, although many questions remain. The most recent reviews on this subject can be found in [11–13]. Neglecting the terms of order  $M^2/Q^2$  in Eqs. (2a)–(2d), the longitudinal polarized structure function  $g_1(x, Q^2)$  receives only twist-2 contributions. On the other hand, the structure function  $g_2(x, Q^2)$  contains not only twist-2 but also twist-3 contributions corresponding to the matrix element  $d^{(2)}(Q^2)$ . The twist-3 contributions coming from spin-dependent quark gluon correlations do not vanish even in the large  $Q^2$  limit [7,14–16].

To separate the twist-2 and twist-3 contributions of  $g_2(x, Q^2)$ , one can write (Wandzura-Wilczek [17])

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2), \quad (4)$$

\*Electronic address: xs3e@virginia.edu

where the twist-2 piece (see, however, discussion in Sec. III) is

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \quad (5a)$$

and the twist-3 piece is

$$\bar{g}_2(x, Q^2) = g_1(x, Q^2) + g_2(x, Q^2) - \int_x^1 \frac{dy}{y} g_1(y, Q^2). \quad (5b)$$

It is easy to check that, to order  $O(M^2/Q^2)$ , we have

$$\int_0^1 g_2^{WW}(x, Q^2) dx = 0, \quad \int_0^1 x^2 g_2^{WW}(x, Q^2) dx = -\frac{1}{3} a^{(2)}, \quad (6a)$$

$$\int_0^1 \bar{g}_2(x, Q^2) dx = 0, \quad \int_0^1 x^2 \bar{g}_2(x, Q^2) dx = \frac{1}{3} d^{(2)}; \quad (6b)$$

hence, the twist-2 and twist-3 contributions of  $g_2(x, Q^2)$  are separated. Measuring  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  as functions of  $x$  and  $Q^2$ , one can obtain  $g_2^{WW}(x, Q^2)$ ,  $\bar{g}_2(x, Q^2)$ , and their third and even higher moments. Up to order  $M^2/Q^2$ ,  $g_2(x, Q^2)$  uniquely measures the twist-3 contributions without involving any model-dependent analysis. Hence,  $g_2$  provides more detailed information about nucleon structure than does  $g_1$ . Recently, a preliminary experimental result of  $g_2$  has been published [18] and more precise data obtained by E143 experiment [19] at SLAC have been published very recently. Since  $g_2$  was discussed only briefly in a previous paper [20], it is appropriate to give a detailed analysis in the modified center-of-mass (c.m.) bag model, using newly obtained data on  $g_2$ . In Sec. II, the c.m. bag model is briefly reviewed and the results for  $g_2(x)$ ,  $g_2^{WW}(x)$ , and  $\bar{g}_2(x)$ , and their moments  $a^{(0,2)}(Q^2)$ ,  $d^{(0,2)}(Q^2)$  are presented. Discussion and comparison with most recent data and other model results are given in Sec. III. A brief summary is given in Sec. IV.

## II. CENTER-OF-MASS BAG MODEL

As mentioned in a previous paper [20], the bag model does not contain gluon fields explicitly, but the boundary of the bag-confined quarks simulates the binding effect coming from quark-gluon and gluon-gluon interactions (the gluon contribution to the proton spin in the bag model has been discussed by Jaffe [21] recently). Hence, the structure function  $g_2$  calculated in the bag model does include higher twist effects. We have reported the c.m. bag model results [20] for the unpolarized and polarized structure functions  $F_1$ ,  $F_2$ , and  $g_1$ , and briefly for  $g_2$ . In this paper, we will focus our attention on  $g_2$ . For reader's convenience, we give a brief review for c.m. bag model calculation of the structure functions. One needs to calculate the hadron tensor

$$W_{\mu\nu}(P, q, S) = \frac{1}{4\pi} \int d^4y e^{iqy} \langle P, S | [J_\mu(y), J_\nu(0)] | P, S \rangle$$

and separate the antisymmetric part to obtain  $W_{\mu\nu}^A$ , which can be expressed, in general, as  $W_{\mu\nu}^A = i \epsilon_{\mu\nu\sigma\rho} (q^\rho/\nu) I^\sigma(x, Q^2)$  in the Bjorken limit, where  $I^\sigma(x, Q^2)$  depends on the model of the nucleon and the approximations used in the calculation.

The basic assumptions and approximations of the c.m. bag model are as follows. (i) The nucleon electromagnetic current  $J_\mu = \gamma NN$  vertex, see (2.7) in [20] — can be approximately expressed by incoherent sum of single quark electromagnetic currents. It implies that the virtual photon interacts with only one quark at a time and the other two quarks are spectators; this is an impulse approximation. The current includes not only the contribution of the struck quark but also those of the spectator quarks. Since the current satisfies translational invariance, four-momentum is conserved. (ii) The nucleon consists of three-valence quarks in their  $S$ -wave state; higher excited states and higher Fock states which include gluons and sea quark pairs in addition to three-valence quarks are neglected. (iii)  $SU(3)_{\text{flavor}} \otimes SU(2)_{\text{spin}}$  wave functions for the proton and neutron are used. Symmetry-breaking effect is described in terms of a parameter  $\xi \equiv R_d^P/R_d^N < 1$ , which simulates the smaller spatial size for the scalar  $u$ - $d$  quark pair than that for the vector  $u$ - $u$  and  $d$ - $d$  quark pairs in the nucleon [22]. (iv) The effect of quark confinement due to nonperturbative quark-gluon and gluon-gluon interactions is described in terms of bound-state quark spatial wave functions, for instance the quark bag wave function in the cavity approximation in the MIT bag model or Gaussian-type quark wave function used in some other models. All necessary formulas for the c.m. bag model calculation can be found in [20]. A formal and general discussion on the theoretical basis of the c.m. bag model has been given in [23]. We note that in addition to the c.m. bag model calculation, the transverse spin structure functions  $g_2$  has also been computed in the original MIT bag model by Jaffe and Ji [8], and other modified versions of the MIT bag model by Schreiber, Signal, and Thomas (SST bag model [24]), and by Stratmann (MOD model [25]).

Experimentally,  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  are measured by combining two different cases of deep inelastic scattering of polarized leptons on polarized nucleons: (i) the beam and target spin orientations are parallel, and (ii) the beam and target spin orientations are perpendicular. Theoretically,  $W_{\mu\nu}^A = i \epsilon_{\mu\nu\sigma\rho} (q^\rho/\nu) I^\sigma(x, Q^2)$  can be calculated from various models of the nucleon. In this case it is convenient to choose suitable projection operators to extract  $g_1$  and  $g_2$  from model results of  $I^\sigma(x, Q^2)$ . One of possible projections is to extract  $g_1 \equiv g_L$  and  $g_1 + g_2 \equiv g_T$  by choosing the nucleon spin parallel ( $L$ ) or perpendicular ( $T$ ) to the virtual photon momentum as we did in [20]. It should be noted that it is not necessary to choose the same projection as that used in the experimental analysis.

