Pair production and correlated decay of heavy Majorana neutrinos in e^+e^- collisions

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We consider the process $e^+e^-\to N_1N_2$, where N_1 and N_2 are heavy Majorana particles, with relative CP given by $\eta_{CP}=+1$ or -1, decaying subsequently via $N_1,N_2\to W^\pm e^\mp$. We derive the energy and angle correlation of the dilepton final state, both for like-sign ($e^\mp e^\mp$) and unlike-sign (e^-e^+) configurations. Interesting differences are found between the cases $\eta_{CP}=+1$ and -1. The characteristics of unlike-sign e^+e^- dileptons originating from a Majorana pair N_1N_2 are contrasted with those arising from the reaction $e^+e^-\to N\overline{N}\to W^+e^-W^-e^+$, where $N\overline{N}$ is a Dirac particle-antiparticle pair. [S0556-2821(96)04015-5]

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I. INTRODUCTION

In an interesting paper [1], Kogo and Tsai have analyzed the reaction $e^+e^-\to N_1N_2$, where $N_{1,2}$ are heavy Majorana neutrinos, and compared the cases where the relative CP of N_1 and N_2 is $\eta_{CP}=+1$ and -1. It was found that the two cases differ in threshold behavior, in angular distribution, and in the dependence on the spin directions of N_1 and N_2. A comparison was also made between the Majorana process and the Dirac process $e^+e^-\to N\overline{N}$, where $N\overline{N}$ is a Dirac particle-antiparticle pair. A related analysis was carried out in Ref. [2]. [The contrast between Majorana neutrinos and Dirac neutrinos has been the subject of several other papers (e.g. [3–6]), and monographs [7,8].]

In the present paper, we examine how the differences between the cases $\eta_{CP}=+1$ and -1 propagate to the decay products of N_1 and N_2, assuming the decays to take place via $N_{1,2}\to W^\pm e^\mp$. We focus on the like-sign lepton pair created in the reaction chain $e^+e^-\to N_1N_2\to W^+e^-W^+e^-$, which is a characteristic signature of Majorana pair production. We derive, in particular, the correlation in the energies of the e^-e^- pair, and in their angles relative to the e^+e^- axis. Interesting differences are found between the cases $\eta_{CP}=+1$ and -1. We also examine the behavior of the unlike-sign dileptons e^+e^-, comparing the Majorana cases with dileptons created in the production and decay of a Dirac $N\overline{N}$ pair, i.e., $e^+e^-\to N\overline{N}\to W^+e^-W^-e^+$.

II. CHARACTERISTICS OF THE REACTION $e^+e^-\to N_1N_2$

The analysis of Ref. [1] was carried out in the context of the simple production mechanism for $e^+e^-\to N_1N_2$, shown in Fig. 1, and we begin by recapitulating the essential results. The interaction Lagrangian is taken to be

$$\mathcal{L}_1(x)=-\frac{g}{2\cos\theta_W}[\overline{e(x)}\,\gamma_\mu(c_V-c_A\gamma_5)e(x)$$
$$+\alpha_N\overline{N_1}(x)\,\gamma_\mu\,\tfrac{1}{2}\,(1-\gamma_5)N_2(x)$$
$$+\alpha_N\overline{N_2}(x)\,\gamma_\mu\,\tfrac{1}{2}\,(1-\gamma_5)N_1(x)]Z^\mu(x), \quad (2.1)$$

where c_V, c_A, and α_N may be regarded as real phenomenological parameters. (For the standard Z boson, $c_V=-1/2+2\sin^2\theta_W$, $c_A=-1/2$.) The matrix element for Majorana neutrinos (with momenta and spins as indicated in Fig. 1) is

$$\mathcal{M}_m=-i\,\alpha_N\left(\frac{g}{2\cos\theta_W}\right)^2 j^e_\mu\Delta_Z^{\mu\nu}[\overline{u_{t_1}}(q_1)\,\gamma_\nu\,\tfrac{1}{2}\,(1-\gamma_5)$$
$$\times v_{t_2}(q_2)\lambda_2-\overline{u_{t_2}}(q_2)\,\gamma_\nu\,\tfrac{1}{2}\,(1-\gamma_5)v_{t_1}(q_1)\lambda_1], \quad (2.2)$$

where

$$j^e_\mu=\overline{v_{s_2}}(p_2)\,\gamma_\mu(c_V-c_A\gamma_5)u_{s_1}(p_1) \quad (2.3)$$

and

$$\Delta_Z^{\mu\nu}=\frac{g^{\mu\nu}-q^\mu q^\nu/m_Z^2}{q^2-m_Z^2+im_Z\Gamma_Z}. \quad (2.4)$$

Assuming CP invariance, the factors λ_1, λ_2 in Eq. (2.2) are such that $\lambda_1\lambda_2^*=+1(-1)$ when N_1 and N_2 have the same (opposite) CP parity [9]. Rewriting the second term in Eq. (2.2) as

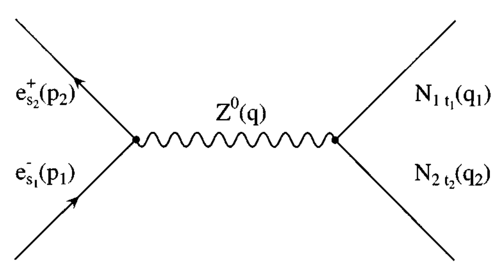

FIG. 1. Feynman diagram for the reaction $e^+e^-\to N_1N_2$.

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$$\overline{u_{t_2}}(q_2)\gamma_\nu \tfrac{1}{2}(1-\gamma_5)v_{t_1}(q_1)=\overline{u_{t_1}}(q_1)\gamma_\nu \tfrac{1}{2}(1+\gamma_5)v_{t_2}(q_2), \tag{2.5}$$

we observe that the current of the Majorana neutrinos is pure axial vector when N_1 and N_2 have the same CP parity ($\eta_{CP}=\lambda_1\lambda_2^*=+1$), and pure vector when they have opposite CP ($\eta_{CP}=\lambda_1\lambda_2^*=-1$). In comparison, the matrix element for the Dirac process $e^+e^-\to N\bar{N}$ is

$$\mathcal{M}_d=-i\alpha_N\left(\frac{g}{2\cos\theta_W}\right)^2 j^e_\mu \Delta_Z^{\mu\nu}\overline{u_{t_1}}(q_1)\gamma_\nu \tfrac{1}{2}(1-\gamma_5)v_{t_2}(q_2). \tag{2.6}$$

The differential cross section for $e^+e^-\to N_1N_2$, for general masses m_1 and m_2 and for arbitrary polarizations $\vec{n}$ and $\vec{n}'$ of the two neutrinos, is given in the Appendix. In Sec. V we compare our formulas with those of Ref. [1], and with special cases treated in other papers. Here, we specialize to the case $m_1=m_2=m_N$, for which the cross section $(d\sigma/d\Omega)$ in the cases $\eta_{CP}=+1$ and -1 is

$$\left(\frac{d\sigma}{d\Omega}\right)_+=\tfrac{1}{2}\sigma_0\beta^3\{f_1[(n_yn_y'-n_xn_x')S^2 + (1+n_zn_z')(1+C^2)]-f_2 2(n_z+n_z')C\}, \tag{2.7}$$

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_-=\sigma_0\beta(f_1\{&2-\beta^2+C^2\beta^2+n_zn_z'[\beta^2+C^2(1/\gamma^2+1)]\\ &+n_xn_x'S^2(1/\gamma^2+1)-n_yn_y'S^2\beta^2\\ &+(n_xn_z'+n_x'n_z)2SC/\gamma^2\}-f_2\{2(n_x+n_x')S/\gamma\\ &+2(n_z+n_z')C\}).\end{aligned} \tag{2.8}$$

For comparison, the differential cross section of the Dirac process $e^+e^-\to N\bar{N}$ is

