PHYSICAL REVIEW D VOLUME 54, NUMBER 2 15 JULY 1996

Oscillons in a hot heat bath

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(Received 12 February 1996)

In models of real scalar fields with degenerate double-well potentials, spherically symmetric, large amplitude fluctuations away from the vacuum are unstable. Neglecting interactions with an external environment, the evolution of such configurations may entail the development of an oscillon, a localized, nonsingular, time-dependent configuration which is *extremely* long lived. In the present study we investigate numerically how the coupling to a heat bath influences the evolution of collapsing bubbles. We show that the existence and lifetime of the oscillon stage is extremely sensitive to how strongly the field is coupled to the heat bath. By modeling the coupling through a Markovian Langevin equation with viscosity coefficient γ, we find that for $\gamma \gtrsim 5\times10^{-4}m$, where m is the typical mass scale in the model, oscillons are not observed. [S0556-2821(96)01914-5]

PACS number(s): 11.10.Lm, 05.70.Ln, 98.80.Cq

I. INTRODUCTION

It is well known that nonlinear field theories allow for the existence of static, regular, and localized configurations [1]. In $1+1$ dimensions, one finds the kink solutions to the Klein-Gordon equation with sine-Gordon or ϕ^4 potentials. Unfortunately, for a higher number of spatial dimensions, the only static and stable solutions involve either two or more fields with some topological conservation law, as in the case of the 't Hooft–Polyakov monopole or the Nielsen-Olesen vortices [2], or a conserved global charge, as in the case of nontopological solitons [3]. The interest in such static solutions stems from a myriad of possible applications, from the modeling of particles and cosmological topological defects, to the propagation of information in optical fibers and defects in liquid crystals and superfluids.

Given the justified interest in static solutions, unstable, time-dependent solutions have received considerably less attention in the literature. However, as recent work has emphasized, the presence of nonlinearities can lead to the existence of extremely long lived (i.e., nearly nondissipative) solutions of the Klein-Gordon equation [4,5]. These spherically symmetric solutions, known as oscillons, are characterized by a rapid oscillation of the field at small radial values (the core) of the configuration, very much like the $(1+1)$-dimensional breathers which form from kink-antikink collisions [6]. In fact, oscillons can be thought of as a possible stage during the collapse of an unstable bubble; as the bubble collapses radiating its initial energy to infinity, it may (or may not) settle into an oscillon before completely disappearing. (The detailed conditions for a bubble to settle into an oscillon configuration are given in Ref. [5].)

Apart from adding to our knowledge of time-dependent coherent nonlinear phenomena in field theories, the fact that collapsing bubbles may settle into long-lived configurations may have important consequences for our understanding of the dynamics of phase transitions. As an example, consider a system in a metastable state which is being cooled down at some rate per unit volume, $\Gamma_{\rm cool}$. For a strong enough first order phase transition, where homogeneous nucleation theory is applicable, the system will supercool in this metastable state before it decays by nucleating a critical radius bubble. The rate per unit volume for nucleating a critical bubble is dominated by the Boltzmann factor, $\Gamma_{\rm CB}\sim\exp[-F_{\rm CB}/T]$, where $F_{\rm CB}$ is the cost in free energy for nucleating the critical bubble, and T is the temperature. Critical bubble nucleation is suppressed for $\Gamma_{\rm CB}/\Gamma_{\rm cool}<1$. Unless the barrier disappears below a certain temperature, the free energy cost for nucleating a critical bubble typically decreases as the system is cooled down, reaching a minimum value before it starts increasing again [7]. Apart from critical bubbles, smaller free energy, unstable (subcritical) bubbles are also nucleated. Since subcritical bubbles may fall into an oscillon stage during their collapse, they may exist for a much longer time than one would naively estimate. If their lifetime $\tau_{\rm osc}$ is longer than the cooling time scale $[\tau_{\rm osc}>(\Gamma_{\rm cool}V)^{-1}]$, it is possible that as the temperature decreases, they will overcome the free-energy barrier and initiate the decay of the metastable state. In other words, oscillons could become critical bubbles.

In practice, this possibility has not yet been analyzed in much detail. Estimates in the context of the electroweak phase transition show that oscillons are sufficiently suppressed to be of negligible impact [8]. However, based on the qualitative arguments above, one expects oscillons to be of greater relevance in earlier (in a cosmological context), and thus faster, phase transitions. In fact, in the work of Copeland, Gleiser, and Müller [5], it was remarked that for a grand-unified-theory- (GUT)-scale transition oscillons are efficiently produced by thermal processes if $F_{\rm osc}/T<10$. This condition may be satisfied by, e.g., Coleman-Weinberg potentials. Also, apart from their potential relevance to the dy-

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namics of both cosmological and laboratory phase transitions, it remains to be seen if oscillons exist in the nonrelativistic limit of the Klein-Gordon equation, as time-dependent solutions of the nonlinear Schrödinger equation.