Several parameters have been used in the c.m. bag model calculation. They are (a) ‘‘bag’’ radius  $R = 5 \text{ GeV}^{-1}$ , (b)  $SU(3)$  symmetry-breaking parameter  $\xi = 0.85$ , and (c) maximum momentum of quarks inside the nucleon  $|\mathbf{p}_{\text{max}}| = 0.6 \text{ GeV}/c$ .  $R$  and  $\xi$  were determined from the fit of the rms radius of the neutron and proton, and the ratio  $\mu_n/\mu_p$  [22]. The model with these two parameters gives a fairly good result for the electromagnetic form factors of the nucleon and the magnetic moments of octet baryons. In particular, the

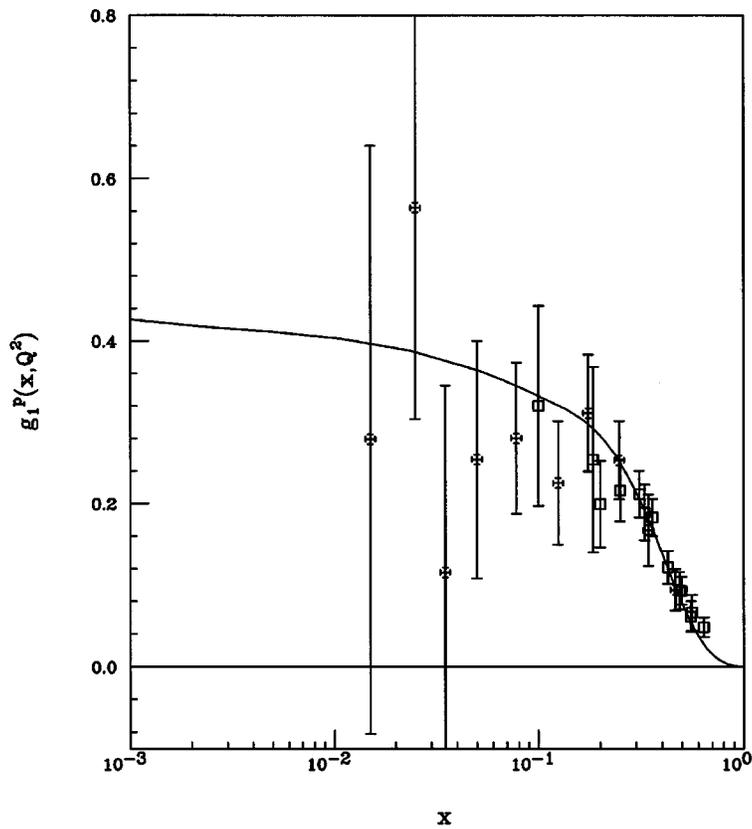


FIG. 1. c.m. bag model result [20] for  $g_1^p(x, Q^2)$  at  $Q^2=5.0$  (GeV/c) $^2$ , data from [1,2,4,34].

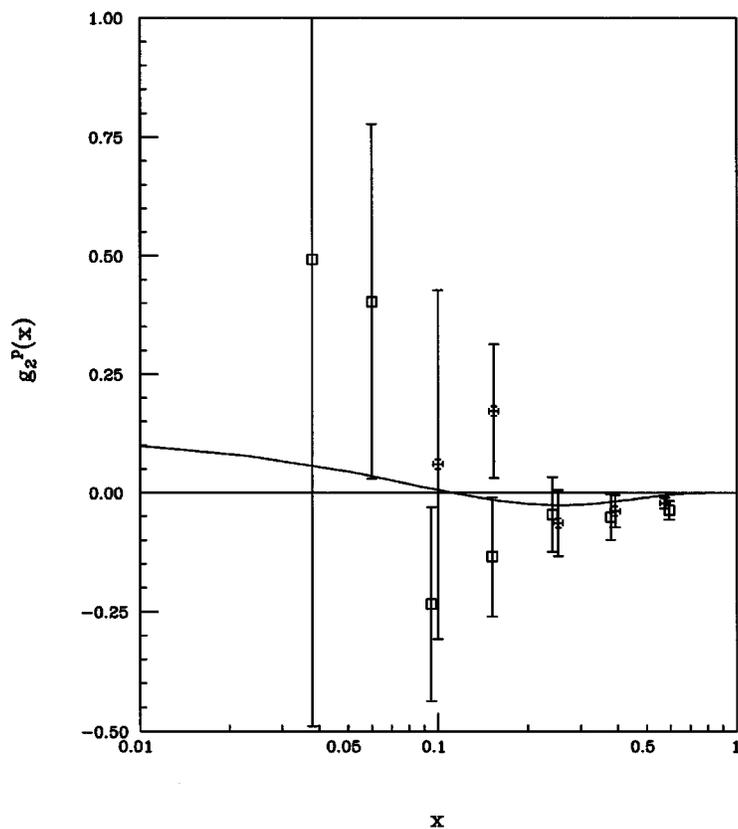


FIG. 2. c.m. bag model prediction for  $g_2^p(x, Q^2)$  at  $Q^2=5.0$  (GeV/c) $^2$ , data from [19].

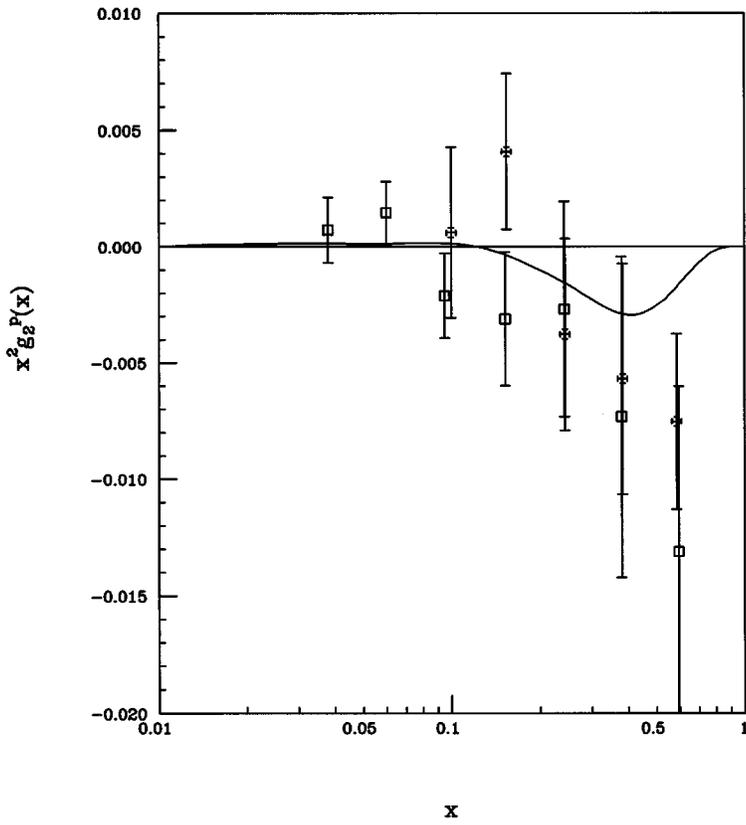


FIG. 3. Same as Fig. 2, but for  $x^2 g_2^p(x, Q^2)$  at  $Q^2 = 5.0$  (GeV/c)<sup>2</sup>, data from [19].