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_d=\tfrac{1}{2}\sigma_0\beta\{&f_1[(1+C^2\beta^2)-(n_z+n_z')\beta(1+C^2)\\ &-(n_x+n_x')SC\beta/\gamma+n_zn_z'(C^2+\beta^2)\\ &+(n_xn_z'+n_zn_x')SC/\gamma+n_xn_x'S^2/\gamma^2]\\ &+f_2[2C\beta-(n_z+n_z')C(1+\beta^2)-(n_x+n_x')S/\gamma\\ &+2n_zn_z'C\beta+(n_xn_z'+n_x'n_z)S\beta/\gamma]\}.\end{aligned} \tag{2.9}$$

The symbols in Eqs. (2.7)–(2.9) are defined as

$$\sigma_0=\frac{G_F^2\alpha_N^2}{512\pi^2}\left|\frac{m_Z^2}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 s,\quad \beta=(1-4m_N^2/s)^{1/2},$$
$$\gamma=(1-\beta^2)^{-1/2},\quad C=\cos\theta,\quad S=\sin\theta,$$
$$f_1=2(c_V^2+c_A^2),\quad f_2=4c_Vc_A, \tag{2.10}$$

θ being the scattering angle of N_1 (or N) with respect to the initial e^- direction. The coordinate axes are defined so that the momentum and spin vectors of N_1 and N_2 in the e^+e^- c.m. frame have the components

$$q_1^\mu=(\gamma m,0,0,\gamma\beta m),\quad t_1^\mu=(\gamma\beta n_z,n_x,n_y,\gamma n_z),$$
$$q_2^\mu=(\gamma m,0,0,-\gamma\beta m),\quad t_2^\mu=(-\gamma\beta n_z',n_x',n_y',\gamma n_z'). \tag{2.11}$$

Inspection of Eqs. (2.7)–(2.9) reveals several interesting features.

(a) The Majorana cases "+" and "−" have different dependences on the spin vectors $\vec{n}$ and $\vec{n}'$, and different angular distributions, even after the spins $\vec{n}$ and $\vec{n}'$ are summed over. These differences stem from the fact that the matrix element $\mathcal{M}_m$ in Eq. (2.2) effectively involves an axial-vector current $\bar{N}_1\gamma_\mu\gamma_5N_2$ when $\lambda_1\lambda_2^*=+1$ and a vector current $\bar{N}_1\gamma_\mu N_2$ when $\lambda_1\lambda_2^*=-1$.

(b) The Majorana cases "+" and "−" differ from the Dirac case "d," in which the current of the $N\bar{N}$ pair has a $V-A$ structure $\bar{N}\gamma_\mu\frac{1}{2}(1-\gamma_5)N$. This difference persists even if the spins of the heavy neutrinos are summed over, in which case

$$\sum_{\vec{n},\vec{n}'}\left(\frac{d\sigma}{d\Omega}\right)_+=2\sigma_0\beta^3[f_1(1+C^2)],$$
$$\sum_{\vec{n},\vec{n}'}\left(\frac{d\sigma}{d\Omega}\right)_-=4\sigma_0\beta[f_1(2-\beta^2+C^2\beta^2)], \tag{2.12}$$
$$\sum_{\vec{n},\vec{n}'}\left(\frac{d\sigma}{d\Omega}\right)_d=2\sigma_0\beta[f_1(1+C^2\beta^2)+f_2(2C\beta)].$$

Whereas the spin-averaged Majorana cross sections are forward-backward symmetric, the Dirac process has a term linear in $\cos\theta$, with a coefficient proportional to $f_2=4c_Vc_A$. Equation (2.12) also shows that the threshold behavior is β^3, β, and β for the cases "+," "−," and "d," respectively. In the asymptotic limit $\beta\to1$, the Majorana cases "+" and "−" have the same angular distribution $(1+C^2)$, distinct from that of the Dirac process.

(c) In the high energy limit $\beta\to1$, the Dirac process $e^+e^-\to N\bar{N}$ has a spin dependence given by

$$\left(\frac{d\sigma}{d\Omega}\right)_d=\tfrac{1}{2}\sigma_0\beta[1-(n_z+n_z')+n_zn_z'][f_1(1+C^2)+2f_2C]. \tag{2.13}$$

The fact that only the longitudinal components (n_z and n_z') of the N, $\bar{N}$ spins appear in this expression is consistent with the expectation that relativistic Dirac neutrinos are eigenstates of helicity. The fact that the cross section (2.13) vanishes when $n_z=-1$, $n_z'=+1$ confirms the expectation that for a $V-A$ current the N and $\bar{N}$ are produced in left-handed and right-handed states, respectively. By comparison, the Majorana processes $e^+e^-\to N_1N_2$, for $\eta_{CP}=+1$ and -1, have the high energy behavior ($\beta\to1$)

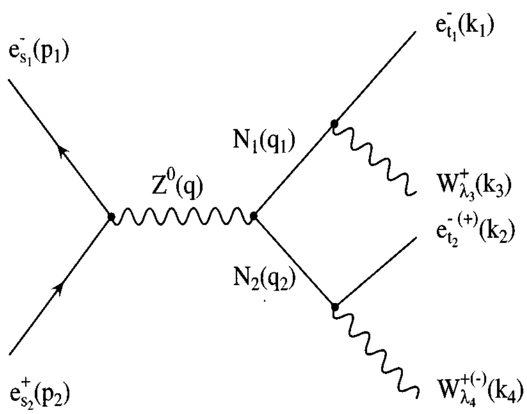

FIG. 2. Diagram showing the sequential process $e^+e^-\to N_1N_2\to e^\pm e^- W^\mp W^+$.

$$\left(\frac{d\sigma}{d\Omega}\right)_+ = \tfrac{1}{2}\,\sigma_0\{f_1[(1+C^2)(1+n_zn_z')+S^2(n_yn_y'-n_xn_x')] -2f_2C(n_z+n_z')\}, \tag{2.14}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_- = \tfrac{1}{2}\,\sigma_0\{f_1[(1+C^2)(1+n_zn_z')+S^2(n_xn_x'-n_yn_y')] -2f_2C(n_z+n_z')\}. \tag{2.15}$$

Contrary to the Dirac case, the Majorana reactions have an explicit dependence on n_x, n_y and n_x', n_y', reflecting the fact that a relativistic Majorana particle with $m_N\neq 0$ is *not* necessarily an eigenstate of helicity, and can have a spin pointing in an arbitrary direction. The Majorana cases "+" and "−" differ in the sign of the term proportional to S^2, which contains the transverse (x and y) components of the neutrino spins. It is with the purpose of exposing the subtle differences in the spin state of the N_1N_2 and NN systems that we investigate in the following sections the dilepton final state created by the decays of the heavy neutrinos via $N_{1,2}\to W^\pm e^\mp$ and $N(\bar{N})\to W^+e^-(W^-e^+)$.

III. LIKE-SIGN DILEPTONS: THE REACTION $e^+e^-\to N_1N_2\to W^+W^+e^-e^-$

As seen in the preceding, the spin state and the angular distribution of the Majorana pair produced in $e^+e^-\to N_1N_2$ depends on the relative CP parity η_{CP} of the two particles. We wish to see how these differences manifest themselves in the decay products of N_1 and N_2. To this end, we assume that $m_N > m_W$, and that the simplest decay mechanism is $N_{1,2}\to W^\mp e^\pm$. In particular, the reaction sequence $e^+e^-\to N_1N_2\to W^+W^+e^-e^-$ leads to the appearance of two like-sign leptons in the final state, an unmistakable signature of Majorana pair production. (For the purpose of this paper we assume that the W bosons decay into quark jets, thus avoiding the complications of final states with three or four charged leptons.)