The above discussion neglected the effects of an external environment on the evolution of shrinking bubbles. However, in most realistic situations, a thermal background will influence the dynamics (and coherence) of any field configuration. In this work, we will examine the evolution of coherent field configurations coupled to heat baths in more detail. The evolution of the scalar field in a thermal bath will be modeled by a Markovian Langevin equation. As we will see, the presence of a thermal bath adds additional constraints on the possible existence and lifetimes of oscillons; even if the collapsing bubble does settle into an oscillon, the bath can affect its lifetime quite appreciably. However, for sufficiently small couplings between the field and the thermal bath, oscillons are still present.

The rest of this work is organized as follows. In the next section, we briefly review the properties of oscillons in the absence of a thermal bath. In Sec. III, we discuss how we study bubble evolution in the presence of a heat bath. In Sec. IV, we present our numerical results. Lifetimes of the oscillons for several sets of parameters are obtained using a method consistent with an ensemble-averaging procedure, together with empirically determined equations of best fit for the data. In Sec. V we summarize our results, pointing to possible directions of future work.

II. COLD OSCILLONS

In this section we briefly review some of the results which established the existence of oscillons as a possible stage during bubble collapse for nonlinear field theories. This discussion will help us set up the notation and concepts which will be useful later when we include thermal effects. More details can be found in Refs. [4,5].

The action for a real scalar field in 3+1 dimensions is

$$S[\phi]=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-V(\phi)\right], \tag{1}$$

where we restrict our investigation to a symmetric double-well potential of the form

$$V(\phi)=\frac{\lambda}{4}\left(\phi^2-\frac{m^2}{\lambda}\right)^2. \tag{2}$$

A solution $\phi(\mathbf{x},t)$ to the equation of motion,

$$\partial^2\phi/\partial t^2-\nabla^2\phi=-\frac{\partial V(\phi)}{\partial\phi}, \tag{3}$$

has energy

$$E[\phi]=\int d^3x\left[\frac{1}{2}(\partial\phi/\partial t)^2+\frac{1}{2}(\nabla\phi)^2+V(\phi)\right]. \tag{4}$$

Since we are only interested in spherically symmetric configurations, it proves convenient to introduce dimensionless variables $\rho=rm$, $\tau=tm$, and $\Phi=(\sqrt{\lambda}/m)\phi$. The nonlinear Klein-Gordon equation is

$$\frac{\partial^2\Phi}{\partial\tau^2}-\frac{\partial^2\Phi}{\partial\rho^2}-\frac{2}{\rho}\frac{\partial\Phi}{\partial\rho}=\Phi-\Phi^3. \tag{5}$$

Oscillons are found by solving the above equation with a bubblelike initial configuration, which so far has been taken to be either a ‘‘Gaussian’’ or a ‘‘tanh’’ bubble:

$$\text{Gaussian: } \Phi(\rho,0)=(\Phi_c-\Phi_0)e^{-\rho^2/R_0^2}+\Phi_0; \tag{6}$$

$$\text{tanh: } \Phi(\rho,0)=\frac{1}{2}[(\Phi_0-\Phi_c)\tanh(\rho-R_0)+\Phi_0+\Phi_c], \quad R_0\gg 1. \tag{7}$$

R_0 is the initial radius of the configuration, Φ_0 is the asymptotic vacuum the configuration decays into (we will take $\Phi_0=-1$), and Φ_c is the value of the field at the configuration's core. (We will take it to be $\Phi_c=1$; i.e., the bubble interpolates between the two vacua.) We also impose the boundary conditions

$$\Phi(\rho\to\infty,\tau)=\Phi_0,\quad \Phi'(0,\tau)=0,\quad \dot{\Phi}(\rho,0)=0\,. \tag{8}$$

Equation (5) is then numerically solved by a finite difference routine second order accurate in time and fourth order accurate in space. (We used the same discrete steps as in Ref. [5], that is, a spatial step $h=0.1$ and a time step $\delta\tau=0.05$.) In order to examine the rate at which the initial configuration decays into the vacuum, we measure the total energy within a spherical shell surrounding it. As discussed in Refs. [4,5], the oscillon configuration is identified by the presence of an almost flat plateau in the plot of energy vs time, signaling a regime during which almost no energy is radiated away. Typically, lifetimes range from 10^3m^{-1} to 10^4m^{-1}. In Fig. 1, we show the energy of a few oscillons as a function of time for several Gaussian bubbles of differing initial radii.