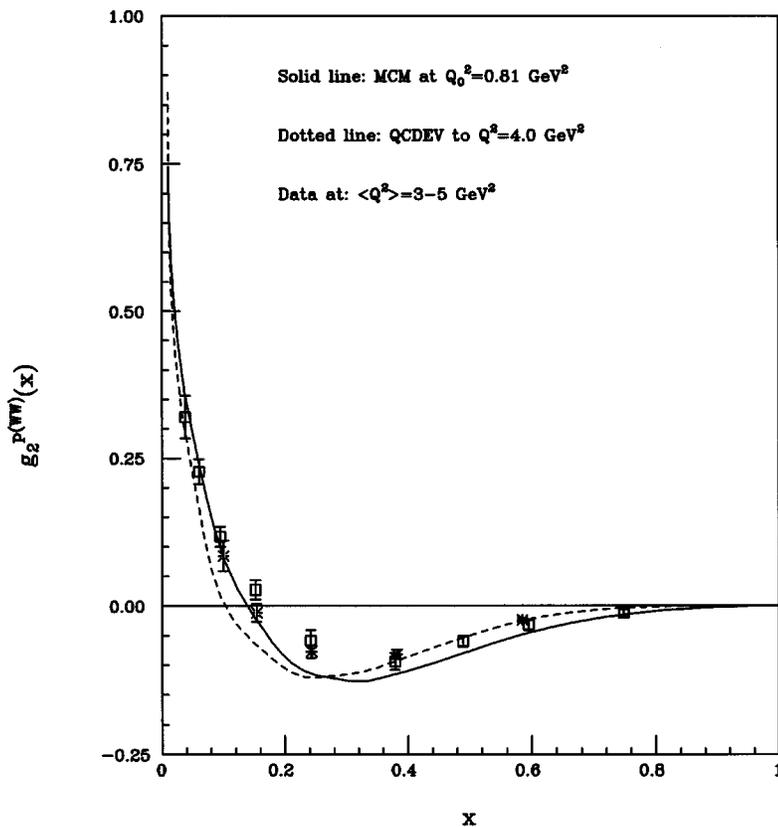


FIG. 4. Evolution of twist-2 piece  $g_2^{p(WW)}(x, Q^2)$  in the c.m. bag model at  $Q^2 = 0.81$  and  $4.0$  (GeV/c)<sup>2</sup>, data from [19].

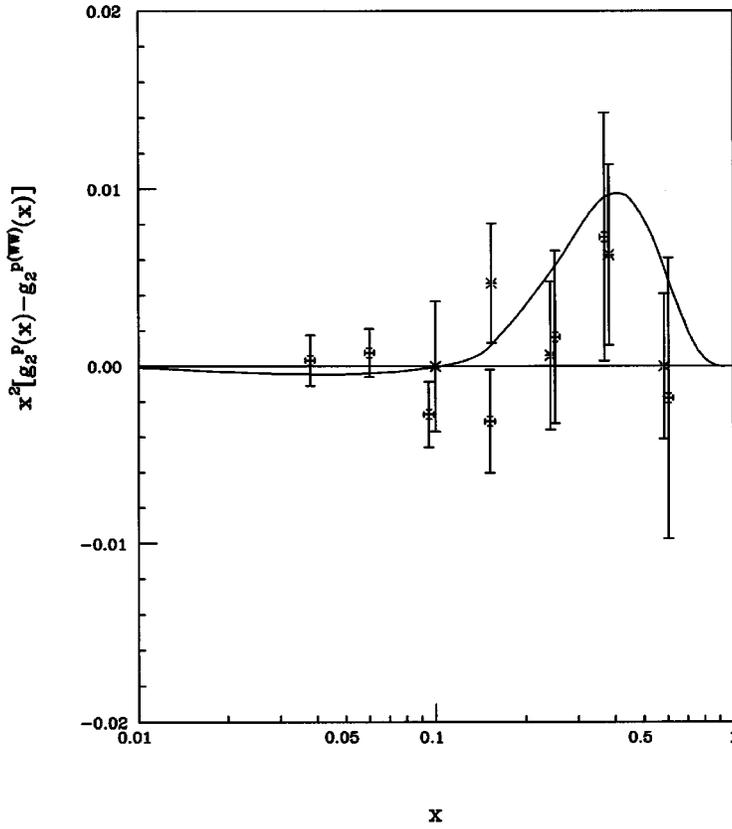


FIG. 5. The twist-3 contribution  $x^2 \bar{g}_2(x, Q^2)$  calculated in the c.m. bag model and evolved to  $Q^2 = 5.0 \text{ (GeV}/c^2)^2$ , data from [19].

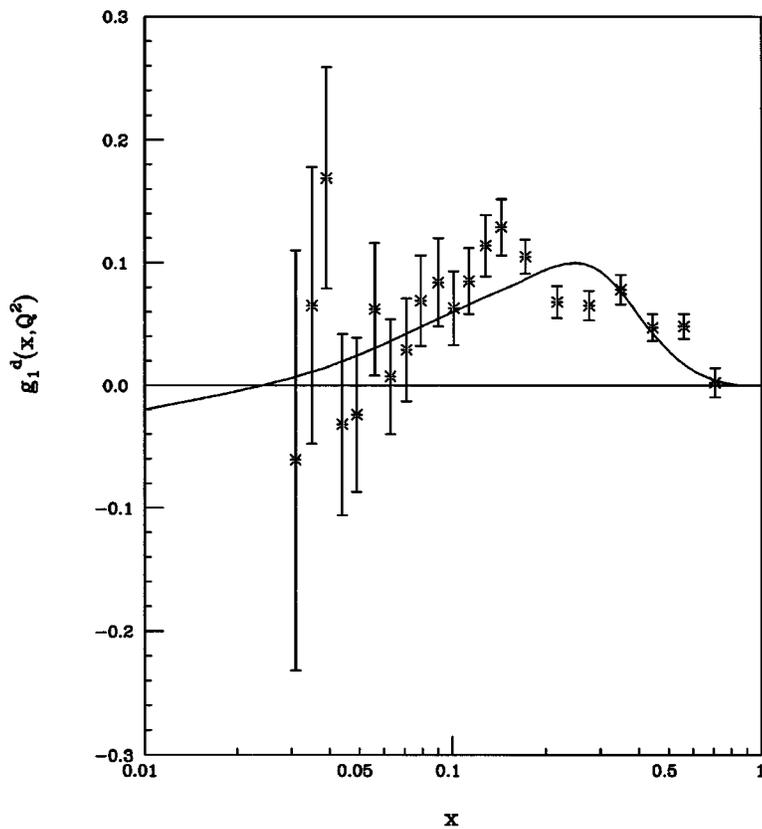


FIG. 6. The deuteron structure function  $g_1^d(x, Q^2)$  calculated in the c.m. bag model and evolved to  $Q^2 = 3.0 \text{ (GeV}/c^2)^2$ , data from [34].

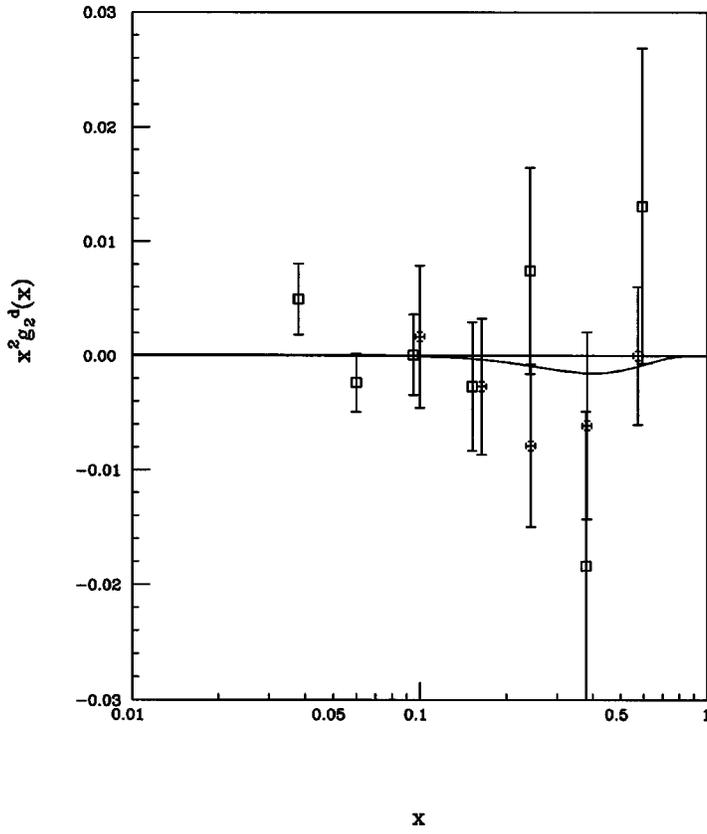


FIG. 7. The deuteron transverse structure function  $x^2 g_2^d(x, Q^2)$  calculated in the c.m. bag model and evolved to  $Q^2=5.0 \text{ (GeV/c)}^2$ , data from [19].

