Consistency conditions for orientifolds and D-manifolds

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We study superstrings with orientifold projections and with generalized open string boundary conditions (D branes). We find two types of consistency condition, one related to the algebra of Chan-Paton factors and the other to cancellation of divergences. One consequence is that the Dirichlet five branes of the type I theory carry a symplectic gauge group, as required by string duality. As another application we study the type I theory on a $K3$ Z_2 orbifold, finding a family of consistent theories with various unitary and symplectic subgroups of U(16)$\times$U(16). We argue that the $K3$ orbifold with spin connection embedded in gauge connection corresponds to an interacting conformal field theory in the type I theory. [S0556-2821(96)05114-4]

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I. INTRODUCTION

One of the notable features of string duality has been the convergence of many previously disjoint lines of development. For example, certain once-obscure string backgrounds, namely orientifolds [1–3] and D manifolds [3], have proven to be dual to more familiar backgrounds [4–8]. In order to find the nonperturbative structure underlying string duality it is important to understand as fully as possible all limits of the theory. The purpose of the present paper is to develop the consistency conditions for orientifolds and D manifolds.

Orientifolds are generalized orbifolds. In the orbifold construction, discrete internal symmetries of the world-sheet theory are gauged. In the orientifold, products of internal symmetries with world-sheet parity reversal are also gauged. D manifolds are manifolds with special submanifolds (D branes) on which strings are allowed to end. These are labeled by a generalized Chan-Paton index, each value of which corresponds to restricting the string end point to a given submanifold of spacetime.

We will discuss consistency conditions of two types. The first comes from closure of the operator product expansion, which restricts the action of the discrete gauge symmetries on the Chan-Paton index. One consequence is that D five branes in type I string theory must have a symplectic rather than orthogonal gauge projection: this is a world-sheet derivation of a result previously found from string duality [7]. Also, D three branes and seven branes are inconsistent in the type I superstring, while D one branes have an orthogonal gauge projection.

The second condition is cancellation of divergences and anomalies at one loop [9], which can be recast in terms of consistency of the field equations [10]. Here we focus on a simple example, the type I theory on a $K3$ orbifold. We find all solutions to the consistency conditions, leading to gauge groups which are various unitary and symplectic subgroups

of U(16)$\times$U(16). Rather surprisingly, we do not find a solution with the spin connection embedded in the gauge connection. We argue that this theory, while it must exist, does not correspond to a free conformal field theory. Finally, we discuss various related work.

II. ORIENTIFOLDS AND CHAN-PATON FACTORS

The orientifold group contains elements of two kinds. The first are purely internal symmetries g of the world-sheet theory, forming a subgroup G_1. For the purposes of the present paper we will think of these as spacetime symmetries, though more generally (as in asymmetric orbifolds) one could consider symmetries whose spacetime interpretation is less clear. The second are elements of the form Ωh, where Ω is the world-sheet parity transformation and h is again a spacetime symmetry, now chosen from a set G_2. Closure implies that $\Omega h \Omega h' \in G_1$ for $h, h' \in G_2$, and if all elements of G_2 commute with Ω this is simply $G_2 G_2 = G_1$. The full orientifold group is $G = G_1 + \Omega G_2$.

In the orientifold construction this group is gauged, meaning that one sums over all group elements around any nontrivial path on the world sheet. This projects onto states invariant under G_1 and ΩG_2. Elements of G_1 also lead to twisted closed strings, from a gauge transformation in going around a closed string. The factor ΩG_2 means that orientation reversal (combined with a G_2 action on the fields) is now part of the local symmetry group, so that unoriented world sheets are included. The elements of ΩG_2 do not give rise directly to new (twisted) sectors of the string Hilbert space; we will discuss later the extent to which it is useful to think of the open strings as being these twisted states.

The Chan-Paton index i labels a set of a submanifolds (D branes) M_i, with a string end point in state i constrained to lie on M_i. Some of the M_i may be coincident. Each element of the discrete gauge group will have some action on the Chan-Paton index. Denote a general open string state by $|\psi, ij\rangle$, where ψ is the state of the world-sheet fields and i and j are the Chan-Paton states of the left and right end points; the boundary conditions on the fields in ψ are of

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course i,j dependent. The elements g act on this as

$$g:|\psi,ij\rangle\rightarrow(\gamma_g)_{ii'}|g\cdot\psi,i'j'\rangle(\gamma_g^{-1})_{j'j}. \quad (2.1)$$

for some matrix γ_g associated with g. This form is determined by the requirement that a general trace of products of wave functions be invariant. The action on the Chan-Paton factors must also be consistent with the action on the fields. That is, for each D brane M_i, the spacetime-transformed D brane $g\cdot M_i$ must appear, and the only nonzero elements of γ_g are those connecting M_i and $g\cdot M_i$. If M_i is left fixed by g then diagonal elements are allowed. Similarly,

$$\Omega h:|\psi,ij\rangle\rightarrow(\gamma_{\Omega h})_{ii'}|\Omega h\cdot\psi,j'i'\rangle(\gamma_{\Omega h}^{-1})_{j'j}. \quad (2.2)$$

Note that the orientation reversal transposes the two end points. The γ_g and $\gamma_{\Omega h}$ are unitary.

To derive further constraints on the matrices γ_g and $\gamma_{\Omega h}$, let us first demonstrate that the discrete gauge group may not include pure gauge twists, those with $g=1\in G_1$ with γ_1 nontrivial. The point is that the allowed Chan-Paton wave functions must form a complete set: the set of string wave functions $|\psi,ij\rangle$ must include nontrivial states for all pairs ij. One can see this heuristically by noting that if there are states ik and jl for some k and l [and, therefore, also lj by CPT invariance], then by a splitting-joining interaction one obtains also ij and lk. This interaction occurs in the interior of the string and so by locality cannot depend on the values of the end points. One can make this precise by requiring that the annulus factorize correctly on the closed string poles, so this is actually a one-loop condition—at tree level it would be consistent to truncate to block-diagonal wave functions. Now, if the identity appears in G_1, we have the projection

$$|\psi,ij\rangle=(\gamma_1)_{ii'}|\psi,i'j'\rangle(\gamma_1^{-1})_{j'j}. \quad (2.3)$$

Since this holds for a complete set, Schur's lemma implies that $\gamma_1\propto1$; we may as well set $\gamma_1=1$ because the overall phase is irrelevant.