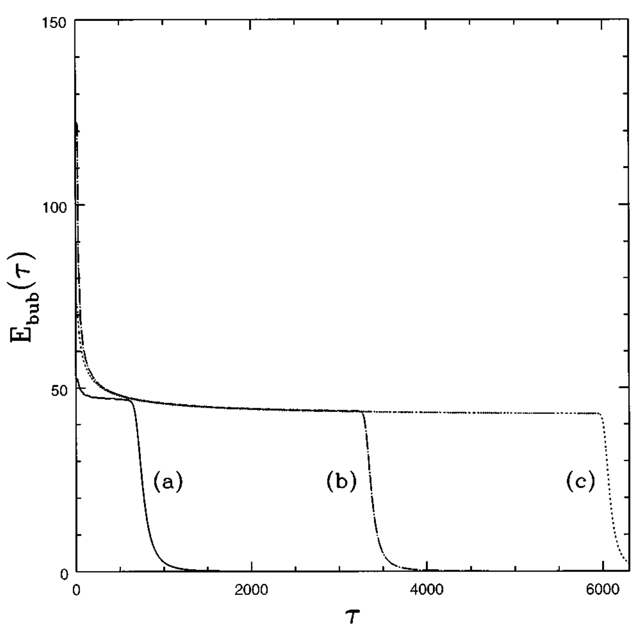
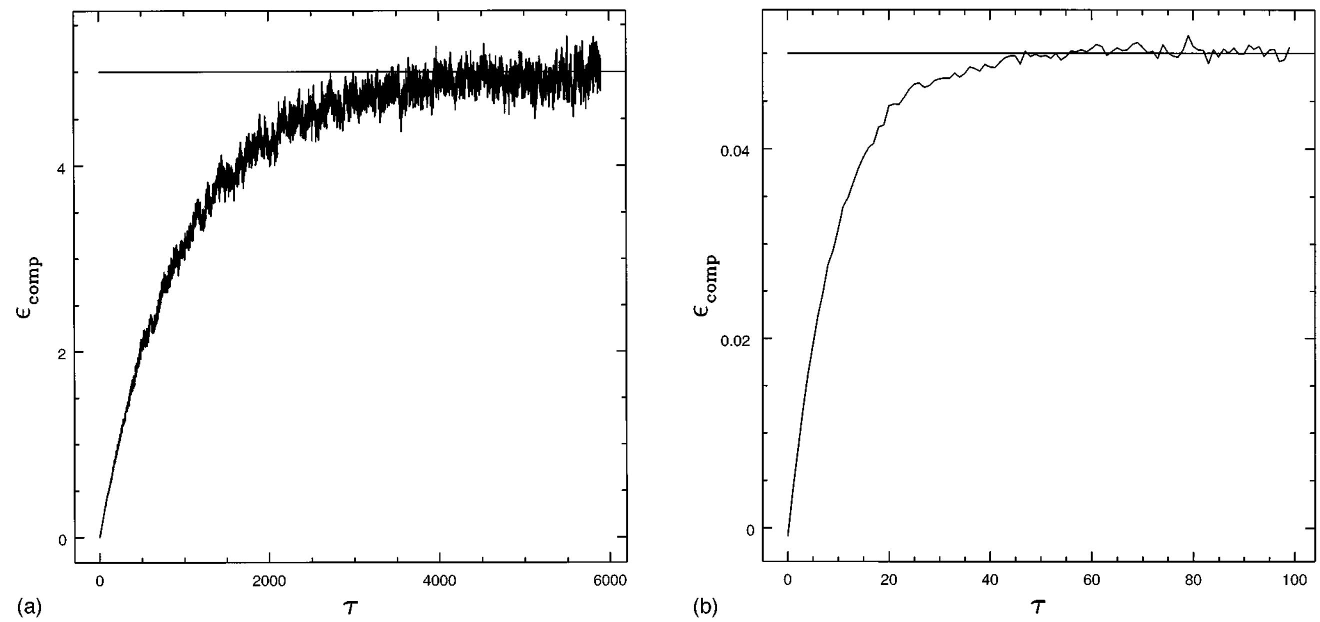


FIG. 1. Bubble energy vs time for $\gamma=0$ and $T=0$ in the case of (a) $R=2.3$, (b) $R=3.4$, and (c) $R=2.7$.



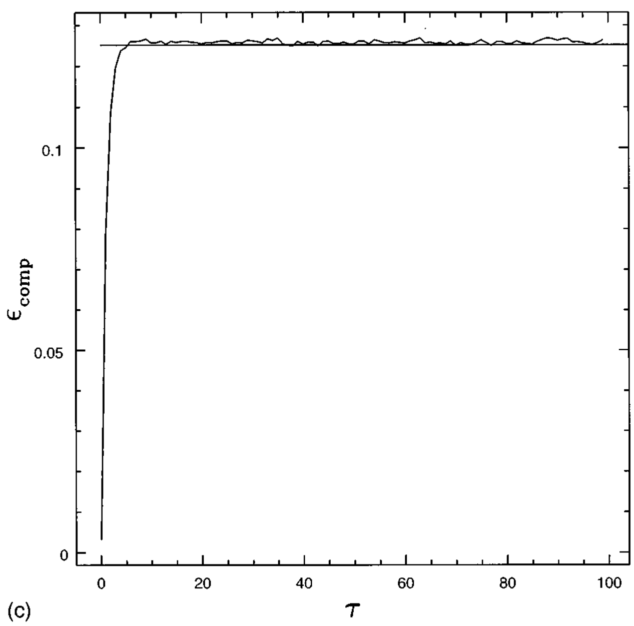


FIG. 2. Computed energy per degree of freedom, $\epsilon_{\rm comp}$, vs time for (a) $\gamma = 10^{-3}$ and $T = 10.0$, (b) $\gamma = 0.1$ and $T = 0.1$, (c) $\gamma = 1.0$ and $T = 0.25$. The horizontal lines in each figure represent the expected values from the equipartition theorem.