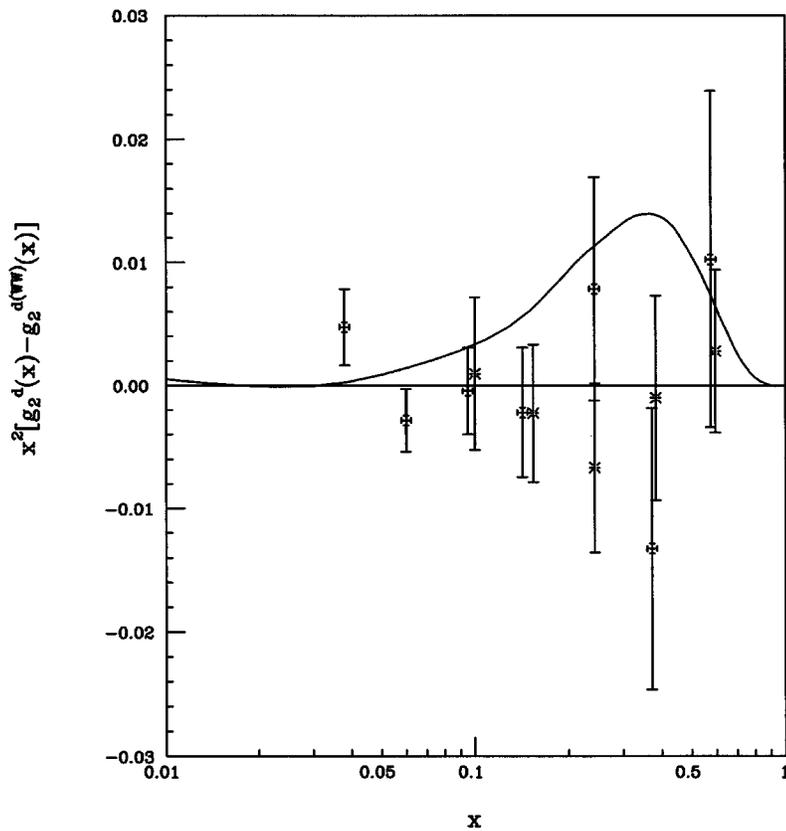


FIG. 8. Same as Fig. 7 but for deuteron twist-3 piece  $x^2 \bar{g}_2^d(x, Q^2)$ , data from [19].

neutron charge form factor is well reproduced. For the DIS parton distributions, the third parameter  $|\mathbf{p}_{\max}|=0.6$  GeV/ $c$  has been introduced. It constrains the unpolarized valence quark distributions to satisfy the sum rules

$$\int_0^1 u_v(x)dx=2, \quad \int_0^1 d_v(x)dx=1. \quad (7)$$

As mentioned in [20], to compare the model results with data, the QCD evolution technique has to be used to evolve the parton distributions from the renormalization scale  $Q_0^2$  to higher  $Q^2$  where the experiments are performed. We choose  $Q_0^2=0.81$  (GeV/ $c$ )<sup>2</sup> and QCD scale parameter  $\Lambda=0.3$  (GeV/ $c$ ). The c.m. bag model with these parameters gives a good description for both unpolarized and polarized structure functions at  $0.3 < x < 1$ , where the valence quark contributions dominate. For small- $x$  region,  $0 < x < 0.3$ , the sea quark contributions are necessary. Using some QCD-inspired phenomenological sea distributions [see (4.19) in [20]], we found that the first moment of  $g_1^p(x, Q^2)$  is consistent with the experimental value and the first moment of  $g_1^p(x, Q^2) - g_1^n(x, Q^2)$  satisfies the Bjorken sum rule. For the higher moments ( $n \geq 2$ ), the sea contributions coming mainly from the small  $x$  region are highly suppressed by the factor  $x^n$  and are thus less important. Hence, we neglect possible sea contributions for the higher moments of  $g_2$ ,  $g_2^{WW}$ , and  $\bar{g}_2$ .

For QCD evolution of the structure function  $g_1$  and twist-2 piece  $g_2^{WW}$  from  $Q_0^2$  to  $Q^2 > Q_0^2$ , the ordinary Altarelli-Parisi equations [26] can be used. For twist-3 part of  $g_2$ , however, it has been shown [27] that due to mixing of twist-3 quark operators and quark-gluon operators with same twist and quantum numbers, the number of independent operators contributing to  $\bar{g}_2$  increases with  $n$ , where  $n$  refers to the  $n$ th moment. It implies that one cannot write down an Altarelli-Parisi-type evolution equation for  $g_2$ . This feature has been confirmed by several later calculations in [28–32] and most recently in [33]. Hence, there is no simple evolution equation for  $g_2$  in the general case. One has to look for some approximate solutions. Two approximate evolution approaches under the limits  $N_c \rightarrow \infty$  or power  $n \rightarrow \infty$  were suggested by Ali, Braun, and Hiller [32]. We use the approach in the large  $N_c$  limit rather than the approximation in the large  $n$  limit, which only provides the asymptotic behavior of  $g_2(x, Q^2)$  in the region  $x \rightarrow 1$ . We also note that as far as the third moments  $\int_0^1 x^2 g_2(x, Q^2) dx$  are concerned, the  $Q^2$  evolution is straightforward, i.e., a single power behavior of  $\ln Q^2$  (for instance, see [33]).

In Figs. 1–5, we present the results of  $g_1^p(x, Q^2)$ ,  $g_2^p(x, Q^2)$ ,  $x^2 g_2^p(x, Q^2)$ ,  $g_2^{pWW}(x, Q^2)$  [which is determined by  $g_1^p(x, Q^2)$ ], and  $x^2 \bar{g}_2^p(x, Q^2)$ , respectively. For the deuteron target,  $g_1^d(x, Q^2)$ ,  $x^2 g_2^d(x, Q^2)$ , and  $x^2 \bar{g}_2^d(x, Q^2)$  are shown in Figs. 6–8. All theoretical curves are calculated in the c.m. bag model at  $Q_0^2=0.81$  (GeV/ $c$ )<sup>2</sup> and evolved to  $Q^2=5.0$  (GeV/ $c$ )<sup>2</sup> except for Figs. 4 and 6, where  $Q^2=4.0$  (GeV/ $c$ )<sup>2</sup> and  $Q^2=3.0$  (GeV/ $c$ )<sup>2</sup>, respectively. Comparisons of our results for the third moments of  $g_1$  and  $g_2$  with recent data and other model predictions are listed in Tables I–III. The data for  $g_1$  are taken from [1,2,4,34] and those for  $g_2$  are taken from [19].