We have calculated the amplitude of the process $e^+e^-\to N_1N_2\to W^+W^+e^-e^-$, depicted in Fig. 2, assuming a decay interaction (α_N' and α_N'' being real parameters)

$$\begin{aligned}\mathcal{L}_2(x) = -\frac{g}{\sqrt{2}}[\,&\alpha_N'\overline{e(x)}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)N_1(x)W^{\mu-}(x)\\ &+\alpha_N'\overline{N_1}(x)\gamma_\mu\tfrac{1}{2}(1-\gamma_5)e(x)W^{\mu+}(x)\\ &+\alpha_N''\overline{e(x)}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)N_2(x)W^{\mu-}(x)\\ &+\alpha_N''\overline{N_2}(x)\gamma_\mu\tfrac{1}{2}(1-\gamma_5)e(x)W^{\mu+}(x)].\end{aligned} \tag{3.1}$$

This amplitude has the form (see Appendix for details)

$$\begin{aligned}\mathcal{M} = iAj^e_\mu\Delta_Z^{\mu\nu}&\frac{1}{q_1^2-m_1^2+im_1\Gamma_1}\,\frac{1}{q_2^2-m_2^2+im_2\Gamma_2}\\ &\times[m_2\lambda_2\overline{u_{t_1}}(k_1)\gamma_\rho\not{q}_1\gamma_\nu\gamma_\sigma\tfrac{1}{2}(1+\gamma_5)v_{t_2}(k_2)\\ &-m_1\lambda_1\overline{u_{t_2}}(k_2)\gamma_\sigma\not{q}_2\gamma_\nu\gamma_\rho\tfrac{1}{2}(1+\gamma_5)v_{t_1}(k_1)]\\ &\times\epsilon^{*\rho}_{\lambda_3}(k_3)\epsilon^{*\sigma}_{\lambda_4}(k_4),\end{aligned} \tag{3.2}$$

where

$$A=\alpha_N\alpha_N'\alpha_N''\frac{g^4}{8\cos^2\theta_W}. \tag{3.3}$$

Using the narrow-width approximation for the N_1, N_2 propagators, and specializing to the case $m_1=m_2=m_N$, we obtain the following expression for the squared matrix element (summed over final and averaged over initial spins), the subscript in $\mathcal{M}_\pm$ denoting $\eta_{CP}=\pm 1$ ($q=p_1+p_2$, $l=p_1-p_2$):

$$\begin{aligned}\overline{|\mathcal{M}_\pm|^2} = \frac{|A|^2}{2}\frac{1}{(s-m_Z^2)^2}\frac{\pi}{m_N\Gamma_N}\delta(q_1^2-m_N^2)\frac{\pi}{m_N\Gamma_N}\delta(q_2^2-m_N^2)\frac{m_N^2}{m_W^4}(f_1\{&\mp(m_N^2-m_W^2)^2(m_N^2+2m_W^2)^2s\mp4(m_N^2\\ -2m_W^2)^2[s(k_1k_2)(q_1q_2)-s(k_1q_2)(k_2q_1)-(k_1k_2)(q_1q)(q_2q)&+(k_1k_2)(q_1l)(q_2l)+(k_1q_2)(k_2q)(q_1q)\\ -(k_1q_2)(k_2l)(q_1l)+(k_1q)(k_2q_1)(q_2q)-(k_1l)(k_2q_1)(q_2l)&-(k_1q)(k_2q)(q_1q_2)+(k_1l)(k_2l)(q_1q_2)\\ \pm m_N^2((k_1q)(k_2q)-(k_1l)(k_2l))]+2(m_N^2-m_W^2)(m_N^2-2m_W^2)^2&[(k_1q)(q_2q)-(k_1l)(q_2l)+(k_2q)(q_1q)-(k_2l)\\ \times(q_1l)]+8m_W^2(m_N^2-m_W^2)^2[(q_1q)(q_2q)-(q_1l)(q_2l)]\}&-2f_2(m_N^2-m_W^2)(m_N^4-4m_W^4)[\pm(k_1q)(q_1l)\\ -(k_1q)(q_2l)\mp(k_1l)(q_1q)+(k_1l)(q_2q)\pm(k_2q)(q_2l)-(k_2q)(q_1l)&\mp(k_2l)(q_2q)+(k_2l)(q_1q)]).\end{aligned} \tag{3.4}$$

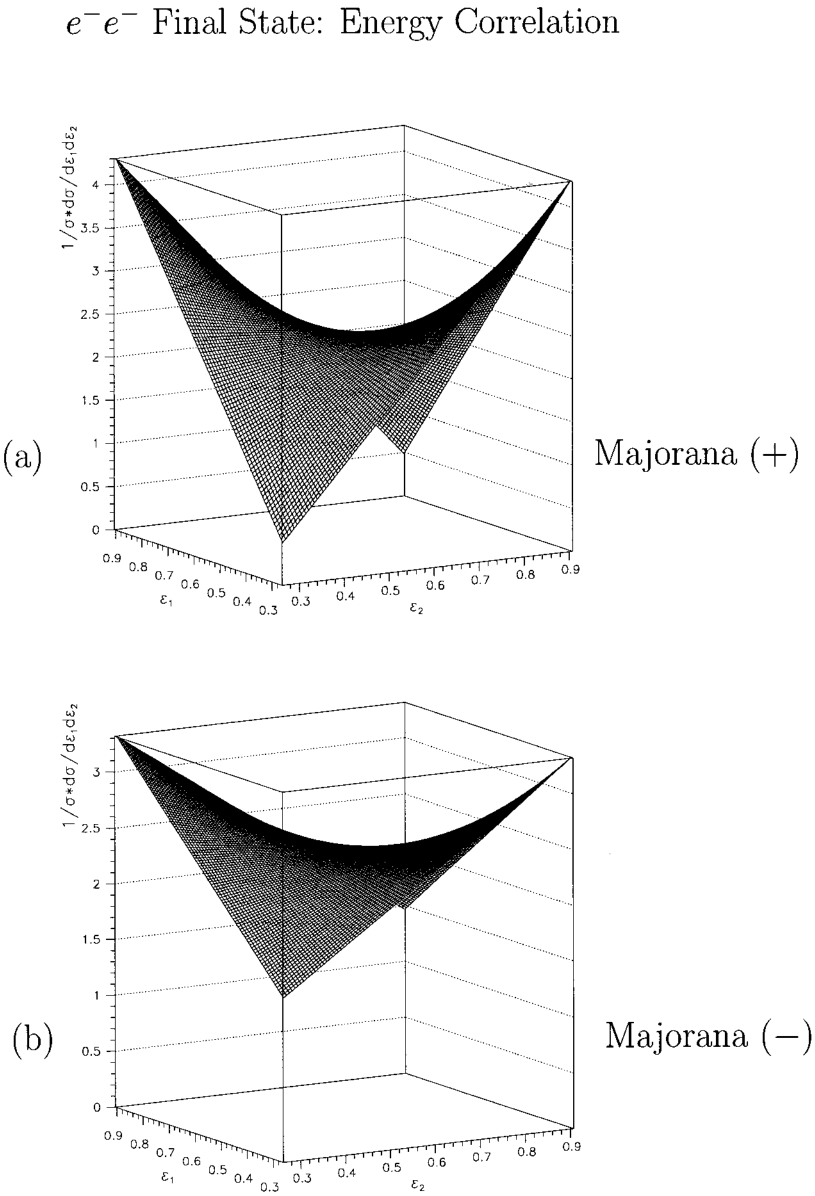

FIG. 3. Energy correlation of the e^-e^- lepton pair in the reaction $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^-W^+$, for the cases (a) $\eta_{CP}=+1$, (b) $\eta_{CP}=-1$. (Parameters for this and succeeding figures: $\sqrt{s}=1.2$ TeV, $m_N=500$ GeV.)

If the final state is e^+e^+ instead of e^-e^-, we replace $f_2 \rightarrow -f_2$ in the above equation.

The expression for $|\overline{\mathcal{M}_\pm}|^2$ can be integrated over the phase space of the two W^+'s (i.e., over the momenta $k_3(=q_1-k_1)$ and $k_4(=q_2-k_2)$, in order to obtain the spectra in the lepton variables k_1 and k_2. Defining the four-vectors k_1 and k_2 in the e^+e^- c.m. frame by

$$k_1^\mu = E_1(1,\sin\theta_1\cos\phi_1,\sin\theta_1\sin\phi_1,\cos\theta_1),$$

$$k_2^\mu = E_2(1,\sin\theta_2\cos\phi_2,\sin\theta_2\sin\phi_2,\cos\theta_2), \qquad (3.5)$$

we have been able to derive the correlated distribution of the energies E_1 and E_2, as well as the correlation of the variables $\cos\theta_1$ and $\cos\theta_2$ measured relative to the e^- beam direction.