III. HOT OSCILLONS

We would like to extend the previous analysis for fields interacting with thermal baths. The simplest way to do this is to couple the scalar field to a thermal bath via a generalized Langevin equation, which assumes the bath to be Markovian (white noise) and that the field couples to the noise additively. The bath is characterized by a viscosity coefficient γ and by a random noise $\xi(\mathbf{x},t)$, which are related by the fluctuation-dissipation theorem:

$$\langle \xi(\mathbf{x},t)\xi(\mathbf{x}',t')\rangle = 2\gamma T \delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'). \tag{9}$$

γ represents the coupling strength between field and thermal bath. For simple models, it can be expressed perturbatively in terms of the coupling constants in the system. (See, e.g., Ref. [9].) However, we will treat it here as a free parameter. Note that γ^{-1} defines the thermalization time scale. More complicated nonlocal forms of the Langevin equation could be used, although we think it wise to defer such approaches to later studies. (More details can be found in Refs. [9,10] and references therein.)

In terms of the dimensionless coordinates defined before, the Langevin equation is given by

$$\frac{\partial^2\Phi}{\partial\tau^2} + \tilde{\gamma}\frac{\partial\Phi}{\partial\tau} - \frac{\partial^2\Phi}{\partial\rho^2} - \frac{2}{\rho}\frac{\partial\Phi}{\partial\rho} = \Phi - \Phi^3 + \tilde{\xi}, \tag{10}$$

where $\tilde{\gamma} = \gamma/m$ and $\tilde{\xi} = \sqrt{\lambda}\,\xi/m^3$ are the dimensionless viscosity and noise, respectively.

Here, one last simplification comes in. In general, one should be solving the fully $(3+1)$-dimensional Langevin equation, as the bath carries no symmetries with it. However, we will assume that the dominant interactions of the bubble with the thermal bath are in the radial direction. We must rewrite the fluctuation-dissipation relation consistently with spherical symmetry, effectively keeping the problem one di-

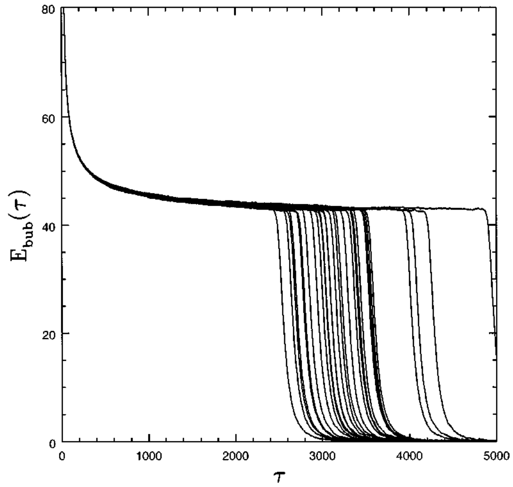


FIG. 3. Bubble energy vs time for $R=3.2$, $\gamma=10^{-5}$, and $T=0.1$ with different seed values for each curve. The figure represents only 30 runs from a total of 150.

mensional. The full (3+1)-dimensional problem proves to be very CPU intensive and of limited interest. (Note that in the presence of a thermal bath, results are obtained after an ensemble averaging procedure. In other words, each measurement can involve hundreds of runs. As it is, the results obtained here already required a network of workstations running for a total of 6000 days of CPU time.) This approximation is not only economical, but also makes sense physically. For bubblelike configurations, it is reasonable to expect that the effect of the bath will be dominant in the radial direction. In any case, the results here should be considered as an upper bound on the lifetimes; including perturbations in all three dimensions will not help increase the oscillon's lifetime.

The spherically symmetric fluctuation-dissipation relation is

$$\langle \tilde{\xi}(\rho,\tau)\tilde{\xi}(\rho',\tau')\rangle = \frac{1}{2\pi\rho^2}\tilde{\gamma}\theta\delta(\tau-\tau')\delta(\rho-\rho'), \quad (11)$$

where $\theta=\lambda T/m$ is the dimensionless temperature.

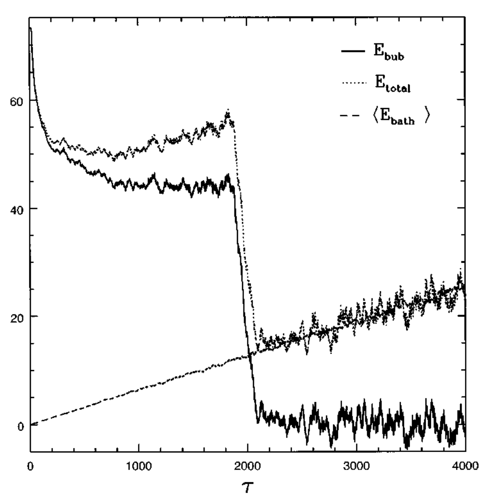


FIG. 4. Bubble energy, total energy, and bath energy as functions of time for $R_0=2.7$, $\gamma=10^{-5}$, and $T=5.0$.