### III. RESULTS AND DISCUSSION

(1) For the leading term of first moments of  $g_1(x)$ , we obtain

$$a_{\text{proton}}^{(0)}=0.252, \quad a_{\text{neutron}}^{(0)}=-0.112, \quad a_{\text{deuteron}}^{(0)}=0.064, \quad (8)$$

which can be compared with SMC data [2] at  $\langle Q^2 \rangle=5$  (GeV/ $c$ )<sup>2</sup>:  $a_{\text{proton}}^{(0)}=0.252 \pm 0.036$ ,  $a_{\text{neutron}}^{(0)}=-0.056 \pm 0.024$ ,  $a_{\text{deuteron}}^{(0)}=0.046 \pm 0.050$  and E143 data [4,5] at  $\langle Q^2 \rangle=3$  (GeV/ $c$ )<sup>2</sup>:  $a_{\text{proton}}^{(0)}=0.254 \pm 0.022$ ,  $a_{\text{neutron}}^{(0)}=-0.074 \pm 0.027$ ,  $a_{\text{deuteron}}^{(0)}=0.084 \pm 0.010$ .

(2) For the leading term of third moments of  $g_1(x)$ , we get

$$a_{\text{proton}}^{(2)}=2.10 \times 10^{-2}, \quad a_{\text{neutron}}^{(2)}=-1.86 \times 10^{-3},$$

$$a_{\text{deuteron}}^{(2)}=8.74 \times 10^{-3}, \quad (9)$$

while preliminary data at  $\langle Q^2 \rangle=5$  (GeV/ $c$ )<sup>2</sup> [19] show:  $a_{\text{proton}}^{(2)}=(2.42 \pm 0.20) \times 10^{-2}$  and  $a_{\text{deuteron}}^{(2)}=(8.0 \pm 1.6) \times 10^{-3}$ . Comparisons with other models are listed in Tables I and III. We note that no  $g_2^n$  data is available yet.

(3) Since  $g_1^p(x, Q^2)$  is always positive in the range  $0 < x < 1$ , hence  $\int_0^1 x^2 g_1^p(x) dx$  must be positive. From Eq. (2c), the leading term of  $\int_0^1 x^2 g_1^p(x) dx$ , i.e.,  $a_{\text{proton}}^{(2)}$  must also be positive as shown in Eq. (9). However, the  $g_1^n(x)$  as function of  $x$  is mostly negative except for large  $x$  region, where  $g_1^n(x)$  is positive but very small. Hence, its third moment, or  $a_{\text{neutron}}^{(2)}$ , is negative. For the deuteron, since the positive contribution from the proton is larger than the negative contribution from the neutron in the  $\int_0^1 x^2 g_1^d(x) dx$ , hence our model predicts a positive  $a_{\text{deuteron}}^{(2)}$  as shown in Eq. (9) and Table III.

(4) As mentioned in Sec. I, in the OPE approach including twist-2 and twist-3 operators in the presence of QCD corrections, the Burkhardt-Cottingham [36] sum rule  $\int_0^1 g_2(x) dx=0$  is known to hold. The c.m. bag model predicts

$$\int_0^1 g_2^p(x) dx = -0.0016, \quad \int_0^1 g_2^n(x) dx = -0.0047 \quad (10a)$$

comparing to the numerical values for  $\int_0^1 g_1^{(p,n)} dx$  in Eq. (8), they are numerically consistent with zero. Hence, in our model, the Burkhardt-Cottingham sum rule is satisfied within numerical errors. Most recent data [19] give

$$\int_{0.03}^1 g_2^p(x) dx = -0.013 \pm 0.028,$$

$$\int_{0.03}^1 g_2^n(x) dx = -0.033 \pm 0.082 \quad (10b)$$

which are consistent with zero. It should be noted that in the MIT bag model with SU(6) symmetry,  $g_2^n(x)$  is identically

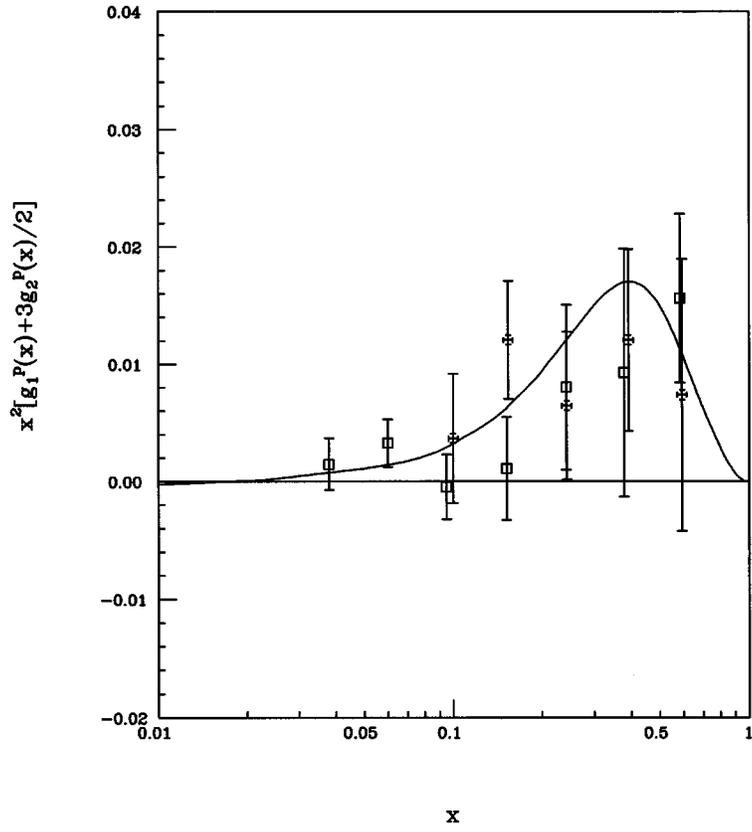


FIG. 9. c.m. bag model prediction for  $x^2[g_1^p(x, Q^2) + \frac{3}{2}g_2^p(x, Q^2)]$  at  $Q^2=5.0$   $(\text{GeV}/c)^2$ , data from [19].

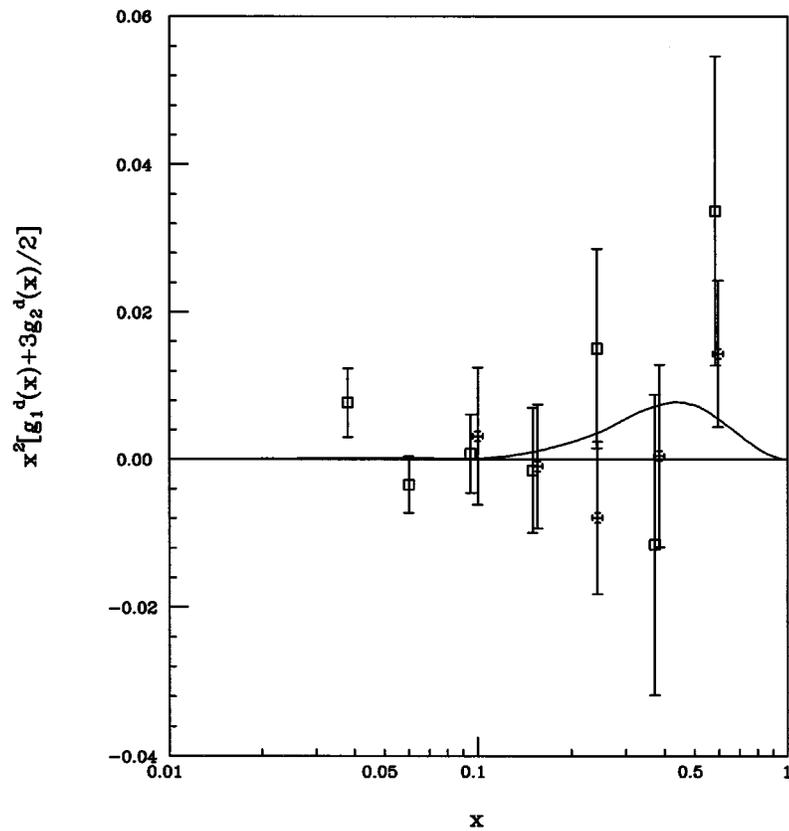


FIG. 10. Same as Fig. 9 but for  $x^2[g_1^d(x, Q^2) + \frac{3}{2}g_2^d(x, Q^2)]$  at  $Q^2=5.0$   $(\text{GeV}/c)^2$ .