A. Energy correlation

The normalized spectrum in the energies of the dilepton pair e^-e^- is ($\mathcal{E}_{1,2}=E_{1,2}/m_N$)

$$\frac{1}{\sigma}\left(\frac{d\sigma}{d\mathcal{E}_1 d\mathcal{E}_2}\right) = \mathcal{N}[a+b(\mathcal{E}_1+\mathcal{E}_2)+c(\mathcal{E}_1+\mathcal{E}_2)^2-c(\mathcal{E}_1-\mathcal{E}_2)^2], \qquad (3.6)$$

where $\mathcal{N}$ is a normalization factor,

$$\mathcal{N}=[\mathcal{W}^2\beta^2(a+b\mathcal{W}+c\mathcal{W}^2)]^{-1}, \qquad (3.7)$$

with $\mathcal{W}=\sqrt{s}(m_N^2-m_W^2)/2m_N^3$, $\beta=(1-4m_N^2/s)^{1/2}$. The coefficients a, b, c depend on the relative CP of the N_1N_2 system, and take the values

$$a^+=m_N^2m_W^2(m_N^2-m_W^2)^2\left(2s-\frac{(m_N^2+2m_W^2)^2}{2m_N^2}\right),$$
$$b^+=\sqrt{s}m_N^3(m_N^2-2m_W^2)^2(m_N^2-m_W^2), \tag{3.8}$$
$$c^+=-m_N^6(m_N^2-2m_W^2)^2,$$
$$a^-=2m_W^2(m_N^2-m_W^2)^2\left(s(s-2m_N^2)-\frac{m_N^2}{m_W^2}(m_N^2+2m_W^2)^2\right),$$
$$b^-=\sqrt{s}m_N(m_N^2-2m_W^2)^2(m_N^2-m_W^2)(s-2m_N^2) \tag{3.9}$$
$$c^-=-m_N^4(m_N^2-2m_W^2)^2(s-2m_N^2).$$

Notice that the ratios b^+/a^+ and b^-/a^- are unequal (likewise the ratios c^+/a^+ and c^-/a^-), although $b^+/c^+=b^-/c^-$. Thus, the energy correlation of the two electrons in the final state is different for the cases $\eta_{CP}=\pm1$. This is illustrated in Fig. 3 for the hypothetical parameters $m_N=500$ GeV, $\sqrt{s}=1200$ GeV. It may be noted that the factor $f_2=4c_Vc_A$ does not appear in the spectrum $(d\sigma/d\mathcal{E}_1d\mathcal{E}_2)$, so that the energy correlation of e^+e^+ dileptons is the same as that of e^-e^-. In the limit $\beta\to1$, the term $a^\pm$ dominates and the "+" and "−" cases are no more distinguishable.

B. Angular correlation

Equation (3.4) also allows a calculation of the correlated angular distribution of the final state e^-e^- system. Defining the angles $\theta_{1,2}$ as in Eq. (3.5), and integrating over all other variables, we find

$$\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^{\pm}\sim\beta\int d\cos\theta_N\Bigg\{f_1\{\mp(m_N^2+2m_W^2)^2s\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}+(m_N^2-2m_W^2)^2s[\pm\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_N\beta-\cos\theta_1)(\cos\theta_N\beta$$
$$+\cos\theta_2)+\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\beta\cos\theta_N\cos\theta_1)+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\beta\cos\theta_N\cos\theta_2)]+2m_W^2s^2\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\cos^2\theta_N\beta^2)$$
$$-4m_N^2(m_N^2-2m_W^2)^2\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\cos\theta_1\cos\theta_2)\}$$
$$+f_2 2(m_N^4-4m_W^4)s\begin{bmatrix}\cos\theta_n\beta(\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}-\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}) & : & +\\ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}\cos\theta_1+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}\cos\theta_2 & : & -\end{bmatrix}\Bigg\}, \tag{3.10}$$

with

$$\mathcal{K}_1^{\theta_{1(2)}}=\frac{2A_{1(2)}}{(A_{1(2)}^2-B_{1(2)}^2)^{3/2}},\quad \mathcal{K}_2^{\theta_{1(2)}}=\frac{2A_{1(2)}^2+B_{1(2)}^2}{(A_{1(2)}^2-B_{1(2)}^2)^{5/2}}, \tag{3.11}$$

$$A_{1(2)}=1-(+)\beta\cos\theta_N\cos\theta_{1(2)},\quad B_{1(2)}=\beta\sin\theta_N\sin\theta_{1(2)}.$$

The correlation (3.10) has been evaluated for $m_N=500$ GeV and $\sqrt{s}=1200$ GeV (using $f_1=1+4\sin^2\theta_W+8\sin^4\theta_W$, $f_2=1-4\sin^2\theta_W$) and is plotted in Fig. 4. There is a clear difference between the cases $\eta_{CP}=\pm1$. The angular correlation in Eq. (3.10) becomes particularly transparent near the threshold $\beta\to0$, where we obtain the analytic results

$$\frac{1}{\sigma^+}\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^+_{\beta\to0}\approx\frac{1}{4}\left[1+\frac{1}{2}\frac{f_2}{f_1}\frac{m_N^2-2m_W^2}{m_N^2+2m_W^2}\times(\cos\theta_1+\cos\theta_2)\right], \tag{3.12}$$

$$\frac{1}{\sigma^-}\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^-_{\beta\to0}\approx\frac{1}{4}\left[1+\frac{(m_N^2-2m_W^2)^2}{(m_N^2+2m_W^2)^2}\cos\theta_1\cos\theta_2+\frac{f_2}{f_1}\frac{(m_N^2-2m_W^2)}{(m_N^2+2m_W^2)}\times(\cos\theta_1+\cos\theta_2)\right]. \tag{3.13}$$

Notice that the distributions in the variables $\cos\theta_1$ and $\cos\theta_2$ become flat in the case $\eta_{CP}=+1$ when f_2/f_1 is neglected. By contrast, there remains a nontrivial correlation for $\eta_{CP}=-1$, even in the absence of f_2. As before, the above results for e^-e^- hold for e^+e^+ if one replaces $f_2\to-f_2$.

IV. UNLIKE-SIGN DILEPTONS: THE REACTION $e^+e^-\to N_1N_2\to W^+W^-e^+e^-$

Proceeding as in Sec. III, the matrix element for the reaction $e^+e^-\to N_1N_2\to W^+W^-e^+e^-$ (Fig. 2) is