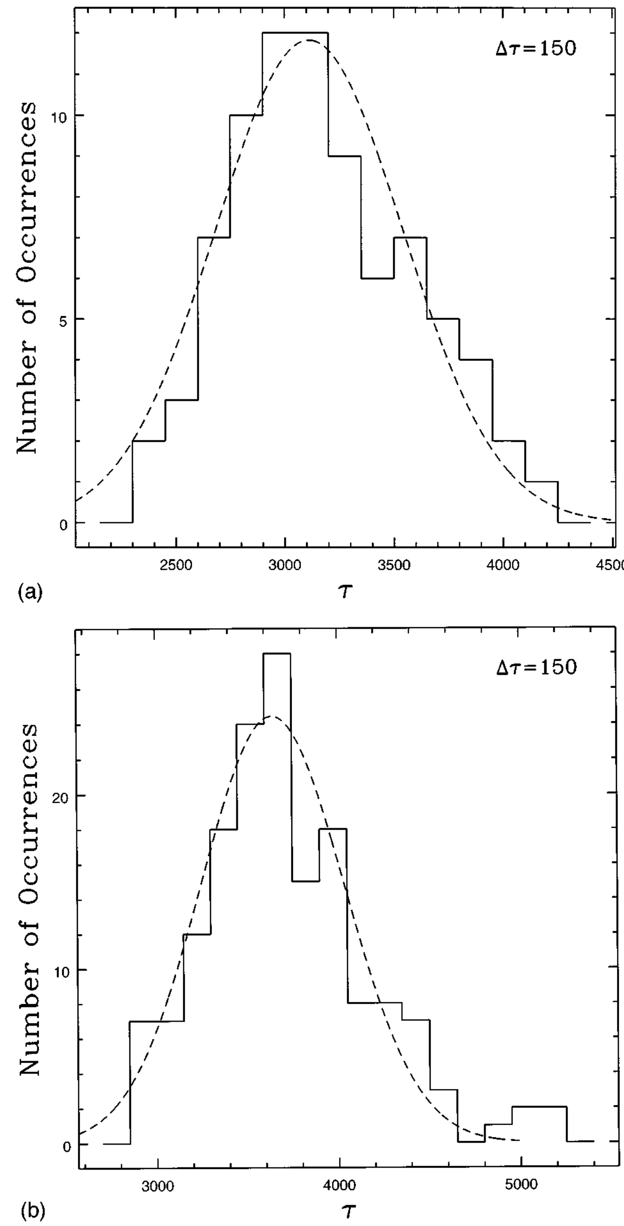


FIG. 5. Histograms for (a) $R=2.5$ and (b) $R=3.0$, for $\gamma=10^{-5}$ and $T=0.1$. $\Delta\tau$ indicates the bin size used in the specific cases.

Before proceeding, we must make sure that our expression for the fluctuation-dissipation relation is physical. A simple way to do this is to consider the evolution of a free field in a parabolic potential and measure the energy per degree of freedom E/N, where N is the total number of spatial lattice points. Since the problem is one dimensional, the equipartition theorem states that, in equilibrium, $E/N=T/2$. We have confirmed that this relation is satisfied for a wide range of parameters, as shown in Fig. 2.

IV. NUMERICAL RESULTS

Now that we have an evolution equation in the presence of a thermal bath consistent with the assumption of spherical symmetry, we can start investigating how the bath influences the formation and longevity of oscillons.

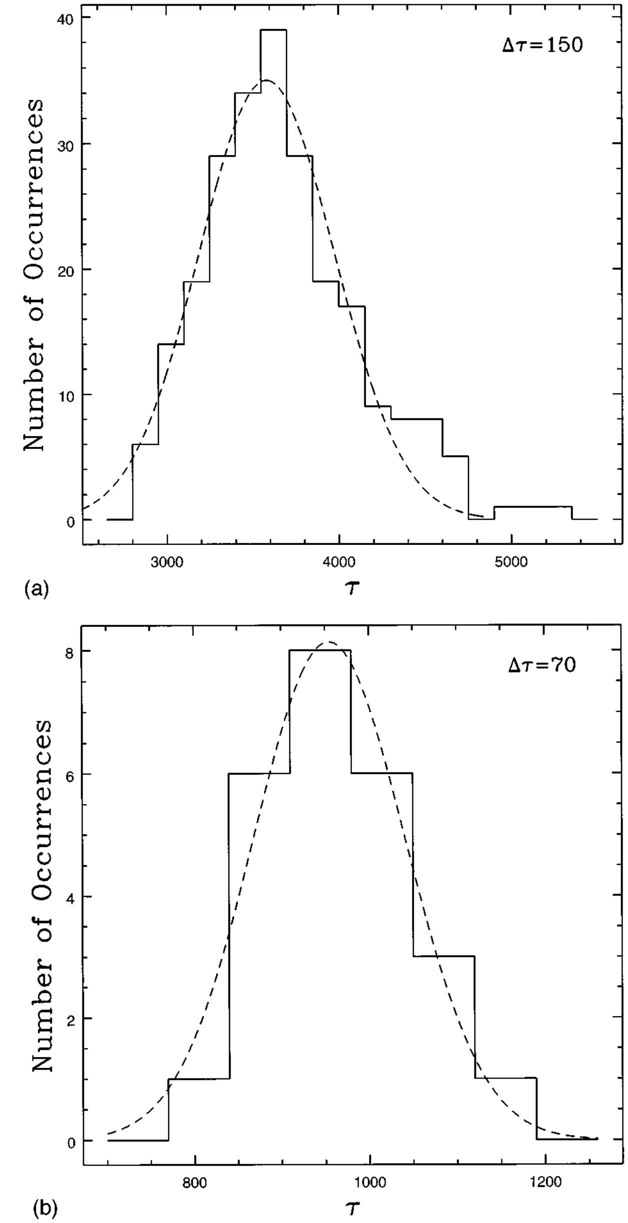


FIG. 6. Histograms for (a) $\gamma=10^{-5}$ and (b) $\gamma=3\times10^{-4}$, for $R_0=2.7$ and $T=0.1$. $\Delta\tau$ indicates the bin size used in the specific cases.