This implies a further restriction on the γ_g and $\gamma_{\Omega h}$: they must satisfy the algebra of the corresponding symmetries, up to a phase. For example, $\gamma_{g_1}\gamma_{g_2}\gamma_{g_2^{-1}g_1^{-1}}\propto1$, else we would contradict the result in the previous section. As another example, suppose that G_1 includes an element of order 2, $g^2=1$. Then on a string state,

$$g^2:|\psi,ij\rangle\rightarrow(\gamma_g^2)_{ii'}|\psi,i'j'\rangle(\gamma_g^{-2})_{j'j}, \quad (2.4)$$

and so (by choice of phase)

$$\gamma_g^2=1. \quad (2.5)$$

Similarly, if G_2 includes an element of order 2, then $(\Omega h)^2$ acts as

$$(\Omega h)^2:|\psi,ij\rangle\rightarrow[\gamma_{\Omega h}(\gamma_{\Omega h}^T)^{-1}]_{ii'}|\psi,i'j'\rangle(\gamma_{\Omega h}^T\gamma_{\Omega h}^{-1})_{j'j}, \quad (2.6)$$

implying that

$$\gamma_{\Omega h}^T=\pm\gamma_{\Omega h}. \quad (2.7)$$

Let us apply this to the type I theory. The type I theory is an orientifold of the type IIB theory with the single non-trivial element Ω; that is, $h=h^2=1$. Tadpole cancellation, to be reviewed in the next section, requires that the orientifolding be accompanied by the inclusion of n nine branes, corresponding to purely Neumann boundary conditions. If γ_Ω is symmetric, we can choose a basis such that $\gamma_\Omega=I$. If γ_Ω is antisymmetric, we can choose a basis such that γ_Ω is the symplectic matrix

$$M=\begin{bmatrix}0 & iI\\-iI & 0\end{bmatrix}, \quad (2.8)$$

where n must be even. For the massless open string vector, the Ω eigenvalue of the oscillator state is -1. For γ_Ω symmetric, the Chan-Paton wave function of the vector is then antisymmetric, giving the gauge group SO(n). For γ_Ω antisymmetric, the massless vectors form the adjoint of USp(n). Tadpole cancellation requires SO(32).

Now let us consider adding five branes. The Chan-Paton index runs over both nine branes and five branes. The only freedom in Eq. (2.7) is the overall sign. Since we are required to take the SO projection on the nine branes it appears that we are required to take the same projection on the five branes. This is in contradiction with Ref. [7], where it was found that string duality requires a symplectic gauge group on the type I five brane. To understand this we need to be somewhat more careful.

The point is that, although Ω^2 acts trivially on the world-sheet fields, it may be a nontrivial phase in various sectors of the Hilbert space. The phase of Ω is determined by the requirement that it be conserved by the operator product of the corresponding vertex operators. Thus, the massless vector state, with vertex operator $\partial_t X^\mu$, necessarily has $\Omega=-1$ because Ω changes the orientation of the tangent derivative ∂_t; we have used this fact two paragraphs previously. In the 55 sector (that is, strings with both ends on a five brane), for the massless vertex operator is $\partial_t X^\mu$ ($\Omega=-1$) for μ parallel to the five brane, and $\partial_n X^\mu$ ($\Omega=+1$) for μ perpendicular. On these states, $\Omega^2=1$, and the same is true for the rest of the 99 and 55 Hilbert spaces. To see this, use the fact that Ω multiplies any mode operator ψ_r by $\pm e^{i\pi r}$. (Details of the mode expansions are given in Sec. III C.) In the Neveu-Schwarz sector this is $\pm i$, but the Gliozzi-Scherk-Olive (GSO) projection requires that these modes operators act in pairs.[1] So $\Omega=\pm1$, and this holds in the R sector as well by supersymmetry.

Now consider the Neveu-Schwarz 59 sector. The four X^μ with mixed Neumann-Dirichlet boundary conditions, say $\mu=6,7,8,9$, have a half-integer mode expansion. Their superconformal partners ψ^μ then have an integer mode expansion and the ground state is a representation of the corresponding Clifford algebra. The vertex operator is thus a spin field: the periodic ψ^μ contribute a factor $V=e^{i(H_3+H_4)/2}$, where $H_{3,4}$ are from the bosonization of the four periodic $\psi^{6,7,8,9}$ [11]. We need only consider this part of the vertex operator, as the rest is the same as in the 99 string and so has

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[1]The operator product expansion (OPE) is single valued only for GSO-projected vertex operators.

$\Omega^2 = +1$. Now, the operator product of V with itself (which is in the 55 or 99 sector) involves $e^{i(H_3+H_4)}$, which is the bosonization of $(\psi^6+i\psi^7)(\psi^8+i\psi^9)$. This in turn is the vertex operator for the state $(\psi^6+i\psi^7)_{-1/2}(\psi^8+i\psi^9)_{-1/2}|0\rangle$. Finally we can deduce the Ω eigenvalue. For $|0\rangle$ it is $+1$, because its vertex operator is the identity, while each $\psi_{-1/2}$ contributes either $-i$ (for a 99 string) or $+i$ (for a 55 string), for an overall -1. That is, the Ω eigenvalue of $V \cdot V$ is -1, so therefore is the Ω^2 eigenvalue of V.

Returning to Eq. (2.6), in the 59 sector there is an extra -1 from the above argument. Separate γ_Ω into a block $\gamma_{\Omega,9}$ which acts on the nine branes and a block $\gamma_{\Omega,5}$ which acts on the five branes. We have $\gamma_{\Omega,9}^T = +\gamma_{\Omega,9}$ from tadpole cancellation. To cancel the sign in the 59 sector we then need $\gamma_{\Omega,5}^T = -\gamma_{\Omega,5}$, giving symplectic groups on the five brane as found in Ref. [7]. This argument seems roundabout, but it is faithful to the logic that the actions of Ω in the 55 and 99 sectors are related because they are both contained in the 59 $\times$ 95 product. Further, there does not appear to be any arbitrariness in the result.

Let us briefly review the consequences of this projection [7]. Consider a pair of coincident five branes, since the symplectic projection requires an even number. The world-brane vectors $\partial_t X^\mu$ (μ parallel to the five brane) have Chan-Paton wave functions σ_{ij}^a, gauge group $USp(2) = SU(2)$. The world-brane scalars $\partial_n X^\mu$ (μ perpendicular to the five brane) have Chan-Paton wave function δ_{ij}. Since these are the collective coordinates for five branes [3], the wave function δ_{ij} means that the two five branes move together as a unit. The need for this can also be seen in another way [12]. In the type I theory the force between five branes, and between one branes, is half of that calculated in Ref. [5] because of the orientation projection. The product of the charges of a single one brane and single five brane would then be only half a Dirac-Teitelboim-Nepomechie unit; but since the five branes are always paired the quantization condition is respected.

The IIB theory also contains $p =$ one, three, and seven branes. The above argument gives $\Omega^2 = (\pm i)^{(9-p)/2}$. This requires an SO projection on the one brane, consistent with type I-heterotic duality. On the three and seven branes it leads to an inconsistency. This is a satisfying result, as there is no conserved charge in the type I theory to give rise to such p branes.

We do not know that we have found the complete set of consistency conditions of this type, but no others are evident to us.