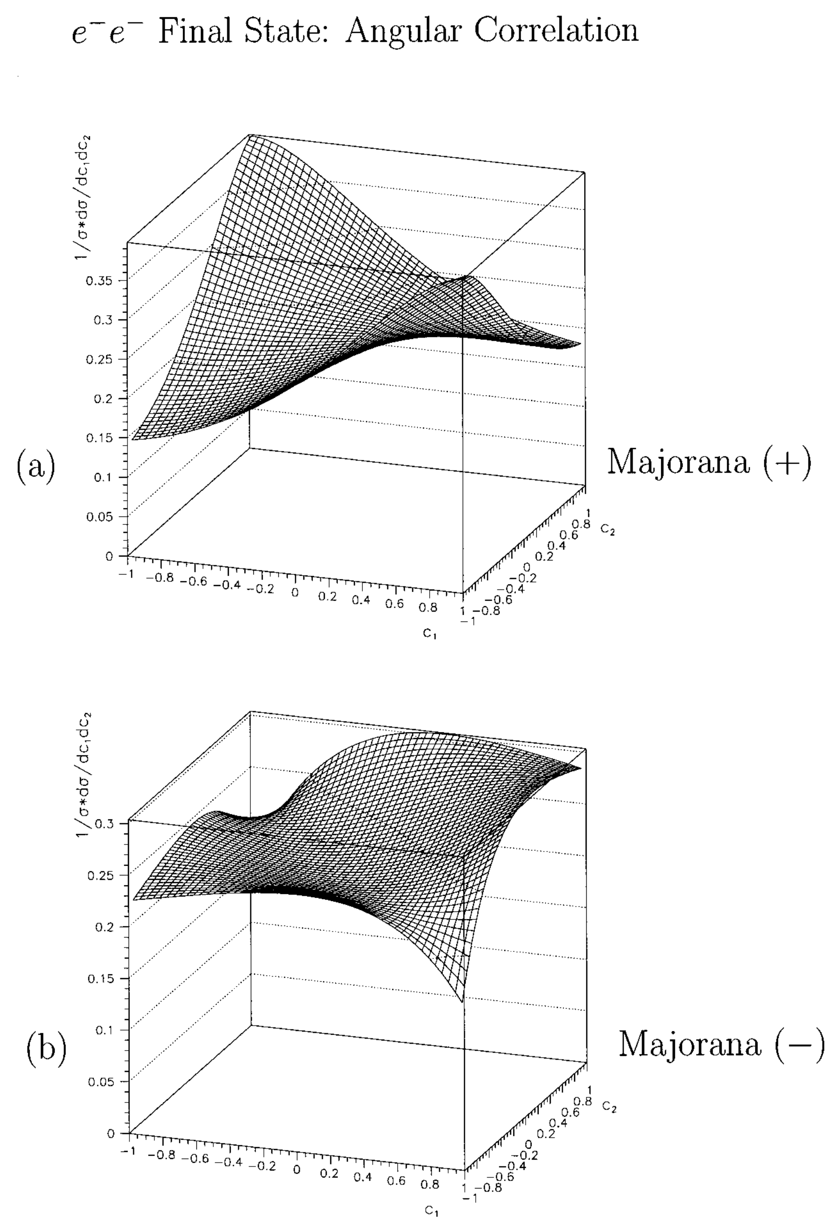

FIG. 4. Angle correlation of e^-e^- dileptons in $e^+e^-\to N_1N_2\to e^-W^+e^-W^+$, for (a) $\eta_{CP}=+1$, (b) $\eta_{CP}=-1$.

$$\mathcal{M}_m = iAj^e_\mu\Delta^{\mu\nu}_Z \frac{1}{q_1^2-m_1^2+im_1\Gamma_1}\,\frac{1}{q_2^2-m_2^2+im_2\Gamma_2} \times[\lambda_2\overline{u_{t_1}}(k_1)\gamma_\rho \not{q}_1\gamma_\nu \not{q}_2\gamma_\sigma \tfrac{1}{2}(1-\gamma_5)v_{t_2}(k_2) + \lambda_1 m_1 m_2\overline{u_{t_1}}(k_1)\gamma_\rho\gamma_\nu\gamma_\sigma \tfrac{1}{2}(1-\gamma_5)v_{t_2}(k_2)] \times\epsilon^{*\rho}_{\lambda_3}(k_3)\epsilon^{*\sigma}_{\lambda_4}(k_4). \tag{4.1}$$

The same final state, produced via a Dirac particle-antiparticle pair ($e^+e^-\to N\overline{N}\to W^+W^-e^+e^-$), has the amplitude

$$\mathcal{M}_d = iAj^e_\mu\Delta^{\mu\nu}_Z \frac{1}{q_1^2-m_1^2+im_1\Gamma_1}\,\frac{1}{q_2^2-m_2^2+im_2\Gamma_2} \times\overline{u_{t_1}}(k_1)\gamma_\rho \not{q}_1\gamma_\nu \not{q}_2\gamma_\sigma \frac{1}{2}(1-\gamma_5)v_{t_2}(q_2) \times\epsilon^{*\rho}_{\lambda_3}(k_3)\epsilon^{*\sigma}_{\lambda_4}(k_4). \tag{4.2}$$

Summing (averaging) over final (initial) polarizations, and using the narrow-width approximation for the N_1, N_2 propagators, we obain the squared matrix elements

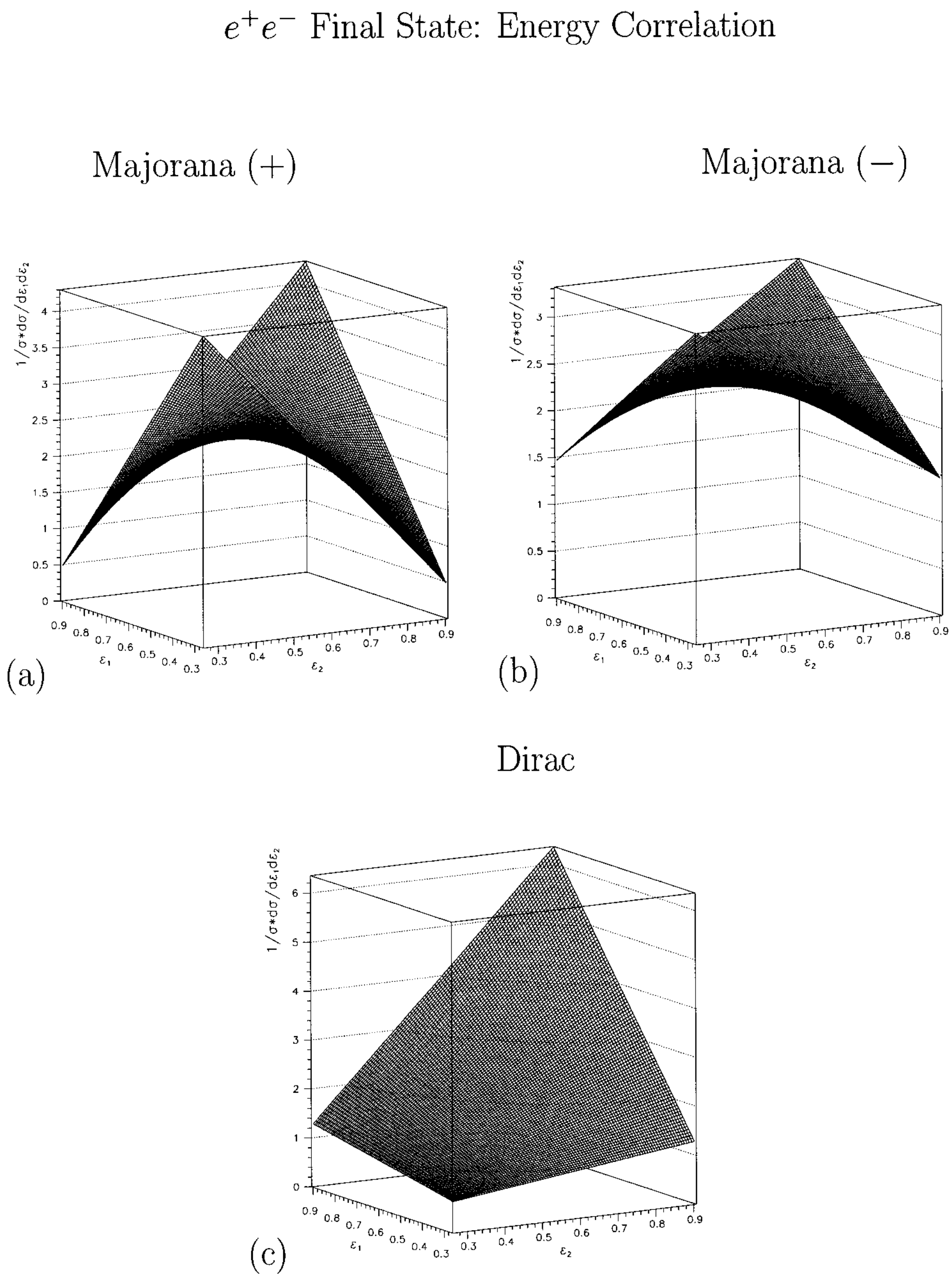

FIG. 5. Energy correlation of e^-e^+ dileptons in $e^+e^- \rightarrow N_1N_2 \rightarrow e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP}=+1$, (b) Majorana pair, $\eta_{CP}=-1$, (c) Dirac $N\underline{N}$-pair.