As in the ''cold oscillon'' case, we measure the energy within a shell surrounding the initial configuration as a function of time. The radius of the shell, $R_{\rm shell}$, should be taken to be sufficiently larger than the radius of the initial configuration, R_0. Typically, we chose $R_{\rm shell}=15$. The presence of temperature, however, immediately introduces two difficulties. First, the bath contributes to the energy within the shell, obscuring the measurement of the bubble's energy. Second, a physical result must be obtained by an ensemble averaging procedure, where different conditions are set by different seeds for the random number generator responsible for the noise term in the equation. However, as soon as we change the seed, the oscillon lifetime changes, as we show in Fig. 3 for a selection of 30 runs out of 150. We must find a consistent procedure for measuring the ensemble-averaged oscillon lifetime. Let us deal with each of these problems in turn.

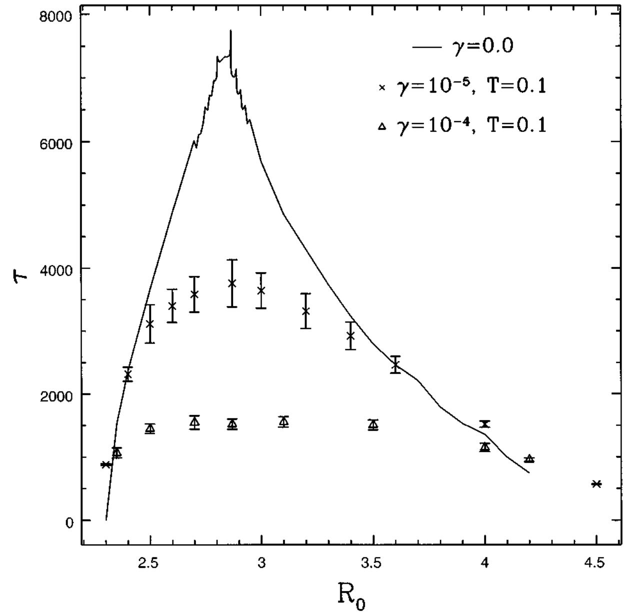


FIG. 7. Oscillon lifetime vs radius for constant viscosity and temperature.

The first problem can be taken care of quite easily; we perform several runs without a bubble [i.e., with a constant initial configuration $\Phi(\rho,0)=-1$], and measure the ensemble-averaged energy of the bath within the shell, $\langle E_{\rm bath}\rangle$. The angular brackets denote ensemble-averaged quantities. We then define the normalized energy of the bubble by

$$E_{\rm bub}=E_{\rm total}-\langle E_{\rm bath}\rangle, \tag{12}$$

where $E_{\rm total}$ is the total energy within the shell. Figure 4 shows a typical case for $E_{\rm bub}$, $E_{\rm total}$, and $\langle E_{\rm bath}\rangle$. As a test of this procedure, we confirmed that $\langle E_{\rm bub}\rangle\rightarrow 0$ after bubbles have decayed. Another test is to measure the plateau energy for small γ and T. The result should be (and is) consistent with the cold oscillon case, $E_{\rm plateau}\sim 43m/\lambda$.

In order to handle the second problem, we first define how we measure the oscillon's lifetime. As opposed to the case of cold oscillons, where we could just estimate the lifetime by checking when the oscillon finally decays (see Fig. 3), here we must automate the procedure, as we will have to handle thousands of runs. First, we must establish if a given initial bubble settles into an oscillon stage. As we remarked earlier, the most important signature of the oscillon is the near constancy of its total energy. In Refs. [4,5], it was measured that during the oscillon stage the plateau energy varies by at most 20%. After an exhaustive search, we found that we can identify an oscillon stage by setting $|dE_{\rm plateau}/dt|\leqslant 0.02m^2/\lambda$. Steeper changes of the plateau energy were incompatible with the nearly nondissipative nature of oscillons. Now that we know how to hunt for oscillons, we must decide how we will measure their lifetimes. Here, the fact that these configurations decay rather quickly allows us to simply set the oscillon lifetime to the time when its energy drops to $0.1E_{\rm plateau}$.