III. TADPOLES

Modular invariance on the torus is one of the central consistency requirements for closed oriented strings. For open and unoriented one-loop graphs there is no corresponding modular group, but cancellation of divergences plays an analogous role in constraining the theory [9]. In Refs. [13,10] these divergences were obtained in the ten-dimensional type I theory from one-loop vacuum amplitudes. In Ref. [10] they were reinterpreted in terms of an inconsistency in the field equation for a Ramond-Ramond (RR) ten-form potential. It is useful to recall the latter interpretation, now generalized to all RR forms. D-branes and orientifold fixed planes are electric and magnetic sources for the RR fields [5]. The n-form field strength H_n thus satisfies

$$dH_n = {}^*J_{9-n}, \quad d{}^*H_n = {}^*J_{n-1}, \qquad (3.1)$$

where J_{9-n} and J_{n-1} are sources of the indicated rank. The field equations are consistent only if

$$\int_{C_k} {}^*J_{10-k} = 0 \qquad (3.2)$$

for all closed curves C_k. In flat $d = 10$ the only nontrivial closed curves are the points C_0, and the corresponding constraint on J_{10} requires the gauge group SO(32). In a compact theory there will be more constraints.

More generally, the right-hand side of the field Eqs. (3.1) will include additional terms from Chern-Simons couplings of the RR fields to curvature and gauge field strengths. In the present work we consider only orientifolds of flat backgrounds, but the more general case will also be interesting.

The tadpole constraints were applied to orientifolds in Refs. [2,15–17]. Many of the results in the present section can be found in Ref. [2], except that our treatment of the Chan-Paton factors will be more general.

A. General framework

The divergences can be determined from the vacuum amplitudes on the Klein bottle (KB), Möbius strip (MS), and cylinder (C). In Fig. 1 these surfaces are all depicted as cylinders of length $2\pi l$ and circumference 2π, with the ends being either boundaries or crosscaps. Taking coordinates $0 \leq \sigma^1 \leq 2\pi l$, $0 \leq \sigma^2 \leq 2\pi$, the periodicity and boundary conditions on generic world-sheet fields ϕ (and their derivatives) are as follows:
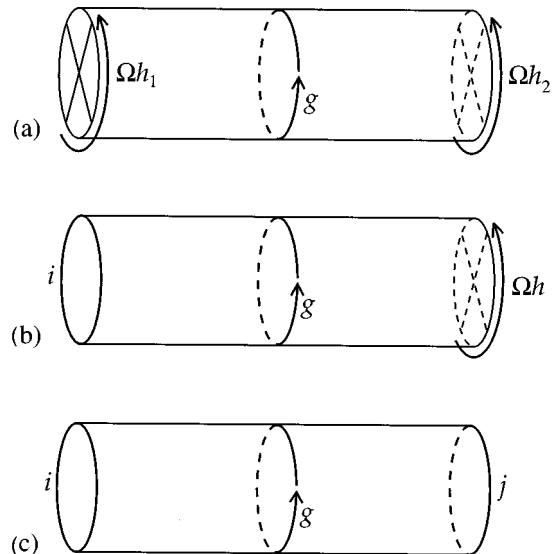

FIG. 1. Riemann surfaces described by Eq. (3.3). (a) Klein bottle. (b) Möbius strip. (c) Cylinder.

FIG. 2. Tadpoles in the g-twisted sector. (a) Crosscap: fields at opposite points differ by an Ωh transformation, where $g = (\Omega h)^2$. (b) Boundary in state i. The manifold M_i must be fixed under g.

KB: $\quad \phi(0, \pi + \sigma^2) = \Omega \widetilde{h}_1 \phi(0, \sigma^2),$

$\quad\quad \phi(2\pi l, \pi + \sigma^2) = \Omega \widetilde{h}_2 \phi(2\pi l, \sigma^2),$

$\quad\quad \phi(\sigma^1, 2\pi + \sigma^2) = \widetilde{g} \phi(\sigma^1, \sigma^2),$

MS: $\quad \phi(0, \sigma^2) \in \widetilde{M}_i, \quad \phi(2\pi l, \pi + \sigma^2) = \Omega \widetilde{h} \phi(2\pi l, \sigma^2),$

$\quad\quad \phi(\sigma^1, 2\pi + \sigma^2) = \widetilde{g} \phi(\sigma^1, \sigma^2),$

C: $\quad \phi(0, \sigma^2) \in \widetilde{M}_i, \quad \phi(2\pi, \sigma^2) \in \widetilde{M}_j,$

$\quad\quad \phi(\sigma^1, 2\pi + \sigma^2) = \widetilde{g} \phi(\sigma^1, \sigma^2). \quad\quad (3.3)$

It is convenient to include in the periodicity or boundary conditions g, h, and M_i, besides the spacetime part discussed earlier, $a \pm 1$ on the world-sheet fermions from the GSO projection; the tilde is a reminder of this additional information. The respective definitions (3.3) are consistent only if

KB: $\quad (\Omega \widetilde{h}_1)^2 = (\Omega \widetilde{h}_2)^2 = \widetilde{g},$

MS: $\quad (\Omega \widetilde{h})^2 = \widetilde{g}, \quad \widetilde{g} \widetilde{M}_i = \widetilde{M}_i,$

C: $\quad \widetilde{g} \widetilde{M}_i = \widetilde{M}_i, \quad \widetilde{g} \widetilde{M}_j = \widetilde{M}_j; \quad\quad (3.4)$

else the corresponding path integral vanishes.

These graphs will have divergences from the tadpoles shown in Fig. 2. If there are m noncompact dimensions, the dangerous tadpoles will be from those massless RR states which are m forms in the noncompact directions. In general there are several such tadpoles, coming from twisted and untwisted sectors.

To write the Klein bottle and Möbius strips in terms of traces, take the alternate coordinate region $0 \leqslant \sigma^1 \leqslant 4\pi l$, $0 \leqslant \sigma^2 \leqslant \pi$ with periodicities[2]

KB: $\phi(\sigma^1, \pi + \sigma^2) = \Omega \widetilde{h}_2 \phi(4\pi l - \sigma^1, \sigma^2),$

$\quad\quad \phi(4\pi l, \sigma^2) = \widetilde{g}' \phi(0, \sigma^2),$

MS: $\phi(\sigma^1, \pi + \sigma^2) = \Omega \widetilde{h} \phi(4\pi l - \sigma^1, \sigma^2), \quad \phi(0, \sigma^2) \in \widetilde{M}_i,$

$\quad\quad \phi(4\pi l, \sigma^2) \in \widetilde{M}_i, \quad\quad (3.5)$

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[2] This is done by taking the upper strip $\pi \leqslant \sigma^2 \leqslant 2\pi$, inverting it from right to left and multiplying the fields by $(\Omega \widetilde{h}_2)^{-1}$, and gluing it to the right side of the lower strip: with this construction the fields are smooth at $\sigma^1 = 2\pi l$.

where $\widetilde{g}' = \Omega \widetilde{h}_2 (\Omega \widetilde{h}_1)^{-1}$. Rescaling the coordinates to standard length (π for open strings and 2π for closed), the respective amplitudes are

KB: $\quad \text{Tr}_{c,g'}(\Omega \widetilde{h}_2 (-1)^{F + \widetilde{F}} e^{\pi(L_0 + \widetilde{L}_0)/2l}),$

MS: $\quad \text{Tr}_{o,ii}(\Omega \widetilde{h} (-1)^F e^{\pi L_0 / 4l}),$

C: $\quad \text{Tr}_{o,ij}(\widetilde{g} (-1)^F e^{\pi L_0 / l}). \quad\quad (3.6)$

The closed string trace is labeled by the spacelike twist g' and the open string traces are labeled by the Chan-Paton states.