$$\begin{aligned}
|\overline{\mathcal{M}_\pm}|^2 = {} & \frac{|A|^2}{2}\frac{1}{(s-m_Z^2)^2}\frac{\pi}{m_N\Gamma_N}\delta(q_1^2-m_N^2)\frac{\pi}{m_N\Gamma_N}\delta(q_2^2-m_N^2)\frac{m_N^2}{m_W^4}\big(f_1\{\mp(m_N^2-m_W^2)^2(m_N^2+2m_W^2)^2 s \pm 4(m_N^2 \\
& -2m_W^2)^2[s(k_1k_2)(q_1q_2)-s(k_1q_2)(k_2q_1)-(k_1k_2)(q_1q)(q_2q)+(k_1k_2)(q_1l)(q_2l)+(k_1q_2)(k_2q)(q_1q)-(k_1q_2) \\
& \times(k_2l)(q_1l)+(k_1q)(k_2q_1)(q_2q)-(k_1l)(k_2q_1)(q_2l)-(k_1q)(k_2q)(q_1q_2)+(k_1l)(k_2l)(q_1q_2)\pm m_N^2((k_1q)(k_2q) \\
& -(k_1l)(k_2l))]-2(m_N^2-m_W^2)(m_N^2-2m_W^2)^2[(k_1q)(q_2q)-(k_1l)(q_2l)+(k_2q)(q_1q)-(k_2l)(q_1l)]+2(m_N^2 \\
& +4m_W^2/m_N^2)(m_N^2-m_W^2)^2[(q_1q)(q_2q)-(q_1l)(q_2l)]\}-2f_2\{(m_N^2-m_W^2)(m_N^4-4m_W^4)[\pm(k_1q)(q_1l)-(k_1q)(q_2l) \\
& \mp(k_1l)(q_1q)+(k_1l)(q_2q)\mp(k_2q)(q_2l)+(k_2q)(q_1l)\pm(k_2l)(q_2q)-(k_2l)(q_1q)]+2(m_N^2+4m_W^4/m_N^2)(m_N^2 \\
& -m_W^2)^2[(q_1q)(q_2l)-(q_2q)(q_1l)]\}\big), \qquad (4.3)
\end{aligned}$$

$$|\overline{\mathcal{M}_d}|^2=|A|^2\frac{1}{(s-m_Z^2)^2}\frac{\pi}{m_N\Gamma_N}\delta(q_1^2-m_N^2)\frac{\pi}{m_N\Gamma_N}\delta(q_2^2-m_N^2)\Bigg\{f_1\Bigg(\frac{m_N^4}{m_W^4}(m_N^2-2m_W^2)^2[(k_1q)(k_2q)-(k_1l)(k_2l)]$$
$$+2\frac{m_N^2}{m_W^2}(m_N^2-m_W^2)(m_N^2-2m_W^2)[(k_1q)(q_2q)-(k_1l)(q_2l)+(k_2q)(q_1q)-(k_2l)(q_1l)]+4(m_N^2-m_W^2)^2[(q_1q)(q_2q)$$
$$-(q_1l)(q_2l)]\Bigg)+f_2\Bigg(\frac{m_N^4}{m_W^4}(m_N^2-2m_W^2)^2[(q_1q)(k_2l)-(k_2q)(k_1l)]+2\frac{m_N^2}{m_W^2}(m_N^2-m_W^2)(m_N^2-2m_W^2)[(k_1q)(q_2l)$$
$$-(k_2q)(q_1l)+(q_1q)(k_2l)-(q_2q)(k_1l)]+4(m_N^2-m_W^2)^2[(q_1q)(q_2l)-(q_2q)(q_1l)]\Bigg)\Bigg\}. \tag{4.4}$$

In complete analogy with the discussion of like-sign leptons (Sec. III), we derive from the above equations the correlation in the energies and angles of the final e^+e^- state.

A. Energy correlation

The distribution in the scaled energies $\mathcal{E}_1$, $\mathcal{E}_2$ has the quadratic form given in Eq. (3.6), where the coefficients in the Majorana cases ''+'' and ''−'' and the Dirac case ''d'' now have the values

$$a^+=\tfrac{1}{2}m_N^2(m_N^2-m_W^2)^2\left(\frac{s}{m_N^2}(m_N^4+4m_W^4)-2(m_N^2+2m_W^2)^2\right),$$
$$b^+=-\sqrt{s}m_N^3(m_N^2-2m_W^2)^2(m_N^2-m_W^2),$$
$$c^+=m_N^6(m_N^2-2m_W^2)^2,$$
$$a^-=\tfrac{1}{2}m_N^2(m_N^2-m_W^2)^2\left[s(s-2m_N^2)\left(1+4\frac{m_W^4}{m_N^4}\right)-4(m_N^2+2m_W^2)^2\right],$$
$$b^-=-\sqrt{s}m_N(m_N^2-2m_W^2)^2(m_N^2-m_W^2)(s-2m_N^2), \tag{4.5}$$
$$c^-=m_N^4(m_N^2-2m_W^2)^2(s-2m_N^2),$$
$$a^d=(m_N^2-m_W^2)^2\left(4(s-m_N^2)(s-4m_N^2)\frac{m_W^4}{m_N^2}-(m_N^2-2m_W^2)\times[2(s-4m_N^2)m_W^2-m_N^2(m_N^2-2m_W^2)]\right),$$
$$b^d=\sqrt{s}m_N(m_N^2-2m_W^2)(m_N^2-m_W^2)[4sm_W^2-(m_N^2+14m_W^2)m_N^2],$$
$$c^d=m_N^4(m_N^2-2m_W^2)^2(s-3m_N^2).$$

The corresponding three distributions are plotted in Fig. 5. As in the case of like-sign dileptons, the e^+e^- pairs have distinct correlations for $\eta_{CP}=\pm 1$. A comparison of the Majorana cases with the Dirac case reveals an interesting difference. In the Majorana cases the total e^+e^- energy $Y=\mathcal{E}_1+\mathcal{E}_2$ is distributed symmetrically around the midpoint of this variable $Y_0=1/2(Y_{\min}+Y_{\max})$. By contrast, e^+e^- pairs resulting from Dirac NN primary state have a total energy distribution that is unsymmetric around the midpoint.

B. Angle correlation

In analogy to the distribution $d\sigma/d\cos\theta_1 d\cos\theta_2$ obtained for e^-e^- pairs [Eq. (3.10)], the result for unlike-sign dileptons e^+e^- is

$$\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^{\pm}\sim\beta\int d\cos\theta_N\Bigg(f_1\{\mp 2m_N^2(m_N^2+2m_W^2)^2 s\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}-2m_N^2(m_N^2-2m_W^2)^2 s[\pm\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_N\beta-\cos\theta_1)$$
$$\times(\cos\theta_N\beta+\cos\theta_2)+\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\beta\cos\theta_N\cos\theta_1)+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\beta\cos\theta_N\cos\theta_2)]$$
$$+(m_N^4+4m_W^4)s^2\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\cos^2\theta_n\beta^2)+8m_N^4(m_N^2-2m_W^2)^2\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\cos\theta_1\cos\theta_2)\}$$
$$+f_2 2(m_N^4-4m_W^4)s\left\{-\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}s\cos\theta_N\beta+2m_N^2\begin{bmatrix}\cos\theta_N\beta(\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}) & : & + \\ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}\cos\theta_1-\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}\cos\theta_2 & : & -\end{bmatrix}\right\}\Bigg), \tag{4.6}$$