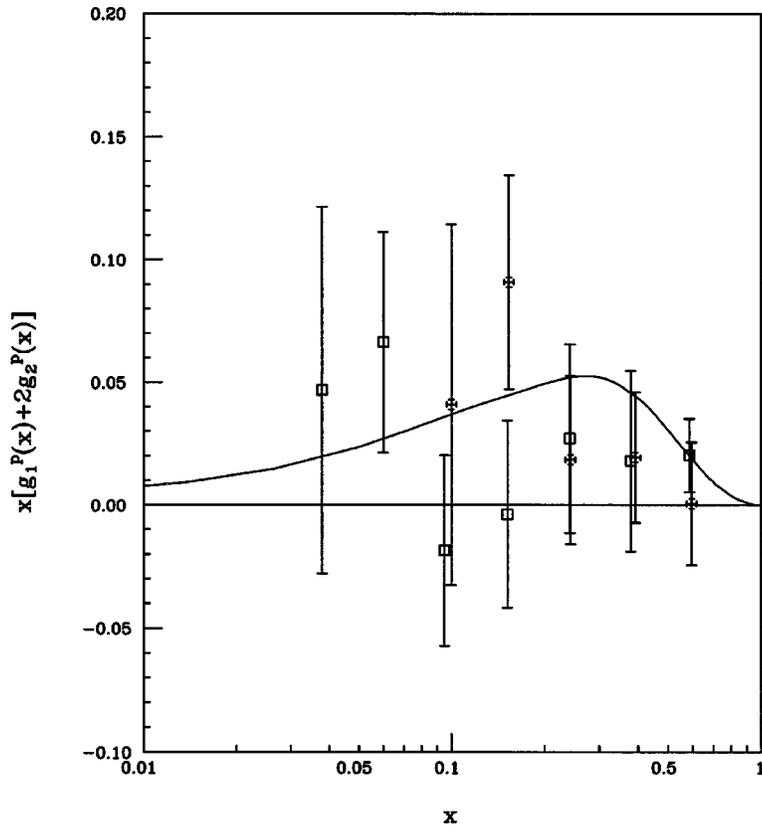


FIG. 11. c.m. bag model prediction for  $x[g_1^p(x, Q^2) + 2g_2^p(x, Q^2)]$  at  $Q^2 = 5.0 (\text{GeV}/c)^2$ , data from [19].

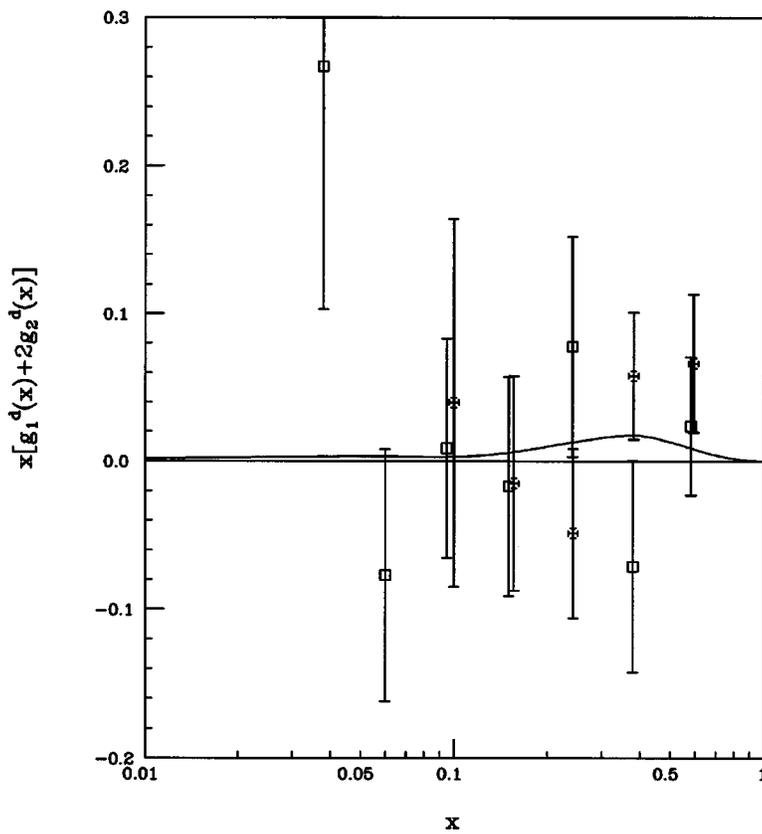


FIG. 12. Same as Fig. 11 but for the deuteron target, data from [19].

TABLE I. Comparison of model results with proton data.

Proton	$\int_0^1 x^2 g_1^p(x, Q^2) dx$ $= \frac{1}{2} a_p^{(2)}$	$\int_0^1 x^2 g_2^p(x, Q^2) dx$ $= \frac{1}{3} (d_p^{(2)} - a_p^{(2)})$	$d_p^{(2)}$
This paper	$1.05 \times 10^{-2}$	$-0.12 \times 10^{-2}$	$1.74 \times 10^{-2}$
MIT bag model [8,35]	—	—	$1.0 \times 10^{-2}$
QCD sum rule [37]	—	—	$-(0.6 \pm 0.3) \times 10^{-2}$
QCD sum rule [38]	—	—	$-(0.3 \pm 0.3) \times 10^{-2}$
Lattice QCD [39]	$(1.50 \pm 0.32) \times 10^{-2}$	$-(2.61 \pm 0.38) \times 10^{-2}$	$-(4.8 \pm 0.5) \times 10^{-2}$
Data [19]	$(1.21 \pm 0.10) \times 10^{-2}$	$-(0.63 \pm 0.18) \times 10^{-2}$	$(0.54 \pm 0.50) \times 10^{-2}$

zero by itself. However, the c.m. bag model with SU(6) symmetry breaking effects ( $\xi \approx 0.85 < 1$ ) predicts a nonzero  $g_2^n(x)$ .

(5) Since  $g_2(x)$  is not identically zero in the model, the Burkhardt-Cottingham sum rule implies that  $g_2(x)$  must change its sign at some  $x=x_0$ , where  $0 < x_0 < 1$ , i.e.,  $g_2(x)$  must have at least one nontrivial zero. The question is whether the  $x$  behavior of  $g_2$  satisfies

$$\text{case1: } g_2(x) < 0 \text{ for } x < x_0, \quad g_2(x) > 0 \text{ for } x > x_0$$

or just opposite

$$\text{case2: } g_2(x) > 0 \text{ for } x < x_0, \quad g_2(x) < 0 \text{ for } x > x_0.$$

For case 1, one has  $\int_0^1 x^2 g_2(x) dx > 0$ , while for case 2,  $\int_0^1 x^2 g_2(x) dx < 0$ . The c.m. bag model predictions for  $\int_0^1 x^2 g_2^{(p,n,d)}(x, Q^2) dx$  are all negative (see Tables I–III or Figs. 3 and 7). This implies the  $x$  behavior of  $g_2(x)$  fits case 2. The preliminary data [19] seem to favor our predictions (see, for instance, Fig. 2 for the proton).