The full one-loop amplitude is

$$\int_0^\infty \frac{dt}{2t} \{ \text{Tr}_c(\mathbf{P}(-1)^{\mathbf{F}} e^{-2\pi t(L_0 + \widetilde{L}_0)}) + \text{Tr}_o(\mathbf{P}(-1)^{\mathbf{F}} e^{-2\pi t L_0}) \},$$

$$(3.7)$$

where $\mathbf{P}$ includes the GSO and G projections, and $\mathbf{F}$ is the spacetime fermion number. The traces are over the transverse oscillator states and include a spacetime momentum sum. The sums in the projection operators and over twisted sectors and Chan-Paton states are equivalent to summing the surfaces in Fig. 1 over all tadpole types. Evaluating the traces, the $t \to 0$ limit produces the divergences of interest. Note that the loop modulus t is related to the cylinder length l differently for each surface:

KB: $\quad t = \dfrac{1}{4l}, \quad$ MS: $\quad t = \dfrac{1}{8l}, \quad$ C: $\quad t = \dfrac{1}{2l}. \quad (3.8)$

B. Type I theory on a K3 orbifold

We will evaluate the tadpoles and solve the consistency conditions for one particular example. This is the type I theory on a $K3$ Z_2 orbifold. The type I theory includes a projection on Ω. The orbifold is formed from the theory on a torus by projecting with R_{6789}, reflection of $X^{6,7,8,9}$; we will abbreviate this as R. Closure gives also the element ΩR. To define R we have to make a specific choice of its action on the fermions; we choose $R = e^{i\pi(J_{67} + J_{89})}$.

This example is of interest for a number of reasons. It is related by T duality to many similar theories. A T-duality transformation on X^μ for given μ (abbreviated T_μ) is a spacetime reflection, but only on the world-sheet right movers. It transforms Ω to ΩR_μ [3,14]. Thus T_6 duality takes the above orientifold group to $\{1, R_{6789}, \Omega R_6, \Omega R_{789}\}$, T_{4567} duality takes it to $\{1, R_{6789}, \Omega R_{4567}, \Omega R_{4589}\}$, and so on. This is the simplest orientifold that is not just the T dual of a toroidal theory.

We can anticipate some of the tadpole calculation. The Ω projection will require nine branes as in the noncompact case [10]. Similarly the ΩR projection, being T_{6789} dual to Ω, will require five branes with fixed $X^{6,7,8,9}$. There is also the possibility of twisted sector tadpoles, and these will indeed appear. In all there are three tadpole types, the ten-form, six-form, and twisted-sector six-form (actually 16 of these last, one for each fixed point) and each receives two contributions. The ten-form receives contributions from the crosscap with $h = 1$ and the nine brane boundary with

$g=1$, the six-form from the crosscap with $h=R$ and the five brane boundary with $g=1$, and the twisted-sector six-forms from five brane and nine brane boundaries with $g=R$.

The IIB theory has $d=10, N=2 \rightarrow d=6, N=4$ spacetime supersymmetry. The Ω projection leaves only the sum of the left- and right-moving supersymmetries, $Q_\alpha + \widetilde{Q}_\alpha$. Similarly the ΩR projection leaves the linear combinations $Q_\alpha + R \widetilde{Q}_\alpha$. The supersymmetries unbroken by both projections correspond to the $+1$ eigenvalues of R; this is half the eigenvalues of R or a quarter of the original supersymmetries, $d=6, N=1$.

Let us work out the massless spectrum of this theory. We focus on the bosons, since the fermions will have the same partition function by supersymmetry. The massless spectrum for the right- or left-moving half of the closed string is

Sector		R	SO(4) rep.
Untwisted NS:	$\psi^\mu_{-1/2}\lvert 0\rangle$	$+$	$(\mathbf{2},\mathbf{2})$
	$\psi^m_{-1/2}\lvert 0\rangle$	$-$	$4(\mathbf{1},\mathbf{1})$
R:	$\lvert s_1 s_2 s_3 s_4\rangle$		
	$s_1=+s_2, s_3=-s_4$	$+$	$2(\mathbf{2},\mathbf{1})$
	$s_1=-s_2, s_3=+s_4$	$-$	$2(\mathbf{1},\mathbf{2})$
Twisted NS:	$\lvert s_3 s_4\rangle, s_3=-s_4$	$+$	$2(\mathbf{1},\mathbf{1})$
R:	$\lvert s_1 s_2\rangle, s_1=-s_2$	$+$	$(\mathbf{1},\mathbf{2})$.

$$(3.9)$$

Here, $\mu \in 2,3,4,5$, $m \in 6,7,8,9$, and SO(4)=SU(2)$\times$SU(2) is the massless little group in six dimensions. We have imposed the GSO projection: all states listed have $(-1)^F = (-1)^{\widetilde{F}} = 1$. This is most easily determined by requiring that the vertex operators be local with respect to the supercharge $e^{-\phi/2} e^{i(H_0+H_1+H_2+H_3-H_4)/2}$ (the minus sign in the exponent is necessary because this must have $R=+$); the ghost times longitudinal part contributes a net Z_2 branch cut in the Neveu-Schwarz (NS) sector and none in the R sector. The bosonic spectrum is given by the product a left-moving state with a right-moving state from the same sector and with the same R. In the NSNS sectors this is symmetrized by the Ω projection, and in the RR sectors it is antisymmetrized because each side is a fermion. Thus, including the degeneracy from the 16 fixed points, the massless closed string spectrum is

Sector	SO(4) rep.
Untwisted NSNS:	$(\mathbf{3},\mathbf{3})+11(\mathbf{1},\mathbf{1})$
RR:	$(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+6(\mathbf{1},\mathbf{1})$
Twisted NSNS:	$48(\mathbf{1},\mathbf{1})$
RR:	$16(\mathbf{1},\mathbf{1})$.

$$(3.10)$$

This is the $d=6$, $N=1$ supergravity multiplet, plus one tensor multiplet, plus 20 $(\mathbf{1},\mathbf{1})$ hypermultiplets.