To summarize so far, we first choose values for the three variables in the model: the initial radius R_0, the viscosity

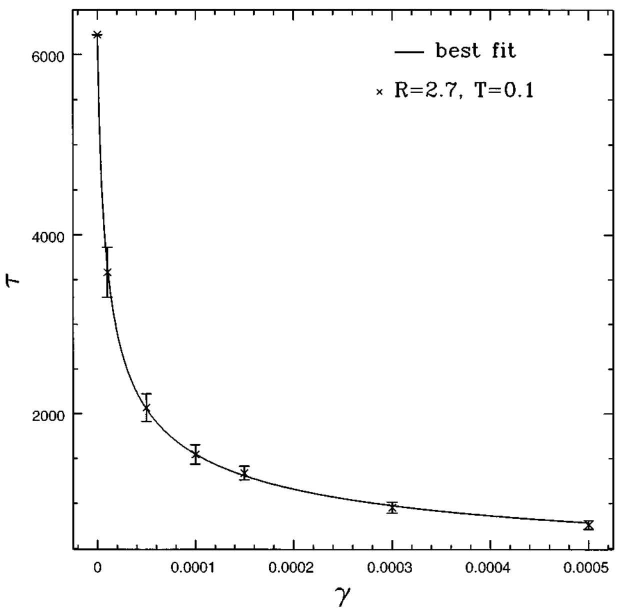


FIG. 8. Oscillon lifetime vs viscosity for constant radius and temperature. The equation providing best fit is $A[(\gamma+B)/\gamma_0]^{-C}$ where $A=2.0\times10^3$, $B=4.0\times10^{-6}$, $C=0.43$, and $\gamma_0=5.8\times10^{-5}$.

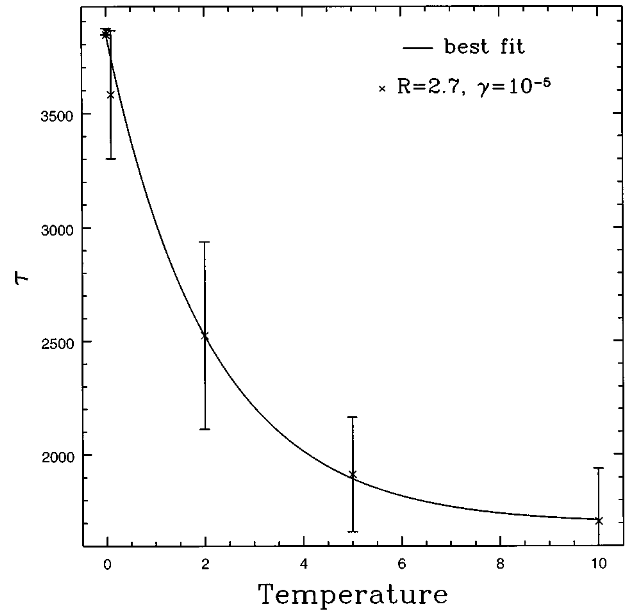


FIG. 9. Oscillon lifetime vs temperature for constant radius and viscosity. The equation providing best fit is $A\exp(-T/T_0)+B$ where $A=2.1\times10^3$, $B=1.7\times10^3$, and $T_0=2.1$.

γ, and the temperature T. Each numerical ''experiment'' consists of evolving the Langevin equation with a choice of variables and a given value for the random number seed. The code then searches for an oscillon by identifying a fairly flat plateau, and then measures its lifetime by recording the time when its energy decays to $0.1E_{\mathrm{plateau}}$. This procedure is repeated a large number of times with different seeds for the random number generator. This way, we obtain a distribution of lifetimes for a given set of variables, as shown in Fig. 3. In order to determine the ensemble-averaged lifetime, we produce a histogram where we bin different decay times vs the number of occurrences. In Fig. 5, we show a few examples of histograms for varying radii and fixed viscosity and temperature. In Fig. 6, we show a few examples of varying viscosity, with fixed radius and temperature.

We can now extract the ensemble-averaged lifetime by fitting a Gaussian to the various histograms, using a least-squares approximation. Since the thermal noise producing the variation in the lifetimes is generated from normal deviates, it is reasonable to assume that the lifetimes will follow the central limit theorem, and produce Gaussian distributed data as well, for a sufficiently large number of runs. The mean gives the desired lifetime and the variance gives the spread in the lifetime measurements. In order to obtain the best possible fit, we used the Levenberg-Marquardt method [11].

The results for the lifetime as a function of radius and different values of viscosity and temperature are shown in Fig. 7. It is clear that the coupling to the bath strongly suppresses the duration of the oscillon stage, when it is at all present; for values of viscosity larger than $\gamma>5\times10^{-4}m$, no oscillons could be found. Bubbles quickly collapse, radiating their coherent energy into the incoherent thermal bath. Since γ^{-1} is the typical thermalization time scale, we obtain a rough criterion for the existence of oscillons in the presence of a Markovian thermal bath,

$$\gamma^{-1}>\tau_0, \tag{13}$$

where τ_0 is the lifetime of the cold oscillon of the same radius. Since typically $10^3\leqslant\tau_0\leqslant10^4$, this result is consistent with the fact that the bath acts to destroy the coherence of the field configuration.

From Fig. 7 we note that although the bath affects the longevity of the oscillons, it does not appreciably affect the range of radii for the initial bubble settling into the oscillon stage; as in Refs. [4,5], we still obtain roughly $2.3\lesssim R_0\lesssim4.5$ for the allowed range. Also, the longest-lived oscillons are the most affected, consistent with the above condition, Eq. (13).