(6) For both the proton and deuteron, we now have  $a^{(2)} > 0$  and  $d^{(2)} - a^{(2)} < 0$ . In the c.m. bag model, the moment  $\int_0^1 x^2 g_2(x) dx (\approx [d^{(2)} - a^{(2)}]/3)$  is about one order of magnitude smaller than  $\int_0^1 x^2 g_1(x) dx (\approx a^{(2)}/2)$ . It implies  $|d^{(2)} - a^{(2)}| \ll a^{(2)}$  (recall  $|g_2| \ll g_1$ , or  $d^{(2)} \approx a^{(2)}$ ). Hence, the twist-3 matrix element  $d^{(2)}$  approximately equals the twist-2 matrix element  $a^{(2)}$  and the sign of  $d^{(2)}$  should also be positive for the proton and deuteron targets. This agrees with data [19] but disagrees with the negative sign predicted by the QCD sum rules [37,38] and quenched lattice QCD [39].

For the neutron, since  $\int_0^1 x^2 g_1^n(x) dx$  is negative and much larger than  $\int_0^1 x^2 g_2^n(x) dx$  in magnitude,  $d_{\text{neutron}}^{(2)}$  is negative. This negative sign is consistent with the results given by

other approaches [37–39]. In magnitude, our result agrees with that given by the quenched lattice QCD, but much less than those given by the QCD sum rules.

(7) Our results show  $d^{(2)} \approx a^{(2)}$ , i.e., the twist-3 contribution is almost of the same magnitude as twist-2 contribution. From Eqs. (6a) and (6b), it implies that the  $g_2^{WW}(x)$  and  $\bar{g}_2(x)$  have opposite signs but approximately same magnitude. They almost cancel each other and lead to a very small  $g_2(x)$  (see also Figs. 12(a) and 12(b) in [20]). For the same reason, the original Wandzura-Wilczek relation

$$g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$

is not a good approximation and the higher twist contributions may not be neglected. As pointed out by Cortes, Pire, and Ralston [40] that the original Wandzura-Wilczek relation was derived by using the Dirac equation for free and massless quarks. Including quark mass effect and gluon-dependent term, an extended Wandzura-Wilczek relation was given (similar formula without quark mass term has been given by Jaffe [7]):

$$g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) - \frac{m_q}{M} \int_x^1 \frac{dy}{y} \frac{\partial h_T}{\partial y} - \int_x^1 \frac{dy}{y} \frac{\partial \xi}{\partial y},$$

where the mass-dependent term is another twist-2 piece which is related to “transversity”  $h_T$ . The last term is the “true” twist-3 piece arising from quark-gluon correlation. As emphasized in [40] that when the quark mass term is

TABLE II. Comparison of model results for the neutron.

Neutron	$\int_0^1 x^2 g_1^n(x, Q^2) dx$ $= \frac{1}{2} a_n^{(2)}$	$\int_0^1 x^2 g_2^n(x, Q^2) dx$ $= \frac{1}{3} (d_n^{(2)} - a_n^{(2)})$	$d_n^{(2)}$
This paper	$-0.93 \times 10^{-3}$	$-0.23 \times 10^{-3}$	$-2.53 \times 10^{-3}$
MIT bag model [8,35]	—	—	0
QCD sum rule [37]	—	—	$-(30 \pm 10) \times 10^{-3}$
QCD sum rule [38]	—	—	$-(25 \pm 10) \times 10^{-3}$
Lattice QCD [39]	$-(1.2 \pm 2.0) \times 10^{-3}$	$-(0.4 \pm 2.2) \times 10^{-3}$	$-(3.9 \pm 2.7) \times 10^{-3}$
Data	—	—	—

TABLE III. Comparison of model results with deuteron data.

Deuteron	$\int_0^1 x^2 g_1^d(x, Q^2) dx$ $= \frac{1}{2} a_d^{(2)}$	$\int_0^1 x^2 g_2^d(x, Q^2) dx$ $= \frac{1}{3} (d_d^{(2)} - a_d^{(2)})$	$d_d^{(2)}$
This paper	$4.37 \times 10^{-3}$	$-0.65 \times 10^{-3}$	$6.79 \times 10^{-3}$
MIT bag model [8,35]	—	—	$5.0 \times 10^{-3}$
QCD sum rule [37]	—	—	$-(17 \pm 5) \times 10^{-3}$
QCD sum rule [38]	—	—	$-(13 \pm 5) \times 10^{-3}$
Lattice QCD [39]	$(6.9 \pm 2.6) \times 10^{-3}$	$-(13.3 \pm 3.0) \times 10^{-3}$	$-(22 \pm 6) \times 10^{-3}$
Data [19]	$(4.0 \pm 0.8) \times 10^{-3}$	$-(1.4 \pm 3.0) \times 10^{-3}$	$(3.9 \pm 9.2) \times 10^{-3}$

included the twist-3 term cannot be isolated in a model-independent way with a measurement of  $g_1$  and  $g_2$ . However, neglecting the strange quark contribution, the quark mass term ( $\sim m_q/M$ ) should be negligible for both up and down quarks. Hence, the separation of twist-2 and twist-3 pieces in Eq. (4) seems to be a reasonable approximation.

(8) According to the OPE analysis, neglecting the  $M^2/Q^2$  corrections, the general formulas for the moments of the structure functions are

$$\int_0^1 x^n g_1(x, Q^2) = \frac{1}{2} a^{(n)}(Q^2) \quad (n=0,2,4, \dots), \quad (11a)$$

$$\int_0^1 x^n g_2(x, Q^2) = -\frac{n}{2(n+1)} [a^{(n)}(Q^2) - d^{(n)}(Q^2)] \quad (n=2,4,6, \dots) \quad (11b)$$

for  $n=2$ , they reduced to Eqs. (2c) and (2d). From Eqs. (11a) and (11b), one obtains

$$\int_0^1 x^n \left[ g_1(x, Q^2) + \frac{n+1}{n} g_2(x, Q^2) \right] dx = \frac{1}{2} d^{(n)}(Q^2) \quad (n=2,4,6, \dots); \quad (12)$$

hence, one has, for  $n=2$ ,

$$d^{(2)} = 2 \int_0^1 x^2 \left[ g_1(x, Q^2) + \frac{3}{2} g_2(x, Q^2) \right] dx. \quad (13)$$

The c.m. bag model prediction for the function  $x^2 [g_1(x, Q^2) + \frac{3}{2} g_2(x, Q^2)]$  for the proton is shown in Fig. 9 and that for the deuteron is shown in Fig. 10. One can see that in both cases, the model predictions for  $d^{(2)}$  are nonzero and positive. This seems to be consistent with recent data [19].