For the open strings consider first the 99 states, with massless bosonic (NS) spectrum

	$R=+$	$\Omega=+$	SO(4) rep.
$\psi^\mu_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=\gamma_{R,9}\lambda\gamma_{R,9}^{-1}$	$\lambda=-\gamma_{\Omega,9}\lambda^T\gamma_{\Omega,9}^{-1}$	$(\mathbf{2},\mathbf{2})$
$\psi^m_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=-\gamma_{R,9}\lambda\gamma_{R,9}^{-1}$	$\lambda=-\gamma_{\Omega,9}\lambda^T\gamma_{\Omega,9}^{-1}$	$4(\mathbf{1},\mathbf{1})$.

$$(3.11)$$

We have indicated the conditions imposed by the R and Ω projections on the Chan-Paton wave functions λ. The subscript 9 indicates the block of γ_R or γ_Ω which acts on the nine branes. For the 55 open strings, let us first consider n_I five branes at the Ith fixed point of R. The massless spectrum is

	$R=+$	$\Omega=+$	SO(4) rep.
$\psi^\mu_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=\gamma_{R,I}\lambda\gamma_{R,I}^{-1}$	$\lambda=-\gamma_{\Omega,I}\lambda^T\gamma_{\Omega,I}^{-1}$	$(\mathbf{2},\mathbf{2})$
$\psi^m_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=-\gamma_{R,I}\lambda\gamma_{R,I}^{-1}$	$\lambda=\gamma_{\Omega,I}\lambda^T\gamma_{\Omega,I}^{-1}4$	$(\mathbf{1},\mathbf{1})$,

$$(3.12)$$

where now $\gamma_{R,I}$ and $\gamma_{\Omega,I}$ are the blocks acting on this set of five branes. The extra sign in the Ω projection follows from the form of the vertex operator, as discussed earlier. Now consider n'_J five branes at a nonfixed point X, which requires also n'_J at $-X$. The massless bosonic strings with both ends at X are

	$\Omega=+$	SO(4) rep.
$\psi^\mu_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=-\gamma'_{\Omega,J}\lambda^T\gamma'^{-1}_{\Omega,J}$	$(\mathbf{2},\mathbf{2})$
$\psi^m_{-1/2}\lvert 0,ij\rangle \lambda_{ji}$	$\lambda=\gamma'_{\Omega,J}\lambda^T\gamma'^{-1}_{\Omega,J}$	$4(\mathbf{1},\mathbf{1})$.

$$(3.13)$$

The R projection relates these wave functions to those of the strings with ends at $-X$, but does not otherwise constrain them. For the 59 strings, we have, in the two cases above,

	$R=+$	SO(4) rep.
$\lvert s_3 s_4, ij\rangle \lambda_{ji}, s_3=-s_4$	$\lambda=\gamma_{R,I}\lambda\gamma_{R,9}^{-1}$	$2(\mathbf{1},\mathbf{1})$

$$(3.14)$$

and

	SO(4) rep.
$\lvert s_3 s_4, ij\rangle \lambda_{ji}, s_3=-s_4$	$2(\mathbf{1},\mathbf{1})$.

$$(3.15)$$

The Ω projection does not constrain these but determines the 95 state in terms of the 59 states.

C. Tadpole calculation

We may now evaluate the sums (3.6) over the closed and open string spectra. The amplitudes are $\int_0^\infty dt/2t$ times

$$\mathrm{KB:} \quad \mathrm{Tr}^{U+T}_{\mathrm{NSNS}+\mathrm{RR}}\left\{\frac{\Omega}{2}\frac{1+R}{2}\frac{1+(-1)^F}{2}e^{-2\pi t(L_0+\widetilde{L}_0)}\right\},$$

$$\mathrm{MS:} \quad \mathrm{Tr}^{99+55}_{\mathrm{NS\text{-}R}}\left\{\frac{\Omega}{2}\frac{1+R}{2}\frac{1+(-1)^F}{2}e^{-2\pi t L_0}\right\},$$

C: $\quad \mathrm{Tr}_{\mathrm{NS\text{-}R}}^{99+95+59+55}\left\{\dfrac{1}{2}\,\dfrac{1+R}{2}\,\dfrac{1+(-1)^F}{2}\,e^{-2\pi tL_0}\right\}.$

$$(3.16)$$

Here U (T) refers to the untwisted (twisted) closed string sector. On the Klein bottle we omit the $\frac{1}{2}(1+(-1)^{\widetilde{F}})$ projector because the left- and right-moving states are identical in the trace. The open-string traces include a sum over Chan-Paton states.

The signs of the operators appearing in the traces, in the various sectors, were given implicitly in the previous section. For completeness we give the action of Ω on the various mode operators; the action of R is obvious. In the closed string,

$$\Omega\alpha_r\Omega^{-1}=\widetilde{\alpha}_r,\quad \Omega\psi_r\Omega^{-1}=\widetilde{\psi}_r,\quad \Omega\widetilde{\psi}_r\Omega^{-1}=-\psi_r$$
$$(3.17)$$

for integer and half-integer r. The minus sign is included in the last equation to give the convenient result $\Omega\psi_M\widetilde{\psi}_M\Omega^{-1}=\psi_M\widetilde{\psi}_M$ for any product ψ_M of mode operators. Alternately this sign can be omitted: this just corresponds to $\Omega\rightarrow(-1)^F\Omega$, which has the same action on physical states. In open string, the mode expansions are

$$X(\sigma,0)=x+i\sqrt{\dfrac{\alpha'}{2}}\sum_{\substack{m=-\infty\\m\neq0}}^{\infty}\dfrac{\alpha_m}{m}(e^{im\sigma}\pm e^{-im\sigma})\quad(3.18)$$

with the upper sign for NN boundaries conditions and lower for DD (N=Neumann, D=Dirichlet). World-sheet parity, $X(\sigma,0)\rightarrow X(\pi-\sigma,0)$, takes

$$\alpha_m\rightarrow\pm e^{i\pi m}\alpha_m.\qquad(3.19)$$

There is no corresponding result for the ND sector, since Ω takes this into a different, DN, sector. For fermions, the mode expansions are

$$\psi(\sigma,0)=\sum_r e^{ir\sigma}\psi_r,\quad \widetilde{\psi}(\sigma,0)=\sum_r e^{-ir\sigma}\psi_r.$$
$$(3.20)$$

Parity, $\psi(\sigma,0)\rightarrow\pm\widetilde{\psi}(\pi-\sigma,0)$, takes

$$\psi_r\rightarrow\pm e^{i\pi r}\psi_r\qquad(3.21)$$

for integer and half-integer r. As in the closed string there is some physically irrelevant sign freedom. In evaluating the traces, note that Ω and R act on the compact momenta p^m and windings $\delta X^m=X^m(2\pi)-X^m(0)$ of the closed string as

$$\Omega p^m\Omega^{-1}=p^m,\quad Rp^mR^{-1}=-p^m,$$

$$\Omega\,\delta X^m\Omega^{-1}=-\delta X^m,\quad R\,\delta X^m R^{-1}=-\delta X^m\quad(3.22)$$

and that only diagonal elements contribute in the traces. Similarly in the open string 99 sector there is an internal momentum, while in the 55 sector with fixed end points there is a winding $\delta X^m=X^m(\pi)-X^m(0)$.