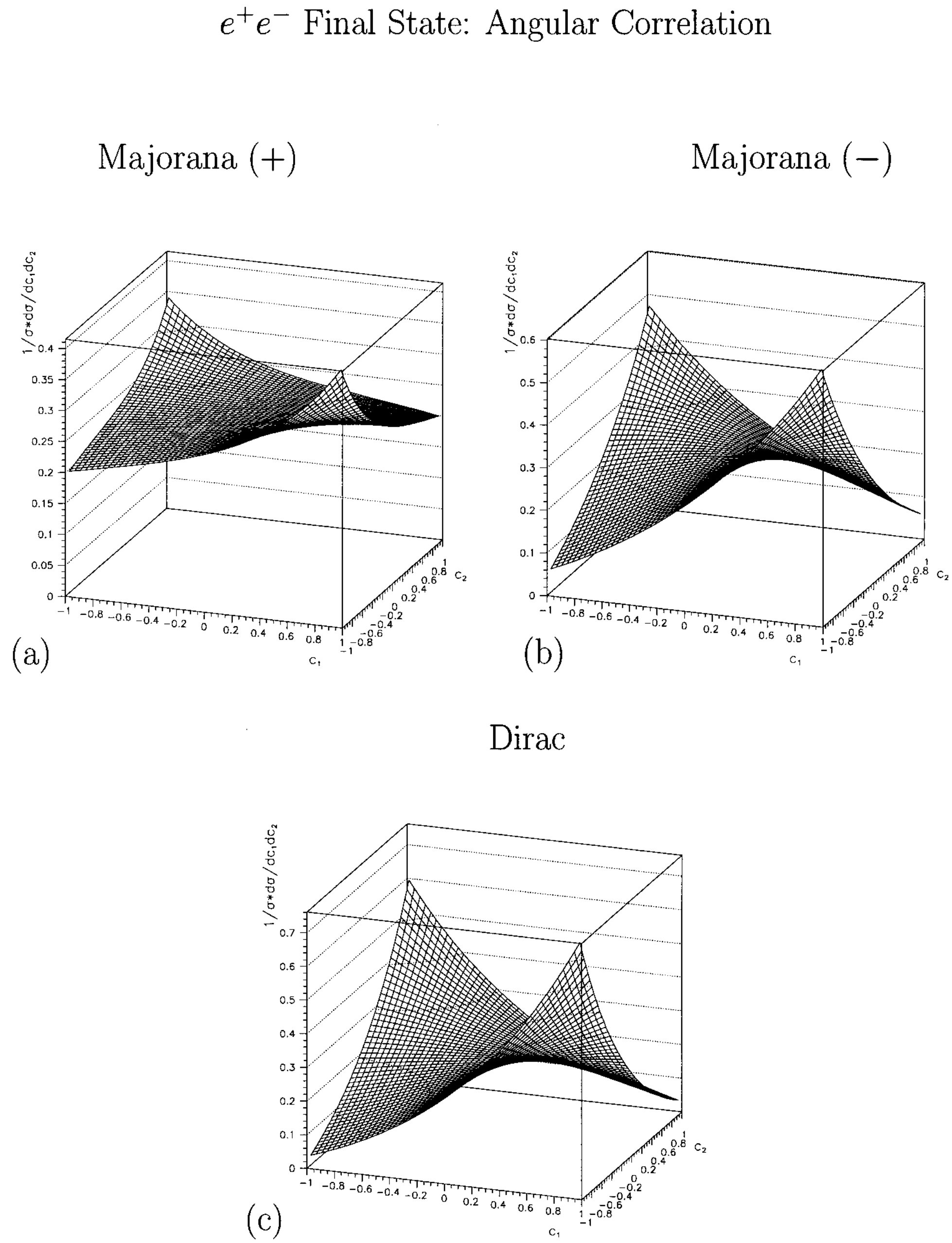

FIG. 6. Angle correlation of e^-e^+ dileptons in $e^+e^-\to N_1N_2 \to e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP}=+1$, (b) Majorana pair, $\eta_{CP}=-1$, (c) Dirac $N\bar{N}$-pair.

$$\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^d \sim \beta\int d\cos\theta_N(f_1\{s^2m_W^4\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\cos\theta_N\beta^2)+m_N^2m_W^2(m_N^2-2m_W^2)s[\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1+\beta\cos\theta_N\cos\theta_1)$$
$$+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\beta\cos\theta_N\cos\theta_2)]+m_N^4(m_N^2-2m_W^2)^2\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\cos\theta_1\cos\theta_2)\}+f_2\{2s^2m_W^4\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}\cos\theta_N\beta$$
$$+m_N^2m_W^2(m_N^2-2m_W^2)s[\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_N\beta+\cos\theta_1)+\mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_N\beta-\cos\theta_2)]$$
$$+m_N^4(m_N^2-2m_W^2)^2\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_1-\cos\theta_2)\}). \quad (4.7)$$

As usual, the indices "+," "−," and "d" differentiate between the Majorana cases $\eta_{CP}=+1, -1$ and the Dirac case. The angle correlations expressed by Eqs. (4.6) and (4.7) are plotted in Fig. 6, where the differences between the three cases are obvious. Close to threshold ($\beta\to 0$), the correlation between $\cos\theta_1$ and $\cos\theta_2$ can be presented in analytic form

$$\frac{1}{\sigma^+}\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^+_{\beta\to 0}\approx\frac{1}{4}\left[1+\frac{1}{2}\frac{f_2}{f_1}\frac{m_N^2-2m_W^2}{m_N^2+2m_W^2}\times(\cos\theta_1-\cos\theta_2)\right], \quad (4.8)$$

$$\frac{1}{\sigma^d}\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^d_{\beta\to 0}=\frac{1}{\sigma^-}\left(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2}\right)^-_{\beta\to 0}\approx\frac{1}{4}\left[1-\frac{(m_N^2-2m_W^2)^2}{(m_N^2+2m_W^2)^2}\cos\theta_1\cos\theta_2+\frac{f_2}{f_1}\frac{(m_N^2-2m_W^2)}{(m_N^2+2m_W^2)}(\cos\theta_1-\cos\theta_2)\right]. \quad (4.9)$$

In this limit, the cases ''+'' and ''−'' remain distinct, but the case $\eta_{CP}=-1$ converges to the Dirac case.

V. COMMENTS

We comment briefly on some other papers which have a partial overlap with the considerations presented above.

(i) Our discussion of the production reaction $e^+e^-\to N_1N_2$ follows very closely that given in Ref. [1]. Our results for $d\sigma/d\Omega$ given in Appendix A [Eqs. (A1)–(A6)] essentially coincide with those in that paper, with two minor differences: The angular distributions for the case of two distinct Majorana particles with the same CP parity, as well as for the case of two distinct Dirac particles [Eqs. (4E) and (4D) in Ref. [1]], are slightly different from our distributions, presented in Appendix A [Eq. (A2) and (A3)].

(ii) The cross section for the Majorana process $e^+e^-\to N_1N_2$, with $m_1=m_2$ and $\eta_{CP}=+1$ calculated in Ref. [2] agrees with that obtained in this paper. However, the Dirac case $e^+e^-\to N\bar{N}$ [Eq. (2) of Ref. [2]] differs from our result [Eq. (A5)], as also noted in Ref. [1].

(iii) The spin-summed differential cross section for the Majorana process $e^+e^-\to N_1N_2$ (with $m_1=m_2$, $\eta_{CP}=+1$) calculated in the present paper, as well as in Refs. [1,2], differs from that given in Ref. [4], but agrees with the results given in Refs. [3,6,10].

(iv) Our analysis of heavy Majorana production and decay has been essentially model independent. Discussions in the context of specific gauge models, based on $SU(2)_L\times SU(2)_R\times U(1)$ or E(6) symmetries, may be found in Refs. [6,10,11].

APPENDIX A: DIFFERENTIAL CROSS SECTION FOR $e^+e^-\to N_1N_2$

Following Ref. [1], we consider the following five cases, where N_1 and N_2 are

(A) Distinct Dirac particles.
(B) Distinct Majorana particles with the same CP parity.
(C) Distinct Majorana particles with opposite CP parity.
(D) Dirac particle-antiparticle pair.
(E) Identical Majorana particles.