(9) If one assumes that Eq. (12) holds also for  $n=1$ , one obtains

$$\int_0^1 x [g_1(x, Q^2) + 2g_2(x, Q^2)] dx = \frac{1}{2} d^{(1)}(Q^2). \quad (14a)$$

To lowest order in  $\alpha_s$ , it was shown [12] by using the field theoretical parton model that  $d^{(1)}(Q^2)$  vanishes in the chiral limit and one can obtain a sum rule for the second moments of  $g_1$  and  $g_2$ . However, because one is dealing with an even

moment it turns out that only the valence parts of the structure functions appear [41]. One has the Efremov-Leader-Teryaev (ELT) sum rule

$$\int_0^1 x [g_1^V(x, Q^2) + 2g_2^V(x, Q^2)] dx = 0, \quad (14b)$$

where the superscript  $V$  denotes the valence part of the structure function. Since our model results of  $g_1$  and  $g_2$  contain only the valence contributions, we plot  $g_1^{(p,d)}(x, Q^2) + 2g_2^{(p,d)}(x, Q^2)$  as functions of  $x$  in Figs. 11 and 12 and compare them with recent proton and deuteron data [19], respectively. One can see that our model predictions are consistent with the SLAC E143 data but do not give the sum rule (14b). However, the ELT sum rule cannot be tested by the existing data of  $g_1$  and  $g_2$  because they contain both the valence and sea parts.

#### IV. SUMMARY

The study of transverse spin structure function  $g_2(x, Q^2)$  has both theoretical and experimental interest. In the most naive parton model, the quark is asymptotic free and has no transverse momentum, and  $g_2(x)$  is identically zero. However, quarks inside the nucleon are not free, the binding effect, which arising from quark-gluon interactions, causes a nonzero transverse momentum for quarks and leads to  $g_2 \neq 0$ . Measurements of  $g_2$  or  $g_T = g_1 + g_2$  allow us to get more information about binding effects, which are mainly formulated as ‘‘higher twist effects.’’ On the other hand, since  $Q^2$  is large in the deep inelastic scattering, quark binding effects should not be significant and  $g_2$  should be small, especially compared to  $g_1$ . This seems to agree with most recent experimental result [19]. Our model calculation is consistent with this conclusion. It should be noted, however, that the theoretical predictions given by different models or approaches seem not to fully agree with one another because of different approximations. In addition, as mentioned in Sec. III that another twist-2 piece which is related to the quark mass and ‘‘transversity’’  $h_T$  has been neglected in our discussion. On the experimental side, the errors of data are still quite large. We hope that several new experiments [42–44] to be performed in the next few years will provide more precise data and tell us more about the quark spin (including longitudinal and transverse) distributions in the nucleon.

## ACKNOWLEDGMENTS

I wish to thank X. Ji for useful comments and suggestions and P. K. Kabir, J. S. McCarthy and H. J. Weber for helpful discussions. I thank E. Leader for useful discussions and for

the clarification on the ELT sum rule. I also thank O. Rondon-Aramayo for providing the updated E143 data and suggestions. This work was supported by the U.S. Department of Energy and the Institute of Nuclear and Particle Physics, University of Virginia.

- 
- [1] J. Ashman *et al.*, Phys. Lett. B **206**, 346 (1988); Nucl. Phys. **B328**, 1 (1989); V. W. Hughes *et al.*, Phys. Lett. B **212**, 511 (1988).
- [2] B. Adeva *et al.*, Phys. Lett. B **302**, 533 (1993); **320**, 400 (1994); D. Adams *et al.*, *ibid.* **329**, 399 (1994).
- [3] P. L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993).
- [4] K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995).
- [5] K. Abe *et al.*, Phys. Rev. Lett. **75**, 25 (1995).
- [6] E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B201**, 141 (1982).
- [7] R. L. Jaffe, Comments Nucl. Part. Phys. **19**, 239 (1990).
- [8] R. L. Jaffe and X. Ji, Phys. Rev. D **43**, 724 (1991).
- [9] X. Ji and P. Unrau, Phys. Lett. B **333**, 228 (1994).
- [10] E. Ehrnsberger, L. Mankiewicz, and A. Schafer, Phys. Lett. B **323**, 439 (1994).
- [11] R. L. Jaffe, Phys. Today **48**, 24 (1995).
- [12] M. Anselmino, A. Efremov, and E. Leader, Phys. Rep. **261**, 1 (1995).
- [13] F. E. Close, Report No. hep-ph/9509251, 1995 (unpublished).
- [14] A. J. G. Hey and J. E. Mandula, Phys. Rev. D **5**, 2610 (1972).
- [15] X. Ji, Nucl. Phys. **B402**, 217 (1993).
- [16] J. Kodaira, S. Matsuda, K. Sasaki, and T. Uematsu, Nucl. Phys. **B159**, 99 (1979).
- [17] S. Wandzura and F. Wilczek, Phys. Lett. **72B**, 195 (1977).
- [18] D. Adams *et al.*, Phys. Lett. B **336**, 125 (1994).
- [19] E143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **76**, 587 (1996).
- [20] X. Song and J. M. McCarthy, Phys. Rev. D **49**, 3169 (1994).
- [21] R. L. Jaffe, Phys. Lett. B **365**, 359 (1996).
- [22] X. Song and J. M. McCarthy, Phys. Rev. C **46**, 1077 (1992).
- [23] X. M. Wang and X. Song, Phys. Rev. C **51**, 2750 (1995).
- [24] A. W. Schreiber, A. I. Signal, and A. W. Thomas Phys. Rev. D **44**, 2653 (1991).
- [25] M. Stratmann, Z. Phys. C **60**, 763 (1993).
- [26] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977); X. Song and J. Du, Phys. Rev. D **40**, 2177 (1989).
- [27] E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B199**, 451 (1982).
- [28] A. P. Bukhvostov, E. A. Kureav, and L. N. Lipatov, JETP Lett. **37**, 482 (1983); Sov. Phys. JETP **60**, 22 (1984).
- [29] P. G. Ratcliffe, Nucl. Phys. **B264**, 493 (1986).
- [30] I. I. Balitsky and V. M. Braun, Nucl. Phys. **B311**, 541 (1988/89).
- [31] X. Ji and C. Chou, Phys. Rev. D **42**, 3637 (1994).
- [32] A. Ali, V. M. Braun, and G. Hiller, Phys. Lett. B **266**, 117 (1991).
- [33] J. Kodaira, Y. Yasui, and T. Uematsu, Phys. Lett. B **344**, 348 (1995).
- [34] T. J. Liu, Ph.D. thesis, University of Virginia, 1995.
- [35] F. M. Steffens, H. Holtmann, and A. W. Thomas, Phys. Lett. B **358**, 139 (1995).
- [36] H. Burkhardt and W. N. Cottingham, Ann. Phys. (N.Y.) **56**, 453 (1970).
- [37] E. Stein *et al.*, Phys. Lett. B **334**, 369 (1995).
- [38] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Phys. Lett. B **242**, 245 (1990); **318**, 648(E) (1993).
- [39] M. Gockeler *et al.*, Phys. Rev. D **53**, 2317 (1996).
- [40] J. L. Cortes, B. Pire, and J. P. Ralston, Z. Phys. C **55**, 409 (1992).
- [41] The sum rule was incorrectly given in [12] where the valence label was left out. See A. Efremov, E. Leader, and O. Teryaev, Birkbeck College Report, London, 1996 (unpublished).
- [42] R. Arnold *et al.*, SLAC-PROPOSAL-E-154, 1993 (unpublished); SLAC-PROPOSAL-E-155, 1993 (unpublished).
- [43] SMC Collaboration, "Measurements of the Spin-dependent Structure Functions of the Neutron and the Proton," Addendum to the NA47 Proposal, Report No. CERN/SPSLC 94-13, 1994 (unpublished).
- [44] M. Dueren and K. Rith, in *Physics at HERA*, Proceedings of the Workshop, Hamburg, Germany, 1991, edited by W. Buchmuller and G. Ingelman (DESY, Hamburg, 1992), Vol. 1, p. 927.