It is useful to define[3]

$$f_1(q)=q^{1/12}\prod_{n=1}^{\infty}(1-q^{2n}),\quad f_2(q)=q^{1/12}\sqrt{2}\prod_{n=1}^{\infty}(1+q^{2n}),$$

$$f_3(q)=q^{-1/24}\prod_{n=1}^{\infty}(1+q^{2n-1}),$$

$$f_4(q)=q^{-1/24}\prod_{n=1}^{\infty}(1-q^{2n-1}).\qquad(3.23)$$

These functions satisfy the "abstruse identity"

$$f_3^8(q)=f_2^8(q)+f_4^8(q)\qquad(3.24)$$

and have the modular transformations

$$f_1(e^{-\pi/s})=\sqrt{s}f_1(e^{-\pi s}),\quad f_3(e^{-\pi/s})=f_3(e^{-\pi s}),$$

$$f_2(e^{-\pi/s})=f_4(e^{-\pi s}).\qquad(3.25)$$

The amplitudes Eqs. (3.16), including the integrals over noncompact momenta, are then found to be $(1-1)v_6/128\int_0^{\infty}dt/t^4$ times

KB: $\quad 8\dfrac{f_4^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t})}\left\{\left(\sum_{n=-\infty}^{\infty}e^{-\pi tn^2/\rho}\right)^4\right.$

$$\left.+\left(\sum_{w=-\infty}^{\infty}e^{-\pi t\rho w^2}\right)^4\right\},$$

MS: $\quad -\dfrac{f_2^8(e^{-2\pi t})f_4^8(e^{-2\pi t})}{f_1^8(e^{-2\pi t})f_3^8(e^{-2\pi t})}\left\{\mathrm{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^T)\right.$

$$\times\left(\sum_{n=-\infty}^{\infty}e^{-2\pi tn^2/\rho}\right)^4$$

$$\left.+\mathrm{Tr}(\gamma_{\Omega R,5}^{-1}\gamma_{\Omega R,5}^T)\left(\sum_{w=-\infty}^{\infty}e^{-2\pi t\rho w^2}\right)^4\right\},$$

C: $\quad \dfrac{f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})}\left\{(\mathrm{Tr}(\gamma_{1,9}))^2\left(\sum_{n=-\infty}^{\infty}e^{-2\pi tn^2/\rho}\right)^4\right.$

$$+\sum_{i,j\in5}(\gamma_{1,5})_{ii}(\gamma_{1,5})_{jj}$$

$$\left.\times\prod_{m=6}^{9}\sum_{w=-\infty}^{\infty}e^{-t(2\pi wr+X_i^m-X_j^m)^2/2\pi\alpha'}\right\}$$

$$-2\dfrac{f_2^4(e^{-\pi t})f_4^4(e^{-\pi t})}{f_1^4(e^{-\pi t})f_3^4(e^{-\pi t})}\mathrm{Tr}(\gamma_{R,5})\mathrm{Tr}(\gamma_{R,9})$$

$$+4\dfrac{f_3^4(e^{-\pi t})f_4^4(e^{-\pi t})}{f_1^4(e^{-\pi t})f_2^4(e^{-\pi t})}\left\{(\mathrm{Tr}(\gamma_{R,9}))^2+\sum_{I=1}^{16}(\mathrm{Tr}(\gamma_{R,I}))^2\right\}.$$

$$(3.26)$$

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[3]The $\sqrt{2}$ in f_2 corrects a typographical error in Ref. [10].

We have defined $v_6 = V_6/(4\pi^2\alpha')^3$ where V_6 is the (regulated) volume of the noncompact dimensions. Also, $2\pi r$ is the periodicity of X^m (assumed for convenience to be independent of m), $\rho = r^2/\alpha'$, and we will later use $v_4 = \rho^2 = V_4/(4\pi^2\alpha')^2$ with V_4 the volume of the torus before the orientifold. The second term in the first brace of the cylinder amplitude includes a sum over five brane pairs $M_i M_j$ and over all ways for an open string to wind from one to the other. In the second term in the second brace of the cylinder, the only diagonal elements of R are those where the open string begins and ends on the same five brane without winding—hence the sum over fixed points I.

Using the modular transformation Eq. (3.25) and the Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} e^{-\pi(n-b)^2/a} = \sqrt{a} \sum_{s=-\infty}^{\infty} e^{-\pi a s^2 + 2\pi i s b}, \quad (3.27)$$

the amplitude becomes $(1-1)\int_0^\infty dt/t^2$ times

KB: $\dfrac{f_2^8(e^{-\pi/2t})}{f_1^8(e^{-\pi/2t})}\left\{ v_6 v_4 \left(\sum_{s=-\infty}^{\infty} e^{-\pi\rho s^2/t} \right)^4 \right.$

$\left. + \dfrac{v_6}{v_4}\left(\sum_{s=-\infty}^{\infty} e^{-\pi s^2/t\rho} \right)^4 \right\},$

MS: $-\dfrac{f_2^8(e^{-\pi/2t})f_4^8(e^{-\pi/2t})}{f_1^8(e^{-\pi/2t})f_3^8(e^{-\pi/2t})}\left\{ \dfrac{v_6 v_4}{32} \text{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^T) \right.$

$\times \left(\sum_{s=-\infty}^{\infty} e^{-\pi\rho s^2/2t} \right)^4 + \dfrac{v_6}{32 v_4}\text{Tr}(\gamma_{\Omega R,5}^{-1}\gamma_{\Omega R,5}^T)$

$\left. \times \left(\sum_{s=-\infty}^{\infty} e^{-\pi s^2/2t\rho} \right)^4 \right\},$

C: $\dfrac{f_2^8(e^{-\pi/t})}{f_1^8(e^{-\pi/t})}\left\{ \dfrac{v_6 v_4}{512}(\text{Tr}(\gamma_{1,9}))^2\left(\sum_{s=-\infty}^{\infty} e^{-\pi\rho s^2/2t} \right)^4 \right.$

$+ \dfrac{v_6}{512 v_4}\sum_{i,j\leq 5}(\gamma_{1,5})_{ii}(\gamma_{1,5})_{jj}$

$\left. \times \prod_{m=6}^{9}\sum_{s=-\infty}^{\infty} e^{-\pi s^2/2t\rho + is(X_i^m - X_j^m)/r} \right\}$

$- \dfrac{f_2^4(e^{-\pi/t})f_4^4(e^{-\pi/t})}{f_1^4(e^{-\pi/t})f_3^4(e^{-\pi/t})}\dfrac{v_6}{64}\text{Tr}(\gamma_{R,5})\text{Tr}(\gamma_{R,9})$

$+ \dfrac{f_3^4(e^{-\pi/t})f_2^4(e^{-\pi/t})}{f_1^4(e^{-\pi/t})f_4^4(e^{-\pi/t})}\dfrac{v_6}{32}$

$\times \left\{ (\text{Tr}(\gamma_{R,9}))^2 + \sum_{I=1}^{16}(\text{Tr}(\gamma_{R,I}))^2 \right\}. \quad (3.28)$

The asymptotics are

KB: $16 v_6 v_4 + 16\dfrac{v_6}{v_4},$

MS: $-\dfrac{v_6 v_4}{2}\text{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^T) - \dfrac{v_6}{2 v_4}\text{Tr}(\gamma_{\Omega R,5}^{-1}\gamma_{\Omega R,5}^T),$

C: $\dfrac{v_6 v_4}{32}(\text{Tr}(\gamma_{1,9}))^2 + \dfrac{v_6}{32 v_4}(\text{Tr}(\gamma_{1,5}))^2$