Choosing the N_1 direction in the e^+e^- c.m. system to be the z axis, and the e^--beam direction to be at an angle θ (= scattering angle), the momenta (q_1, q_2) and spins (t_1, t_2) of N_1 and N_2 have components

$$\begin{aligned}N_1:\ & q_1^\mu=(\gamma m_1,0,0,\gamma\beta m_1),\\ & t_1^\mu=(\gamma\beta n_z,n_x,n_y,\gamma n_z),\\ N_2:\ & q_2^\mu=(\gamma' m_2,0,0,-\gamma'\beta' m_2),\\ & t_2^\mu=(-\gamma'\beta' n_z',n_x',n_y',\gamma' n_z').\end{aligned} \quad (A1)$$

The differential cross sections are [with $\beta=(1-4m_1^2/s)^{1/2}$, $\beta'=(1-4m_2^2/s)^{1/2}$, $\gamma=(1-\beta^2)^{-1/2}$, $\gamma'=(1-\beta'^2)^{-1/2}$, $\lambda(x,y,z)=[x^2+y^2+z^2-2(xy+yz+zx)]^{1/2}$]

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_A=&\frac{G_F^2\alpha_N^2}{512\pi^2}|R(s)|^2[1-(m_1^2-m_2^2)^2/s^2]\lambda(s,m_1^2,m_2^2)\\ &\times\{f_1[(1+C^2\beta'\beta)-(\beta n_z'+\beta' n_z)(1+C^2)\\ &-(\beta' n_x/\gamma+\beta n_x'/\gamma')SC+n_zn_z'(C^2+\beta\beta')\\ &+(n_xn_z'/\gamma+n_x'n_z/\gamma')SC+n_xn_x'S^2/\gamma\gamma']\\ &+f_2[C(\beta+\beta')-(n_z+n_z')C(1+\beta\beta')-(n_x/\gamma\\ &+n_x'/\gamma')S+n_zn_z'C(\beta+\beta')+S(\beta' n_xn_z'/\gamma\\ &+\beta n_x'n_z/\gamma')]\},\end{aligned} \quad (A2)$$

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_B=&\frac{G_F^2\alpha_N^2}{256\pi^2}|R(s)|^2[1-(m_1^2-m_2^2)^2/s^2]\lambda(s,m_1^2,m_2^2)\\ &(f_1\{n_xn_x'S^2(1/\gamma\gamma'-1)+n_yn_y'S^2\beta\beta'+n_zn_z'[\beta\beta'\\ &-C^2(1/\gamma\gamma'-1)]+(n_xn_z'-n_x'n_z)SC(1/\gamma-1/\gamma')\\ &+C^2\beta\beta'-1/\gamma\gamma'+1\}+f_2\{(n_x-n_x')S(1/\gamma'-1/\gamma)\\ &+(n_z+n_z')C(1/\gamma\gamma'-\beta\beta'-1)\}),\end{aligned} \quad (A3)$$

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right)_C=&\frac{G_F^2\alpha_N^2}{256\pi^2}|R(s)|^2[1-(m_1^2-m_2^2)^2/s^2]\lambda(s,m_1^2,m_2^2)\\ &\times(f_1\{n_xn_x'S^2(1/\gamma\gamma'+1)-n_yn_y'S^2\beta\beta'\\ &+n_zn_z'[\beta\beta'+C^2(1/\gamma\gamma'+1)]+(n_xn_z'\\ &+n_x'n_z)SC(1/\gamma+1/\gamma')+C^2\beta\beta'+1/\gamma\gamma'+1\}\\ &-f_2\{(n_x+n_x')S(1/\gamma+1/\gamma')\\ &+(n_z+n_z')C(1/\gamma\gamma'+\beta\beta'+1)\}),\end{aligned} \quad (A4)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_D=\left(\frac{d\sigma}{d\Omega}\right)_{A,m_1=m_2}=\frac{G_F^2\alpha_N^2}{512\pi^2}|R(s)|^2\lambda(s,m^2,m^2)$$
$$\times\{f_1[(1+C^2\beta^2)-(n_z+n_z')\beta(1+C^2)$$
$$-(n_x+n_x')SC\beta/\gamma+n_zn_z'(C^2+\beta^2)$$
$$+(n_xn_z'+n_zn_x')SC/\gamma+n_xn_x'S^2/\gamma^2]$$
$$+f_2[2C\beta-(n_z+n_z')C(1+\beta^2)-(n_x+n_x')S/\gamma$$
$$+2n_zn_z'C\beta+(n_xn_z'+n_x'n_z)S\beta/\gamma]\}, \quad \text{(A5)}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_E=\frac{1}{2}\cdot\left(\frac{d\sigma}{d\Omega}\right)_{B,m_1=m_2}$$
$$=\frac{G_F^2\alpha_N^2}{512\pi^2}|R(s)|^2\lambda(s,m^2,m^2)\beta^2\{f_1[(n_yn_y'-n_xn_x')S^2$$
$$+(1+n_zn_z')(1+C^2)]-f_22(n_z+n_z')C\}. \quad \text{(A6)}$$

APPENDIX B: MATRIX ELEMENTS FOR $e^+e^-\rightarrow N_1N_2\rightarrow e^\pm e^- W^\mp W^+$

1. Like-sign dileptons

The matrix element for the reaction (Fig. 2)

$$e^+(p_2,s_2)+e^-(p_1,s_1)$$
$$\rightarrow e^-(k_1,t_1)e^-(k_2,t_2)W^+(k_3,\lambda_3)W^+(k_4,\lambda_4) \quad \text{(B1)}$$

is

$$\mathcal{M}=iAj_\mu^e\Delta_Z^{\mu\nu}\Bigg\{\lambda_2\overline{u_{t_1}}(k_1)\gamma_\rho\tfrac{1}{2}(1-\gamma_5)\frac{\not q_1+m_1}{q_1^2-m_1^2+im_1\Gamma_1}\gamma_\nu$$
$$\times\tfrac{1}{2}(1-\gamma_5)\frac{-\not q_2+m_2}{q_2^2-m_2^2+im_2\Gamma_2}\gamma_\sigma\tfrac{1}{2}(1+\gamma_5)v_{t_2}(k_2)$$
$$-\lambda_1\overline{u_{t_2}}(k_2)\gamma_\sigma\tfrac{1}{2}(1-\gamma_5)\frac{\not q_2+m_2}{q_2^2-m_2^2+im_2\Gamma_2}\gamma_\nu$$
$$\times\tfrac{1}{2}(1-\gamma_5)\frac{-\not q_1+m_1}{q_1^2-m_1^2+im_1\Gamma_1}\gamma_\rho$$
$$\times\tfrac{1}{2}(1+\gamma_5)v_{t_1}(k_1)\Bigg\}\epsilon_{\lambda_3}^{*\rho}(k_3)\epsilon_{\lambda_4}^{*\sigma}(k_4). \quad \text{(B2)}$$

Upon rearrangement, this gives the matrix element of Eq. (3.2).

2. Unlike-sign dileptons

The matrix element for the Majorana-mediated process (Fig. 2)

$$e^+(p_2,s_2)+e^-(p_1,s_1)$$
$$\rightarrow e^-(k_1,t_1)e^-(k_2,t_2)W^+(k_3,\lambda_3)W^+(k_4,\lambda_4) \quad \text{(B3)}$$

is

$$\mathcal{M}_m=-iAj_\mu^e\Delta_Z^{\mu\nu}\Bigg\{\lambda_2\overline{u_{t_1}}(k_1)\gamma_\rho\tfrac{1}{2}(1-\gamma_5)$$
$$\times\frac{\not q_1+m_1}{q_1^2-m_1^2+im_1\Gamma_1}\gamma_\nu\tfrac{1}{2}(1-\gamma_5)$$
$$\times\frac{-\not q_2+m_2}{q_2^2-m_2^2+im_2\Gamma_2}\gamma_\sigma\tfrac{1}{2}(1-\gamma_5)v_{t_2}(k_2)$$
$$-\lambda_1\overline{u_{t_1}}(k_1)\gamma_\rho\tfrac{1}{2}(1-\gamma_5)\frac{\not q_1+m_1}{q_1^2-m_1^2+im_1\Gamma_1}\gamma_\nu$$
$$\times\tfrac{1}{2}(1+\gamma_5)\frac{-\not q_2+m_2}{q_2^2-m_2^2+im_2\Gamma_2}\gamma_\sigma$$
$$\times\tfrac{1}{2}(1-\gamma_5)v_{t_2}(k_2)\Bigg\}\epsilon_{\lambda_3}^{*\rho}(k_3)\epsilon_{\lambda_4}^{*\sigma}(k_4). \quad \text{(B4)}$$

Upon rearrangement, this gives the matrix element of Eq. (4.1).

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