In Fig. 8 we display the results for lifetime vs viscosity, for $R_0=2.7m^{-1}$ and $T=0.1m/\lambda$. A least-squares fit produces the empirical relation

$$\tau(\gamma)=A\left[\frac{\gamma+B}{\gamma_0}\right]^{-C}, \tag{14}$$

where $A=2.0\times10^3$, $B=4.0\times10^{-6}$, $C=0.43$, and, $\gamma_0=5.8\times10^{-5}$. The error bars are given by the variance of the Gaussian fit the histograms. γ_0 sets the scale for the existence of oscillons.

Finally, in Fig. 9 we display the results of lifetime vs temperature, for $R_0=2.7m^{-1}$ and $\gamma=10^{-5}$. It is clear that the lifetime is less sensitive to temperature than to viscosity. Again, an empirical fit can be obtained:

$$\tau(T)=A\exp\left(-\frac{T}{T_0}\right)+B, \tag{15}$$

where $A=2.1\times10^3$, $B=1.7\times10^3$, and $T_0=2.1$. For higher temperatures, it becomes very difficult to extract meaningful results, as the signal-to-noise ratio sharply decreases.

V. CONCLUSIONS

We investigated the evolution of collapsing bubblelike configurations in the presence of a thermal bath. The bubbles were assumed to have Gaussian profile, and the dynamics were modelled by a generalized Langevin equation with a Markovian thermal bath. In the absence of a thermal bath, it is possible for these configurations to settle into what is known as an oscillon stage, a nearly nondissipative configuration which is thus extremely long lived.

We showed that the presence of a thermal bath affects the existence and longevity of the oscillon stage rather strongly. For values of the viscosity coefficient, $\gamma>5\times10^{-4}m$, no oscillon stage develops, independent of temperature. This is naively analogous to an overdamped harmonic oscillator. For smaller values of γ, an oscillon stage develops, but its lifetime is suppressed. This result can be related to the fact that the time scale for thermalization is $\sim\gamma^{-1}$, while in the absence of a heat bath the typical lifetime of the oscillon stage is $\tau_0\sim(10^3\text{–}10^4)m^{-1}$. This is analogous to an underdamped harmonic oscillator, where the decay time scale is γ^{-1}. When the decay time scale is of order the lifetime of the oscillon, the bath suppresses the duration of the oscillon stage. For even smaller values of γ, the oscillon approaches similar lifetimes as in the cold oscillon case. Thus, the relevance of oscillons in different contexts in which a thermal bath is present will depend on the strength of the coupling between the field and the thermal bath, here modeled by γ. As a rough rule, oscillons are present whenever $\gamma^{-1}>\tau_0$.

Our discussion focused on Markovian thermal baths. However, given the nature of interacting field theories, it is possible that the bath will have more complicated properties such as spatiotemporal correlations which are nonlocal and/or couplings which are multiplicative as oppose to additive. (For example, terms of the form $\xi\phi$ in the equation of motion [9].) It remains to be seen if these nonlocal effects will act as an additional source of coherence for fields. That being the case, coupling to more general thermal baths may have very different consequences for the existence and longevity of oscillons.

ACKNOWLEDGMENTS

We would like to thank Hans-Reinhard Müller for many useful discussions and assistance with the early development of the numerics. M.G. was partially supported at Dartmouth by the National Science Foundation through Grant No. PHY-9453431 and by a NASA Grant No. NAGW-4270.

[1] For reviews see A.C. Scott, F.Y.F. Chiu, and D.W. Mclaughlin, Proc. IEEE **61**, 1443 (1973); S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, England, 1985).

[2] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1987).

[3] T.D. Lee and G.C. Wick, Phys. Rev. D **9**, 2291 (1974); R. Friedberg, T.D. Lee, and A. Sirlin, *ibid.* **13**, 2739 (1976); **13**, 2379 (1976); E. Copeland, E.W. Kolb, and K. Lee, Nucl. Phys. **B319**, 501 (1989).

[4] M. Gleiser, Phys. Rev. D **49**, 2978 (1994). See also I.L. Bogolubsky and V.G. Makhankov, Pis'ma Zh. Éksp. Teor. Fiz. **24**, 15 (1976) [JETP Lett. **24**, 12 (1976)]; **25**, 120 (1977) [**25**, 107 (1977)]; V.G. Makhankov, Phys. Rep. C **35**, 1 (1978).

[5] E.J. Copeland, M. Gleiser, and H.-R. Müller, Phys. Rev. D **52**, 1920 (1995).

[6] D.K. Campbell, J.F. Schonfeld, and C.A. Wingate, Physica **9D**, 1 (1983).

[7] A.H. Guth and E.J. Weinberg, Phys. Rev. D **23**, 876 (1981).

[8] A. Riotto, Phys. Lett. B **365**, 64 (1996).

[9] M. Gleiser and R. Ramos, Phys. Rev. D **50**, 2441 (1994); S. Jeon, *ibid.* **47**, 4586 (1993).

[10] M. Gleiser and J. Borrill, Phys. Rev. D **51**, 4111 (1995).

[11] W.H. Press *et al.*, *Numerical Recipes* (Cambridge University Press, Cambridge, England, 1986).