$- \dfrac{v_6}{16}\text{Tr}(\gamma_{R,5})\text{Tr}(\gamma_{R,9})$

$+ \dfrac{v_6}{8}\left\{ (\text{Tr}(\gamma_{R,9}))^2 + \sum_{I=1}^{16}(\text{Tr}(\gamma_{R,I}))^2 \right\}. \quad (3.29)$

Finally, the total amplitude for large l [noting the relations Eq. (3.8) between t and l] is $(1-1)\int_0^\infty dl$ times

$\dfrac{v_6 v_4}{16}\{32^2 - 64\,\text{Tr}(\gamma_{\Omega,9}^{-1}\gamma_{\Omega,9}^T) + (\text{Tr}(\gamma_{1,9}))^2\}$

$+ \dfrac{v_6}{16 v_4}\{32^2 - 64\,\text{Tr}(\gamma_{\Omega R,5}^{-1}\gamma_{\Omega R,5}^T) + (\text{Tr}(\gamma_{1,5}))^2\}$

$+ \dfrac{v_6}{64}\sum_{I=1}^{16}(\text{Tr}(\gamma_{R,9}) - 4\,\text{Tr}(\gamma_{R,I}))^2. \quad (3.30)$

The $(1-1)$ represents the contributions of NSNS and RR exchange. These $l \to \infty$ divergences are equal and opposite by supersymmetry, but must vanish separately in a consistent theory [10]. The divergences have the expected form. The RR part of the first line, proportional to the total spacetime volume, is from exchange of the ten-form; in the second line, proportional to the T-dual spacetime volume, it is from exchange of the six-form; in the third line, independent of the volume of the internal spacetime, it is from exchange of twisted-sector six-forms, one for each fixed point. Note that $\gamma_1 = 1$ and so $\text{Tr}(\gamma_{1,9}) = n_9$, $\text{Tr}(\gamma_{1,5}) = n_5$, the numbers of nine branes and five branes, respectively.

IV. SOLUTIONS

Now let use solve the consistency conditions we have found, from the algebra of Chan-Paton matrices and the cancellation of divergences. From the algebra we have Eq. (2.7), implying $\gamma_{\Omega,9}^T = \pm\gamma_{\Omega,9}$ and $\gamma_{\Omega R,5}^T = \pm'\gamma_{\Omega R,5}$. The ten-form and six-form divergences are thus proportional to $(32 \mp n_9)^2$ and $(32 \mp' n_5)^2$, and so

$$n_9 = 32, \quad \gamma_{\Omega,9}^T = \gamma_{\Omega,9}, \quad \gamma_{\Omega,5}^T = -\gamma_{\Omega,5},$$

$$n_5 = 32, \quad \gamma_{\Omega R,5}^T = \gamma_{\Omega R,5}, \quad \gamma_{\Omega R,9}^T = -\gamma_{\Omega R,9}. \quad (4.1)$$

The last equality in each line follows from the discussion at the end of Sec. II. By a unitary change of basis, $\gamma_{\Omega h} \to U\gamma_{\Omega h}U^T$, one can take

$$\gamma_{\Omega,9} = I, \quad \gamma_{\Omega R,5} = I. \quad (4.2)$$

The remaining constraints from the algebra are,

$$\gamma_{R,9} = \gamma_{\Omega,9}\gamma_{\Omega R,9} = \gamma_{\Omega R,9},$$

$$\gamma_{R,5} = \gamma_{\Omega,5}\gamma_{\Omega R,5} = \gamma_{\Omega,5},$$

$$\gamma_{R,5}^2 = \gamma_{R,9}^2 = 1, \quad (4.3)$$

where all phases have been set to one by choice of the irrel-evant overall phases of $\gamma_{R,9}$, $\gamma_{R,5}$, $\gamma_{\Omega R,9}$, and $\gamma_{\Omega,5}$. To-gether with the unitarity of these matrices, this implies that they are Hermitian, as well as antisymmetric. The choice Eq. (4.2) leaves the freedom to make real orthogonal transforma-tions. With this, we can take

$$\gamma_{R,9} = \gamma_{R,5} = \gamma_{\Omega R,9} = \gamma_{\Omega,5} = M = \begin{bmatrix} 0 & iI \\ -iI & 0 \end{bmatrix}, \quad (4.4)$$

the blocks being 16×16. Finally, the twisted sector tadpoles vanish. Thus we have found a unique consistent solution for the action of the symmetries on the Chan-Paton factors.

Returning to the massless spectra in Sec. III B, we can now solve for the Chan-Paton wave functions. For the 99 open strings, Eq. (3.11) implies the wave functions

$$\text{vectors:} \quad \lambda = \begin{bmatrix} A & S \\ -S & A \end{bmatrix},$$

$$\text{scalars:} \quad \lambda = \begin{bmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{bmatrix}, \quad (4.5)$$

where S and A refer to symmetric and antisymmetric blocks, respectively. The vectors form the adjoint of U(16), with the Chan-Paton index transforming as $\mathbf{16} + \overline{\mathbf{16}}$. The scalars trans-form as the antisymmetric tensor $\mathbf{120} + \overline{\mathbf{120}}$ of U(16). The scalars are in sets of four, from $m = 6,7,8,9$, which is the content of a hypermultiplet. Thus the 99 sector contains a vector multiplet in the adjoint of U(16) and hypermultiplets in the $\mathbf{120} + \overline{\mathbf{120}}$ (or equivalently, two hypermultiplets in the $\mathbf{120}$).

For 55 open strings, consider first n_I D branes at fixed point I; $n_I \equiv 2m_I$ must be even in order for the matrices (4.4) to have a sensible action. For open strings with both ends at I, Eq. (3.12) gives the same wave functions (4.5) as for the 99 strings, a vector multiplet in the adjoint of $U(m_I)$ and two hypermultiplets in the antisymmetric $\frac{1}{2}\mathbf{m}_I(\mathbf{m}_I-\mathbf{1})$ of this group. Now consider n_J' D branes at a nonfixed point X, where again $n_J' \equiv 2m_J'$ must be even. Equation (3.13) implies that the vector multiplets are in the adjoint representation of $\text{USp}(n_J')$ and the hypermultiplets are in one antisymmetric $\frac{1}{2}\mathbf{n}_J'(\mathbf{n}_J'-\mathbf{1})$ [which is reducible in $\text{USp}(n_J')$, containing one singlet state].

For 59 open strings with the five brane at a fixed point, Eq. (3.14) implies the wave functions

$$\text{scalars:} \quad \lambda = \begin{bmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{bmatrix}, \quad (4.6)$$

with X_1 and X_2 general $16 \times m_I$ matrices. These states trans-form as the $(\mathbf{16},\mathbf{m}_I) + (\overline{\mathbf{16}},\overline{\mathbf{m}}_I)$ of $U(16) \times U(m_I)$, but be-cause there are only two scalar states Eq. (3.14) in this sec-tor, this is a single hypermultiplet in the $(\mathbf{16},\mathbf{m}_I)$. Similarly, for 59 strings with five brane not at a fixed point, Eq. (3.15) gives a hypermultiplet in the $(\mathbf{16},\mathbf{n}_J')$.

The total gauge group is

$$U(16) \times \prod_{I=1}^{16} U(m_I) \times \prod_J \text{USp}(n_J'), \quad \sum_{I=1}^{16} m_I + \sum_J n_J' = 16, \quad (4.7)$$

with hypermultiplets in the representations

$$2(\mathbf{120},\mathbf{1},\mathbf{1}) + \sum_{I=1}^{16} \{2(\mathbf{1}, \tfrac{1}{2}\mathbf{m}_I(\mathbf{m}_I-\mathbf{1}),\mathbf{1})_I + (\mathbf{16},\mathbf{m}_I,\mathbf{1})_I\}$$

$$+ \sum_J \{(\mathbf{1},\mathbf{1}, \tfrac{1}{2}\mathbf{n}_J'(\mathbf{n}_J'-\mathbf{1})-\mathbf{1})_J + (\mathbf{1},\mathbf{1},\mathbf{1}) + (\mathbf{16},\mathbf{1},\mathbf{n}_J')_J\}. \quad (4.8)$$

We have checked that the R^4 and F^4 anomalies cancel for this spectrum.[4] Much of this space of theories is connected. A multiple of four five branes can move away from a fixed point. A single pair forms the basic dynamical five brane and must move together as discussed in Sec. II, and in the orien-tifold an image pair must move in the opposite direction. If $4k$ five branes move away from fixed point I, $U(m_I)$ breaks to $U(m_I-2k) \times \text{USp}(2k)$. The collective coordinate for this motion is one of the antisymmetric tensors of $U(m_I)$, which can indeed break $U(m_I)$ in this way. Because m_I can change only mod2, there are disconnected sectors of moduli space according to whether each of the m_I is odd or even. The largest group is $U(16) \times U(16)$ with all five branes on a single fixed point. Incidentally, we implicitly began with a torus with no Wilson lines, but these Wilson lines [transforming again as the antisymmetric tensor of U(16)] can break the 99 U(16) in the same pattern as the 55.

The spacetime anomalies of this model will be discussed further in a future publication [19]. The spectrum above has U(1) anomalies, which are cancelled by a generalization of the Green-Schwarz mechanism. This also generates masses of order $g_s^{1/2}$ for up to 16 U(1) multiplets, so the above spec-trum is only correct in the formal $g_s \to 0$ limit.

V. DISCUSSION

The surprise is that we have not found the theory that we most expected, the $K3$ orbifold with spin connection embed-ded in the gauge connection. This has gauge group SO(28) $\times$ SU(2), possibly enhanced at special points. This theory must exist because it exists for type I on a smooth $K3$, where the spectrum is the same as for the heterotic string because the low energy supergravities are the same. The question is the nature of the orbifold limit. We believe that what is hap-pening is as follows. For type I on a smooth $K3$, there is only one kind of end point, with Neumann boundary conditions. As we approach the orbifold limit, some wave functions be-come localized at the fixed point while others remain ex-tended. In the limit, end points in localized states become Dirichlet end points, while end points in the extended states remain Neumann. But there is no reason for a transition from one type of state to the other to be forbidden, particularly as we have neglected the coupling of the end point to the rest of

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[4]Though these are not among the anomaly-free theories recently found in Ref. [18].

the string. This would correspond to a term in the world sheet action which changes the boundary condition from five brane to nine brane, which is just a 59 open string vertex operator. So we conjecture that the orbifold limit is a theory with nontrivial 59 backgrounds. This is no longer a free world-sheet theory. In fact, it is rather complicated, similar to an orbifold with a twisted-state background.

Let us pursue this a bit further. Embedding the spin connection in the gauge connection means that $\gamma_{R,9} = \text{diag}(+1^{28}, -1^4)$. Section II then implies that $\gamma_{R,5}$ is antisymmetric. This makes it impossible to cancel the six-form tadpole in the theories we consider, but if we ignore this for now we might guess that we still need 32 five branes, two at each fixed point. This gives an SU(2) at each fixed point, for total gauge group SO(28)×SO(4)×SU(2) [16]. Now, we have noted in the beginning of Sec. III that open string field strengths are also sources for RR fields. So it may be that it is possible to cancel the tadpoles with an appropriate 59 background. Moreover, some 59 strings are in (**2**,**2**)'s of SO(4) and **2**'s of one of the fixed point SU(2)'s making it possible to break down to a diagonal SU(2) and obtain the expected gauge group with hypermultiplet [5] **2**'s [which exist on the smooth $K3$ but cannot be obtained directly from the **32** of SO(32)].

There has been a substantial literature on this and related models. Reference [20] considered the type I theory on $K3$ with spin connection embedded in gauge connection and did not find an anomaly-free theory based on a free CFT. This is consistent with the discussion above. Further, they were led to argue for Dirichlet open strings with SU(2) Chan-Paton factors at each fixed point, as above, and that these SU(2)'s should be identified with an SU(2) in SO(32), again consistent with the discussion of the 59 background. However, we do not see much hope for making this more precise, because of the complicated nature of the world-sheet theory.

References [2,15] also considered type I on $K3$, but without embedding the spin connection in the gauge connection. Both implicitly assumed diagonal γ's, and neither observed the need [7] for a symplectic projection on the five branes.

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[5]It is interesting to ask whether the theories we have found are in the same moduli space as the spin=gauge orbifold, with a different gauge background; we have not answered this.

Reference [15] found it impossible to cancel all tadpoles, though we do not understand their calculation in detail. Reference [14] found a model with group SO(32)×SO(2) [16]. However, because of the orthogonal projection on the five branes, this suffers from the problem observed in Ref. [7]: there are half-hypermultiplets in real (not pseudoreal) representations. Also, it is not clear that the twisted tadpoles cancel for this model: they are discussed but the relative normalization does not seem to agree with our result Eq. (3.30). Reference [17] find anomaly-free orientifold models with gauge groups such as USp(8) [4] that also arise in our construction, though the matter content is different (no antisymmetric tensors of the gauge group, but additional antisymmetric tensor supergravity multiplets). These models are constructed from more abstract conformal field theories (CFT's), and the description is rather inexplicit, so we have not been able to make a detailed comparison. Also, Refs. [16] construct similar models using free fermions. Curiously the gauge groups are smaller than those found elsewhere [e.g., USp(8) in $d=6$]; again we have not been able to make detailed comparisons.

As an aside, there is a strong temptation to regard the Dirichlet open strings as the twisted sector that is otherwise absent for the orientifold [20,1,2]. This is true in a number of formal senses, but we find it somewhat dangerous to think this way, in that it might lead one only to a subset of the consistent theories. Note, too, that it is not always true: a D brane in a noncompact space need not be accompanied by an orientifold—there is no inconsistency in the RR field equation because the flux can escape to infinity. Conversely an orientifold of a noncompact space does not require the introduction of D branes.

In conclusion, we have developed some of the necessary technology of orientifolds and D manifolds, as a step toward trying to uncover the structure underlying string duality. It will be interesting to analyze the duality symmetries of these theories [19].

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